第 29 卷 第 2 期
2003 年 3 月

自 动 化 学 报
ACTA AUTOMATICA SINICA

Vol. 29, No. 2
Mar., 2003

The Concept of Configuration Redundancy and Integrated Evaluation and Disposition of Redundancy in Sensor Systems[1)]

GE Tong
(*Naval Architecture and Ocean Engineering School, Shanghai Jiaotong University, Shanghai 200030*)
(E-mail: tongge@mail.sjtu.edu.cn)

Abstract An important subject in control system design is to actively explore and dispose both functional and physical redundancy, which is abundant in modern control systems and of various fashion and mechanism. We first present a new concept of '*configuration redundancy*', which may provide a common understanding of different kinds of redundancy. An index is then proposed based on the concept to evaluate the amount of functional and physical redundancy in a system, and consequently provides a new basis for redundancy disposition and active survivability design. The concept and index are applied to control sensor systems in detail, and some examples are provided to show the effectiveness.

Key words Redundancy, survivability, disposal, evaluation, configuration

1 Introduction

The technique of physical redundancy has been widely employed in traditional control system design. By disposing multiple physical elements of the same function, system reliability is improved, but complexity and cost are increased. The technique of functional redundancy is being applied more and more in modern control system design. By exploring functional overlap of different elements (or components) in the control system, not only reliability of the system is improved, but also physical complexity and cost are decreased. Functional redundancy is still based on the system hardware, therefore, optimal design of the control system needs integrated exploration and disposition of both functional and physical redundancy. There are abundant results in the area of redundancy exploration. This paper focuses on the integrated redundancy disposition. We start with analyzing the mechanism of redundancy , then propose a new concept of 'configuration redundancy' to generally describe different kinds of redundancy, and define an index of 'configuration redundancy', and finally state the method of redundancy evaluation and disposition.

Integrated redundancy disposition needs a general index for redundancy evaluation. Because the redundancy technique is originally employed to improve reliability, some methods of redundancy disposition directly employ indexes of reliability or based on reliability[1]. However, redundancy is essentially different from reliability, so these indexes cannot reflect the amount of redundancy contained in the system well. Ziha[2] proposed another general index of redundancy based on the concept of 'event'. 'Event' is in some way similar to 'configuration' here, but it does not consider any correlation among system states. Additionally, there are still some other techniques for redundancy evaluation, such as evaluation of sensors' analytical redundancy based on the dimensions of the observable subspace[3], and evaluation of kinematics redundancy based on the dimensions of the Euclid space[4]. These techniques are for their special areas, and are not suitable for integrated redundancy evaluation.

1) Supported by National Natural Science Foundation of P. R. China(60104001), Phosphor Person Plan of Shanghai (01QF14028), and Youth Science Foundation of Shanghai(99QF21)
Received April 9, 2001; in revised form June 13, 2002
收稿日期 2001-04-09; 收修改稿日期 2002-06-13

2 Concept and index of configuration redundancy

Let $\Omega=\{a_i,\ i=1,\cdots,q\}$ be the set of all elements of a system. Different a_i might drop into fault in different order, which makes the system drop into different states, denoted by s_l, $l=1,2,\cdots,m$. States in which the system can perform the expected functions are valid states, and states in which the system cannot perform the expected functions are invalid states.

Definition 1. Suppose the system is in the state s_l. The set of all the normal elements in Ω is called the realization of the system corresponding to s_l, denoted by $\mathrm{N}(s_l)$ in form of $\{\ \bullet\ \}$. The sequence formed by all the fault elements in the order of dropping into the fault is called the sequential realization of the system corresponding to s_l, written in the form of $\langle\ \bullet\ \rangle$.

Denote the set of all possible realizations of the system with INS(Ω), and the set of all possible sequential realizations of the system with TINS(Ω). The pair $\langle \mathrm{INS}(\Omega), \subseteq\rangle$ is a partially ordered set, where '$\subseteq$' is the subset relation on INS(Ω). $\langle \mathrm{INS}(\Omega), \subseteq\rangle$ can be described by its Hasse diagram $\langle \mathrm{INS}(\Omega),\ \mathrm{COV}\rangle$[5], where COV is the 'cover' relation corresponding to '$\subseteq$'. Then in $\langle \mathrm{INS}(\Omega),\ \mathrm{COV}\rangle$, a node $N(s_l)$ denotes a realization of the system, the path from the top to a node $N(s_l)$ denotes a sequential realization, and the set of all such paths is TINS(Ω).

Definition 2. A configuration φ_i of the system is a subset of Ω, which can perform expected functions independently under suitable environment and working patterns. For a valid state s_l, $N(s_l)$ must include some configurations $\varphi_i(i=1,2,\cdots)$, and we say φ_i supports s_l.

Definition 3. Let $\Phi=\{\varphi_i,\ i=1,\cdots,n\}$ be a set of some configurations of the system. If for any valid state s_l, there is in Φ at least a configuration φ_i supporting s_l, Φ is called a configuration base of the system.

Instead of directly improving reliability of the system's elements, the redundancy technology improves reliability of the system by providing multiple system configurations essentially. Thus all systems with redundancy can be considered to have a general structure, which is formed by parallelizing multiple configurations. To distinguish this viewpoint from others, here we call the redundancy in the above structure configuration redundancy. The concept of 'configuration redundancy' shows not only the relationship of reliability and redundancy, but also difference between them: essentially, redundancy is not a statistical characteristic that reflects the reliability of a system, but a structural one that reflects survivability of the system after faults and failures. Studying evaluation of configuration redundancy will provide novel views for system design.

Denote the valid mode and invalid mode of the element a_i with a_i and $\bar{a}_i$, respectively, and denote the valid mode and invalid mode of the configuration φ_i with φ_i and $\bar{\varphi}_i$, respectively. A physical redundancy system with self-diagnosis has the simplest structure of configurations, e.g., the n-redundancy sensor system. Each component is just a configuration, all the configurations change their modes independently, and the system knows its state clearly. Split the n configurations into k subsets, so that the ith subset has q_i elements with the same invalid probability P_i. The invalid probability of the system is

$$P(\bar{\varphi}_1,\bar{\varphi}_2,\cdots\bar{\varphi}_n) = \prod_{i=1}^{n} P(\bar{\varphi}_i) = \prod_{i=1}^{k} P_i^{q_i} \tag{1}$$

Obviously, redundancy contained in the system can be measured by n, the number of the configurations of the configuration base, and

$$\sum_{i=1}^{n} \log_{P(\bar{\varphi}_i)} P(\bar{\varphi}_i) = \sum_{i=1}^{k} \log_{P_i} P_i^{q_i} = n \tag{2}$$

When considering the functional redundancy, modes of the configurations correlate complicatedly, and the amount of redundancy in the system decreases. To measure the contained redundancy generally, we start once more from the invalidation probability.

Let $\{\langle j_1, j_2, \cdots, j_n\rangle \mid j=1,\cdots,m,\ m=n!\}$ be the set of all the permutations of $\{1,2,\cdots,n\}$. For any $\langle j_1, j_2, \cdots, j_n\rangle$,

$$P(\bar{\varphi}_{j_1}, \bar{\varphi}_{j_2}, \cdots, \bar{\varphi}_{j_n}) = \prod_{i=1}^{n} P(\bar{\varphi}_{j_i} \mid \bar{\varphi}_{j_{(i+1)}}, \bar{\varphi}_{j_{(i+2)}}, \cdots, \bar{\varphi}_{j_n}) = \prod_{i=1}^{n} \bar{P}_j(\bar{\varphi}_i),\ j = 1,2,\cdots,m \tag{3}$$

where

$$\bar{P}_j(\bar{\varphi}_i) = P(\bar{\varphi}_{j_k} \mid \bar{\varphi}_{j_{(k+1)}}, \bar{\varphi}_{j_{(k+2)}}, \cdots, \bar{\varphi}_{j_n}),\ \forall i = j_k \tag{4}$$

Multiplying the m equations in (3), we have

$$P(\bar{\varphi}_1, \bar{\varphi}_2, \cdots, \bar{\varphi}_n) = \prod_{i=1}^{n} \hat{P}(\bar{\varphi}_i) \tag{5}$$

$$\hat{P}(\bar{\varphi}_i) = \prod_{j=1}^{m} \bar{P}_j^{1/m}(\bar{\varphi}_i) \tag{6}$$

Equation (5) and (1) are in the similar form. Similar to (2), we give the following definition.

Definition 4. For a system with the configuration base $\Phi=\{\varphi_i,\ i=1,\cdots,n\}$, define the following index to evaluate the redundancy in the system,

$$\mathrm{Re}(\Phi) = \sum_{i=1}^{n} \mathrm{Re}(\Phi_i) = \sum_{i=1}^{n} \log_{P(\bar{\varphi}_i)} \hat{P}(\bar{\varphi}_i) \tag{7}$$

which is called the configuration redundancy degree.

Physically, $\mathrm{Re}(\varphi_i)$ can be looked as the contribution of φ_i to the system redundancy in consideration of correlation among modes of the configurations. The total amount of system redundancy is the sum of contributions of all φ_i's. Mathematically, the configuration base forms an abstract space. The system redundancy is the sum of its projections to all the dimensions. By comparing (5) and (7), it can be seen that the reliability is the combination of redundancy and base vectors.

Direct deduction gives the following properties.

Property 1. For systems with the structure of parallel multiple configurations, $\mathrm{Re}(\Phi)$ is determined and $R(\Phi)\in[1,n]$.

Property 2. If the configuration base Φ can be divided into two independent subsets Φ_1 and Φ_2, i. e. , for any $\varphi_1\in\Phi_1$ and $\Phi_0\subseteq\Phi$,

$$P(\bar{\varphi}_1 \mid \bar{a}_i \mid a_i \in \Phi_0) = P(\bar{\varphi}_1 \mid \{\bar{b}_i \mid b_i \in \Phi_0 - \Phi_2\}) \tag{8}$$

and there is a similar equation for any $\varphi_2\in\Phi_2$ and $\Phi_0\subseteq\Phi$, then

$$\mathrm{Re}(\Phi) = \mathrm{Re}(\Phi_1) + \mathrm{Re}(\Phi_2) \tag{9}$$

Corollary 1. If all the configurations in Φ change their modes independently, then $\mathrm{Re}(\Phi)$ of the system is n. Equations (2) and (7) coincide with each other. On the other side, if all the configurations overlap, i. e. , the conditional probability $\bar{P}_j(\bar{\varphi}_i)=1$, $\mathrm{Re}(\Phi)$ of the system is 1.

3 Calculation of the configuration redundancy measure

Denote the current system state with a random variable s. Then the set of all possible system states $S=\{s_1, s_2, \cdots\}$ is the sample space of s. The distribution of s is a prior information. Only some states in S are valid and form the subset S_e. For $s=s_l\in S-S_e$, all configurations in Φ are invalid, and for $s=s_l\in S_e$, there is a nonempty subset of valid configurations $\{\varphi_i \mid \varphi_i\in\Phi,\ \varphi_i\subseteq N(s_l)\}$. Therefore, the conditional probability

$$P(\varphi_{j_i} \mid \bar{\varphi}_{j_{(i+1)}}, \bar{\varphi}_{j_{(i+2)}}, \cdots, \bar{\varphi}_{j_n}) = \Big[\sum_{\substack{s_l \in S_e \\ \varphi_{j_i} - N(s_l) = \varnothing \\ \varphi_{j_{(i+1)}} - N(s_l) \neq \varnothing \\ \vdots \\ \varphi_{j_n} - N(s_l) \neq \varnothing}} P(s_l) \Big] \Big/ \Big[\sum_{\substack{s_l \in S_e \\ \varphi_{j_i} - N(s_l) = \varnothing \\ \varphi_{j_{(i+1)}} - N(s_l) \neq \varnothing \\ \vdots \\ \varphi_{j_n} - N(s_l) \neq \varnothing}} P(s_l) + \sum_{s_l \notin S_e} P(s_l) \Big],$$

$$j = 1, 2, \cdots, m, \quad i = 1, 2, \cdots, n \tag{10}$$

Two conditions are necessary for a states s_l to be in $S_e (s_l \in S_e)$:

1) There is at least a configuration to be a subset of $N(s_l)$;

2) When $s = s_l$, the system knows its current state well enough.

Condition 1) is the basic requirement for the system to perform the expected functions, while condition 2) relates to the order in which the elements fall into fault and the capability of the system to detect and diagnose the faults. Obviously, $\mathrm{Re}(\Phi)$ will get its maximum value without consideration of condition 2). In this case, $\mathrm{Re}(\Phi)$ reflects the total amount of redundancy contained in the system, and we face a problem of maximum redundancy evaluation. When considering both the above conditions, $\mathrm{Re}(\Phi)$ will get a smaller value that reflects the amount of redundancy usable under current system intelligence; we face a problem of practical redundancy evaluation.

For the problem of maximum redundancy evaluation, S is just $\mathrm{INS}(\Omega)$. In the Hasse diagram $\langle \mathrm{INS}(\Omega), \mathrm{COV} \rangle$, if a node is valid then all its parent nodes are also valid. Therefore, S_e is the complète sub graph $\langle \mathrm{INS}', \mathrm{COV}' \rangle$of $\langle \mathrm{INS}(\Omega), \mathrm{COV} \rangle$ between the top node and a subset of the configuration base. For the problem of practical redundancy evaluation, the order of faults must be considered, and S is $\mathrm{TINS}(\Omega)$. If an edge e_i is invalid, then all paths including it are also invalid. Therefore, the set of valid states can be determined as follows. In the Hasse diagram $\langle \mathrm{INS}(\Omega), \mathrm{COV} \rangle$, go down from the top node to determine the validation of every node in turn, and get the valid complete sub graph $\langle \mathrm{INS}', \mathrm{COV}' \rangle$. INS' is Se for the problem of maximum redundancy evaluation. In the sub graph $\langle \mathrm{INS}', \mathrm{COV}' \rangle$, go down from the top to determine validation of every edge $e_i \in \mathrm{COV}'$. All the valid paths from the top node in $\langle \mathrm{INS}', \mathrm{COV}' \rangle$ give Se for the problem of practical redundancy evaluation.

Notice that calculation of $\mathrm{Re}(\Phi)$ depends on selection of Φ. In the frame of definition 3, selection of Φ is not unique. In the extreme condition, it can be chosen as the set formed by the minimum elements of Φ.

4 Maximum redundancy evaluation for sensor systems

Maximum redundancy evaluation needs to determine validation of the realities. Without consideration of the dynamics of sensors, a system with l sensors can be described as

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + \boldsymbol{\xi}_w(k) \tag{11}$$

$$\boldsymbol{y}_s(k) = C_s\boldsymbol{x}(k) + \boldsymbol{\xi}_s(k) \tag{12}$$

In the normal case, $\xi_w(k)$ and $\xi_s(k)$ are mutually uncorrelated zero mean Gaussian white noise sequences, and by suitable normalizing, they satisfy

$$E\{\boldsymbol{\xi}_s(k)\boldsymbol{\xi}_s^{\mathrm{T}}(j)\} = \sigma_s^2\delta_{kj}I,\ E\{\boldsymbol{\xi}_w(i)\boldsymbol{\xi}_w^{\mathrm{T}}(j)\} = Q\delta_{kj} \tag{13}$$

where δ_{kj} is the Kronecker function, $\boldsymbol{I}$ is the identical matrix, and the covariance matrix $\boldsymbol{Q}$ is positive definite.

The functional requirement for the sensor system is that the system is able to give estimates of a group of expected variables with certain accuracy. Denote the group of expected variables as

$$\boldsymbol{y}_a(k) = C_a\boldsymbol{x}(\mathrm{k}) \tag{14}$$

In the case of direct redundancy[6], there is a matrix $\boldsymbol{H}$ of maximum rank such that

$$C_s = HC_a \tag{15}$$

Then the smallest error covariance of all the estimates is

$$E[\tilde{\boldsymbol{y}}_a(k)\tilde{\boldsymbol{y}}_a^{\mathrm{T}}(k)] = \sigma_s^2(H^{\mathrm{T}}H)^{-1} \tag{16}$$

In the case of indirect signal redundancy[6], estimates of $\boldsymbol{y}_a(k)$ can be derived from (11), (12) and (14). The smallest error covariance is given by the Kalman filter[7]. Let P_x be the steady-state error covariance of $\boldsymbol{x}$. The steady-state error covariance of $\boldsymbol{y}_a$ *is*

$$E[\tilde{\boldsymbol{y}}_a(k)\tilde{\boldsymbol{y}}_a^{\mathrm{T}}(k)] = C_aP_xC_a^{\mathrm{T}} \tag{17}$$

5 Practical redundancy evaluation for sensor systems

To determine validation of an edge $e_i \in \mathrm{COV}'$ is actually a problem to evaluate the capability of the system to detect and diagnose its faults, which can be further described as a problem of multiple hypotheses test in a limited interval with expected false alarm rate α and missed alarm rate β. For the traditional physical redundancy systems, there exist multiple sensor elements for any variable in $\boldsymbol{y}_a$ to form a redundant sensor component, and fault detection and diagnosis (FDD) are limited in every component. To explore functional redundancy, FDD among components are necessary.

5.1 FDD in a component

For an n-redundancy sensor component without self-diagnosis, the maximum number of fault elements that can be detected and located by the normal noting technique is only $n-2$. Therefore the maximum redundancy degree is not achieved. To identify the system state when faults happen in the residual 2 elements, functional redundancy among different components must be explored to provide a reference estimate of the observed variable.

For the observed variable $\boldsymbol{y}_{si}(k)$ in (12), its reference estimate $\hat{y}_{si}(k) \sim N(y_{si}(k), \hat{\sigma}_i^2)$, where $\hat{\sigma}_i^2$ is provided by (16) or (17). Denote the output of the jth sensor element observing $y_{si}(k)$ as $y_{si}^j(k)$. In the normal case $y_{si}^j(k) \sim N(y_{si}(k), \sigma_s^2)$, while in the fault case $y_{si}^j(k) \sim N(y_{si}(k) \pm \Delta_i, \sigma_s^2)$, where $\Delta_i > 0$ is the given error bound of the sensor. We need perform the following dual hypotheses test for the jth sensor element:

$$\gamma_j(k) = y_{si}^j(k) - \hat{y}_{si}(k) \sim N(\mu_j, \sigma_s^2 + \hat{\sigma}_i^2),\ k = 1, 2, \cdots, N \tag{18a}$$

$$H_0: \mu_j = 0,\ H_1: \mu_j = \pm\Delta_i \tag{18b}$$

Notice that the number of samples, N, can be used as the measure of the time interval. The following theorem gives the expression of N.

Theorem 1. The minimum N for the above problem is

$$N = \frac{(Z_\alpha + Z_\beta)^2(\sigma_s^2 + \hat{\sigma}_i^2)}{\Delta_i^2} \tag{19}$$

where Z_α and Z_β are the α and β confidence bounds of the normal Gaussian distribution, respectively.

5.2 FDD among components

5.2.1 Case of Direct Redundancy

Consider (11)~(15). In the normal case, $\xi_s(k) \sim N(0, \sigma_s^2 I)$. When the ith element drops into fault, $\xi_s(k) \sim N(\pm\Delta_i \boldsymbol{e}_i^l, \sigma_s^2 I)$, where $\boldsymbol{e}_i^l$ is the ith unit vector of the l-dimensional space.

Let

$$V^{\mathrm{T}}H = 0,\quad V^{\mathrm{T}}V = I \tag{20}$$

We have

$$y_s'(k) = V^{\mathrm{T}}y_s(k) = V^{\mathrm{T}}\boldsymbol{\xi}_s(k) = \boldsymbol{\xi}_s'(k),\ E\{\boldsymbol{\xi}'_s(k)\boldsymbol{\xi}'^{\mathrm{T}}_s(k)\} = \sigma_s^2 I \tag{21}$$

Let $\boldsymbol{v}_i$ be the ith row of V. Fault diagnosis for the ith sensor component need perform the following dual hypotheses tests.

$$y_s'(k) \sim N(\boldsymbol{\mu}', \sigma_s^2 I),\ k = 1, 2, \cdots, N \tag{22a}$$

$$H_0: \boldsymbol{\mu}' = \pm\Delta_i \boldsymbol{v}_i^{\mathrm{T}},\quad H_1: \boldsymbol{\mu}' = \pm\Delta_j \boldsymbol{v}_j^{\mathrm{T}}, j \neq i \tag{22b}$$

Theorem 2. The minimum N for the above problem with false alarm rate α and missed alarm rate β is

$$N = \max_{j\neq i} \frac{(Z_\alpha + Z_\beta)^2 \sigma_s^2}{\Delta_i^2 \|v_i\|^2 + \Delta_j^2 \|v_j\|^2 - 2\Delta_i\Delta_j \mid v_i v_j^T \mid} \tag{23}$$

5.2.2 Case of Indirect Redundancy

Consider the system described by (11) and (12). For the observation $y_{sa}(k) \in \boldsymbol{y}_s(k)$, there exists a group of different observations $\boldsymbol{y}_{sb}(k) = [y_{sb1}(k), y_{sb2}(k), \cdots, y_{sbp}(k)]^T \subset \boldsymbol{y}_s(k)$, which imply indirect redundancy of $y_{sa}(k)$. Let C_{sa} and C_{sb} be the observation matrix of $y_{sa}(k)$ and $\boldsymbol{y}_{sb}(k)$, respectively. Then from $\boldsymbol{y}_{sb}(k)$ we can get a reference estimate $\hat{y}_{sa}(k)$ of $y_{sa}(k)$. The minimum mean square estimate of $y_{sa}(k)$ is given by the steady-state Kalman filter, with the Kalman gain $\bar{K}$ and the single-state error-covariance matrix $\bar{P}$.

For the sensor component observing $y_{sa}(k)$, the fundamental way of fault diagnosis is to do statistical test on the following sequence:

$$\gamma_{sa}(k) = y_{sa}(k) - \hat{y}_{sa}(k) \tag{24}$$

Because if some of the sensor components observing y_{sb} are fault, the characteristic of $\gamma_{sa}(k)$ can also change, we need test the following Kalman innovations sequence additionally,

$$\boldsymbol{\gamma}_{sb}(k) = \boldsymbol{y}_{sb}(k) - \hat{\boldsymbol{y}}_{sb}(k) \tag{25}$$

As a result, fault diagnosis in this case need perform p dual hypotheses tests as follows:

$$\boldsymbol{\gamma}(k) = [\boldsymbol{\gamma}_{sb}^T(k) \quad \gamma_{sa}(k)]^T \sim N(\boldsymbol{\mu}(k), R), \quad k = 1, 2, \cdots \tag{26a}$$

$$\begin{cases} H_0(y_{sa} \text{ sensor fault}): \boldsymbol{\mu}(k) = \pm \Delta_{sa}\boldsymbol{u}_{sa}(k) \\ H_1(y_{sbi} \text{ sensor fault}): \boldsymbol{\mu}(k) = \pm \Delta_{sbi}\boldsymbol{u}_{sbi}(k) \end{cases}, \quad i = 1, \cdots, p \tag{26b}$$

where Δ_{sa} and Δ_{sbi} are the given error bounds for the sensors of y_{sa} and y_{sbi}, respectively, and $k=1$ at the time fault happens. $\boldsymbol{u}_{sa}(k)$ and $\boldsymbol{u}_{sbi}(k)$ are the corresponding fault vectors, and

$$\boldsymbol{u}_{sa}(k) = \begin{bmatrix} 0_{p\times 1} \\ 1 \end{bmatrix}, R = [C_{sb}^T \quad C_{sa}^T]^T \bar{P} [C_{sb}^T \quad C_{sa}^T] + \sigma_s^2 I \tag{27}$$

$\boldsymbol{u}_{sbi}(k)$ is given by the following theorem.

Theorem 3. Let e_i^p be the ith unit vector of p-dimensional space. Then

$$\boldsymbol{u}_{sbi}(k) = \begin{bmatrix} -[C_{sb}\boldsymbol{m}_i(k) + e_i^p] \\ -C_{sa}\boldsymbol{m}_i(k) \end{bmatrix}, \quad k = 1, 2, \cdots \tag{28}$$

$$\boldsymbol{m}_i(k) = A(\boldsymbol{m}_i(k-1) - \bar{K}(C_{sb}\boldsymbol{m}(k-1) + e_i^p)), \quad \boldsymbol{m}_i(0) = 0 \tag{29}$$

Proof. Notice that the observation noise $\boldsymbol{\xi}_{sb}(k) \sim N(\pm\Delta_{sbi}e_i^p, \ \sigma_s^2 I)$, $k = 1, 2, \cdots$. Substituting it into the steady-state Kalman filter leads to (29). □

Theorem 4. For the above problem, the minimum capacity N with false alarm rate α and missed alarm rate β is determined by

$$\sigma_{Li}^2 > (Z_\alpha + Z_\beta)^2, \quad i = 1, 2, \cdots, p \tag{30}$$

$$\sigma_{Li}^2 = \sum_{j=1}^{N} [\Delta_{sbi}^2 \boldsymbol{u}_{sbi}^T(j) R^{-1} \boldsymbol{u}_{sbi}(j) + \Delta_{sa}^2 \boldsymbol{u}_{sa}^T(j) R^{-1} \boldsymbol{u}_{sa}(j) - 2 \mid \Delta_{sbi}\Delta_{sa} \boldsymbol{u}_{sbi}^T(j) R^{-1} \boldsymbol{u}_{sa}^T(j) \mid] \tag{31}$$

6 Examples for redundancy evaluation

6.1 Example 1

To show the effectiveness of the 'configuration redundancy' concept in the integrated redundancy evaluation, consider the system with 3 redundant elements, A, B and C. The set of realizations is shown as figure 1, in which the sets of nodes above different specific curves give different S_e's. Assume that all the elements have the same invalidation probability P, Re(Φ) for all the Se's are shown in table 1.

S_{e1} represents the standard physical 3-redundancy system with self-diagnosis. Its configuration set is $\{\{A\},\{B\},\{C\}\}$, and the configuration redundancy degree is always 3. This is also the maximum Re(Φ) possible for a 3-redundancy system. With the capability of self-diagnosis decreasing, we face the problem of practical redundancy evaluation and get smaller Re(Φ). For example, S_{e2} represents the standard physical 3-redundancy system without self-diagnosis, and S_{e3}, in which A and B are not allowed to be fault at the same time, represents the physical 2-redundancy system (A and B) with self-diagnosis. Let the configuration set for them be $\{\{AB\},\{BC\},\{CA\},\{A\},\{B\}\}$. It is shown from table 1 that the reliability of S_{e3} is always higher than that of S_{e2}, while the practical redundancy degree of S_{e3} is always lower than that of S_{e2}. This verifies once more that redundancy and reliability are essentially different, and that they give us different viewpoints of system design. Table 1 also shows that changing invalidation probability of elements does not bring about large disturbance of Re(Φ), the slight disturbance is because that changing invalidation probability of elements would change the level in which the configurations correlate each other.

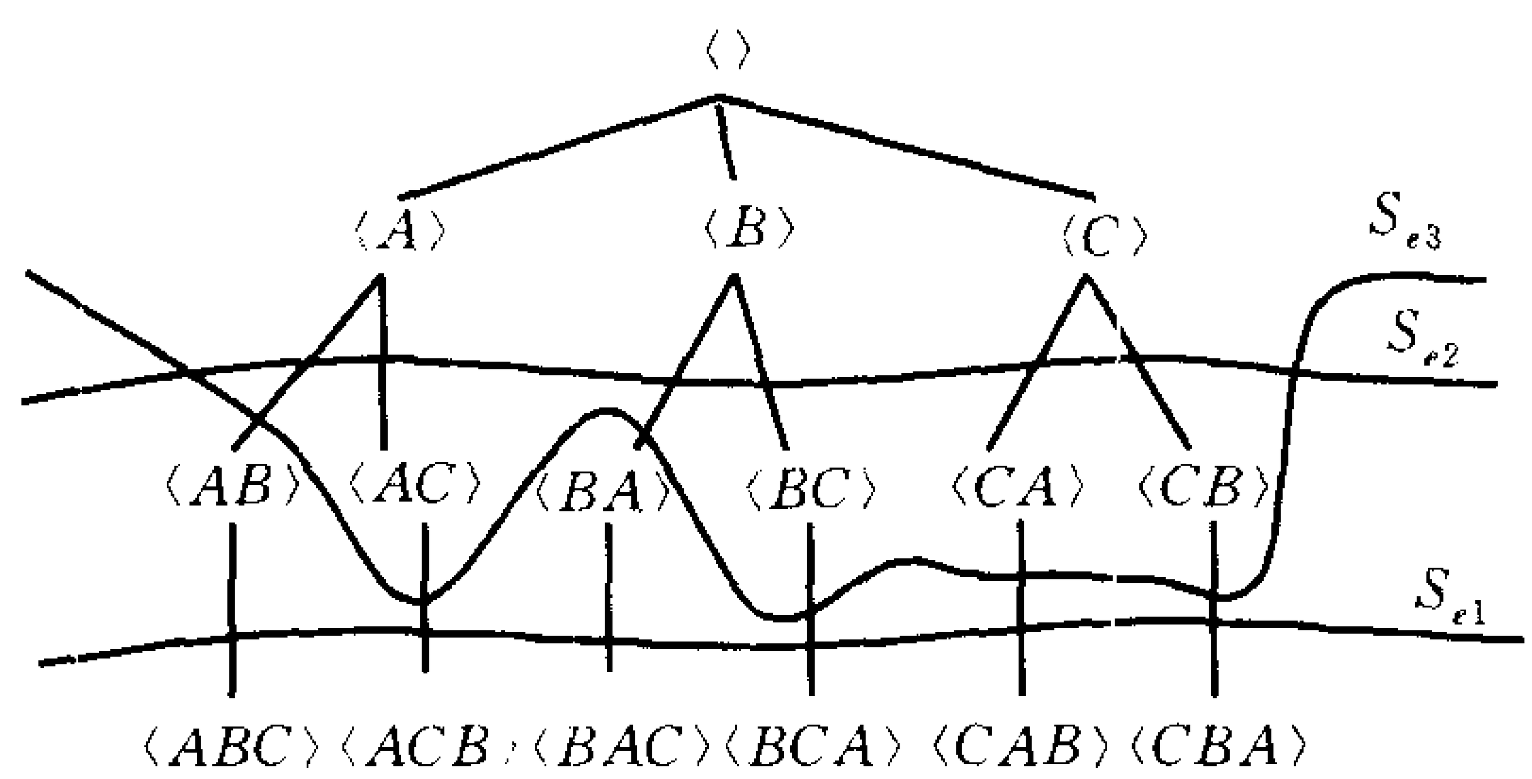

Fig. 1 Realizations of the system: Example 1

Table 1 Result of redundancy evaluation: Example 1

P	Re(S_{e1})	Re(S_{e2})	Re(S_{e3})	R(S_{e2})	R(S_{e3})
0.3	3.0	2.15	2.0	0.78	0.91
0.2	3.0	2.22	2.0	0.89	0.96
0.1	3.0	2.28	2.0	0.97	0.99

6.2 Example 2

Consider a flight sensor system. The longitudinal dynamics of the aircraft is in the form of (11), and

$$\boldsymbol{x} = \begin{bmatrix} \alpha \\ q \\ \theta \end{bmatrix}, \quad A = \begin{bmatrix} -1.1 & 1.0 & 0 \\ -3.16 & -1.67 & 0 \\ 0 & 1.0 & 0 \end{bmatrix}, \quad Q = 10^{-5} \cdot I_{3\times 3},$$

where θ is the pitch angle , q the pitch rate, and α the attack angle.

Control augment and attitude control need feedback of all the state variables, i. e. $\boldsymbol{y}_a = [\alpha, q, \theta]^{\mathrm{T}}$. Additionally, it is usually necessary to observe the normal acceleration n_z. Dispose a single sensor element for each variable above, and normalize them according to (13). We have the observation vector

$$\boldsymbol{y}_s = [\alpha/0.03 \quad n_z/0.007 \quad q/0.01 \quad \theta/0.01]^{\mathrm{T}}, \quad n_z = 14.85\alpha, \quad \sigma_s = 1.$$

For the problem of maximum redundancy evaluation, let the required estimation accuracy for $\boldsymbol{y}_a$ be $\sigma_q \leqslant 0.05$(rad/s), $\sigma_\alpha \leqslant 0.05$(rad), $\sigma_\theta \leqslant 0.03$(rad). Calculate the actual estimation accuracy of $\boldsymbol{y}_a$ for every realization according to (16) and (17). Then the set of the minimum elements of S_e is $S_{e_{\min 1}} = \{\theta\}$. For the problem of practical redundancy evaluation, let $\Delta = 2$, $\alpha = \beta = 0.005$ and $N = 8$, and calculate σ_{Li}^2 for every sequent realization according to (30). Then the set of the minimum elements of S_e is $S_{e_{\min 2}} = \{\langle n_z, \alpha\rangle, \langle \alpha, n_z\rangle,$

$\langle n_z, q\rangle, \langle q, n_z\rangle, \langle q, \alpha\rangle, \langle \alpha, q\rangle\}$. According to $S_{e_{min1}}$ and $S_{e_{min2}}$, let the configuration base be $\{\{\theta\}, \{q, \theta\}, \{\alpha, \theta\}, \{n_z, \theta\}\}$. The results of redundancy evaluation are as shown in table 2. It is shown that, when considering physical/functional redundancy generally, even a normal system with single physical redundancy can provide configuration redundancy degree greater than 1. This is just the foundation of all the techniques of functional redundancy exploration. The practical redundancy degree is usually smaller than the maximum one, however the two values can be made closer by improving the system capability to diagnose its faults.

The concept of configuration redundancy also provides the criteria of integrated disposition of physical/functional redundancy for system design. In this example, if it is needed to add another sensor element to make the system more fault-tolerant, we can get an optimized program according to the configuration redundancy degree. Chose a general set of elements, $\{\alpha_1, n_{z1}, q_1, \theta_1, \alpha_2, n_{z2}, q_2, \theta_2\}$, and a general configuration base $\{\{\theta_1\}, \{\theta_2\}, \{q_1, \theta_1\}, \{\alpha_1, \theta_1\}, \{n_{z1}, \theta_1\}\{q_2, \theta_1\}, \{\alpha_2, \theta_1\}, \{n_{z2}, \theta_1\}, \{q_1, \theta_2\}, \{\alpha_1, \theta_2\}, \{n_{z1}, \theta_2\}\}$. The results of redundancy evaluation are shown in table 3, in which the program 4 is most fault-tolerant.

Table 2 Results of redundancy evaluation: Example 2

Practical	Maximum
1.662903	1.677644

Table 3 Optimized disposition: Example 2

编号	Ω	$Re(\Omega)$
1	$\{\alpha_1, \alpha_2, n_{z1}, q_1, \theta_1\}$	1.633853
2	$\{\alpha_1, n_{z1}, n_{z2}, q_1, \theta_1\}$	1.713906
3	$\{\alpha_1, n_{z1}, q_1, q_2, \theta_1\}$	1.826798
4	$\{\alpha_1, n_{z1}, q_1, \theta_1, \theta_2\}$	2.3092440

7 Conclusion

The concept of configuration redundancy gives us a general point to understand different kinds of redundancy existing in a system, and shows both relationship and difference between reliability and redundancy. Essentially, redundancy is not a statistical characteristic which reflects reliability of a system, but a structural one which reflects survivability of the system after fault and failure. Studying evaluation of configuration redundancy will provide novel criteria for system design.

Based on the provided index of configuration redundancy degree, it is feasible to evaluate different kinds of redundancy in the system, consequently to guide the integrated disposition of them. The provided methods and examples for sensor systems have verified the effectiveness of the concept and index.

References

1 Chen Kaisheng, Zhang Shengkun. Semi-probabilistic method for evaluating system redundancy of existing offshore structures. *Ocean Engineering (Pergamon)*, 1996, **23**(6):455~464

2 Ziha K. Redundancy and robustness of systems of events. *Probabilistic Engineering Mechanics*, 2000, **15**(4):347~357

3 Luc B, Jorge A. Measurement subspaces of parallel manipulators under sensor redundancy. *American Society of Mechanical Engineers, Design Engineering Division (Publication)*, 1994, **69**(2):467~474

4 Veljko P, Goran D. Kinematic redundancy and sensor redundancy for enhancement of robot tracking performance. *Journal of Intelligent and Robotic Systems: Theory & Applications*, 1996, **15**(3): 263~289

5 Richard J. Discrete Mathematics. New York: Macmillan Publishing Company, 1984. 248~305

6 Cheng Y. Functional Redundancy Analysis of Actuators and Sensors in Linear Multiple Variable Systems. *Information and Control*, 1990, **19**(1):56~60(in Chinese)

7 Mendel J M. Lessons in Digital Estimation Theory. New Jersey: Prentice-Hall, 1987. 149~160

GE Tong Received his master degree and Ph. D. degree from Northwestern Polytechnic University. He is now an associate professor of Shanghai Jiaotong University. His research interests include intelligent control and reconfigurable control of aircraft and underwater vehicles.

构形冗余概念及传感器系统冗余综合评估与配置

葛 彤

（上海交通大学船舶与海洋工程学院 上海 200030）

（E-mail：tongge@mail. sjtu. edu. cn）

摘 要 复杂控制系统中存在大量不同机理和作用方式的功能和硬件冗余，主动开发和配置这种冗余是控制系统设计的重要课题. 本文提出新的“构形冗余”概念，对不同的系统冗余提供了统一的理解角度和通用冗余指标，可以综合评价系统中蕴涵的各种冗余，因而指导对这些冗余的统一配置，提供新的系统主动生存性设计依据. 构形冗余概念被具体用于传感器系统，给出了其构形冗余指标的具体计算方法，并通过若干实例验证了上述方法在冗余综合评估与配置中的应用效果.

关键词 冗余，生存性，配置，评估，构形

中图分类号 TP202；TP11