Comments on "H_∞ Output Feedback Control Design of Fuzzy Dynamic Systems Via LMI"[1)]

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Abstract We point out a mistake and the incorrect conclusion of the paper "H_∞ output feedback control design of fuzzy dynamic systems via LMI" in Acta Automatica Sinica 2001, 27(4). We also give the correction of the mistake.

Key words Fuzzy dynamic systems, H_∞ output feedback control, linear matrix inequality (LMI), asymptotical stability

1 Introduction

In the paper "H_∞ output feedback control design of fuzzy dynamic systems via LMI" in Acta Automatica Sinica 2001, 27(4)[1], the authors discussed the H_∞ output feedback control problem for fuzzy dynamic systems. Unfortunately, the conclusion of its Theorem 2, which refers to asymptotical stability of H_∞ output feedback control problem for fuzzy dynamic systems, is incorrect.

2 Description of the problem

The mistake of the paper is found in the proof of its Theorem 2 regarding the asymptotical stability of the closed loop system, where the following equation is used (see equ. (37) of [1]):

$$\varepsilon_l^{-1}\Delta\hat{A}_l^T\Delta\hat{A}_l=\varepsilon_l^{-1}\begin{bmatrix}I&0\\0&C_{cl}\end{bmatrix}^T\begin{bmatrix}\Delta A_l&\Delta B_l\\0&0\end{bmatrix}^T\begin{bmatrix}I&0\\0&B_{cl}\end{bmatrix}^T\begin{bmatrix}I&0\\0&B_{cl}\end{bmatrix}\begin{bmatrix}\Delta A_l&\Delta B_l\\0&0\end{bmatrix}\begin{bmatrix}I&0\\0&C_{cl}\end{bmatrix}\tag{1}$$

Directly calculating the right hand side of (1) yields

$$\varepsilon_l^{-1}\begin{bmatrix}I&0\\0&C_{cl}\end{bmatrix}^T\begin{bmatrix}\Delta A_l&\Delta B_l\\0&0\end{bmatrix}^T\begin{bmatrix}I&0\\0&B_{cl}\end{bmatrix}^T\begin{bmatrix}I&0\\0&B_{cl}\end{bmatrix}\begin{bmatrix}\Delta A_l&\Delta B_l\\0&0\end{bmatrix}\begin{bmatrix}I&0\\0&C_{cl}\end{bmatrix}=\begin{bmatrix}\Delta A_l^T\Delta A_l&\Delta A_l^T\Delta B_lC_{cl}\\C_{cl}^T\Delta B_l^T\Delta A_l&C_{cl}^T\Delta B_l^T\Delta B_lC_{cl}\end{bmatrix}\tag{2}$$

However, since

$$\varepsilon_l^{-1}\Delta\hat{A}_l^T\Delta\hat{A}_l=\begin{bmatrix}\Delta A_l^T\Delta A_l+\Delta C_l^TB_{cl}^TB_{cl}\Delta C_l&\Delta A_l^T\Delta B_lC_{cl}\\C_{cl}^T\Delta B_l^T\Delta A_l&C_{cl}^T\Delta B_l^T\Delta B_lC_{cl}\end{bmatrix}\tag{3}$$

it immediately follows from (2) and (3) that (1) is incorrect. Thus, the corresponding conclusion of Theorem 2 of [1] becomes untenable and, of course, should be corrected. In fact, from the above analysis it is easy to see that Theorem 2 of [1] is valid only when $\Delta C_l=0$. It should also be pointed out that the simulation provided in [1] exactly satisfies this condition and thus does not reveal the limitation.

3 Correction of Theorem 2 of [1]

If we directly use the following estimate of $\Delta\hat{A}_l^TP_l+P_l\Delta\hat{A}_l$:

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$$\Delta\hat{A}_l^{\mathrm{T}}P_l + P_l\Delta\hat{A}_l \leqslant \varepsilon_l P_l P_l + \varepsilon_l^{-1}\Delta\hat{A}_l^{\mathrm{T}}\Delta\hat{A}_l \tag{4}$$

then the term $\Delta C_l^{\mathrm{T}}B_{cl}^{\mathrm{T}}B_{cl}\Delta C_l$, which appears in (2) and leads to the incorrect conclusion in [1], will prevent us from further processing. Instead of using the method in [1], we can first decompose $\Delta\hat{A}_l$ in (4) as follows:

$$\Delta\hat{A}_l = \Delta\hat{A}_{l1} + \Delta\hat{A}_{l2} = \begin{bmatrix}\Delta A_l & \Delta B_l C_{cl}\\ 0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ B_{cl}\Delta C_l & 0\end{bmatrix} =$$
$$\begin{bmatrix}\Delta A_l & \Delta B_l\\ 0 & 0\end{bmatrix}\begin{bmatrix}I & 0\\ 0 & C_{cl}\end{bmatrix} + \begin{bmatrix}I & 0\\ 0 & B_{cl}\end{bmatrix}\begin{bmatrix}0 & 0\\ \Delta C_l & 0\end{bmatrix} \tag{5}$$

Then using lemma 2 of [1], we can estimate $\Delta\hat{A}_{l1}^{\mathrm{T}}P_l + P_l\Delta\hat{A}_{l1}$ and $\Delta\hat{A}_{l2}^{\mathrm{T}}P_l + P_l\Delta\hat{A}_{l2}$, respectively, without introducing the term $\Delta C_l^{\mathrm{T}}B_{cl}^{\mathrm{T}}B_{cl}\Delta C_l$. Now we give a correction to Theorem 2 of [1] as follows.

Given a fuzzy dynamic system described as in (21) of [1], its feedback controller described as in (22) of [1], an approximate upper bound of the uncertainties of (21) of [1] defined by (29) of [1], and the H_∞ performance bound $\gamma>0$ as a constant, if there exist a set of constants $\varepsilon_{l1}>0$, $\varepsilon_{l2}>0$, and a set of variables $\bar{A}_l$, $\bar{B}_l$, $\bar{C}_l$, $\bar{E}_{l0}$, $\bar{E}_{l1}$, $\bar{E}_{l2}$, X_l and Y_l satisfying the following inequalities:

$$\begin{bmatrix}M_{1l} & M_{2l}\\ M_{2l}^{\mathrm{T}} & M_{3l}\end{bmatrix} < 0,\ \begin{bmatrix}X_l & I\\ I & Y_l\end{bmatrix} > 0,\ P(\tau_i^+) \leqslant P(\tau_i^-),\ i = 1,2,\cdots,\bar{T}$$

where

$$M_{1l} = \begin{bmatrix}\Omega_{l11} & \Omega_{l21}^{\mathrm{T}}\\ \Omega_{l21} & \Omega_{l22}\end{bmatrix}$$

$$M_{3l} = \mathrm{diag}\left\{-\varepsilon_l^{-1}\begin{bmatrix}\bar{E}_{l1} & X_l\\ X_l & I\end{bmatrix}, -\varepsilon_l\begin{bmatrix}I & Y_l\\ Y_l & \bar{E}_{l2}\end{bmatrix}, \frac{-\gamma^2 I}{2}, \frac{-\gamma^2 I}{2}, \frac{-I}{2}, \frac{-I}{2}, \frac{-I}{\varepsilon_{l2}}, \frac{-I}{\varepsilon_{l2}}, -\varepsilon_{l2}I\right\}$$

$$M_{2l} = \begin{bmatrix}X_l & I & \Omega_{l51}^{\mathrm{T}} & \bar{E}_{l0}^{\mathrm{T}} & B_{wl} & E_{Bwl} & (C_{zl}X_l)^{\mathrm{T}} & (E_{Czl}X_l)^{\mathrm{T}} & I & 0 & (E_{cl}X_l)^{\mathrm{T}}\\ I & Y_l & E_{Al}^{\mathrm{T}} & (Y_l E_{Al})^{\mathrm{T}} & Y_l B_{wl} & Y_l E_{Bwl} & C_{zl}^{\mathrm{T}} & E_{Cl}^{\mathrm{T}} & Y_l & \bar{B}_l & E_{cl}^{\mathrm{T}}\end{bmatrix}$$

$$\Omega_{l11} = A_l X_l + X_l A_l^{\mathrm{T}} + B_l\bar{C}_l + (B_l\bar{C}_l)^{\mathrm{T}},\ \Omega_{l22} = A_l^{\mathrm{T}}Y_l + Y_l A_l + \bar{B}_l C_l - (\bar{B}_l C_l)^{\mathrm{T}}$$

$$\Omega_{l21} = \bar{A}_l + A_l^{\mathrm{T}},\ \Omega_{l51} = E_{Al}X_l + E_{Bl}\bar{C}_l,\ l = 1,2,\cdots,m$$

then the closed loop system (23) of [1] is asymptotically stable with H_∞ performance bound γ. Furthermore, if the matrices M_l and N_l are in the form of full rank decomposition

$$M_l N_l^{\mathrm{T}} = I - X_l Y_l$$

then a parameterization of H_∞ output feedback controller can be determined as

$$C_{cl} = \bar{C}_l M_l^{+^{\mathrm{T}}},\ B_{cl} = N_l^{+}\bar{B}_l,\ A_{cl} = N_l^{+}(\bar{A}_l - (N_l B_{cl} C_l X_l + Y_l B_l C_{cl} M_l^{\mathrm{T}} + Y_l A_l X_l))M_l^{+^{\mathrm{T}}}$$

where M_l^{+} and N_l^{+} denote the Moore-Penrose general inverses of M_l and N_l, respectively.

4 Conclusion

In this note a possible careless mistake which leads to the incorrect conclusion of [1] is pointed out, and its correction is given.

There is much work that needs to be discussed for the problem of stability and performance of the closed system by using switching control. Some classical results have been discussed in detail in [2] and the related results and techniques in [3] and [4] are beneficial to this issue.

References

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与"基于 LMI 的模糊动态系统 H_∞ 输出反馈控制设计"一文商榷

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摘　要　本文指出了《自动化学报》2001 年 27 卷 4 期上刊登的文章"基于 LMI 的模糊动态系统 H_∞ 输出反馈控制设计"中某些结论的不妥之处,并给出了修改.

关键词　模糊动力系统,H_∞ 输出反馈控制,线性矩阵不等式(LMI),渐近稳定性

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