

非马尔可夫随机 Petri 网模型的 混合状态分析法¹⁾

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摘 要

讨论具有发射时间任意分布之变迁的随机 Petri 网的解析问题。定义了随机 Petri 网的混合状态和混合状态密度,提出混合状态分析法,并给出具有一步转移关系标识下的混合状态密度的递推公式。使非马尔可夫型随机 Petri 网的分析成为可能。通过算例说明了混合状态分析法在系统性能评估中的应用。

关键词: 随机 Petri 网,混合状态,性能评估。

1 引言

作为 Petri 网^[1]的扩展形式之一,随机 Petri 网 (SPN) 已在各种离散事件系统的性能评估中获得广泛的应用^[2-5]。根据 SPN 标识过程 $\{M(\tau), \tau \geq 0\}$ 的随机特性,可将 SPN 分为三类,即马尔可夫型 SPN^[2,3]、半马尔可夫型 SPN^[4,5] 和非马尔可夫型 SPN。只要 SPN 的某可达标识下存在发射时间为连续的非负指数分布的并发型使能变迁,则该 SPN 就是非马尔可夫型。由于传统的分析方法将 SPN 的标识定义为 SPN 的随机状态变量,因而对于非马尔可夫 SPN 的分析就显得无能为力。而客观上,又存在这样的离散事件系统,它们中事件发生的持续时间服从诸如截尾高斯分布之类的连续型非负指数分布。为此,有必要提出既适合于马尔可夫型和半马尔可夫型 SPN,也同样适合于非马尔可夫型 SPN 的混合状态分析法。

这里的讨论基于以下基本假定:

(B1) 任意变迁 $t_k \in T$ (T 为 SPN 的变迁集),发射时间 θ_k 独立于标识 M , 即其分布函数 $\Phi_k(x|M) = \Phi_k(x)$ 。 θ_k 的分布密度记为 $\varphi_k(x)$;

(B2) 所有 T 中变迁的发射时间互相独立;

(B3) 若 $M_i \geq k \cdot I(t_j)$, 即标识 M_i 下变迁 t_j 是 k -使能的, $k \geq 2$, 则规定 t_j 在 M_i 下发射后, t_j 的使能时间在新的标识 (设为 M_*) 下自零值随时间递增,并将 t_j 视为 M_* 下新产生的使能变迁;

(B4) 任意 $M_i, M_j \in R$ (R 为 SPN 的可达标识集), 至多只存在一个使能变迁 t_k ,

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使得 $M_i \xrightarrow{t_k} M_j$.

2 SPN 的混合状态密度

设 τ 时刻 SPN 处于标识 $\tilde{M}(\tau)$, 任意 $t_i \in T$ 的使能时间为 $\tilde{x}_i(\tau)$, 且 $|T| = m$, 则

$$\{(\tilde{M}(\tau), \tilde{x}_1(\tau), \tilde{x}_2(\tau), \dots, \tilde{x}_m(\tau))^T, \tau \geq 0\}$$

构成了一个混合状态的随机过程. 其中 $\tilde{M}(\tau)$ 为离散型状态分量, 其值域为可达集 R ; $\tilde{x}_i(\tau)$ 为连续型状态分量, 其值域为非负实数域. 记任意标识 M_k 下使能变迁的集合为 $E(M_k)$. 易知, 在 $\tilde{M}(\tau) = M_k$ 的条件下, 对于任意 $t_i \in T \setminus E(M_k)$, 有

$$P_{\text{rob}}\{\tilde{x}_i(\tau) = 0\} = 1. \tag{1}$$

为简便计, 这里将 SPN 的标识及标识下使能变迁的使能时间一起定义为 SPN 的混合状态.

设 $M_k \in R, E(M_k) = \{t_{k1}, t_{k2}, \dots, t_{kr}\}$, 并令

$$p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr}) = \lim_{\Delta x_{k1} \rightarrow 0} \lim_{\Delta x_{k2} \rightarrow 0} \dots \lim_{\Delta x_{kr} \rightarrow 0} \frac{P_{\text{rob}}\{\tilde{M}(\tau) = M_k, x_{ki} \leq x_{ki}(\tau) < x_{ki} + \Delta x_{ki}, i = 1, 2, \dots, r\}}{\Delta x_{k1} \Delta x_{k2} \dots \Delta x_{kr}}, \tag{2}$$

称函数 $p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr})$ 为 SPN 的混合状态密度.

由定义(2)式可知, 混合状态密度 $p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr})$ 是指 SPN 在 τ 时刻处于标识 M_k , 且其使能变迁 $t_{k1}, t_{k2}, \dots, t_{kr}$ 的使能时间依次为 $x_{k1}, x_{k2}, \dots, x_{kr}$ 的概率密度.

鉴于变迁的使能时间总为非负值, 约定

$$p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr}) = 0, \text{ 当 } \min\{x_{k1}, x_{k2}, \dots, x_{kr}\} < 0. \tag{3}$$

3 混合状态密度的计算

3.1 标识下使能变迁的划分

设 $t_q \in E(M_j)$, 则 $E(M_j)$ 中与 t_q 为竞争关系的变迁集为

$$C'(M_j, t_q) = \{t_l | t_l \in E(M_j) \setminus \{t_q\}, I(t_q) + I(t_l) > M_j\}. \tag{4}$$

令
$$C(M_j, t_q) = C'(M_j, t_q) \cup \{t_q\}. \tag{5}$$

若 $M_j \xrightarrow{t_q} M_k$, 则记

$$E(M_k) = E_{j1}(M_k) + E_{j2}(M_k). \tag{6}$$

其中

$$E_{j1}(M_k) = E(M_j) \cap E(M_k) \setminus C(M_j, t_q), \tag{7}$$

$$\begin{aligned} E_{j2}(M_k) &= E(M_k) \setminus [E(M_j) \cap E(M_k) \setminus C(M_j, t_q)] \\ &= E(M_k) \setminus [E(M_j) \setminus C(M_j, t_q)]. \end{aligned} \tag{8}$$

这样, 即将标识 $M_k \in R$ 的使能变迁集 $E(M_k)$ 划分为两部分: 第一部分 $E_{j1}(M_k)$ 为标识 M_k 下新产生的使能变迁集, 包括在 M_k 下使能, 但在 M_j 下不使能的变迁; 在 M_j ,

M_k 下都使能,但在 M_j 下与 t_q 成竞争关系的变迁,后者的使能时间在 t_q 的发射时刻回归零值(基本假定 (B3));第二部分 $E_{j2}(M_k)$ 为从 M_j 保留下来的使能变迁集,即 $E_{j2}(M_k)$ 中变迁在 M_j 下为使能变迁,且与 t_q 成并发关系.显然,对应于 M_k 的不同的直接前继标识, $E(M_k)$ 的划分也不尽相同.

3.2 混合状态密度的递推方程

记 M_1 为 SPN 的初始标识.对于标识 $M_k \in R \setminus \{M_1\}$,将 M_k 的直接前继标识集 $U(M_k)$ 分解为两个互补子集 $U_1(M_k)$ 和 $U_2(M_k)$,使得

$$E_{j1}(M_k) \neq \emptyset, \forall M_j \in U_1(M_k), \quad (9)$$

$$E_{j1}(M_k) = \emptyset, \forall M_j \in U_2(M_k). \quad (10)$$

即当 SPN 从任意 $U_1(M_k)$ 中标识跃迁到 M_k 时,必产生新的使能变迁;而从任意 $U_2(M_k)$ 中标识跃迁到 M_k 时,必不产生新的使能变迁.

先假定 $U_2(M_k) = \emptyset, U_1(M_k) = \{M_j\}$, 并且 $M_j \xrightarrow{t_q} M_k$.

设 $\min\{x_{k1}, x_{k2}, \dots, x_{kr}\} > \varepsilon > 0$, 且 $\Delta\tau$ 充分小,则时间段 $[\tau - \Delta\tau, \tau]$ 内无变迁发射,而有

$$\begin{aligned} & p(\tau - \Delta\tau; M_k, x_{k1} - \Delta\tau, x_{k2} - \Delta\tau, \dots, x_{kr} - \Delta\tau) \\ &= p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr}) \prod_{i=1}^r \left[1 - \frac{\varphi_{ki}(x_{ki})}{1 - \Phi_{ki}(x_{ki})} \Delta\tau \right] \\ &= p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr}) \left[1 - \sum_{i=1}^r \frac{\varphi_{ki}(x_{ki})}{1 - \Phi_{ki}(x_{ki})} \Delta\tau + o(\Delta\tau) \right]. \quad (11) \end{aligned}$$

若 $p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr})$ 对每个 $x_{ki} \in \{x_{k1}, x_{k2}, \dots, x_{kr}\}$ 都是可微的,则由(11)式可得

$$\begin{aligned} & \frac{\partial \ln p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr})}{\partial \tau} + \sum_{i=1}^r \frac{\partial \ln p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr})}{\partial x_{ki}} \\ &= - \sum_{i=1}^r \frac{\varphi_{ki}(x_{ki})}{1 - \Phi_{ki}(x_{ki})}. \quad (12) \end{aligned}$$

解(12)式可得

$$\begin{aligned} & p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr}) \\ &= p(\tau - x_{(k)}; M_k, x_{k1} - x_{(k)}, x_{k2} - x_{(k)}, \dots, x_{kr} - x_{(k)}) \\ & \quad \cdot \prod_{i=1}^r \frac{1 - \Phi_{ki}(x_{ki})}{1 - \Phi_{ki}(x_{ki} - x_{(k)})}, \quad (13) \end{aligned}$$

其中 $x_{(k)} = \min\{x_{k1}, x_{k2}, \dots, x_{kr}\}$. (14)

因已假定标识 M_k 下必产生新的使能变迁,故 $x_{(k)}$ 代表 SPN 跃迁到 M_k 后逝去的时间,从而 $\tau - x_{(k)}$ 即为 SPN 跃迁到 M_k 的时刻.所以,(13)式给出了标识 M_k 下任意时刻与刚跃入 M_k 的那个时刻的 SPN 混合状态密度之间的关系.

下面考虑标识更新时刻 M_j 和 M_k 标识下混合状态密度的关系.因为 $M_j \xrightarrow{t_q} M_k$, 故有

$$E(M_j) = E_{j2}(M_k) + C(M_j, t_q). \quad (15)$$

$$\text{令 } E_{j_2}(M_k) = \{t_{k(s+1)}, t_{k(s+2)}, \dots, t_{kr}\}, s \geq 1, \quad (16)$$

$$C(M_j, t_q) = \{t_{j_1}, t_{j_2}, \dots, t_{j_b}, t_q\}, \quad (17)$$

$$\text{则 } E(M_j) = \{t_{k(s+1)}, t_{k(s+2)}, \dots, t_r, t_{j_1}, t_{j_2}, \dots, t_{j_b}, t_q\}, \quad (18)$$

$$E_{j_1}(M_k) = \{t_{k_1}, t_{k_2}, \dots, t_{k_s}\}. \quad (19)$$

因为 $M_j \in U_1(M_k)$, 故 $E_{j_1}(M_k)$ 非空.

考虑小区间 $\left[\tau_0 - \frac{\Delta\tau}{2}, \tau_0 + \frac{\Delta\tau}{2}\right]$, 事件

$$\left\{ \tilde{M}\left(\tau_0 + \frac{\Delta\tau}{2}\right) = M_k, 0 \leq \tilde{x}_{k_1}\left(\tau_0 + \frac{\Delta\tau}{2}\right) < \Delta\tau, \right.$$

$$\left. x_{ki}^0 \leq \tilde{x}_{ki}\left(\tau_0 + \frac{\Delta\tau}{2}\right) < x_{ki}^0 + \Delta\tau, i = s+1, s+2, \dots, r \right\}$$

蕴含了在区间 $\left[\tau_0 - \frac{\Delta\tau}{2}, \tau_0 + \frac{\Delta\tau}{2}\right]$ 中发生标识更新, 更新后 $t_{k_1} \in E_{j_1}(M_k)$ 开始使能.

由概率论知识可知

$$P_{\text{rob}}\left\{ \tilde{M}\left(\tau_0 + \frac{\Delta\tau}{2}\right) = M_k, 0 \leq \tilde{x}_{k_1}\left(\tau_0 + \frac{\Delta\tau}{2}\right) < \Delta\tau, \right.$$

$$\left. x_{ki}^0 \leq \tilde{x}_{ki}\left(\tau_0 + \frac{\Delta\tau}{2}\right) < x_{ki}^0 + \Delta\tau, i = s+1, s+2, \dots, r \right\}$$

$$= \sum_{i_{j_1}=1}^{\infty} \sum_{i_{j_2}=1}^{\infty} \dots \sum_{i_{j_b}=1}^{\infty} \sum_{i_q=1}^{\infty} \left[P_{\text{rob}}\left\{ \tilde{M}\left(\tau_0 - \frac{\Delta\tau}{2}\right) = M_j; \right.$$

$$\left. x_{ki}^0 - \Delta\tau \leq \tilde{x}_{ki}\left(\tau_0 - \frac{\Delta\tau}{2}\right) < x_{ki}^0, i = s+1, s+2, \dots, r; \right.$$

$$\left. x_{j_l}^{i_{j_l}} - \Delta\tau \leq \tilde{x}_{j_l}\left(\tau_0 - \frac{\Delta\tau}{2}\right) < x_{j_l}^{i_{j_l}}, l = 1, 2, \dots, b; \right.$$

$$\left. x_q^{i_q} - \Delta\tau \leq \tilde{x}_q\left(\tau_0 - \frac{\Delta\tau}{2}\right) < x_q^{i_q} \right\} \cdot \prod_{i=s+1}^r \frac{1 - \Phi_{ki}(x_{ki}^0)}{1 - \Phi_{ki}(x_{ki}^0 - \Delta\tau)}$$

$$\cdot \prod_{i=1}^b \frac{1 - \Phi_{j_l}(x_{j_l}^{i_{j_l}})}{1 - \Phi_{j_l}(x_{j_l}^{i_{j_l}} - \Delta\tau)} \cdot \frac{\varphi_q(x_q^{i_q} - \Delta\tau)}{1 - \Phi_q(x_q^{i_q} - \Delta\tau)} \Delta\tau \Big]. \quad (20)$$

其中 $x_{j_l}^{i_{j_l}} = i_{j_l} \cdot \Delta\tau, i_{j_l} = 1, 2, \dots, l = 1, 2, \dots, b; x_q^{i_q} = i_q \cdot \Delta\tau, i_q = 1, 2, \dots$. 由式 (20) 可得

$$\begin{aligned} & p(\tau_0^+; M_k, x_{k_1}^0 = 0, x_{k(s+1)}^0, x_{k(s+2)}^0, \dots, x_{kr}^0) \\ &= \int_0^{\infty} \int_0^{\infty} \dots \int_0^{\infty} p(\tau_0^-; M_j, x_{k(s+1)}^0, x_{k(s+2)}^0, \dots, x_{kr}^0, x_{j_1}, x_{j_2}, \dots, x_{j_b}, x_q) \\ & \quad \cdot \frac{\varphi_q(x_q)}{1 - \Phi_q(x_q)} dx_{j_1} dx_{j_2} \dots dx_{j_b} dx_q. \end{aligned} \quad (21)$$

易知, 在时刻 τ_0^+ , 有 $x_{k_2}^0 = x_{k_3}^0 = \dots = x_{k_s}^0 = x_{k_1}^0 = 0$, 且在标识 M_k 下, 对任意 $t_{ki} \in E(M_k)$, 有 $x_{ki} = x_{ki}^0 + x_{k_1}$. 因此,

$$\begin{aligned}
& p(\tau - x_{k1}; M_k, x_{k1} - x_{k1}, x_{k2} - x_{k1}, \dots, x_{kr} - x_{k1}) \\
&= \int_0^\infty \int_0^\infty \dots \int_0^\infty p(\tau - x_{k1}; M_j, x_{k(s+1)} - x_{k1}, \dots, x_{kr} - x_{k1}, x_{j1}, \dots, x_{jb}, x_q) \\
&\quad \cdot \frac{\varphi_q(x_q)}{1 - \Phi_q(x_q)} dx_{j1} \dots dx_{jb} dx_q \cdot \prod_{i=2}^s \delta(x_{ki} - x_{k1}). \tag{22}
\end{aligned}$$

这里 $\delta(\cdot)$ 是狄拉克函数。

令 $x_{(k)} = x_{k1}(x_{(k)})$ 显然满足式(14)。由(22)式和(13)式, 可得在假定 $U(M_k) = U_1(M_k) = \{M_j\}$ 时, 标识 M_j 和 M_k 的混合状态密度的递推方程为

$$\begin{aligned}
p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr}) &= \prod_{i=1}^r \frac{1 - \Phi_{ki}(x_{ki})}{1 - \Phi_{ki}(x_{ki} - x_{k1})} \\
&\cdot \int_0^\infty \int_0^\infty \dots \int_0^\infty p(\tau - x_{k1}; M_j, x_{k(s+1)} - x_{k1}, \dots, x_{kr} - x_{k1}, x_{j1}, \dots, x_{jb}, x_q) \\
&\cdot \frac{\varphi_q(x_q)}{1 - \Phi_q(x_q)} \cdot dx_{j1} \dots dx_{jb} dx_q \cdot \prod_{i=2}^s \delta(x_{ki} - x_{k1}). \tag{23}
\end{aligned}$$

现假定 $U_1(M_k) = \emptyset, U_2(M_k) = \{M_l\}$, 且 $M_l \xrightarrow{t_u} M_k$. 此时, $E_{l1}(M_k) = \emptyset, E_{l2}(M_k) = E(M_k)$. 记 $E(M_l) = E(M_k) \cup \{t_{l1}, t_{l2}, \dots, t_{le}, t_u\}$, 用类似上述分析方法可得

$$\begin{aligned}
p(\tau; M_k, x_{k1}, x_{k2}, \dots, x_{kr}) &= \int_0^\infty \prod_{i=1}^r \frac{1 - \Phi_{ki}(x_{ki})}{1 - \Phi_{ki}(x_{ki} - x_{(k)})} \\
&\cdot \int_0^\infty \int_0^\infty \dots \int_0^\infty p(\tau - x_{k1}; M_l, x_{k1} - x_{(k)}, \dots, x_{kr} - x_{(k)}, x_{l1}, \dots, x_{le}, x_u) \\
&\cdot \frac{\varphi_u(x_u)}{1 - \Phi_u(x_u)} \cdot dx_{l1} dx_{l2} \dots dx_{le} dx_u dx_{(k)}. \tag{24}
\end{aligned}$$

由(23), (24)两式可得一般情况下 M_k 及其直接前继标识的混合状态密度之递推计算公式为

$$\begin{aligned}
& p(\tau; x_k, x_{k1}, x_{k2}, \dots, x_{kr}) \\
&= \sum_{M_j \in U_1(M_k)} \left[\prod_{i=1}^r \frac{1 - \Phi_{ki}(x_{ki})}{1 - \Phi_{ki}(x_{ki} - x_{(k)})} \int_0^\infty \int_0^\infty \dots \int_0^\infty p(\tau - x_{k1}; M_j, x_{k(s+1)} \right. \\
&\quad \left. - x_{k1}, \dots, x_{kr} - x_{k1}, x_{j1}, \dots, x_{jb}, x_q) \cdot \frac{\varphi_q(x_q)}{1 - \Phi_q(x_q)} \right. \\
&\quad \left. \cdot dx_{j1} \dots dx_{jb} dx_q \cdot \prod_{i=2}^s \delta(x_{ki} - x_{k1}) \right] \\
&+ \sum_{M_l \in U_2(M_k)} \left[\int_0^\infty \prod_{i=1}^r \frac{1 - \Phi_{ki}(x_{ki})}{1 - \Phi_{ki}(x_{ki} - x_{(k)})} \right. \\
&\quad \cdot \int_0^\infty \int_0^\infty \dots \int_0^\infty p(\tau - x_{(k)}; M_l, x_{k1} - x_{(k)}, \dots, x_{kr} - x_{(k)}, x_{l1}, \dots, x_{le}, x_u) \\
&\quad \left. \cdot \frac{\varphi_u(x_u)}{1 - \Phi_u(x_u)} dx_{l1} \dots dx_{le} dx_u dx_{(k)} \right]. \tag{25}
\end{aligned}$$

3.3 初始混合状态密度

设 SPN 在时刻 $\tau = 0$ 以概率 1 处于初始标识 M_1 , 且此时 $E(M_1)$ 中各使能变迁自零值起计使能时间. 又假定当 SPN 离开 M_1 后, 便永远不再返回 M_1 . 记 $E(M_1) = \{t_{11}, t_{12}, \dots, t_{1s}\}$. 显然, τ 时刻 SPN 处于 M_1 , 等价于 SPN 在 $[0, \tau]$ 内无变迁发射, 其概率为

$$P_{\text{rob}}\{\tilde{M}(\tau) = M_1\} = \prod_{i=1}^s [1 - \Phi_{1i}(\tau)]. \quad (26)$$

在 τ 时刻 SPN 处于 M_1 的条件下, 各使能变迁的使能时间必为 τ , 即 τ 时刻使能变迁 $t_{11}, t_{12}, \dots, t_{1s}$ 的使能时间为 $x_{11}, x_{12}, \dots, x_{1s}$ 的条件密度为

$$p(\tau; x_{11}, x_{12}, \dots, x_{1s} | \tilde{M}(\tau) = M_1) = \delta(x_{11} - \tau) \delta(x_{12} - \tau) \cdots \delta(x_{1s} - \tau). \quad (27)$$

因此, 标识为 M_1 时 SPN 的混合状态密度为

$$\begin{aligned} & p(\tau; M_1, x_{11}, x_{12}, \dots, x_{1s}) \\ &= p(\tau; x_{11}, x_{12}, \dots, x_{1s} | \tilde{M}(\tau) = M_1) \cdot P_{\text{rob}}\{\tilde{M}(\tau) = M_1\} \\ &= \prod_{i=1}^s [1 - \Phi_{1i}(x_{1i})] \cdot \prod_{i=1}^s \delta(x_{1i} - \tau). \end{aligned} \quad (28)$$

从式(28)出发, 利用式(25), 即可递推计算出 SPN 在任意时刻的混合状态密度.

4 混合状态分析法的应用

图1(a)所示 SPN 变迁集为 $T = \{t_1, t_2, t_3\}$, 位置集为 $P = \{p_1, p_2, p_3, p_4, p_5\}$, 初始标识 $M_1 = (1, 1, 0, 0, 0)^T$. 可达图 (reachable graph) 如图 1(b) 所示, 其中 $M_2 = (0, 1, 1, 0, 0)^T$, $M_3 = (1, 0, 0, 1, 0)^T$, $M_4 = (0, 0, 1, 1, 0)^T$, $M_5 = (0, 0, 0, 0, 1)^T$.

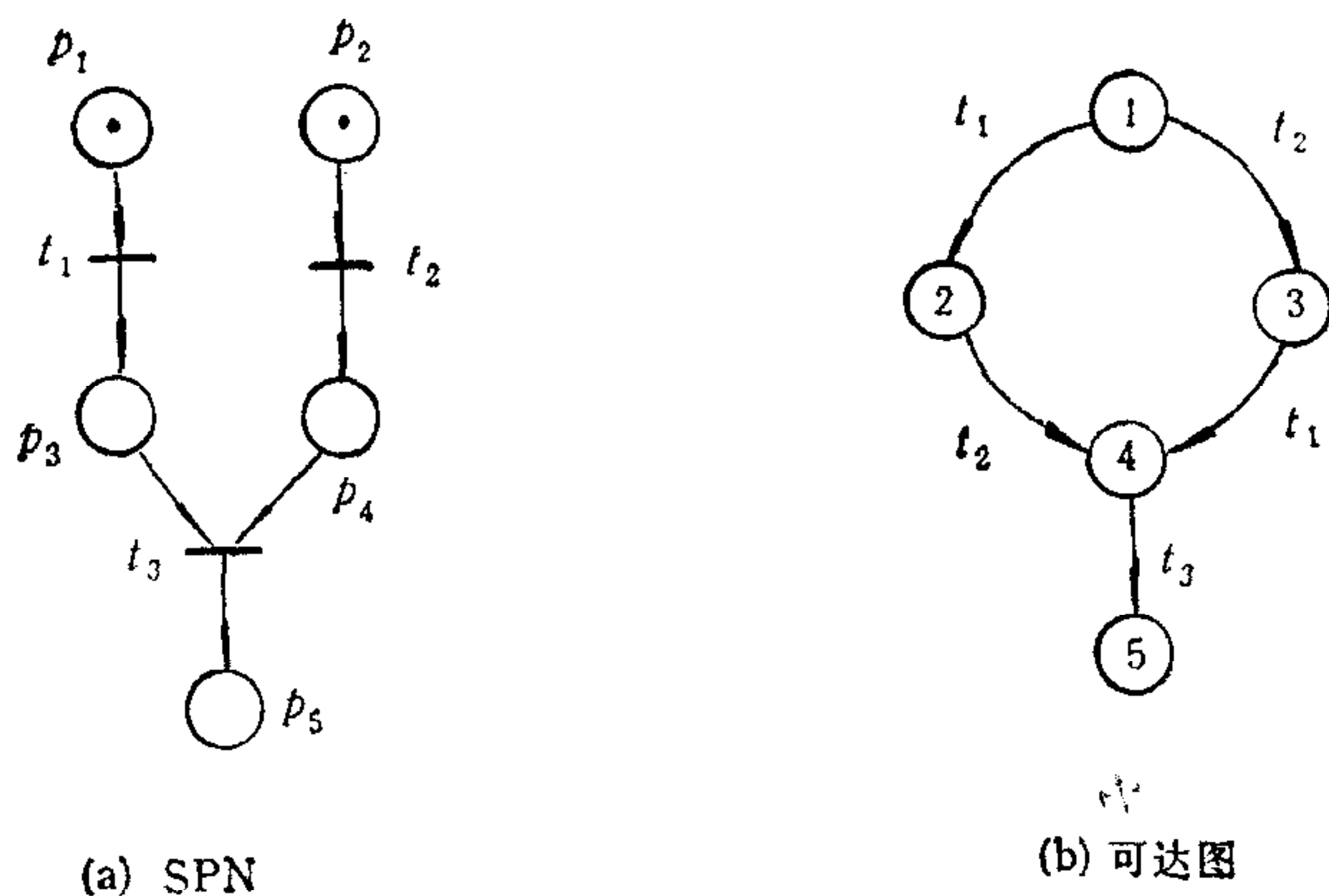


图 1 一简单 SPN 模型

初始标识 M_1 下的两个使能变迁 t_1, t_2 互为并发型变迁, 且 $C'(M_1, t_1) = C'(M_1, t_2) = \emptyset$. 由式(28)可得

$$p(\tau; M_1, x_1, x_2) = [1 - \Phi_1(x_1)][1 - \Phi_2(x_2)]\delta(x_1 - \tau)\delta(x_2 - \tau).$$

M_2, M_3 同为 M_1 的直接后继标识, 且 $U(M_2) = U_2(M_2) = U(M_3) = U_2(M_3) = \{M_1\}$; t_2, t_1 分别为 M_2, M_3 的互斥型使能变迁. 由式(25)可得

$$p(\tau; M_2, x_2) = [\Phi_1(\tau) - \Phi_1(\tau - x_2)][1 - \Phi_2(x_2)]\delta(x_2 - \tau);$$

$$p(\tau; M_3, x_1) = [\Phi_2(\tau) - \Phi_2(\tau - x_1)][1 - \Phi_1(x_1)]\delta(x_1 - \tau).$$

对于 $M_4, U(M_4) = U_2(M_4) = \{M_2, M_3\}$, 且 $E(M_4) = E_{21}(M_4) = E_{31}(M_4) = \{t_3\}$, 故可得

$$p(\tau; M_4, x_3) = [1 - \Phi_3(x_3)][\Phi_1(\tau - x_3)\varphi_2(\tau - x_3) + \Phi_2(\tau - x_3)\varphi_1(\tau - x_3)].$$

因为 $E(M_5) = \emptyset$, 亦即 M_5 是一吸收标识, M_5 下的混合状态密度即为 SPN 在 τ 时刻处于 M_5 的概率, 即

$$\begin{aligned} p(\tau; M_5) &= P_{\text{rob}}\{\tilde{M}(\tau) = M_5\} \\ &= \int_0^\infty \left\{ \frac{d}{d\tau} [\Phi_1(\tau - x_{(5)})\Phi_2(\tau - x_{(5)})] \cdot \varphi_3(\tau - x_{(5)}) \right\} dx_{(5)}. \end{aligned}$$

利用混合状态密度可直接求得 SPN 模拟系统的一些性能指标. 例如, τ 时刻 SPN 处于标识 M_2 的概率为

$$\begin{aligned} P_{\text{rob}}\{\dot{M}(\tau) = M_2\} \\ = \int_0^\infty p(\tau; M_2, x_2) dx_2 = \Phi_1(\tau)[1 - \Phi_2(\tau)]; \end{aligned}$$

又如, 因为该 SPN 模型系一时延网模型^[6], 其时延密度函数(亦即网模拟系统运行时延的密度函数) $f(\tau)$ 可利用 $p(\tau; M_4, x_3)$ 计算如下:

$$\begin{aligned} f(\tau) &= \int_0^\infty p(\tau; M_4, x_3) \frac{\varphi_3(x_3)}{1 - \Phi_3(x_3)} dx_3 \\ &= \frac{d}{d\tau} \{ \Phi_1(\tau)\Phi_2(\tau) \} * \Phi_3(\tau), \end{aligned}$$

其中“*”是卷积号.

5 结束语

传统的 SPN 分析方法都是将网的标识定义为状态, 再利用现有随机过程理论分析网模拟系统的时间特性. 因此, 这些方法只适用于研究那些标识过程具有马尔可夫性和半马尔可夫性的网, 而对那些更为一般的网的分析却显得无能为力. 混合状态分析法以标识和标识下使能变迁的使能时间共同构成网的状态, 使非马尔可夫 SPN 的分析成为可能. 算例说明了该法的正确性.

对于复杂 SPN, 宜结合集结方法^[7]进行分析.

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A HYBRID STATE ANALYSIS METHOD TO NON-MARKOV STOCHASTIC PETRI NET MODELS

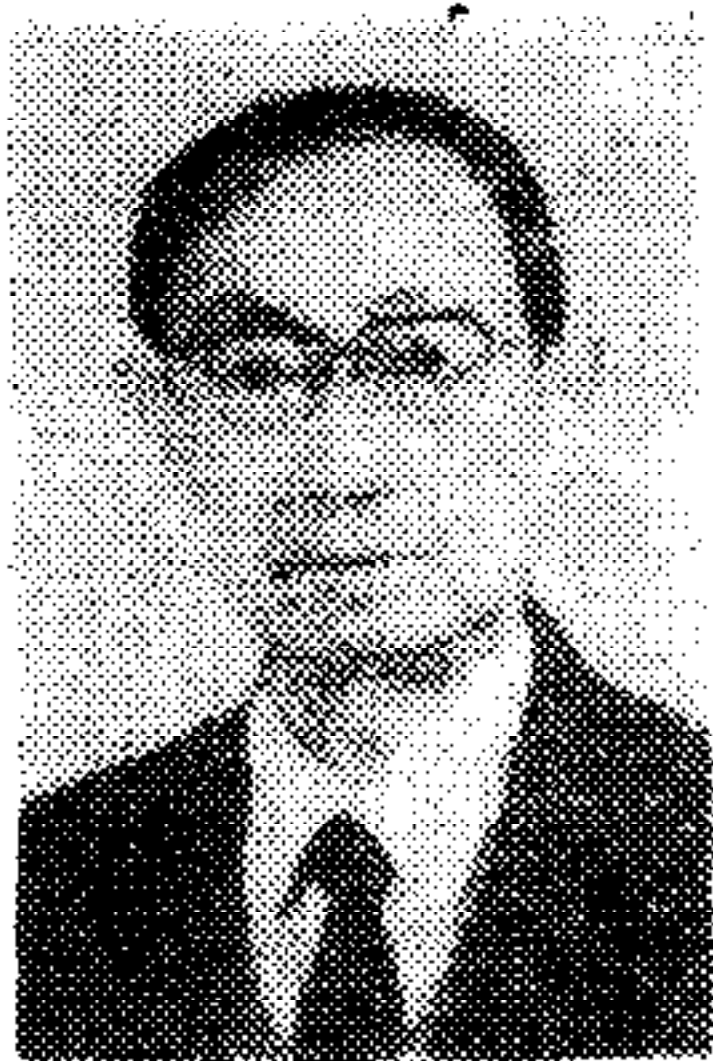
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ABSTRACT

This paper focuses on analysis of stochastic Petri net (SPN) models with arbitrarily distributed transition firing times. Concepts of hybrid states and hybrid state density functions are introduced, and a hybrid state analysis (HSA) method is developed systematically. A recursive equation of hybrid state density functions is also given. The HSA method makes analysis of non-Markovian SPNs possible. An example shows the application of this method to systems' performance evaluation.

Key words: Stochastic Petri nets, hybrid state, performance evaluation.



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