

关于 Hybrid 线性二次最优控制问题¹⁾

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摘 要

本文证明了文[3]所讨论的具有 Hybrid 指标的线性二次最优控制问题实际上可被归入经典的线性二次最优控制问题, 并且利用本文的方法还可把更一般的 Hybrid 线性二次最优控制问题也归入经典的线性二次最优控制问题, 从而可借助关于后者的现成的理论推出针对前者的结论.

关键词: Hybrid, 线性二次, 最优控制, 无限维.

1 引言

自60年代 Kalman 开创线性二次最优控制问题以来, 许多学者相继进行了深入的研究, 形成了线性二次最优控制问题的经典理论(包括对无限维系统). 对于有限时区上的线性二次最优控制问题, 如果不仅对系统的终端状态逼近定值的程度有比较严格的要求, 而且对系统在某些中间时刻的状态接近给定值的程度亦有较为严格的要求, 则必须考虑所谓的 Hybrid 线性二次最优控制问题, 即考虑线性控制系统

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)u(t) + f(t), & t \in [t_0, t_N], \\ x(t_0) = x_0 \end{cases} \quad (1.1)$$

(其中 $u(\cdot): [t_0, t_N] \rightarrow U$ 为系统(1.1)的控制函数)关于如下 Hybrid 型二次目标泛函

$$\begin{aligned} J(u(\cdot)) = & \left\langle W \left[D \begin{pmatrix} x(t_1) \\ \vdots \\ x(t_N) \end{pmatrix} - q \right], D \begin{pmatrix} x(t_1) \\ \vdots \\ x(t_N) \end{pmatrix} - q \right\rangle \\ & + \int_{t_0}^{t_N} \{ \langle Q(t)[C(t)x(t) - p(t)], C(t)x(t) - p(t) \rangle \\ & + \langle R(t)u(t), u(t) \rangle \} dt \end{aligned} \quad (1.2)$$

($t_0 < t_1 < \dots < t_N$ 是一些固定的实数)的最优控制问题 (HP) 寻找 $\hat{u}(\cdot) \in \mathcal{U} \equiv L^2(t_0, t_N; U)$, 使得 $J(\hat{u}(\cdot)) = \min\{J(u(\cdot)) | u(\cdot) \in \mathcal{U}\}$.

这里的 $A(t): \mathcal{D}(A(t)) \subseteq X \rightarrow X$ 生成 Hilbert 空间 X 上的强连续发展算子 $\Phi(\cdot, \cdot)$; $B(\cdot) \in C([t_0, t_N], \mathcal{L}(U, X))$ (U 是另一 Hilbert 空间, $\mathcal{L}(E_1, E_2)$ 表示映 Banach 空

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间 E_1 入 Banach 空间 E_2 的有界线性算子全体), 当 $E_1 = E_2$ 时, 简记 $\mathcal{L}(E_1, E_2)$ 为 $\mathcal{L}(E_1)$; $f(\cdot) \in L^2(t_0, t_N; X)$ 与 $x_0 \in X$ 给定; $W \in \mathcal{L}(Z)$ (Z 亦是一 Hilbert 空间), $D \in \mathcal{L}(X^N, Z)$, $q \in Z$; $Q(\cdot) \in C([t_0, t_N], \mathcal{L}(Y))$ (Y 是又一 Hilbert 空间), $C(\cdot) \in C([t_0, t_N], \mathcal{L}(X, Y))$, $p(\cdot) \in C([t_0, t_N], Y)$, $R(\cdot) \in C([t_0, t_N], \mathcal{L}(U))$; 且 W 及 $\forall t \in [t_0, t_N]$, $Q(t)$, $R(t)$ 均自伴. 它们满足所谓的 Hamilton 条件:

$$\left. \begin{aligned} W &\geq 0, \\ Q(t) &\geq 0, \forall t \in [t_0, t_N], \\ \exists \delta > 0, \forall t \in [t_0, t_N], R(t) &\geq \delta I_U. \end{aligned} \right\} \quad (1.3)$$

文[3]依照目标泛函所涉及的诸时刻点构成的时区剖分, 逐段使用动态规划方法导出非交叉型(即 $q = 0$ 且 D^*WD 为对角阵, 对角元取值于 $\mathcal{L}(X)$ 的情形) Hybrid 线性二次最优控制问题 (HP). (文[3]中的目标泛函显得还要特殊一些, 但其所提供的方法原则上已可用于求解 (HP)), 得到了最优控制由在这些时刻点上的状态量测表示的闭环结果, 给出了确定反馈算子的关系式. 但文[3]未指出应如何处理交叉型 Hybrid 线性二次最优控制问题. 为此, 本文用添设辅助状态方程的办法, 把 (HP) 等价地化为经典的线性二次最优控制问题, 然后利用关于后者的已有结果推出有关前者的结论.

2 结果的表述与证明

定理 2.1. (HP) 等价于如下标准的线性二次最优控制问题 (P) 求 $\hat{v}(\cdot) \in \mathcal{U}$, 使得下式成立:

$$\begin{aligned} \tilde{J}(\hat{v}(\cdot)) = \min_{u(\cdot) \in \mathcal{U}} \{ &\tilde{J}(u(\cdot)) \equiv \langle W[Dz(t_N) - q], Dz(t_N) - q \rangle \\ &+ \int_{t_0}^{t_N} \{ \langle Q(t)[\tilde{C}(t)z(t) - p(t)], \tilde{C}(t)z(t) - p(t) \rangle \\ &+ \langle R(t)u(t), u(t) \rangle \} dt \}, \end{aligned} \quad (2.1)$$

即
$$\hat{v}(\cdot) = \hat{u}(\cdot). \quad (2.2)$$

式(2.1)中

$$z(t) := \begin{pmatrix} y_1(t) \\ \vdots \\ y_{N-1}(t) \\ y_N(t) \end{pmatrix}, \quad \tilde{C}(t) := (0, \dots, 0, C(t)), \quad t \in [t_0, t_N].$$

其中 $y_k(\cdot)$ 为下列控制系统

$$\begin{cases} \dot{y}_k(t) = A_k(t)y_k(t) + B_k(t)u(t) + f_k(t), & t \in [t_0, t_N], \\ y_k(0) = x_0 \end{cases}$$

的状态函数.

这里 $A_k(\cdot) := \begin{cases} A(t), & t \in [t_0, t_k], \\ 0, & t \in (t_k, t_N], \end{cases} \quad B_k(t) := \begin{cases} B(t), & t \in [t_0, t_k], \\ 0, & t \in (t_k, t_N], \end{cases}$

$$f_k(t) := \begin{cases} f(t), & t \in [t_0, t_k], \\ 0, & t \in (t_k, t_N], \end{cases} \quad k = 1, \dots, N,$$

而 $y_N(\cdot) := x(\cdot)$ (系统(1.1)对应于控制函数 $u(\cdot)$ 的状态函数).

证. 令

$$\left. \begin{aligned} \tilde{\Phi}(t,s) &= \text{diag}(\Phi_1(t,s), \dots, \Phi_N(t,s)), \\ \forall (t,s) \in \bar{\Delta} &\equiv \{(\tau, \sigma) \in \mathbf{R}^2 \mid t_0 \leq \sigma \leq \tau \leq t_N\}, \end{aligned} \right\} \quad (2.3)$$

其中

$$\Phi_k(t,s) := \begin{cases} \Phi(\min(t, t_k), s), & s \leq t_k, \\ I_X, & s > t_k, \end{cases} \quad \forall (t,s) \in \bar{\Delta}, \quad k = 1, \dots, N.$$

由 $\Phi_k(\cdot, \cdot)$ 的定义式及 $\Phi(\cdot, \cdot)$ 在 $\bar{\Delta}$ 上的连续性、有界性可知:

i) $\Phi_k(t, \cdot) \in C^0([t_0, t], \mathcal{L}(X)) (C^0([a, b], \mathcal{L}(E))$ 表示映 \mathbf{R}^1 中的闭区间 $[a, b]$ 入 Banach 空间 E 上有界线性算子全体所成的 Banach 代数 $\mathcal{L}(E)$ 的强连续算子值函数全体, $\Phi_k(\cdot, s) \in C^0([s, t], \mathcal{L}(X))$.

ii) $\sup_{(t,s) \in \bar{\Delta}} \|\Phi_k(t,s)\|_{\mathcal{L}(X)} \leq \sup_{(t,s) \in \bar{\Delta}} \|\Phi(t,s)\|_{\mathcal{L}(X)} < \infty$.

iii) $\forall t_0 \leq r \leq s \leq t \leq t_N$, 若 $r > t_k$, 则

$$\Phi_k(t,s)\Phi_k(s,r) = I_X \cdot I_X = I_X = \Phi_k(t,r);$$

若 $s \leq t_k$, 则

$$\begin{aligned} \Phi_k(t,s)\Phi_k(s,r) &= \Phi(\min(t, t_k), s)\Phi(\min(s, t_k), r) \\ &= \Phi(\min(t, t_k), s)\Phi(s, r) \\ &= \Phi(\min(t, t_k), r) = \Phi_k(t, r); \end{aligned}$$

若 $r \leq t_k$, 但 $s > t_k$, 则

$$\begin{aligned} \Phi_k(t,s)\Phi_k(s,r) &= I_X \cdot \Phi(\min(s, t_k), r) \\ &= \Phi(t_k, r) = \Phi(\min(t, t_k), r) \\ &= \Phi_k(t, r), \quad k = 1, \dots, N. \end{aligned}$$

i)—iii) 表明: $\tilde{\Phi}(\cdot, \cdot)$ 亦是 $\bar{\Delta}$ 上强连续发展算子; 并且, 容易看出 $\Phi_k(\cdot, \cdot)$ 的生成元为 $A_k(\cdot)$, $k = 1, \dots, N$ (规定 $A_N(\cdot) := A(\cdot)$). 所以,

$$\begin{aligned} y_k(t) &= \Phi_k(t, t_k)x_0 + \int_{t_0}^t \Phi_k(t, \tau)[B_k(\tau)u(\tau) + f_k(\tau)]d\tau \\ &= \Phi_k(\min(t, t_k), t_0)x_0 + \int_{t_0}^{\min(t, t_k)} \Phi_k(t, \tau)[B(\tau)u(\tau) + f(\tau)]d\tau \\ &= \Phi(\min(t, t_k), t_0)x_0 + \int_{t_0}^{\min(t, t_k)} \Phi(\min(t, t_k), \tau)[B(\tau)u(\tau) + f(\tau)]d\tau \\ &= x(\min(t, t_k)), \quad \forall t \in [t_0, t_N], k = 1, \dots, N. \end{aligned} \quad (2.4)$$

上式蕴涵

$$z(t_N) = \begin{pmatrix} x(\min(t_N, t_1)) \\ \vdots \\ x(\min(t_N, t_{N-1})) \\ x(t_N) \end{pmatrix} = \begin{pmatrix} x(t_1) \\ \vdots \\ x(t_{N-1}) \\ x(t_N) \end{pmatrix}. \quad (2.5)$$

故 (HP) 等价于 (P).

证毕.

对于问题(P), 由标准的无限维线性二次最优控制理论(参见文[1]或[2])立即可以获得如下引理.

定理 2.2. 设前述 Hamilton 条件(1.3)成立. 则 (P) 之最优控制 $\hat{u}(\cdot)$ 与最优状态轨线 $\hat{x}(\cdot)$ 适合

$$\hat{u}(t) = -R(t)^{-1}\tilde{B}(t)^*[\hat{K}(t)\hat{x}(t) + \hat{l}(t)], \quad \forall t \in [t_0, t_N], \quad (2.6)$$

其中 $\hat{K}(\cdot)$ 与 $\hat{l}(\cdot)$ 依次是下列 Riccati 积分方程

$$\begin{aligned} K(t) = & \tilde{\Phi}(t_N, t)^* D^* W D \tilde{\Phi}(t_N, t) \\ & + \int_t^{t_N} \tilde{\Phi}(\tau, t)^* [\tilde{C}(\tau)^* Q(\tau) \tilde{C}(\tau) \\ & - K(\tau) \tilde{M}(\tau) K(\tau)] \tilde{\Phi}(\tau, t) d\tau, \quad t \in [t_0, t_N] \end{aligned} \quad (2.7)$$

与 Volterra 积分方程

$$\begin{aligned} l(t) + & \int_t^{t_N} \tilde{\Phi}(\tau, t)^* K(\tau) \tilde{M}(\tau) l(\tau) d\tau \\ = & \int_t^{t_N} \tilde{\Phi}(\tau, t)^* [K(\tau) \tilde{f}(\tau) - \tilde{C}(\tau)^* Q(\tau) p(\tau)] d\tau \\ & - \tilde{\Phi}(t_N, t)^* D^* W q, \quad t \in [t_0, t_N] \end{aligned} \quad (2.8)$$

的(唯一)强连续整体解, 并且 $\hat{K}(\cdot)$ 满足

$$\hat{K}(t)^* = \hat{K}(t), \quad \forall t \in [t_0, t_N]. \quad (2.9)$$

在式(2.6)–(2.8)中,

$$\tilde{B}(\cdot) := \begin{pmatrix} B_1(\cdot) \\ \vdots \\ B_{N-1}(\cdot) \\ B(\cdot) \end{pmatrix}, \quad \tilde{M}(\cdot) = \tilde{B}(\cdot) R(\cdot)^{-1} \tilde{B}(\cdot)^*, \quad \tilde{f}(\cdot) = \begin{pmatrix} f_1(\cdot) \\ \vdots \\ f_{N-1}(\cdot) \\ f(\cdot) \end{pmatrix}.$$

为叙述简单起见, 置

$$\begin{aligned} \bar{p}(\cdot) & := C(\cdot)^* Q(\cdot) p(\cdot), \\ \tilde{Q}(\cdot) & := \tilde{C}(\cdot)^* Q(\cdot) \tilde{C}(\cdot) = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ I_X \end{pmatrix} \bar{Q}(\cdot) (0 \cdots 0 I_X), \end{aligned}$$

其中 $\bar{Q}(\cdot) \equiv C(\cdot)^* Q(\cdot) C(\cdot)$.

注意到式(2.4), (2.6), 由引理 2.1–2.2 可知, 一旦获得了 Riccati 方程(2.7)在 $[t_0, t_N]$ 上的强连续整体解 $\hat{K}(\cdot) \equiv (\hat{K}_{ij}(\cdot))_{N \times N}$, $\hat{K}_{ij}(\cdot) \in \mathcal{L}(X)$, $i, j = 1, \dots, N$, 并进而求得了 Volterra 积分方程(2.8)的解 $\hat{l}(\cdot) \equiv (l_i(\cdot))_N$, $l_i(\cdot) \in C([t_0, t_N], X)$, $i = 1, \dots, N$ (它们在理论上的存在唯一性由引理 2.2 保证), 则从式(2.6)可得

$$\begin{aligned} \hat{u}(t) = & -R(t)^{-1} B(t)^* \sum_{i=k}^N \left\{ \sum_{j=1}^{k-1} \hat{K}_{ij}(t) \hat{x}(t_j) + \left[\sum_{i=k}^N \hat{K}_{ij}(t) \right] \hat{x}(t) + \hat{l}_i(t) \right\}, \\ & \forall t \in [t_{k-1}, t_k], \quad k = 1, \dots, N. \end{aligned} \quad (2.10)$$

其中 $\hat{x}(\cdot)$ 是系统(1.1)对应于最优控制 $\hat{u}(\cdot)$ 的状态函数.

由前所述可以看出, 方程(2.7)与(2.8)的未知函数分别是在 $\mathcal{L}(X^N)$ 与 X^N 中取值的函数. 而原问题的状态空间是 X , 因此, 很自然地提出一个问题: 是否能够做到通过依次求解若干个未知函数在 X 中取值的方程, 便可据以决定式(2.10)中的诸反馈算子与非

齐次(关于状态)项? 从式(2.10)可看出: 为决定这些反馈算子及非齐次项, 实际上并不需要确知 $\hat{K}(\cdot)$ 和 $\hat{l}(\cdot)$, 只要能求得 $\sum_{i=k}^N \hat{K}_{ij}(\cdot)$ 与 $\sum_{i=k}^N \sum_{j=k}^N \hat{K}_{ij}(\cdot)$ 在 $[t_{k-1}, t_k]$ 上的限制 ($j = 1, \dots, k-1$) 及 $\sum_{i=k}^N \hat{l}_i(\cdot)$ 在 $[t_{k-1}, t_k]$ 上的限制 ($k = 1, \dots, N$) 即可。答案是肯定的, 这就是下面的定理。

定理 2.3. 假设条件同引理 2.2, 则最优对 $(\hat{u}(\cdot), \hat{x}(\cdot))$ 适合如下反馈关系式

$$\hat{u}(t) = -R(t)^{-1}B(t)^* \left[\sum_{j=1}^{k-1} \hat{L}_k^{(j)}(t)^* \hat{x}(t_j) + \hat{K}_k(t) \hat{x}(t) + \hat{l}^{(k)}(t) \right], \quad \forall t \in [t_{k-1}, t_k], \quad k = 1, \dots, N. \quad (2.11)$$

其中 $\{\hat{K}_k(\cdot), \hat{L}_k^{(j)}(\cdot) (j = 1, \dots, k-1), \hat{l}^{(k)}\}_{k=1}^N$ 分别是方程组

$$\begin{aligned} K_k(t) = & \sum_{ij=k}^N \Phi(t_i, t)^* \tilde{W}_{ij} \Phi(t_k, t) + \int_t^{t_k} \Phi(\tau, t)^* [\bar{Q}(\tau) \\ & - K_k(\tau) \bar{M}(\tau) K_k(\tau)] \Phi(\tau, t) d\tau \\ & + \sum_{q=k+1}^N \int_{t_{q-1}}^{t_q} \left\{ \Phi(\tau, t)^* \bar{Q}(\tau) \Phi(\tau, t) \right. \\ & - \left[\Phi(\tau, t)^* K_q(\tau) + \sum_{i=k}^{q-1} \Phi(t_i, t)^* L_q^{(i)}(\tau) \right] \bar{M}(\tau) \\ & \left. \times \left[\Phi(\tau, t)^* K_q(\tau) + \sum_{i=k}^{q-1} \Phi(t_i, t)^* L_q^{(i)}(\tau) \right]^* \right\} d\tau, \end{aligned} \quad (2.12)$$

$$\begin{aligned} L_k^{(i)}(t) = & \sum_{i=k}^N \tilde{W}_{ij} \Phi(t_j, t) + \int_t^{t_k} L_k^{(i)}(\tau) \bar{M}(\tau) K_k(\tau) \Phi(\tau, t) d\tau \\ & + \sum_{q=k+1}^N \int_{t_q}^{t_{q+1}} L_q^{(i)}(\tau) \bar{M}(\tau) \left[\Phi(\tau, t)^* K_q(\tau) \right. \\ & \left. + \sum_{v=k}^{q-1} \Phi(t_v, t)^* L_q^{(v)}(\tau) \right]^* d\tau, \quad i = 1, \dots, k-1, \end{aligned} \quad (2.13)$$

$$\begin{aligned} l^{(k)}(t) + & \int_t^{t_k} \Phi(\tau, t)^* K_k(\tau) \bar{M}(\tau) l^{(k)}(\tau) d\tau \\ = & \int_t^{t_k} \Phi(\tau, t)^* [K_k(\tau) f(\tau) - C(\tau)^* Q(\tau) p(\tau)] d\tau \\ & + \sum_{q=k+1}^N \int_{t_{q-1}}^{t_q} \left\{ \left[\Phi(\tau, t)^* K_q(\tau) + \sum_{i=k}^{q-1} \Phi(t_i, t)^* L_q^{(i)}(\tau) \right] \right. \\ & \left. \times [f(\tau) - \bar{M}(\tau) l^{(q)}(\tau)] - \Phi(\tau, t)^* \bar{p}(\tau) \right\} d\tau \\ & - \sum_{i=k}^N \Phi(t_i, t)^* \tilde{q}_i, \quad t_{k-1} < t \leq t_k, \quad k = N, \dots, 1 \end{aligned} \quad (2.14)$$

的解。其中 $\tilde{W}_{ij} (\in \mathcal{L}(X))$ 是 N 阶算子阵 $\tilde{W} \equiv D^* W D$ 的 (i, j) 一个元素; $\tilde{q}_i (\in X) := D^* W_q$ 的第 i 个分量, $i, j = 1, \dots, N$; $\bar{M}(t) := B(t) R^{-1}(t) B(t)^*, \forall t \in [t_0, t_N]$ 。

证。设 $\hat{K}(\cdot)$ 是方程(2.7)的解, 令

$$\hat{K}_k(t) = \sum_{i,j=k}^N \hat{K}_{ij}(t) = \left(\sum_{i=k}^N T_i \right) \hat{K}(t) \left(\sum_{j=k}^N T_j \right)^*, \tag{2.15}$$

$$\hat{L}_k^{(i)}(t) = \sum_{j=k}^N \hat{K}_{ij}(t) = T_i \hat{K}(t) \left(\sum_{j=k}^N T_j \right)^*, \quad i = 1, \dots, k-1, \tag{2.16}$$

$$\hat{l}^{(k)} = \sum_{i=k}^N \hat{l}_i(t) = \left(\sum_{i=k}^N T_i \right) \hat{l}(t), \quad \forall t \in (t_{k-1}, t_k], \quad k = 1, \dots, N. \tag{2.17}$$

其中 $T_i = (T_i^{(1)}, \dots, T_i^{(N)})$; $T_i^{(i)} = I_X$; $T_i^{(1)} = \dots = T_i^{(i-1)} = T_i^{(i+1)} = \dots = T_i^{(N)} = \mathcal{L}(X)$ 中的零元素, $i = 1, \dots, N$. 则容易验证: 式(2.15)–(2.17)所定义的函数 $\{\hat{K}_k(\cdot), \hat{L}_k^{(i)}(\cdot), i = 1, \dots, k-1, \hat{l}^{(k)}(\cdot)\}_{k=1}^N$ 构成式(2.12)–(2.14)的一组解. 从而由式(2.10)与方程组(2.12)–(2.16)之解的唯一性(这可通过逐段利用 Gronwall 引理来得到)便可得出本定理的结论. 证毕.

下面要提出的问题是: 如果仅设式(1.3)中的 $R > 0$ 成立, 但却已知方程组(2.12)–(2.14)有解, 则此时式(2.11)是否仍是 (HP) 的最优状态反馈解? 由以下两个定理即可得到肯定的答复.

定理 2.4. 设 R 正定, 但不要求 W 与 Q 半正定, 其余假定及所引进的记号的含义均保持不变. 并设由下列诸方程

$$K_k(t) = \Phi(t_k, t)^* \tilde{W}_{kk}^{(k)} \Phi(t_k, t) + \int_t^{t_k} \Phi(\tau, t)^* [\bar{Q}(\tau) - K_k(\tau) \bar{M}(\tau) K_k(\tau)] \Phi(\tau, t) d\tau, \tag{2.18}$$

$$L_k^{(i)}(t) + \int_t^{t_k} L_k^{(i)}(\tau) \bar{M}(\tau) K_k(\tau) \Phi(\tau, t) d\tau = \tilde{W}_{ik}^{(k)} \Phi(t_k, t), \tag{2.19}$$

$$i = 1, \dots, k-1,$$

$$l^{(k)}(t) + \int_t^{t_k} \Phi(\tau, t)^* \{K_k(\tau) [\bar{M}(\tau) l^{(k)}(\tau) + f(\tau)] - \bar{p}(\tau)\} d\tau = \Phi(t_k, t)^* \tilde{q}_k^{(k)}, \quad t_{k-1} < t \leq t_k, \quad k = N, \dots, 1 \tag{2.20}$$

构成的方程组有解 $\{\hat{K}_k(\cdot), \hat{L}_k^{(i)}(\cdot), i = 1, \dots, k-1, \hat{l}^{(k)}(\cdot)\}_{k=1}^N$. 式(2.18)–(2.20)中

$$\left. \begin{aligned} \tilde{W}_{ij}^{(k-1)} &:= \tilde{W}_{ij}^{(k)} - \int_{t_{k-1}}^{t_k} L_k^{(i)}(t) \bar{M}(t) L_k^{(j)}(t)^* dt, \\ \tilde{W}_{i,k-1}^{(k-1)} &:= \tilde{W}_{i,k-1}^{(k)} + L_k^{(i)}(t_{k-1}) - \int_{t_{k-1}}^{t_k} L_k^{(i)}(t) \bar{M}(t) L_k^{(k-1)}(t)^* dt, \\ \tilde{W}_{k-1,j}^{(k-1)} &:= \tilde{W}_{j,k-1}^{(k-1)*}, \\ \tilde{q}_i^{(k-1)} &:= \tilde{q}_i^{(k)} - \int_{t_{k-1}}^{t_k} L_k^{(i)}(t) \bar{M}(t) l^{(k)}(t) dt, \quad i, j = 1, \dots, k-2, \\ \tilde{W}_{k-1,k-1}^{(k-1)} &:= \tilde{W}_{k-1,k-1}^{(k)} + L_k^{(k-1)}(t_{k-1}) + L_k^{(k-1)}(t_{k-1})^* + K_k(t_{k-1}) \\ &\quad - \int_{t_{k-1}}^{t_k} L_k^{(k-1)}(t) \bar{M}(t) L_k^{(k-1)}(t)^* dt, \\ \tilde{q}_{k-1}^{(k-1)} &:= \tilde{q}_{k-1}^{(k)} + l^{(k)}(t_{k-1}) - \int_{t_{k-1}}^{t_k} L_k^{(k-1)}(t) \bar{M}(t) l^{(k)}(t) dt, \quad k = N, \dots, 2, \\ \tilde{W}_{ij}^{(N)} &:= \tilde{W}_{ij}, \quad \tilde{q}_i^{(N)} = \tilde{q}_i, \quad i, j = 1, \dots, N. \end{aligned} \right\} \tag{2.21}$$

则 (HP) 有如下最优状态反馈解

$$\hat{u}(t) = -R(t)^{-1}B(t)^* \left[\sum_{j=1}^{k-1} \hat{L}_k^{(j)}(t)^* \hat{x}(t_j) + \hat{K}_k(t) \hat{x}(t) + \hat{l}^{(k)}(t) \right],$$

$$\forall t \in (t_{k-1}, t_k], k = 1, \dots, N, \quad (2.22)$$

其中 $\hat{x}(\cdot)$ 是对应于最优控制 $\hat{u}(\cdot)$ 的状态轨线; 且有

$$\begin{aligned} \min_{u(\cdot) \in \mathcal{U}} J(u(\cdot)) &= \langle Wq, q \rangle + \int_{t_0}^{t_N} \langle Q(t)p(t), p(t) \rangle dt + \langle \hat{K}_1(t_0)x_0, x_0 \rangle \\ &\quad - 2 \left[\langle \hat{l}^{(1)}(t_0), x_0 \rangle + \sum_{k=1}^N \int_{t_{k-1}}^{t_k} \langle \hat{l}^{(k)}(t), f(t) \rangle dt \right] \\ &\quad - \sum_{k=1}^N \int_{t_{k-1}}^{t_k} \langle \bar{M}(t) \hat{l}^{(k)}(t), \hat{l}^{(k)}(t) \rangle dt. \end{aligned} \quad (2.23)$$

定理 2.5. 方程组(2.12)—(2.14)与方程组(2.18)—(2.20)同解。

限于篇幅, 定理 2.4—2.5 的证明从略。

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HYBRID LINEAR-QUADRATIC OPTIMAL CONTROL PROBLEM

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ABSTRACT

In this paper we proved that the linear-quadratic optimal control problem with hybrid performance index can be reduced to the standard case. Thus we obtained a feedback solution of the hybrid problem mentioned above by means of the standard linear-quadratic optimal control theory. The problem considered here is much more general than the problem discussed in [3].

Key words: Hybrid, linear-quadratic, optimal control, infinite dimensional.



潘立平 1964年4月生于上海。1985年7月毕业于复旦大学数学系计算数学专业,获理学学士学位。同年9月考入复旦大学数学研究所为该所运筹学与控制论专业攻读硕士学位的研究生。1991年取得理学博士学位并留校任教至今。主要研究领域为分布参数系统的最优控制。已在国外有关专业刊物上发表了数篇论文。

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