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Traveling Wave Equations and Restoration of Motion-Blurred Images[1)]

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Abstract The relative motion between a camera and the scene will lead to blurred images. Note that the linear motion of an object can be described by the traveling wave equation. This model may be used for explaining the formation of the above image blur and for its restoration in spatial domain as well. Investigations in this paper are first made on the removal of image blur caused by a uniform, linear motion along the horizontal direction, including the detection and measurement of the motion direction and displacement, and a recursive solution of the original image; then on the generalization of the method to 2-D case so as to suit the uniform linear motion along an arbitrary direction. The experimental results show that the ring-bell effect inherent from traditional frequency domain processing as well as the expensive iterations in solution can be avoided by such a spatial domain processing.

Key words Motion blur, image restoration, traveling wave, Hough transformation

1 Introduction

During the process of image formation, the quality of images may degrade due to a variety of causes, such as out of focus, distortion of optical systems, the relative motion between the camera and the scene, etc. The task of image restoration is to recover the original image with the most fidelity.

Suppose $f(x,y)$ is the original image, $g(x,y)$ the blurred image, $h(x,y)$ the degradation function and $n(x,y)$ the noise. With a reference to the traditional model of image degradation $g(x,y)=h(x,y) * f(x,y)+n(x,y)$, many methods[1~6], such as Inverse filtering, Wiener filter, constrained least-squares restoration, Maximum entropy, Lucy-Richardson iteration, etc., have been proposed to remove image blur either using Fourier transformation in the frequency domain $G(u,v)=H(u,v)F(u,v)+N(u,v)$ or using the optimization techniques in algebraic forms $g=hf+n$. Indeed, degraded images are restored, however, with ring-bell effects and at high computing costs.

As an alternative approach, this paper proposes a spatial domain processing which starts from the traveling wave equation to restore motion-blurred images. In appearance, this approach gives an analytical solution same to Gonzalez's method[1], it actually works with the wave process, i. e., the traveling wave equation, thus being more intuitive and clear in physical sense, easy to analyze and explain the formation of image blurring, and convenient to utilize sophisticated means already developed in solution. Investigation within this approach includes a traveling wave model for image blur caused by motion, the removal of image blur due to horizontal uniform linear motion, the determination of the direction and displacement of motion along arbitrary directions, the deblurring of these images without rotation operations, etc. Experimental results are given for a comparison with the results of traditional Lucy-Richardson frequency domain processing.

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2 Motion blur and traveling wave equation

Suppose that an object $f(x)$ moves at a rate v along the horizontal direction x under surveillance of a stationary camera. The total exposure $\tilde{g}(x)$ at any point of the recording medium (e. g. , film) is obtained by integrating the instantaneous exposure over the time interval $0 \leqslant t \leqslant T$ during which the camera shutter is open. To isolate the motion effect, it is assumed that shutter opening and closing takes place instantaneously, and that the optical imaging process is perfect. For the duration of exposure, there will be

$$\tilde{g}(x) = \int_0^T f(x - vt)\mathrm{d}t \tag{1}$$

By which, a frequency domain processing can be done by applying Fourier transformation to both sides of Eq. (1), and it is possible to completely reconstruct the original image using knowledge about the blurred image $\tilde{g}(x)$ inside and outside the interval $0 \leqslant x \leqslant L$ as addressed in [1].

The above motion blurring process can be described with a 1-D traveling wave equation[7,8] as follows:

$$\begin{cases} \left(\dfrac{\partial}{\partial t} + v\dfrac{\partial}{\partial x}\right)w(x,t) = 0 \\ w(x,0) = f(x) \end{cases} \tag{2}$$

where $w(x,t)$ is the instantaneous exposure of the moving object at time t, and $f(x)$ its initial value at $t=0$.

As the solution of Eq. (2), the so-called D'Alembert solution takes the following form[7,8]:

$$w(x,t) = f(x - vt) \tag{3}$$

It can be seen that $w(x,t)$ keeps the wave front invariant along each characteristic line $x - vt = const$ on $x-t$ plane, that is, the image moves as a rigid object along the horizontal direction. So, when $t > 0$, the accumulated exposure effects (or the blurred image) on the film should be

$$g(x,t) = \int_0^t w(x,\tau)\mathrm{d}\tau = \int_0^t f(x - v\tau)\mathrm{d}\tau \tag{4}$$

together with the constraints at both ends of the time interval respectively:

$$g(x,0) = 0, \quad g(x,T) = \int_0^T f(x - v\tau)\mathrm{d}\tau = \tilde{g}(x) \tag{5}$$

Therefore, it can be seen from Eq. (4) that for a stationary object ($v=0$) there will be

$$g(x,t) = \int_0^t f(x)\mathrm{d}\tau = t \cdot f(x), \quad \tilde{g}(x) = g(x,T) = T \cdot f(x) \tag{6}$$

Obviously, the images $g(x,t)$ and $\tilde{g}(x)$ will not be blurred since the accumulated exposure is simply the initial exposure multiplied by the exposure time at each stationary position; whereas, the image of a moving object ($v \neq 0$), as an equivalent to the stationary exposure of an averaged $f(x)$ over a neighborhood, must be a blur one:

$$g(x,t) = t \cdot \frac{1}{vt}\int_{x-vt}^{x} f(\zeta)\mathrm{d}\zeta = t \cdot f(\bar{x}), \quad \bar{x} \in [x - vt, x] \tag{7}$$

$$\tilde{g}(x) = T \cdot \frac{1}{vT}\int_{x-vT}^{x} f(\zeta)\mathrm{d}\zeta = T \cdot f(\bar{x}), \quad \bar{x} \in [x - vT, x] \tag{8}$$

In addition, a 1st temporal derivative to Eq. (4) gives

$$\frac{\partial}{\partial t}g(x,t) = f(x - vt) \tag{9}$$

and from Eqs. (2)~(4) there will be

$$v\frac{\partial}{\partial x}g(x,t)=\int_0^t v\frac{\partial w(x,\tau)}{\partial x}\mathrm{d}\tau=\int_0^t -\frac{\partial w(x,\tau)}{\partial \tau}\mathrm{d}\tau=-w(x,\tau)\Big|_0^t=f(x)-f(x-vt) \tag{10}$$

This makes clear that the motion-blurred image $g(x,t)$ itself subjects to a non-homogeneous traveling wave equation with the unknown initial image $f(x)$ as the right hand term:

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)g(x,t)=f(x) \tag{11}$$

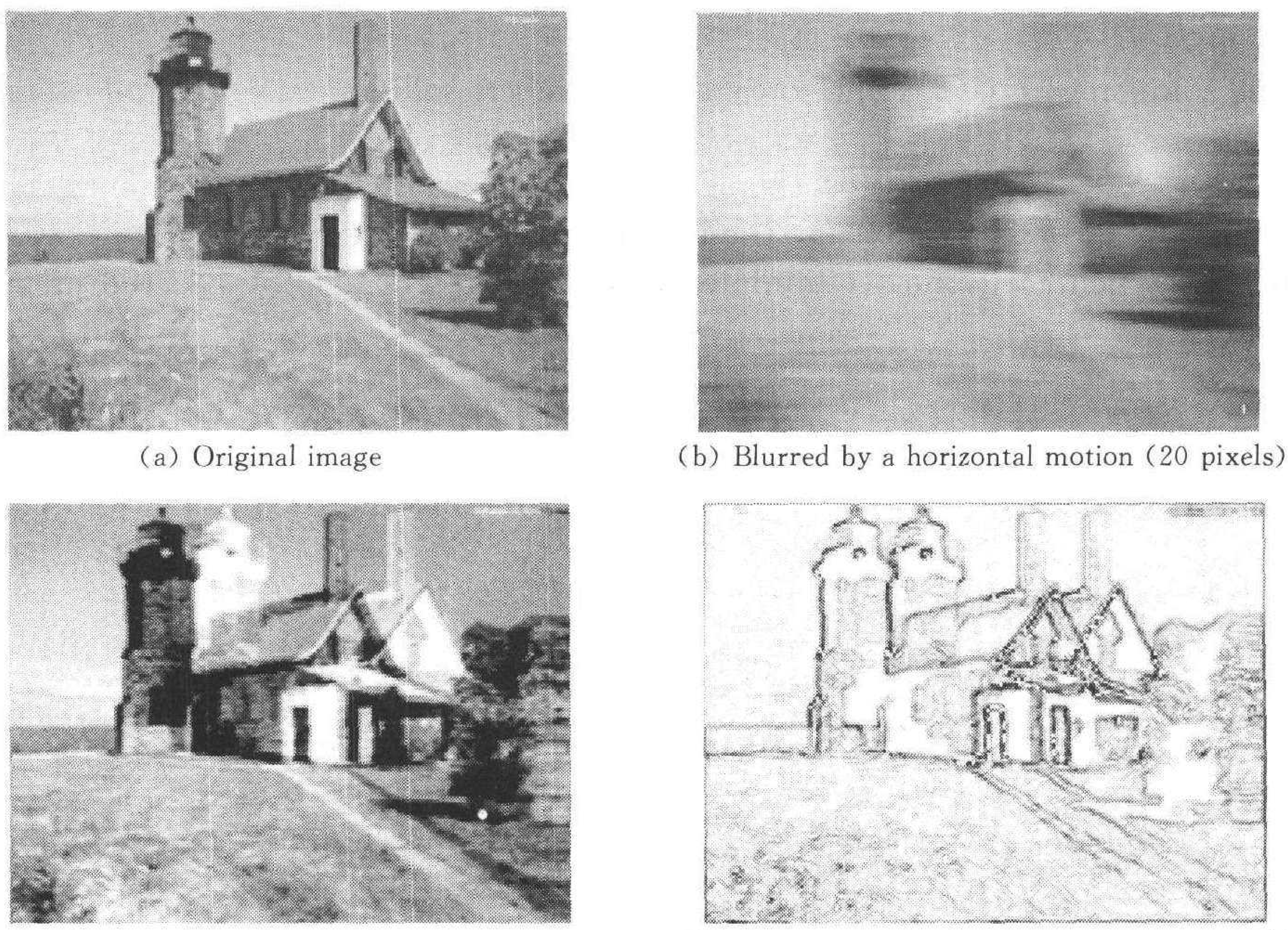
This suggests a uniform criterion for objective assessments to performance of different restoration methods for motion blurred images.

3 Removal of image blur caused by horizontal motion

Eq. (10) provides clues for estimation of the total displacement $D=vT$. Note that

$$v\frac{\partial}{\partial x}\tilde{g}(x)=f(x)-f(x-D) \tag{12}$$

This shows the 1st spatial derivative of the blurred image $\tilde{g}(x)$ is an overlapping of the initial image $f(x)$ and its negative and shifted version $-f(x-D)$, where the shift is just the total displacement D between points in one version to their counterparts in other version, so it can be determined by interactive or heuristic matching as shown in Fig. 1.



(a) Original image (b) Blurred by a horizontal motion (20 pixels)

(c) 1st derivative to (b) along horizontal direction (d) Processed (c) using Canny edge detector[9]

Fig. 1 A conceptual illustration for detection of the horizontal shift using matching

Eq. (12) can be written as

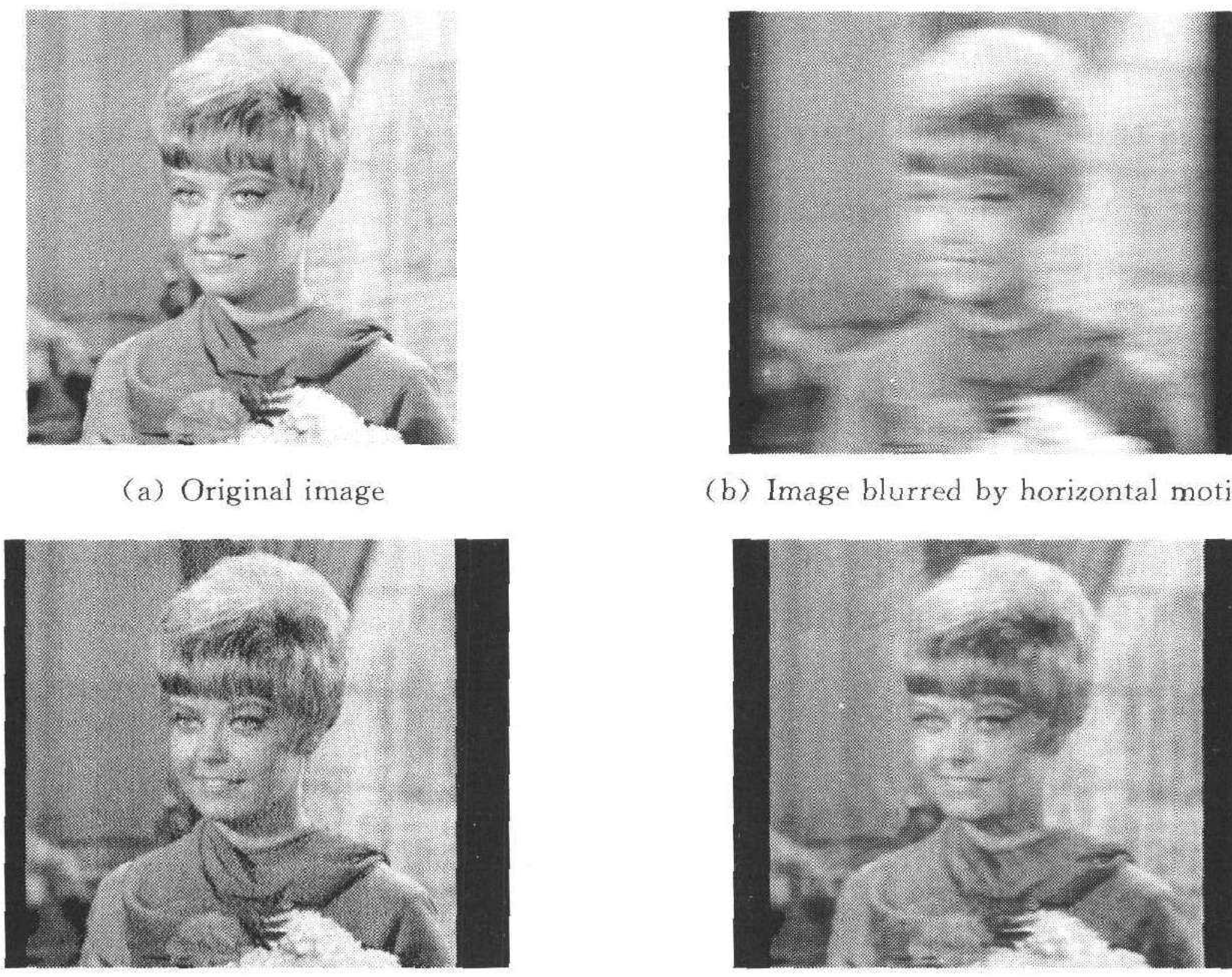
$$f(x)=v\frac{\partial}{\partial x}\tilde{g}(x)+f(x-D) \tag{13}$$

So, when the total displacement $D=vT$ is known and the image $f(x)$ has only a limited width $L>0$, i. e., when $f(x)=0, \forall x\notin[0,L]$, $f(x)$ can be obtained from $\tilde{g}(x)$ by a successive forward solution as follows:

$$f(x)=v\frac{\partial}{\partial x}\tilde{g}(x) \qquad x\in[0,D) \tag{14a}$$

$$f(x) = v \frac{\partial}{\partial x} \tilde{g}(x) + f(x - D) \quad x \in [D, L) \tag{14b}$$

Here the unknown speed v can be omitted in computation as a scalar to $\tilde{g}(x)$ without loss of generality. The image produced by Eq. (14) is shown in Fig. 2(c), compared with the image produced by a typical frequency domain method[4] as shown in Fig. 2(d).



(a) Original image (b) Image blurred by horizontal motion

(c) Traveling wave deblurring (d) L-R deblurring (100 iterations)

Fig. 2 Traveling wave method compared with Lucy-Richardson method in removal of horizontal motion blur

4 Oblique uniform motion and Hough transformation

So far, it has been assumed that the uniform linear motion goes along the horizontal direction. When the motion goes along an arbitrary direction, the directional angle φ should first be measured. This can be done by edge detection to the motion blurred image, followed by a Hough transformation[1] to find out all straight lines expressed by the normal representation below:

$$x\cos\theta + y\sin\theta = \rho \tag{15}$$

Note that during the uniform linear motion all points of the rigid object leave a lot of parallel lines as their traces. They make a majority in the set of all lines. So, the histogram of the directional angle θ of lines in the motion-blurred image can be used to identify the direction of motion $\varphi = \theta - \pi/2$.

5 Removal of oblique motion blur

Oblique motion blur can be removed by an image rotation of angle φ:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{16}$$

This transforms an oblique motion into the horizontal one so that image blur can be removed using Eq. (13), and followed by an inverse rotation in order to put the de-blurred image onto the proper position. Such a processing is simple and feasible. However it needs considerable computation for twice rotations per pixel, hence being better to avoid.

Let $\boldsymbol{k}=(\cos\varphi, \sin\varphi)^{\mathrm{T}}$ be the direction of an oblique motion, and $\boldsymbol{p}$ a moving point along this direction; then corresponding to Eq. (13), the oblique motion can be expressed as:

$$f(\boldsymbol{p}) = v\frac{\partial}{\partial k}\tilde{g}(\boldsymbol{p}) + f(\boldsymbol{p} - D\boldsymbol{k}) \tag{17}$$

or

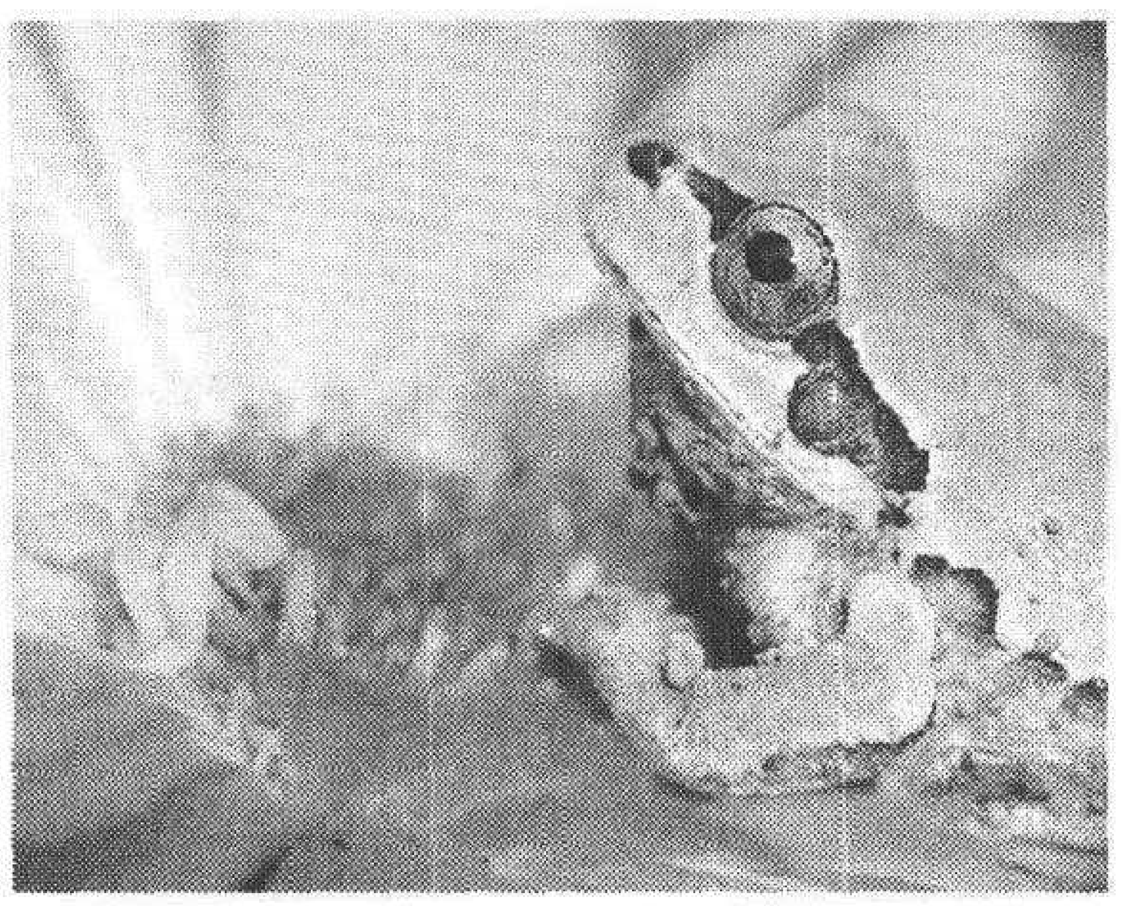
$$f(x,y) = v\frac{\partial}{\partial k}\tilde{g}(x,y) + f(x - D\cos\varphi, y - D\sin\varphi) \tag{18}$$

where $\frac{\partial}{\partial k}\tilde{g}(\boldsymbol{p})$ is the directional derivative of $\tilde{g}(\boldsymbol{p})$ along vector $\boldsymbol{k}$, thus we have a recursive solution without rotation transformation:

$$f(x,y) = v\frac{\partial}{\partial k}\tilde{g}(x,y), \quad 0 \leqslant x < D\cos\varphi \cup 0 \leqslant y < D\sin\varphi \tag{19a}$$

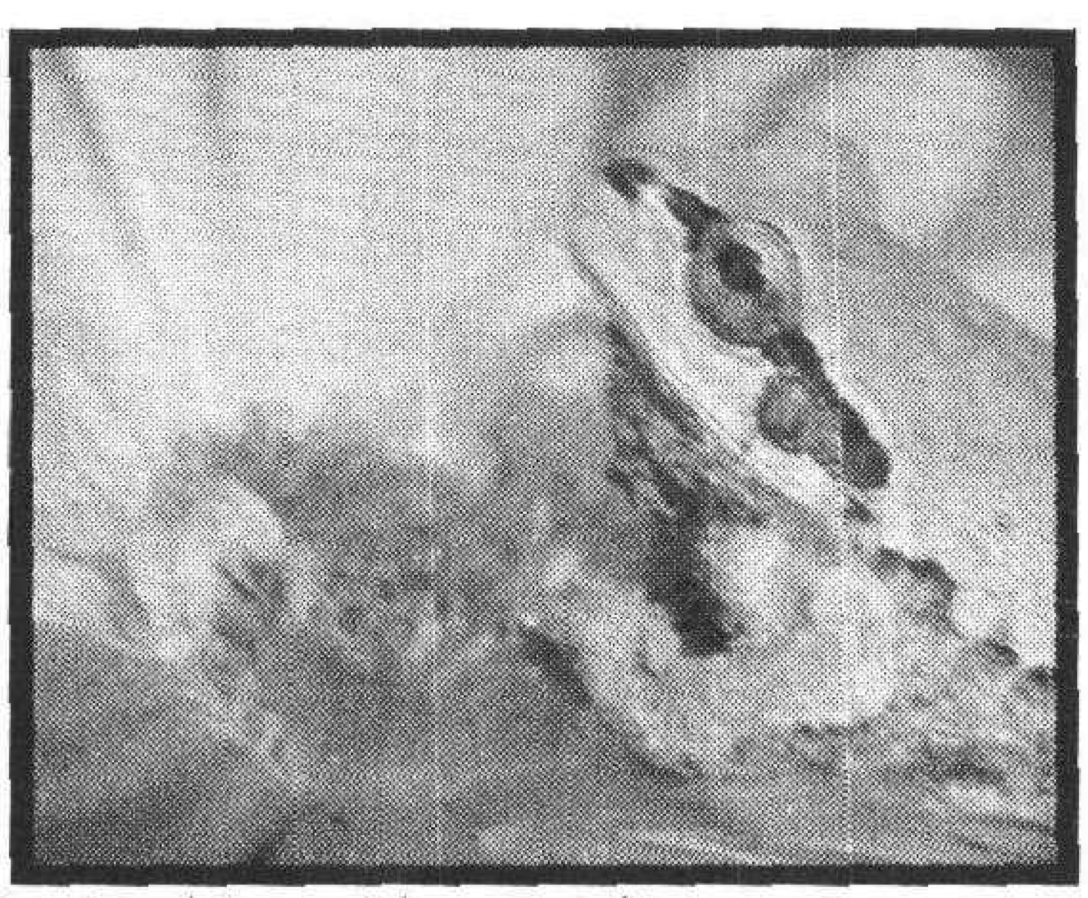
$$f(x,y) = v\frac{\partial}{\partial k}\tilde{g}(x,y) + f(x - D\cos\varphi, y - D\sin\varphi),$$
$$D\cos\varphi \leqslant x < L_x \cap D\sin\varphi \leqslant y < L_y \tag{19b}$$

Fig. 3 gives an example where the original image is blurred due to an oblique motion with 20 pixels in shift along the angle of 145 degrees. It is shown that the traveling wave deblurring gets better results than that of Lucy-Richardson deblurring.

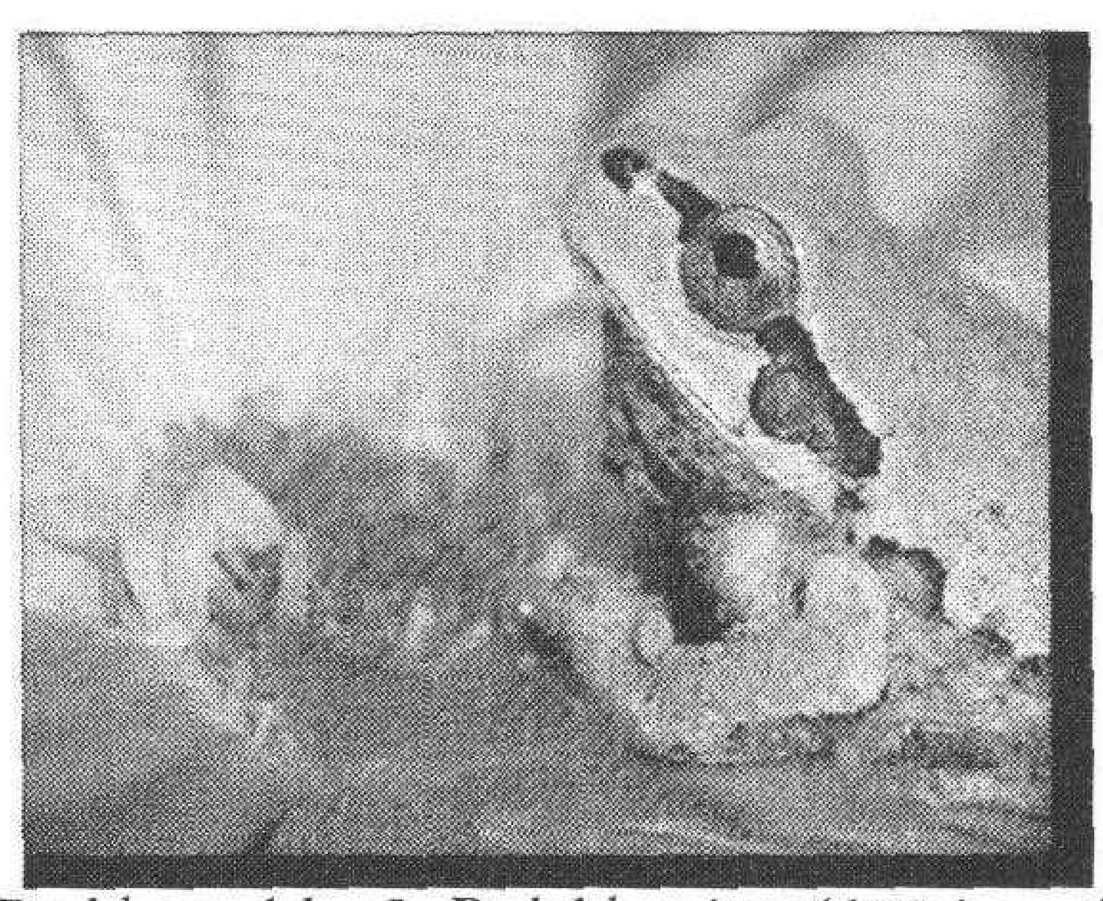


(a) Original image

(b) Blurred by oblique motion(145degrees, 20 pixels)



(c) De-blurred by traveling wave equation



(d) De-blurred by L-R deblurring (100 iterations)

Fig. 3 Traveling wave method compared with Lucy-Richardson method in removal of oblique motion blur

6 Conclusion

An image deblurring method using traveling wave equation is proposed and implemented in this paper for recovering images blurred by uniform linear motion. Discussion are made on three key issues including the detection of motion direction, the measurement of displacement and the inverse solution of the traveling wave equation through recursion. Experimental results show that as a spatial domain processing, the traveling wave deblurring produces better quality images than the traditional frequency domain processing such as Lucy-Richardson methods etc. while avoids the high cost of iterative computation and the ring-bell effect which more or less appears in the frequency domain processing.

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传播波方程与运动模糊图像恢复

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摘　要　首选讨论水平方向匀速直线运动所导致的图像模糊的去除，包括运动方向和位移量的探测和原始图像的递推求取，然后推广到任意方向的匀速直线运动上．实验结果表明这样的空间域处理可以避免传统的频率域处理中必有的振铃效应和迭代求解的高计算量．

关键词　运动模糊，图像恢复，传播波，Hough 变换

中图分类号　TP391.41