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Stability Analysis of Generalized Predictive Control with Input Nonlinearity Based-on Popov's Theorem[1)]

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Abstract For systems with input saturation constraint and invertible static input nonlinearity, a two step generalized predictive control (TSGPC) strategy is adopted. An intermediate variable representing the desired control action is obtained by applying linear GPC (LGPC), then the invertible static nonlinearity is compensated by solving nonlinear algebraic equation (NAE) and the input saturation constraint is satisfied by desaturation. TSGPC has low computational burden and is especially suitable for fast control application. The closed-loop block diagram of this system is turned into a static nonlinear feedback form, and Popov's theorem is applied to the closed-loop stability analysis. The sufficient stability conditions are obtained. Effective algorithms for determining controller parameters are given to make the stability conclusions applicable and an example is given for illustration.

Key words Input nonlinearity, two-step scheme, generalized predictive control, Popov's theorem

1 Introduction

Input nonlinearity includes input saturation, deadzone, relay cycle, etc. Moreover, systems represented by Hammerstein model are often met as input nonlinear system. [1～4] utilize the two-step control scheme for Hammerstein nonlinear systems with "polynomial +CARIMA" model. They first obtain intermediate variable through LGPC, and then obtain the control action by solving NAE. Moreover, desaturation is often applied to systems with input saturation, which is a special form of two-step control scheme. Desaturation is especially favored for fast control, and can save time in adaptive case. The TSGPC studied in this paper is shown in Figure 1, which deals with both Hammerstein nonlinearity and input saturation, and utilizes both NAE and desaturation. In a word, the advantage of TSGPC is that it designs a controller within the scope of linear systems, so it is much simpler than the scheme that solves the nonlinear control problem with nonlinear system model and/or nonlinear objective function[5]. If the desired intermediate variable is precisely implemented by actual control action, the effect of middle three blocks in Figure 1 disappears and the stability could be designed by LGPC. But this ideal case can not be generally guar-

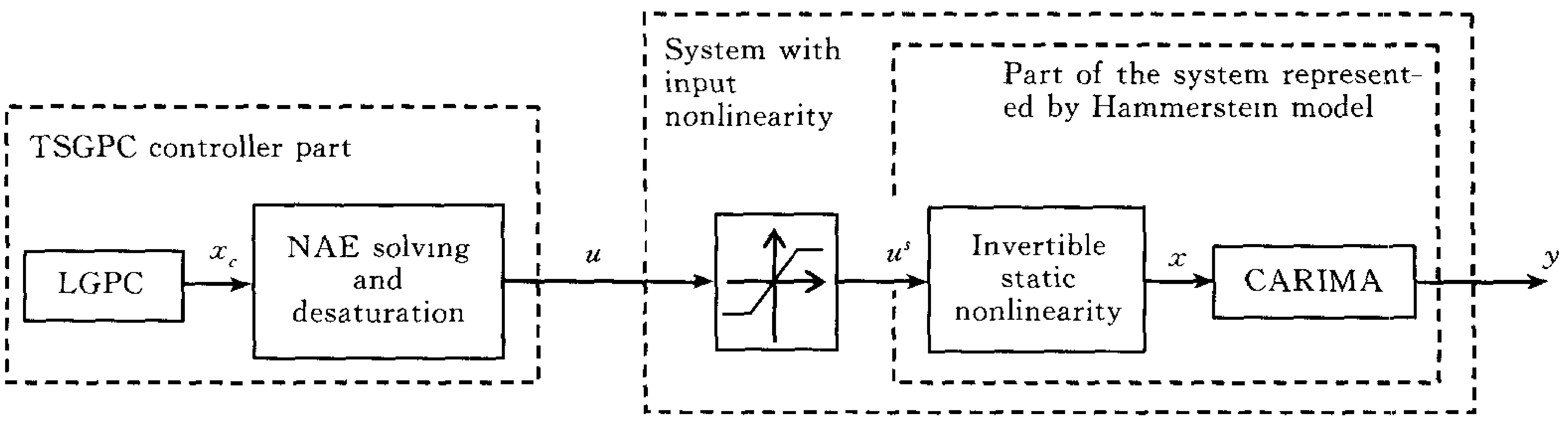

Fig. 1 Schematic structure of TSGPC for constrained Hammerstein model
x_c: desired intermediate variable; x: intermediate variable; u: input; y: output

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anteed. The control action may be saturated, and the solution of NAE may have error. In all these cases, the stability analysis of TSGPC becomes difficult, to which there isn't clear conclusion.

In this paper, the original closed-loop block diagram of TSGPC is turned into an equivalent static nonlinear feedback form. This shows that the stability of TSGPC can be deduced from Popov's theorem. So a stability conclusion could be obtained, which does not need the stability of linear control law and can serve as guidance to the parameter design in TSGPC.

2 The description of TSGPC

The invertible static nonlinearity is represented by $x(t)=f(u(t))$ where $f(0)=0$ and the CARIMA (controlled auto regressive integrated moving average) model is represented by

$$a(z^{-1})y(t) = b(z^{-1})x(t-1) + \xi(t)/\Delta, \quad \Delta = 1 - z^{-1} \tag{1}$$

where z^{-1} is the backward shift operator; $\xi(t)$ is the white noise with zero mean value; $\Delta a(z^{-1})=\Delta(1+a_1z^{-1}+\cdots+a_{na}z^{-na})=1+\bar{a}_1z^{-1}+\cdots+\bar{a}_{na+1}z^{-na-1}$ and $b(z^{-1})=b_1+b_2z^{-1}+\cdots+b_{nb}z^{-nb+1}$, with $a_{na}\neq 0$ and $b_{nb}\neq 0$; (a,b) are irreducible pair. The input constraint is $|u(t)|\leqslant U$.

TSGPC firstly obtains the desired intermediate variable by applying LGPC to Equation(1), adopting the following objective function:

$$J(t) = \sum_{i=N_1}^{N_2}[y(t+i)-\omega]^2 + \sum_{j=1}^{N_u}\lambda\Delta x^2(t+j-1) \tag{2}$$

where ω is the set-point. The following two Diophantine equations are introduced into LGPC:

$$1 = E_j a\Delta + z^{-j}F_j, \quad G_j = E_j b = \widetilde{G}_j + z^{-(j-1)}H_j \tag{3}$$

where $E_j=e_1+e_2z^{-1}+\cdots+e_jz^{-(j-1)}$, $F_j=f_{j,0}+f_{j,1}z^{-1}+\cdots+f_{j,na}z^{-na}$, $\widetilde{G}_j=g_1+g_2z^{-1}+\cdots+g_jz^{-(j-1)}$ and $H_j=h_{j,1}z^{-1}+h_{j,2}z^{-2}+\cdots+h_{j,nb-1}z^{-(nb-1)}$. Define $\boldsymbol{H}=[H_{N_1}, H_{N_1+1},\cdots, H_{N_2}]^{\mathrm{T}}$, $\boldsymbol{F}=[F_{N_1},F_{N_1+1},\cdots,F_{N_2}]^{\mathrm{T}}$, $\boldsymbol{M}=[1,1,\cdots,1]^{\mathrm{T}}$ and $\boldsymbol{d}^{\mathrm{T}}=[1,0,\cdots,0](\lambda I+G^{\mathrm{T}}G)^{-1}G^{\mathrm{T}}$, where $\boldsymbol{G}=\begin{bmatrix} g_{N1} & g_{N1-1} & \cdots & g_{N1-Nu+1} \\ g_{N1+1} & g_{N1} & \cdots & g_{N1-Nu+2} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N2} & g_{N2-1} & \cdots & g_{N2-Nu+1} \end{bmatrix}$ with $g_j=0$ for all $j\leqslant 0$. The control law of LGPC is $\Delta x(t)=\boldsymbol{d}^{\mathrm{T}}(\vec{\boldsymbol{\omega}}-\vec{\boldsymbol{f}})$ where $\vec{\boldsymbol{\omega}}=[\omega,\omega,\cdots,\omega]^{\mathrm{T}}$, while $\vec{\boldsymbol{f}}$ is composed of the past intermediate variable, past output and current output[6].

The second step of TSGPC is to obtain the real control action $u(t)$ from $x_c(t)=x_c(t-1)+\Delta x(t)$ by one of the following two approaches.

Approach A. Solve NAE $f(\tilde{u}(t))-x_c(t)=0$ for $\tilde{u}(t)$, which is formalized as $\tilde{u}(t)=\tilde{f}^{-1}(x_c(t))$. Then $u(t)$ is obtained by desaturation: $u(t)=sat\{\tilde{u}(t)\}$, where $sat\{v\}=\mathrm{sign}\{v\}\min\{|v|,U\}$, and this is formalized as $u(t)=\hat{f}^{-1}(x_c(t))$. Approach A is of low computational burden, but $f(\tilde{u}(t))-x_c(t)=0$ may have no real-valued solution[1~5]. If f is non-monotonic and the system has an inherent static error, approach A may be unable to achieve the smallest static error[4]. If f is an odd-ordered monotonic polynomial, approach A is preferable.

Approach B. Transform input constraint into the constraint on $x(t)$[4], i.e., $x_{\min}\leqslant x(t)\leqslant x_{\max}$. Let $\tilde{x}(t)=\begin{cases} x_{\min}, & x_c(t)\leqslant x_{\min} \\ x_c(t), & x_{\min}<x_c(t)<x_{\max} \\ x_{\max}, & x_c(t)\geqslant x_{\max} \end{cases}$. Solve NAE $f(u(t))-\tilde{x}(t)=0$ to obtain $u(t)=\tilde{f}^{-1}(\tilde{x}(t))$, also formalized as $u(t)=\hat{f}^{-1}(x_c(t))$. By Approach B the NAE always has a real-valued solution and the smallest static error can be achieved.

Figure 2 is the block diagram of TSGPC[4] with notations above, where "plant" represents the real system. Assume the real static nonlinearity is f_0, then TSGPC is equivalent to LGPC if and only if $f_0 = \hat{f}$[4]. However, it is difficult to achieve $f_0 = \hat{f}$ if $f_0 \neq f$ and/or $f \neq \hat{f}$. Actually, it is generally impossible to achieve $f_0 = \hat{f}$. It is the aim of this paper to study the closed-loop stability of TSGPC in the case $f_0 \neq \hat{f}$. As $\hat{f}^{-1}$ has incorporated desaturation computation, the nonlinear item in the closed-loop system of TSGPC becomes $f_0 \hat{f}^{-1}$. Since the uncertainty in f and the nonlinearity of the actuator[7] can be incorporated into f_0, the stability result for TSGPC is also the robustness result of this kind of system.

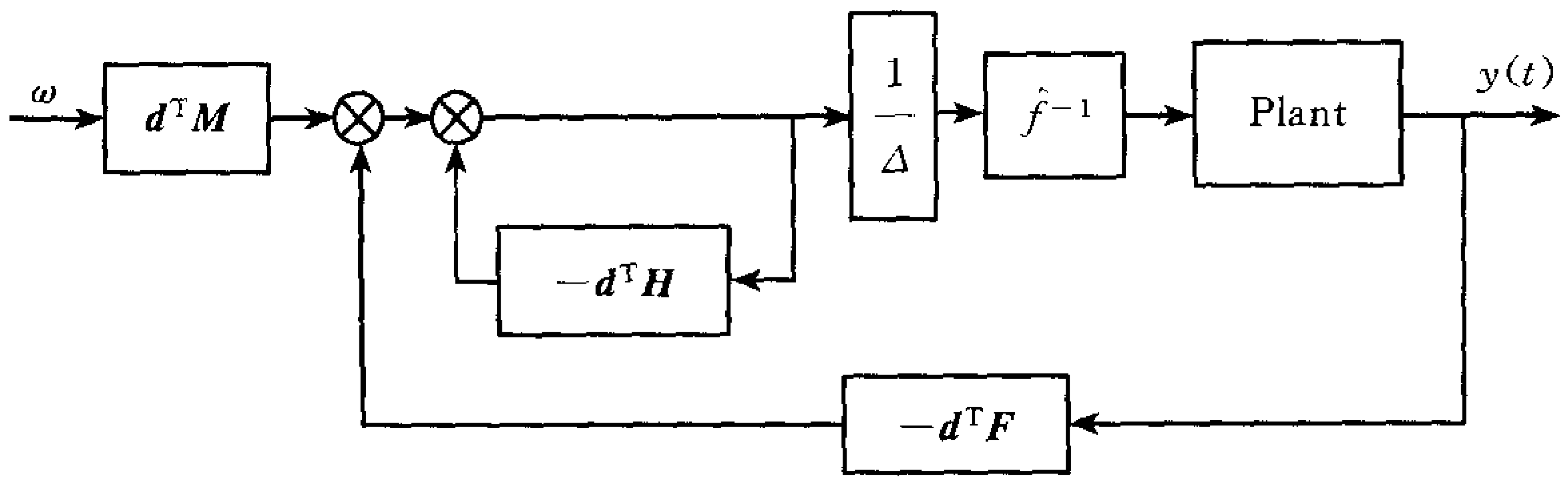

Fig. 2 The original block diagram of TSGPC

3 Applying Popov's theorem to TSGPC

3.1 The stability conclusion

Lemma 1 (Popov's Theorem). Suppose that $G(z)$ in Figure 3 is stable and $0 \leqslant \Phi(\theta)\theta \leqslant K\theta^2$. Then the closed-loop system is stable if $1/K + \mathrm{Re}\{G(z)\} > 0$ for all $|z| = 1$.

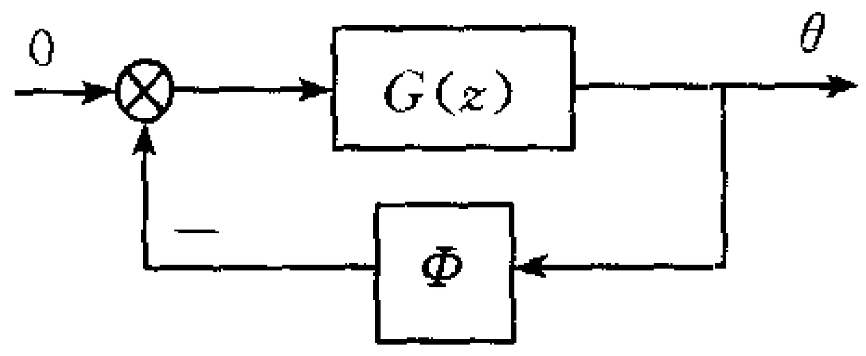

Fig. 3 The static nonlinear feedback form, 1

Applying Lemma 1 we can obtain the following stability result of TSGPC.

Theorem 1. Suppose that the linear part of the model is accurate and there exist two positive constants k_1 and k_2 such that i) the roots of $a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta + (1+k_1)z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b = 0$ are all located in the unit circle; and ii)

$$\frac{1}{k_2 - k_1} + \mathrm{Re}\left\{\frac{z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b}{a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta + (1+k_1)z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b}\right\} > 0, \quad \forall\, |z| = 1 \tag{4}$$

Then the closed-loop system of TSGPC is stable if the nonlinear item satisfies

$$k_1\theta^2 \leqslant (f_0\hat{f}^{-1} - 1)(\theta)\theta \leqslant k_2\theta^2 \tag{5}$$

Proof. Assume, without loss of generality, that $\omega = 0$ in Figure 2. Transform Figure 2 into Figures 4 (a), (b) and (c). Assume that the feedback item $f_0\hat{f}^{-1} - 1$ in Figure 4(c)

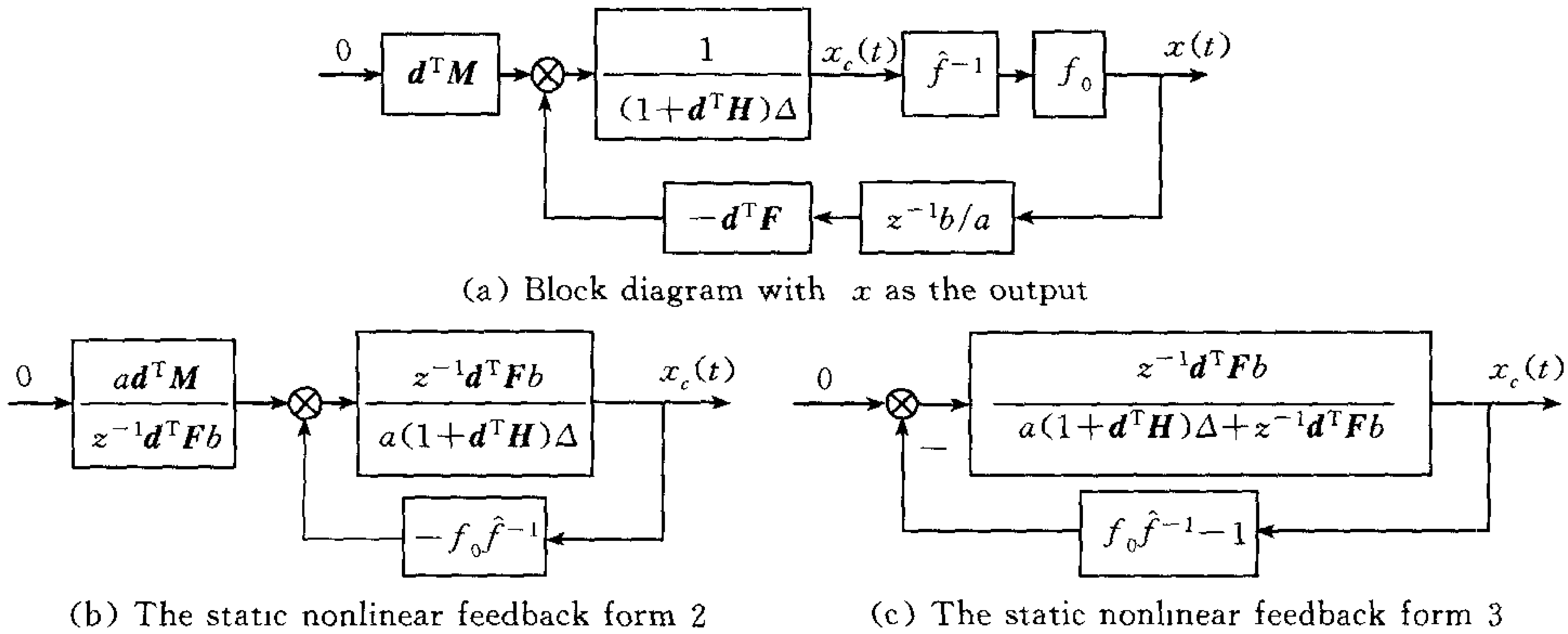

Fig. 4 The block diagram transformation

satisfies $k_1\theta^2 \leqslant (f_0\hat{f}^{-1}-1)(\theta)\theta \leqslant k_2\theta^2$ where k_1 and k_2 are constants. For utilizing Lemma 1, take $0 \leqslant \psi(\theta)\theta \leqslant (k_2-k_1)\theta^2 = k\theta^2$ where $\psi(\theta)=(f_0\hat{f}^{-1}-1-k_1)(\theta)$ and transform Figure 4(c) into Figure 5. Now, the characteristic equation of the linear part becomes $a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta+(1+k_1)\cdot z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b=0$. Therefore, the theorem holds by Lemma 1. □

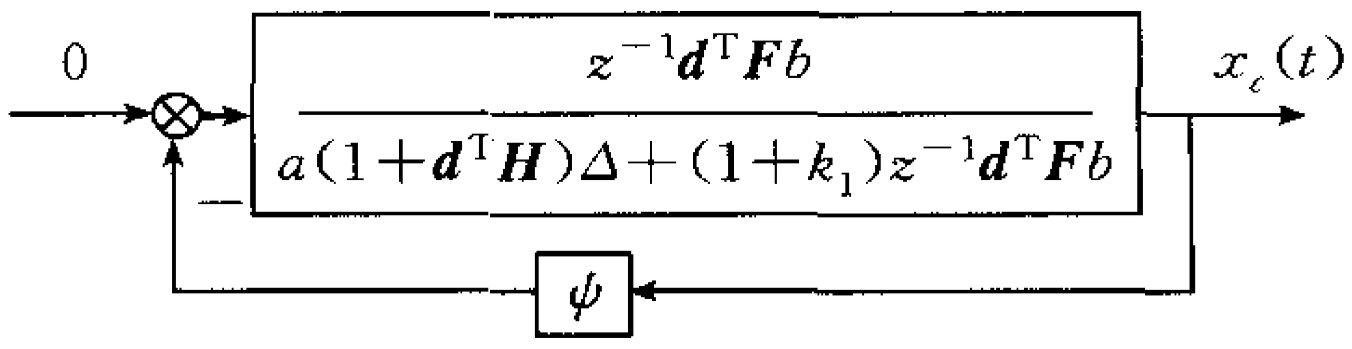

Fig. 5 The static nonlinear feedback form, 4

Remark 1. For given λ, N_1, N_2 and N_u, we may find multiple sets of (k_0, k_3) such that for all $k_1 \in [k_0, k_3]$, the roots of $a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta+(1+k_1)z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b=0$ are all located in the unit circle. In this way, the number of sets (k_1, k_2) satisfying $[k_1, k_2] \subseteq [k_0, k_3]$ and the conditions i), ii) in Theorem 1 may be innumerable. Suppose that in the real system

$$k_1^0\theta^2 \leqslant (f_0\hat{f}^{-1}-1)(\theta)\theta \leqslant k_2^0\theta^2 \tag{6}$$

where k_1^0 and k_2^0 are positive constants. Then Theorem 1 means: given λ, N_1, N_2 and N_u, if any set of (k_1, k_2) satisfies $[k_1, k_2] \supseteq [k_1^0, k_2^0]$, then TSGPC is stable. In fact, with Equation (6) known, verifying the stability can directly apply the following conclusion.

Corollary 1. Suppose that the linear part of the model is accurate and the nonlinear item satisfies Equation (6); then under the following two conditions the closed-loop system of TSGPC will be stable:

i) all the roots of $a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta+(1+k_1^0)z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b=0$ are located in the unit circle,

ii) $$\frac{1}{k_2^0-k_1^0}+\mathrm{Re}\left\{\frac{z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b}{a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta+(1+k_1^0)z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b}\right\}>0, \quad \forall\,|z|=1 \tag{7}$$

Remark 2. Theorem 1 and Corollary 1 do not require the corresponding LGPC to be stable, i.e., they do not require all the roots of $a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta+z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b=0$ to locate in the unit circle. This is an advantage of the stability result in this paper. Considering the relationship between TSGPC and LGPC shown in Figures 1 and 2, $0 \in [k_1^0, k_2^0]$ will have many advantages, but this means that the corresponding LGPC is stable.

3.2 Two algorithms for finding controller parameters

Theorem 1 and Corollary 1 can also be applied to design of the controller parameters λ, N_1, N_2 and N_u to stabilize the system. In the following we discuss two cases in the form of algorithm.

Algorithm 1. Given (k_1^0, k_2^0), design parameters $\{\lambda, N_1, N_2, N_u\}$ to stabilize the system.

Step 1. Search N_1, N_2, N_u and λ by variable alternation method, within their permissible (with respect to computational burden, etc.) ranges. If the search is finished, then terminate the whole algorithm, else choose one set of $\{\lambda, N_1, N_2, N_u\}$ and determine $a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta+z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b$.

Step 2. Apply Jury's criterion to examine whether all roots of $a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta+(1+k_1^0)\cdot z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b=0$ are located in the unit circle. If not, then go to step 1.

Step 3. Transform $-z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b/[a(1+\boldsymbol{d}^{\mathrm{T}}\boldsymbol{H})\Delta+(1+k_1^0)z^{-1}\boldsymbol{d}^{\mathrm{T}}\boldsymbol{F}b]$ into irreducible z-polynomial, $G(k_1^0, z)$.

Step 4. Substitute $z=\sigma+\sqrt{1-\sigma^2}\,i$ into $G(k_1^0, z)$ to obtain $\mathrm{Re}\{G(k_1^0, z)\}=G_R(k_1^0, \sigma)$.

Step 5. Let $M=\max\limits_{\sigma\in[-1,1]} G_R(k_1^0, \sigma)$. If $k_2^0 \leqslant k_1^0+\frac{1}{M}$, then terminate, else go to step 1.

If the open-loop system has no eigenvalues outside of the unit circle, generally Algorithm 1 can obtain satisfactory $\{\lambda, N_1, N_2, N_u\}$. Otherwise, satisfactory $\{\lambda, N_1, N_2, N_u\}$

may not be found for all given (k_1^0, k_2^0). In case no satisfactory $\{\lambda, N_1, N_2, N_u\}$ has been found by Algorithm 1, we can restrict the desaturation level, i. e., try to increase k_1^0 (this will be clear in Section 3.3). The following algorithm can be used to determine a smallest k_1^0.

Algorithm 2. Given desired (k_1^0, k_2^0), determine the controller parameters $\{\lambda, N_1, N_2, N_u\}$ such that (k_{10}^0, k_2^0) satisfies stability requirements and $k_{10}^0 - k_1^0$ is minimized.

Step 1. First let $k_{10}^{0,old} = k_2^0$.

Step 2. Same as step 1 in Algorithm 1.

Step 3. Utilize root locus or Jury's criterion to decide (k_0, k_3) such that $[k_0, k_3] \supset [k_{10}^{0,old}, k_2^0]$ and all roots of $a(1+\boldsymbol{d}^T\boldsymbol{H})\Delta + (1+k_1)z^{-1}\boldsymbol{d}^T\boldsymbol{F}\boldsymbol{b} = 0$ are located in the unit circle, for all $k_1 \in [k_0, k_3]$. If such (k_0, k_3) does not exist, then go to step2.

Step 4. Search k_{10}^0 by increasing it gradually. If the search is finished, then go to step2, else take $k_{10}^0 \in [\max\{k_0, k_1^0\}, k_{10}^{0,old}]$ and transform $-z^{-1}\boldsymbol{d}^T\boldsymbol{F}\boldsymbol{b}/[a(1+\boldsymbol{d}^T\boldsymbol{H})\Delta + (1+k_{10}^0) \cdot z^{-1}\boldsymbol{d}^T\boldsymbol{F}\boldsymbol{b}]$ into irreducible z-polynomial, $G(k_{10}^0, z)$.

Step 5. Substitute $z = \sigma + \sqrt{1-\sigma^2}i$ into $G(k_{10}^0, z)$ to obtain $\mathrm{Re}\{G(k_{10}^0, z)\} = G_R(k_{10}^0, \sigma)$.

Step 6. Let $M = \max_{\sigma \in [-1,1]} G_R(k_{10}^0, \sigma)$. If $k_2^0 \leqslant k_{10}^0 + \frac{1}{M}$ and $k_{10}^0 \leqslant k_{10}^{0,old}$, then take $k_{10}^{0,old} = k_{10}^0$ and go to step 2, else go to step 4.

Step 7. On finishing the search, let $k_{10}^0 = k_{10}^{0,old}$.

Simulation studies show that most searching tasks can be terminated in a few minutes by applying the above two algorithms.

3.3 The determination of (k_1^0, k_2^0) for a real system

In the above, given (k_1^0, k_2^0), we described the algorithms for determining the controller parameters. In the following we briefly illustrate how to decide (k_1^0, k_2^0) so as to bring Theorem 1 and Corollary 1 into play. From the above discussion we know that $f_0\hat{f}^{-1} \neq 1$ may be due to the following reasons:

i) desaturation effect;

ii) solution error of the NAE, including the case where an approximate solution is given since no accurate real-valued solution exists[1];

iii) inaccuracy in modeling of the invertible static nonlinearity;

iv) execution error of the actuator in a real system.

Assume the second step in TSGPC adopts Approach B. Then $f_0\hat{f}^{-1}$ is shown in Figure 6. Further assume that i) no error exists in solving NAE, ii) $k_{0,1}f(\theta)\theta \leqslant f_0(\theta)\theta \leqslant k_{0,2}f(\theta)\theta$ for all $x_{\min} \leqslant x \leqslant x_{\max}$, and iii) the desaturation level satisfies $k_{s,1}\theta^2 \leqslant sat(\theta)\theta \leqslant \theta^2$. Then

A) $f_0\hat{f}^{-1} = f_0 f^{-1} sat$,

B) $k_{0,1} \cdot sat(\theta)\theta \leqslant f_0\hat{f}^{-1}(\theta)\theta \leqslant k_{0,2} \cdot sat(\theta)\theta$,

C) $k_{0,1}k_{s,1}\theta^2 \leqslant f_0\hat{f}^{-1}(\theta)\theta \leqslant k_{0,2}\theta^2$, and finally, $k_1^0 = k_{0,1}k_{s,1} - 1$ and $k_2^0 = k_{0,2} - 1$.

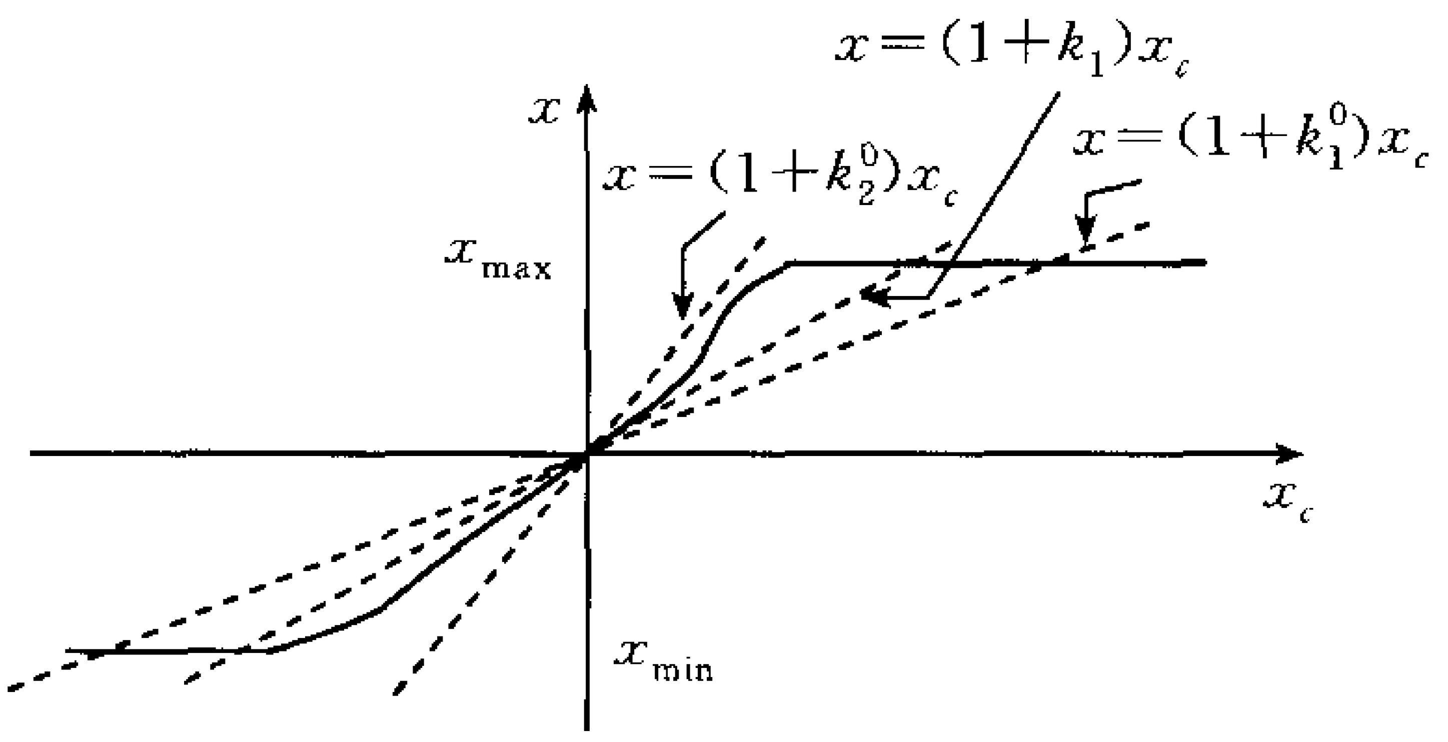

Fig. 6 The sketch map of nonlinear item $f_0\hat{f}^{-1}$

4 Simulation example

Here we only give an example to illustrate Theorem 1. The system adopted is $y(t)-2y(t-1)=x(t-1)$. Choosing $N_1=1$, $N_2=N_u=2$, and $\lambda=10$ we obtain $k_0=0.044$, and $k_3=1.8449$. As $k_1\in[k_0,k_3]$, condition i) in Theorem 1 is satisfied. Further, take $k_1=0.287$, and the largest k_2 satisfying condition ii) in Theorem 1 is $k_2=1.8314$. This choice of (k_1,k_2) is the one that satisfies $[k_1,k_2]\subseteq[k_0,k_3]$ and at the same time makes k_2-k_1 maximized. The initial values are $y(-1)=3$, $y(0)=7$ and $x(-1)=1$, while the set-point is $\omega=5$.

$$\text{Assume } f_0(\theta)=f(\theta)=\begin{cases}1.287\theta, & |\theta^4\sin\theta|\leqslant 1.287|\theta| \\ \text{sign}\{\theta\}|\theta^4\sin\theta|, & 1.287|\theta|\leqslant|\theta^4\sin\theta|\leqslant 2.8314|\theta| \\ 2.8314\theta, & |\theta^4\sin\theta|\geqslant 2.8314|\theta|\end{cases}.$$

Let $\hat{f}^{-1}=1$. Then $f\cdot\hat{f}^{-1}(\theta)=f(\theta)$. As shown in Figure 7, $(1+k_1)\theta^2\leqslant f_0\hat{f}^{-1}(\theta)\theta\leqslant(1+k_2)\theta^2$. All conditions in Theorem 1 are satisfied. The simulation result in Figure 8 shows that the system is stable.

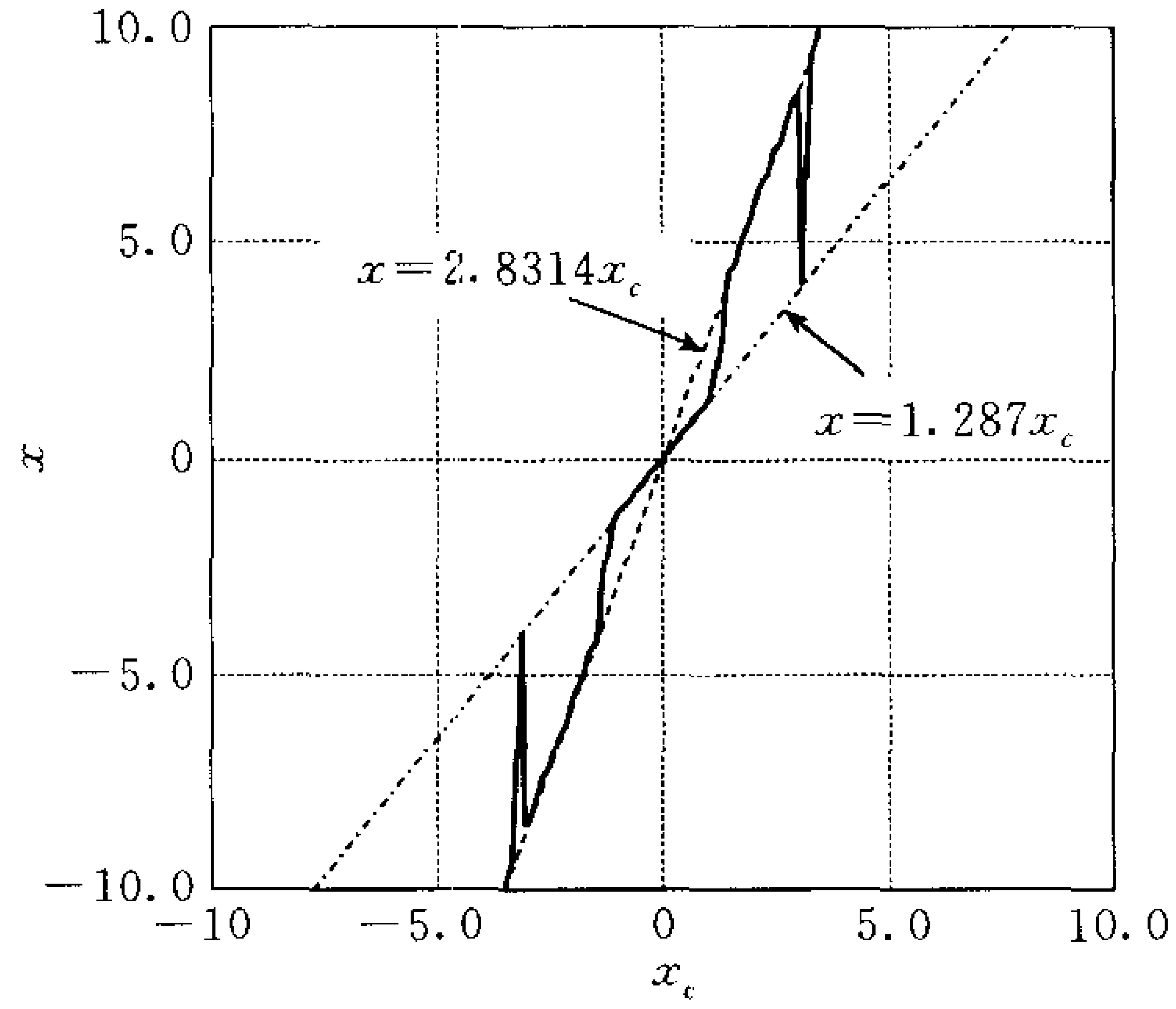

Fig. 7 The sketch map of nonlinear function $f_0\hat{f}^{-1}$

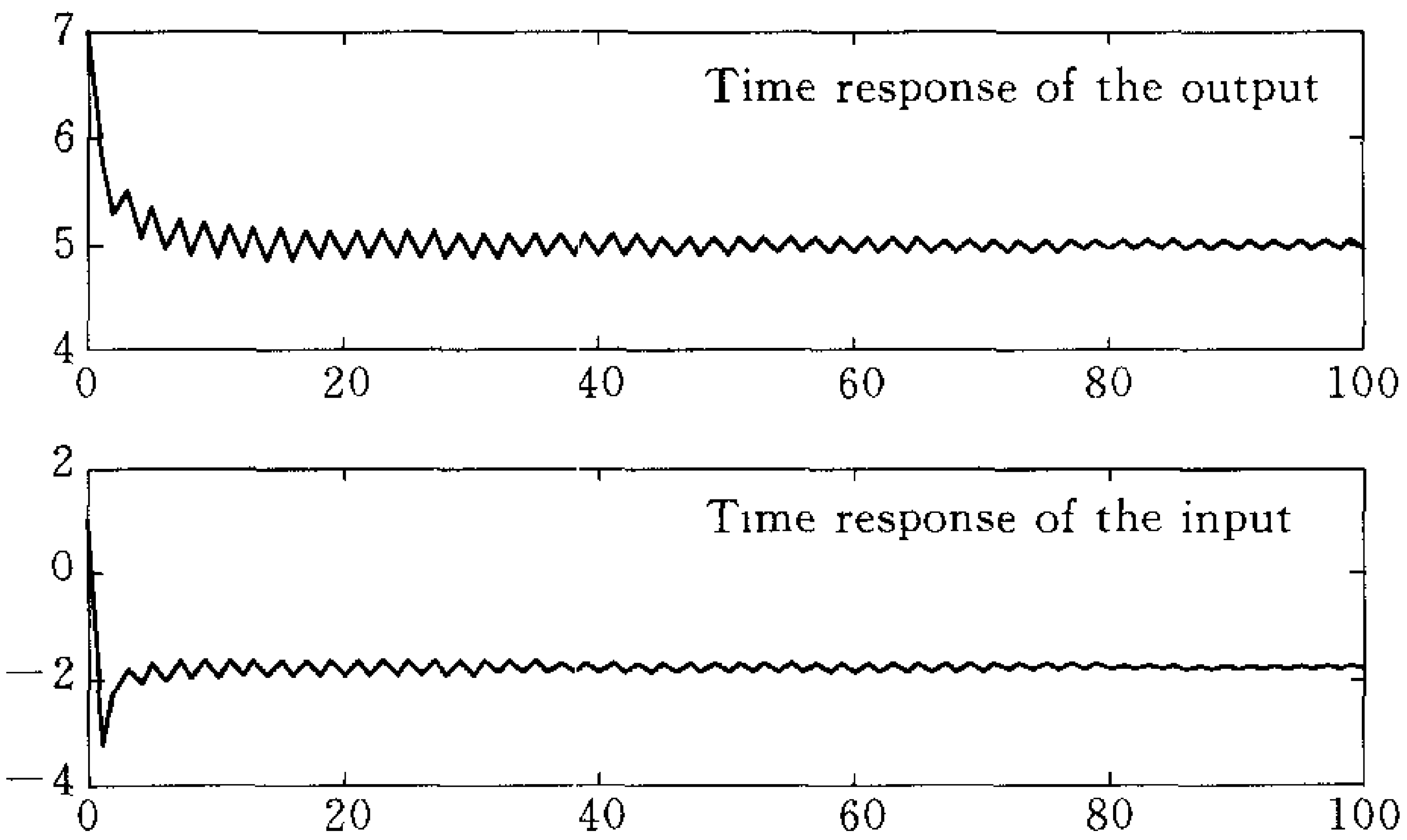

Fig. 8 Time responses of two-step (or input nonlinear) GPC

5 Conclusion

We transform the block diagram of the closed-loop system for two-step generalized predictive control with input nonlinearity into the static nonlinear feedback form, and utilize Popov's theorem to analyze the stability property of this kind of system. The paper gives the stability design method and illustrates the theoretical result with simulation. The significance of the nonlinear item in the closed-loop system of TSGPC is manifold, and by giving its variation range in the 2-dimensional space it can incorporate in varying degrees the various kinds of nonlinearities and uncertainties. This is very useful in real application.

The domain of attraction for TSGPC deserves further investigation. Although we have shown how to decide (k_1^0, k_2^0), to determine in which state set $k_1^0\theta^2 \leqslant (f_0 \hat{f}^{-1} - 1)(\theta)\theta \leqslant k_2^0\theta^2$ can be always satisfied is also very important. This problem can be considered under the state-space framework and will be discussed in another paper.

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基于 Popov 定理的输入非线性广义预测控制系统的稳定性分析

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摘　要　对存在输入饱和约束和输入可逆静态非线性的系统，采用两步法广义预测控制策略。首先用线性广义预测控制策略得到中间变量，代表期望的控制作用，然后用解方程方法补偿可逆静态非线性并用解饱和方法满足饱和约束，得到实际的控制作用。两步法计算简单，特别适用于快速控制的场合。将该控制系统闭环结构转化为静态非线性增益反馈结构，利用 Popov 定理分析了该系统的闭环稳定性，得到了稳定的充分条件，并具体给出了有效的控制器参数确定算法使得稳定性结论具备实用的价值。给出了算例验证了稳定条件。

关键词　输入非线性，两步法，广义预测控制，Popov 定理

中图分类号　TP273