# Fault Detection Approach for Networked Control System Based on a Memoryless Reduced-Order Observer<sup>1)</sup>

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Abstract The networked control system (NCS) deals with how to realize close-loop control through network. It is a multi-subject research region involving control science, computer network, etc. In this paper, the networked control system is regarded as a sampled-data control system with time-delay. A memoryless reduced-order state observer is constructed. Then the residual

is generated by comparing the output of the observer with the output of the practical system, and faults of the NCS is thus detected. Simulation result shows the approach is feasible.

**Key words** Fault detection, networked control system, network-induced delay, reduced-order observer, sampled-data control system

#### 1 Introduction

In control systems for civil and military use, for example, modern complicate industry process, high-performance automobile, air craft and space shuttle, nuclear power station, etc, computer network is often adopted to realize information exchange and control signal transmission between such components as monitor computer, controller, intelligent sensor, actuator located in different areas. If close-loop control of the whole system is achieved by network, such system is called a networked control system(NCS). Comparing with the traditional point-to-point feedback control system, the NCS can be installed and maintained easily; it can improve the system reliability and save space, energy and wiring.

Presently, some scholars have worked on the modeling and analysis of the networked control system. Halevi and Ray treat NCS as a continuous plant and a discrete controller. They use a discrete method to study communication of the components, set a clock-driven controller to adjust the asynchronization of the controller and the plant, and add the past input and output of the plant into the system model. Finally they establish a finite-dimensional, time-variant discrete augmented model<sup>[1]</sup>. Nilsson also deals with NCS in discrete-domain. He analyses the network-induced delay in detail, treats them as constant, independently random, or random of Markov chain. Then he solves the different LOG optimal problem according to different delay model<sup>[2]</sup>. Unlike above, Walsh *et al.* study the NCS in which network only exists between the sensor and controller. They regard it as a continuous plant and continuous controller. They set up a notion of maximum allowable transfer interval(MATI) to limit the interval of sensor data pockets and achieve the ideal performance of NCS<sup>[3]</sup>.

As in Figure 1, in this paper we regard the NCS as a sampled-data control system that consists of continuous plant and discrete controller. Data is sampled by the controller

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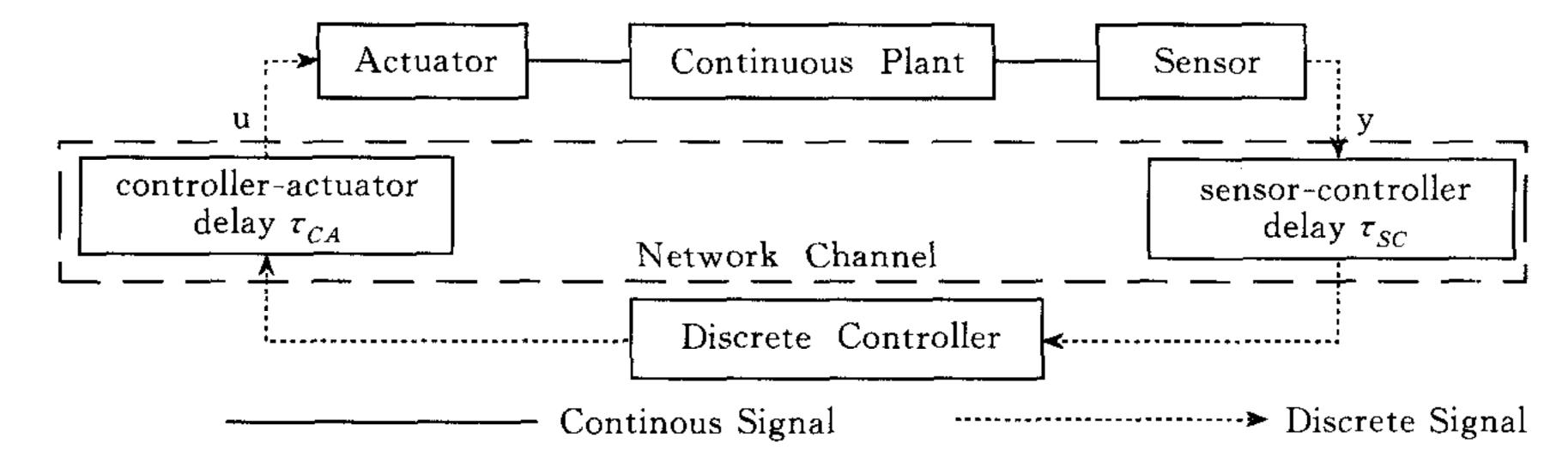


Fig. 1 The structure of networked control system

through network channel, and it will be sent to the actuator after computation. It can be inferred from Figure 1 that the whole system discrete model can be set up after discretizing the continuous part of the NCS.

Because of the limited communication bandwidth and lots of information sources, the network-induced delay is inevitable during information transmission and receiving. The delay brings many control difficulties, and makes conventional controller and observer design methods ineffective. There are two kinds of time-delays: sensor-controller delay  $\tau_{SC}$  and controller-actuator delay  $\tau_{CA}$ . They can be lumped together as the total network-induced delay  $\tau = \tau_{SC} + \tau_{CA}$ . It can be constant, time-variant, or even random. And it will be constant by some network protocols (for example, token ring, token bus), or by algorithm that compensates for the delay. In this paper we make use of the characteristic that the eigenvalue of delay system is relevant to the delay to establish a memoryless reduced-order state observer to study and analyze the fault problem of NCS with constant delay.

For control system the observer-based fault diagnosis method has been one of the main topics in fault diagnosis region. Its basic principle is to set up the residual r(t) as the error of a prior value provided by the system model and the observer measurement to detect and isolate system faults. Generally, r(t) is equal or close to zero for the normal case; otherwise it will be nonzero.

The article is organized as follows. After introduction, we present the problem and set up the mathematical model of NCS. In Section 3, we will construct the observer described as above. Section 4 is about generation of residual and realization of fault detection approach. An example and simulation result are presented in Section 5. At last, conclusion and perspective are given in Section 6.

#### 2 Mathematical model of networked control system

As in Figure 1,  $\tau_{CA}$  and  $\tau_{SC}$  are random and tine-variant, which increase the complexity of NCS. If the controller node is clock-driven<sup>[1]</sup>, then at sample instant kT(T) is the sample period) the message arriving at the controller/actuator and its sequence will not be determined. So it is impossible to get the distinct state model of the plant.

Therefore, we choose another control mode for controller—event-driven<sup>[5]</sup>. As in Figure 2, once the sensor data reaches the controller, the controller starts immediately and figures out the control vector and send it to the actuator. To eliminate data loss and guarantee its sequence, we set up transmission buffers at the sensor and the controller, the lengths of which are longer than the maximum delay periods of  $\tau_{SC}$  and  $\tau_{CA}$ , respectively. If there are data pockets in the buffer, they will be sent out as soon as the bus is idle. Thus the integrity and sequence of the information transmission is guaranteed.

From Figure 1 we can get the dynamics of the continuous linear plant

Fig. 2 The network-induced delay time diagram of event-driven mode

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) + B_d\mathbf{d}(t) + B_f f_a(t)$$

$$\mathbf{y}(t) = C\mathbf{x}(t) + f_s(t)$$
(1)

and the discrete controller dynamics

$$u(kT) = -Dx(kT) \tag{2}$$

where  $x(t) \in \mathbb{R}^n$  and  $x(kT) \in \mathbb{R}^n$  are the state vectors,  $u(t) \in \mathbb{R}^p$  and  $u(kT) \in \mathbb{R}^p$  are the control vectors, y(t) is the plant output, d(t) is the disturbance vector,  $f_a(t)$  denotes actuator fault,  $f_s(t)$  denotes sensor fault,  $f_a(t)$  and  $f_s(t)$  are unknown. Matrices  $A, B, B_d$ ,  $B_f, C$  are of corresponding dimensions.

Discretize (1), and consider the effect of time-delay  $\tau$  between the plant and controller. Since  $\tau$  is constant, so in each period the plant only receives one delayed control data. Assume  $0 \le (l-1)T < \tau < lT$ ,  $l \ge 1$ . Then we might suppose  $\tau' = \tau - (l-1)T$  as well which satisfies  $0 \le \tau' < T$ . So the discrete state model of the system with network-induced delay can be described as

$$\mathbf{x}(k+1) = \hat{A}\mathbf{x}(k) + \hat{B}_{0}\mathbf{u}(k-l) + \hat{B}_{1}\mathbf{u}(k-l+1) + \hat{B}_{d}\mathbf{d}(k) + \hat{B}_{f}\mathbf{f}_{a}(k)$$

$$\mathbf{y}(k) = \hat{C}\mathbf{x}(k) + f_{s}(k)$$
(3)

where

$$\hat{A} = e^{AT}, \quad \hat{B}_{0} = \int_{T-\tau'}^{T} e^{As} B \, ds, \quad \hat{B}_{1} = \int_{0}^{T-\tau'} e^{As} B \, ds, \quad \hat{B}_{d} d = \int_{0}^{T} e^{A(T-s)} B_{d} d(s) \, ds$$

$$\hat{B}_{f} f_{a} = \int_{0}^{T} e^{A(T-s)} B_{f} f_{a}(s) \, ds, \quad \hat{C} = C$$
(4)

# 3 Memoryless reduced-order state observer

[7] introduces a memoryless reduced-order state observer of time-delay system. Based on this, a fault detection observer is constructed for NCS(3) in this paper. The design step is shown as follows.

Firstly, compute unstable and poorly damped eigenvalues of the characteristic matrix  $(zI_n - \hat{A})$  according to the specific critical value  $\gamma$ , and form  $\Omega$  space, i. e.,  $\Omega = \{z: \det(zI_n - \hat{A}) = 0, |z| \ge \gamma\}$ ,  $0 < \gamma < 1$ .

Secondly, compute  $\Lambda_r$  and V.  $\Lambda_r = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_r)$ , where  $\lambda_1, \lambda_2, \dots, \lambda_r \in \Omega$ . If the eigenvalue is a complex number, then  $\Lambda_r$  will be the corresponding Jordan canonical form. And

$$(zI_n - \hat{A})^{-1} = V(zI_r - \Lambda_r)^{-1}W + \tilde{V}(zI_{n-r} - \Lambda_{n-r})^{-1}\tilde{W}$$
(5)

where  $V \in C^{n \times r}$  and  $V = [v_i], v_i \in C^n$ ,  $i = 1, 2 \cdots, r$  satisfies the following equation:

$$[z_i I_n - \hat{A}] v_i = 0, \quad \forall z_i \in \Omega$$
 (6)

Therefore the structure of the rth-order fault observer is derived as follows:

$$x_{r}(k+1) = \Lambda_{r}x_{r}(k) + L[y(k) - \hat{C}Vx_{r}(k)]$$

$$\hat{x}(k) = Vx_{r}(k) + D_{r}u(k)$$

$$\hat{y}(k) = \hat{C}\hat{x}(k)$$
(7)

where  $\hat{x}(k)$  is the observed states.

Suppose e(k) is the error between the observed state and the actual state, i. e.,

$$e(k) = x(k) - \hat{x}(k) \tag{8}$$

Combine (3), (7) and (8), and we have the following result

$$\mathbf{x}_r(k+1) = \Lambda_r \mathbf{x}_r(k) + L \hat{C} \mathbf{e}(k) + L \hat{C} D_r \mathbf{u}(k) + L \mathbf{f}_s(k)$$
(9)

The Z-transform of (3) and (9) is

$$(zI_{n} - \hat{A})[I_{n} + V(zI_{r} - \Lambda_{r})^{-1}L\hat{C}]e(z) = [\hat{B}_{0}z^{-l'} + \hat{B}_{1}z^{-l'+1} - (zI_{n} - \hat{A})(D_{r} + V(zI_{r} - \Lambda_{r})^{-1}L\hat{C}D_{r})]u(z) + \hat{B}_{d}d(z) + \hat{B}_{f}f_{a}(z) - (zI_{n} - \hat{A})V(zI_{r} - \Lambda_{r})^{-1}Lf_{s}(z)$$
(10)

In order for e(z) to approach to 0 within a  $\gamma$ -stability region, two conditions must be satisfied<sup>[7]</sup>: a) e(z) must be  $\gamma$ -stable; b) when  $z \rightarrow 1$ , the coefficient of u(z) must approach to 0. Now we begin step 3 to determine  $L,D_r$  by satisfying the above conditions.

**Theorem.** If system (3) is observable for  $\forall z \in \Omega$ , then condition a) is satisfied.

**Proof.** Take linear transform on (3), and diagonalize A. Then we get

$$x(k+1) = \Lambda_n x(k) + R^{-1} \hat{B}_d d(k) + R^{-1} \hat{B}_f f_a(k) + R^{-1} \hat{B}_0 u(k-l') + R^{-1} \hat{B}_1 u(k-l'+1)$$

$$y(k) = \hat{C}Rx(k) + f_s(k)$$

where R is a non-singular transform matrix.

System (3) is observable for  $\forall z \in \Omega$ .

Decompose the above system according to  $z \in \Omega$  and  $z \notin \Omega$ . The observable subsystem can be derived as

$$\mathbf{x}_{o}(k+1) = \Lambda_{r}\mathbf{x}_{o}(k) + R_{r}^{-1}\hat{B}_{d}\mathbf{d}(k) + R_{r}^{-1}\hat{B}_{f}\mathbf{f}_{a}(k) + R_{r}^{-1}\hat{B}_{0}\mathbf{u}(k-l') + R_{r}^{-1}\hat{B}_{1}\mathbf{u}(k-l'+1)$$

$$\mathbf{y}(k) = \hat{C}R_{r}\mathbf{x}_{o}(k) + \mathbf{f}_{s}(k)$$

where  $x_o$  is the state vector of the observable subsystem. And from equation (6), the definition of V, we conclude  $V=R_r$ .

The observable subsystem of the above system is  $(\Lambda_r, \hat{C}V)$ . We can design L to make  $(\Lambda_r - LCV) \gamma$ -stable.

$$\varphi(z) = \det(zI_r - \Lambda_r + LCV)$$
 is  $\gamma$ -stable.

Namely,  $\varphi(z) = \det(zI_r - \Lambda_r) \det(I_r + (zI_r - \Lambda_r)^{-1}LCV)$  is  $\gamma$ -stable.

Assume  $\phi(z)$  is a function on plane  $|z| > \gamma$ . So it is obvious that  $\det(zI_n - \hat{A}) = \phi(z)$   $\det(zI_r - \Lambda_r)$  exists for  $\forall z \in \Omega$ . And since the matrix formula  $\det(I_n + GH) = \det(I_r + HG)$  exists where G is an  $n \times r$  matrix, H is an  $r \times n$  matrix, the coefficient of  $e(z) : \psi(z) = \phi(z) \det(zI_r - \Lambda_r) \det(I_n + V(zI_r - \Lambda_r)^{-1}LC) = \phi(z)\varphi(z)$  is  $\gamma$ -stable.  $\square$ 

The theorem summarizes the conditions which satisfies a). To b), when  $z \rightarrow 1$ , the coefficient of u(z) in equation (10) is

$$\hat{B}_0 + \hat{B}_1 - (I_n - \hat{A})V(I_r - \Lambda_r)^{-1}LCD_r - (I_n - \hat{A})D_r = 0$$

so we can conclude

$$D_r = [(I_n - \hat{A})(I_n + V(I_r - \Lambda_r)^{-1}L\,\hat{C})]^{-1}[\hat{B}_0 + \hat{B}_1]$$
(11)

A memoryless reduced-order state observer (7) has been set up by the above 3 steps.

#### 4 Generation of residual and fault detection

Take the product of the coefficient Q and the error between NCS actual output and observer output as the residual r(k), viz

$$\mathbf{r}(k) = \mathbf{Q}(\mathbf{y}(k) - \hat{\mathbf{y}}(k)) = \mathbf{Q}\hat{C}\mathbf{e}(k) + \mathbf{Q}\mathbf{f}_{s}(k)$$
 (12)

where the coefficient Q increases the residual design freedom, improves the residual robust performance and reduces its sensitivity to disturbance.

Combining the Z-transform of (12), with (10) and (11), we can get

$$\mathbf{r}(z) = Q\hat{C}P^{-1}\hat{B}_{d}\mathbf{d}(z) + Q\hat{C}P^{-1}\hat{B}_{f}\mathbf{f}_{a}(z) + [Q - Q\hat{C}P^{-1}(zI_{n} - \hat{A})V(zI_{r} - \Lambda_{r})^{-1}L]\mathbf{f}_{s}(z)$$
(13)

where

$$P = (zI_n - \hat{A})[I_n + V(zI_r - \Lambda_r)^{-1}L\hat{C}] = zI_n - [\hat{A} + \hat{A}V(zI_r - \Lambda_r)^{-1}L\hat{C} - zV(zI_r - \Lambda_r)^{-1}L\hat{C}]$$
 Equation (13) indicates that residual  $r$  is only relevant to disturbance  $d$ , actuator fault  $f_a$  and sensor fault  $f_s$ .

To remove the effect of the disturbance, it is required that

$$Q\hat{C}P^{-1}\hat{B}_d = H(zI_n - P')^{-1}\hat{B}_d = z^{-1}H(I + P'z^{-1} + P'^2z^{-2} + \cdots)\hat{B}_d = 0$$
 (14)

where

$$H = Q\hat{C}, P' = \hat{A} + \hat{A}V(zI_r - \Lambda_r)^{-1}L\hat{C} - zV(zI_r - \Lambda_r)^{-1}L\hat{C}.$$

Equation (14) can be achieved<sup>[8]</sup>, if the following condition a) or b) is met:

a) 
$$H\hat{B}_d = 0$$
 and  $HP' = 0$ ; b)  $H\hat{B}_d = 0$  and  $P'\hat{B}_d = 0$ .

The above conditions are the general principle for disturbance de-coupling residual. For practical systems further assistant tool will be required, such as eigenstructure assignment, etc.

The structure of fault observer and generation of residual is shown in Figure 3.

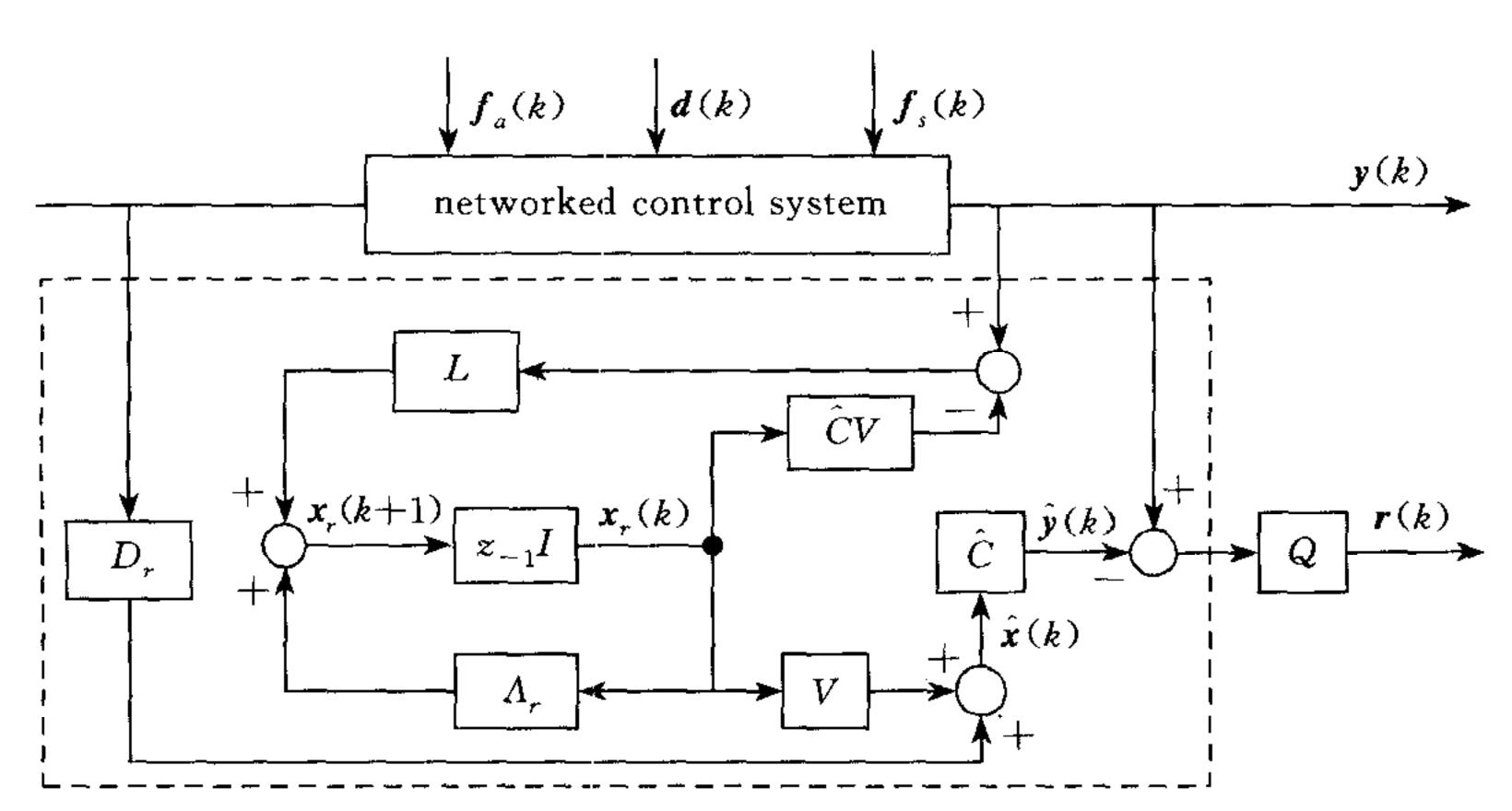


Fig. 3 The fault observer structure and residual generation

### 5 Example

Consider the following continuous plant dynamics

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0.3 & 0.15 \\ 0 & 0.45 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f_a(t)$$

$$\mathbf{y}(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(t) + f_s(t)$$
(15)

Set  $T=1s, \tau=1.1s$ , so  $l=2, \tau'=0.1s$ . Discretizing (15) by (4), we get

$$\mathbf{x}(k+1) = \begin{bmatrix} 1.3499 & 1.1618 \\ 1 & 1.5683 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0.1153 \\ 0.1534 \end{bmatrix} u(k-2) + \\ \begin{bmatrix} 0.9636 \\ 1.1096 \end{bmatrix} u(k-1) + \begin{bmatrix} 1.0789 \\ 1.2629 \end{bmatrix} d(k) + \begin{bmatrix} 1.0789 \\ 1.2629 \end{bmatrix} f_a(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k) + f_s(k)$$

$$(16)$$

The reduced-order fault observer can be established by the three steps described in Section 3 as follows.

Step 1. Figure out the eigenvalues of  $\hat{A}$  which are 0.3757 and 2.5425. Choose  $\gamma = 0.4$ , so  $\Omega = \{2.5425\}$ .

Step 2. Get  $\Lambda_r = 2.5425$ . Derive  $V = [-0.6978 -0.7163]^T$  from (6).

Step 3. Since system (16) is observable, from the theorem we can find L to  $\gamma$ -stablize  $(\Lambda_r - L\hat{C}V)$ . Set the eigenvalue of  $(\Lambda_r - L\hat{C}V)$  to be 0.3, and we can conclude that L = -3.2137. And from (11) we know  $D_r = [1.9544 \ 2.2552]^T$ .

Therefore, the structure of the reduced-order fault observer is

$$\hat{x}_r(k+1) = 0.3x_r(k) - 3.2137y(k)$$

$$\hat{x}(k) = \begin{bmatrix} -0.6978 \\ -0.7163 \end{bmatrix} x_r(k) + \begin{bmatrix} 1.9544 \\ 2.2552 \end{bmatrix} u(k)$$

$$\hat{y}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \hat{x}(k)$$

MATALAB is used for simulation. Set coefficient Q=1. When there is no fault in the system, the simulation curves of system's actual output, observer output and residual are shown in Figure 4. Because NCS is viewed as a discrete system, and data will be changed at only sample instant, the curve is piecewise. The track effect of observer output to system output can be seen in Figure 4(a). And from the magnifier Figure 4(b) we observe that the residual drifts in a small region around 0.

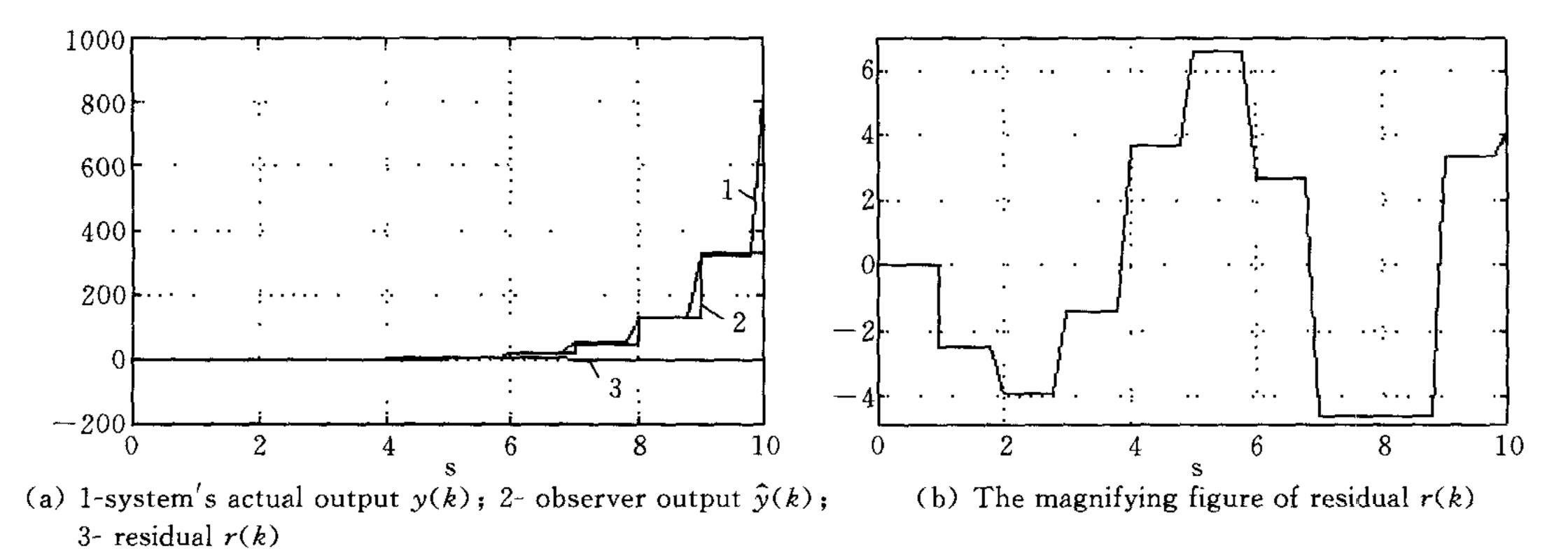


Fig. 4 Simulation result of fault-free case

If a sensor fault occurs at 4s with amplitude 500, the system's actual output, observer output and residual are shown in Figure 5. From it we observe that the residual increases rapidly after 4s, which indicates the system is faulty.

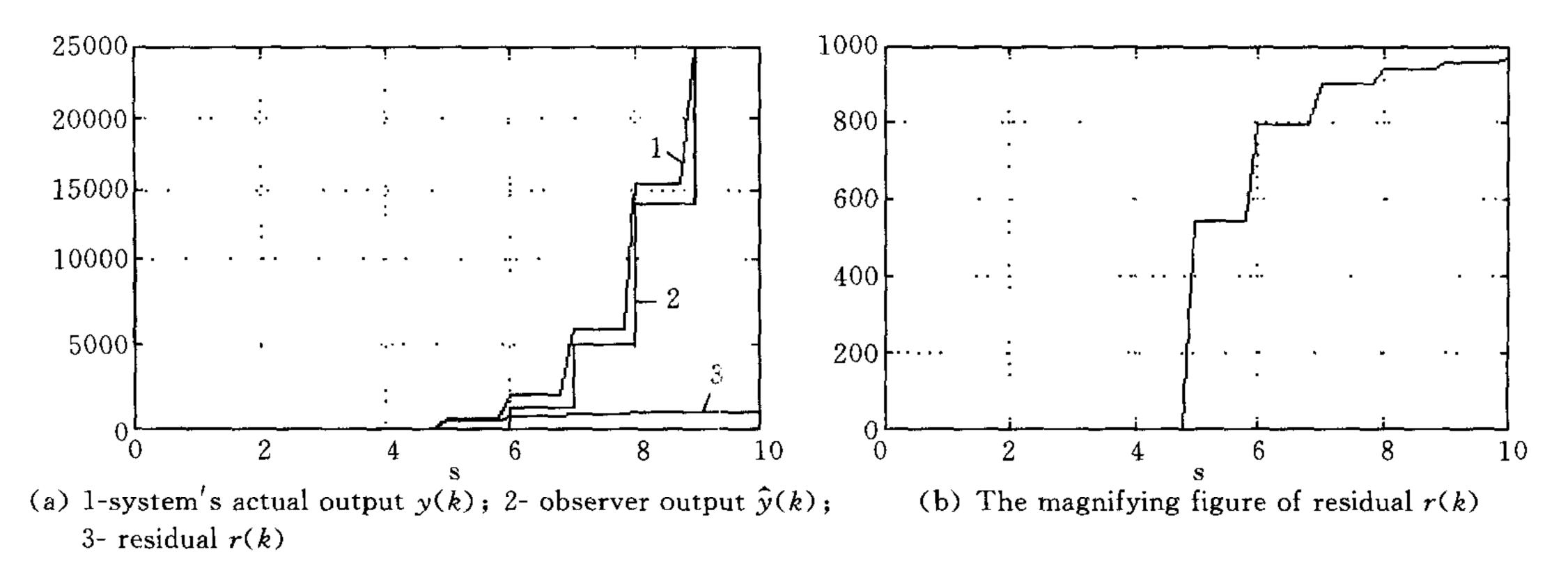


Fig. 5 Simulation result of the case with a 500 amplitude sensor fault at 4s

## 6 Conclusion and perspective

Fault detection for networked control system is a new research topic. In this paper we regard NCS as a sampled-data control system with time-delay, and set up its mathematical model. By constructing a memoryless reduced-order fault observer we achieve the fault detection of NCS.

In Ethernet, ControNet and DeviceNet, network-induced delay can be constant, time-variant or random. In token-bus and token-ring, time-delay is generally constant since the periodicity of the token. Moreover, to adopt some measure such as compensating for the delay also approxmates the delay to a constant. A great deal of present research results are only limited to the case that networked-induced delay is less than a sample period. Except for the premise that it is constant, there is no limitation to the delay in this paper.

Our future work has two directions. One involves fault isolation and identification of NCS, the other is to consider the instance that the network-induced delay is time-variant and random since the network is complex after all.

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# 基于无记忆降阶观测器的网络化控制系统故障检测方法

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摘 要 网络化控制系统(Networked Control System,简称 NCS)研究的是如何通过网络实现闭环控制,是涉及控制科学和计算机网络等多个学科知识的跨学科研究领域.本文将网络化控制系统视为带有时延的采样控制系统,构造了一个无记忆的降阶状态观测器,通过比较观测器输出和系统实际输出来产生残差,从而对网络化系统的控制故障进行了检测.仿真结果表明此方法是可行的.

关键词 故障检测,网络化控制系统,网络时延,降阶观测器,采样控制系统中**图分类号** TP277;TP393