Hierarichical Optimal Procuction Control Policy for Production Systems Involving Aging[1)]

SONG Chun-Yue[1] WANG Hui[2] LI Ping[1]
[1](Institute of Industrial Process Control, Zhejiang University, Hangzhou 310027)
[2](Institute of Systems Engineering, Zhejiang University, Hangzhou 310027)
(E-mail: cysong@iipc. zju. edu. cn)

Abstract A production system consisting of a set of failure-prone equipment is studied, whose state jump rate depends on its aging. A new hierarchical optimal production rate control framework involving setup and aging of the system is discussed. By introducing new state variable and new time horizon of the objective function, the original problem is decomposed and the necessary and sufficient dynamic programming optimality conditions in the lower level are proposed according to the optimal production duration. Simulation results show the feasibility of the proposed approach in practice.

Key words Hierarchical flow control, optimal production, dynamic programming, equipment aging

1 Introduction

Many efforts focused on the optimal production of production systems with stochastic jump Markov disturbances, in which however it is popular to assume that the frequency of system failure only depends on the time, i. e., breakdowns occur independently of whether machines are being used or not. In practice, the occurrence of equipment breakdown depends on many factors, particularly its age. So. it is more practical to take the machine age factor into consideration. [1] and [2] discussed the aging of the machine which affects machine failure. [3] discussed preventive maintenance of flexible manufacturing systems considering the machine age function. In the paper, the aging of the system and setup are taken into consideration and the optimal production of production systems is discussed. A new finite horizon of integral is put forward instead of the deterministic finite horizon discussed in the former work, which is very meaningful in practice. Moreover, the optimal duration of producing one type of products is introduced into our framework as a new state variable. Based on the above, the original problem is decomposed, and a new framework of hierarchical flow control model involving setup and aging of the system is constructed. The necessary and sufficient dynamic programming optimality conditions in the lower level are proposed according to the optimal production duration.

2 Description of the problem

The production system consisting of a set of unreliable equipment can produce n different types of products P_i, $i=1, \cdots, n$ with only one at any given time. A setup (with setup duration and setup cost) is required if production is to be switched from one type of products to another. It is assumed that for $i, j=1, \cdots, n$ and $i \neq j$, $\theta_{ij} \geqslant 0$ and $K_{ij} \geqslant 0$, which denote the setup duration and cost of switching from production of P_i to P_j, respectively, and θ_{ij}, K_{ij} are constant. Moreover, for any $i, j, k=1, \cdots, n$, $i \neq j$ and $j \neq k$, $\max\{\theta_{ij}, K_{ij}\}>0$, $\theta_{ij}+\theta_{jk}-\theta_{ik} \geqslant 0$ and $K_{ij}+K_{jk}e^{-\rho\theta_{ij}}-K_{ik}>0$. If $i=j$, then $\theta_{ij}=K_{ij}=0$.

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Here $0<\rho<1$ denotes the discount rate.

2.1 The dynamic model of the system

For $t\geqslant 0$, let $x_i(t)\in R^1=(-\infty,\infty)$, $u_i(t)\in R^+=[0,\infty)$, and $z_i(t)\in R^+=[0,\infty)$ denote the surplus, production rate, and the rate of demand, respectively, for product P_i at time t, $i=1,\cdots,n$. X, U, and Z are used to denote vectors $[x_1(t), x_2(t), \cdots, x_n(t)]^T \in R^n$, $[u_1(t), u_2(t), \cdots, u_n(t)]^T\in R^{+n}$, and $[z_1(t), z_1(t), \cdots, z_n(t)]^T\in R^{+n}$, respectively. Z is considered as constant here. $h(t)$ is used to represent the age of the equipment at time t, $h(t)\in R^+$. The inventory/shortage levels and equipment age of the system are described by the following dynamic differential equations:

$$\begin{cases}\dot{X}(t) = F(\alpha,U(t), Z(t)) = U^\alpha(t) - Z(t) \\ \dot{h}(t) = f(u^\alpha(t)) \quad (X(0),h(0)) = (X_0,h_0)\end{cases} \tag{1}$$

where $F(\cdot,\cdot)=[F_1, F_2, \cdots, F_n]^T$, $U^\alpha=[u_1^\alpha,\cdots,u_n^\alpha]^T$ and $u^\alpha(t)=\sum_{i=1}^n u_i^\alpha(t)$, $u_i^\alpha(t)$ is the instantaneous production rate of P_i at time t with the equipment state $\zeta(t)=\alpha$ (defined later). The function f in Equation (1) represents the effect of the production rate $u^\alpha(t)$ on the equipment age. The equipment states can be classified as (i) operational, denoted by state 1; (ii) under repair, denoted by state 0. Under the operational state, any type of products can be produced; under the breakdown state, nothing is produced. Let $\zeta(t)$ denote the state of the equipment, and it is a finite-state Markov process. Let $E=\{0,1\}$ be the state space of the process $\zeta(t)$, $\zeta(t)\in E$.

Let $q_{\alpha\beta}(h(t))$ be the jump rate of the process $\zeta(t)$ from state α to state β at time t. These jump rates are defined by

$$P[\zeta(t+\mathrm{d}t) = \beta \mid \zeta(t) = \alpha] = q_{\alpha\beta}(h(t))\mathrm{d}t + o(\mathrm{d}t) \tag{2}$$

$$P[\zeta(t+\mathrm{d}t) = \alpha \mid \zeta(t) = \alpha] = 1 + q_{\alpha\alpha}(h(t))\mathrm{d}t + o(\mathrm{d}t) \tag{3}$$

where $\lim_{\mathrm{d}t\to 0} o(\mathrm{d}t)/\mathrm{d}t=0$, $q_{\alpha\alpha}(\cdot)=-\sum_{\beta\neq\alpha}q_{\alpha\beta}(\cdot)$. It is assumed that the jump rate $q_{\alpha\beta}(h(t))$ is bounded and satisfies the following conditions: $|q_{\alpha\beta}(h(t))-q_{\alpha\beta}(h'(t))|\leqslant C|h(t)-h'(t)|$, $\forall h(t), h'(t)\in \mathrm{R}^+$, for some constant C and $|q_{\alpha\alpha}|\geqslant c_0>0$, $q_{\alpha\beta}(h(t))\geqslant 0$.

When the equipment has a breakdown, it goes through a repair process. Repaired equipment is considered renewed, i. e., $f(u^\alpha(t))=0$, which means the age of the equipment is reset to 0. Since $f=0$ when the equipment is under repair, for convenience, the age $h(t)$ is reset to 0 at the beginning instead of the end of the repair process. Thus, according to our notation, if there is a jump from state α to state β, then the age function $h(t)$ jumps to $h'(t)=\beta h(t)$. According to the above, the following holds

$$h(t+\Delta t) = \begin{cases}0, & \text{If } \zeta(t+\Delta t) = 1 \text{ and } \zeta(t-\Delta t) \neq 1; \\ h(t), & \text{Other.}\end{cases} \quad \Delta t > 0 \text{ is small enough.} \tag{4}$$

2.2 The cost function and constraints

The optimality problem of finding a production control policy is to minimize the following cost function:

$$J(i,X,s,\Xi,U(\cdot),h) = \int_0^s \mathrm{e}^{-\rho t}G(X(t),0)\mathrm{d}t + E\Big(\int_s^\infty \mathrm{e}^{-\rho t}G(X(t),U(t))\mathrm{d}t + \sum_{l=0}^\infty \mathrm{e}^{-\rho\tau_l}K_{i_l i_{l+1}}\Big) \tag{5}$$

where s denotes the remaining setup time, $0\leqslant s\leqslant\theta_{ij}$. The decision variables are the rates of production $U(\cdot)$ over time and a sequence of setups denoted by $\Xi=\{(\tau_0, i_0 i_1),(\tau_1,i_1 i_2),\cdots\}$, where a setup (τ, ij) is defined by the starting time τ and a pair ij denoting that the equipment was set up to produce P_i and is being switched to produce P_j. Let $G(X(t), U(t))$ denote the running cost function of surplus and production. Usually $G(X(t), U(t)) = \sum_{i=1}^n c_i^+ x_i^+ + c_i^- x_i^-$. It is supposed that a positive surplus incurs a holding cost of c_i^+ per unit commodity per unit time, while a negative a cost of c_i^-, with $c_i^+>0$, $c_i^->0$. $x_i^+ := \max(x_i, 0)$, $x_i^- := \max(-x_i, 0)$.

For $t \geqslant 0$, the production constraints are given as follows:

$$\begin{cases} 0 \leqslant u_i(t) \leqslant \zeta(t) r_i, & i = 1, 2, \cdots, n \\ u_j(t) = 0, & j \neq i \end{cases} \tag{6}$$

where r_i denotes the maximum production rate of P_i. For each $\alpha \in E$, the control set is $\Gamma_i(\alpha) = \{U=(u_1, \cdots, u_n) \geqslant 0,\ u_i^a \leqslant \alpha r_i,\ u_j = 0,\ j \neq i\}$, $i=1, 2, \cdots, n$.

Let $\Gamma(\alpha)$, a closed subset of R^{+n}, denote the control constraints, $\forall \alpha \in E$. Any measurable function $U(t)$ defined on $\Gamma(\alpha)$, for each $\alpha \in E$, is called an admissible control. The set $\Theta = \{U(t) : t \geqslant 0\}$ is admissible policy. The admissible control function $U(t)$ is supposed to be piecewise continuous in t and continuously differentiable with bounded partial derivatives in X. Let $(X(t), \alpha, ij)$ denote the system state at time t, and the space of the system state be $R^n \times M \times \{ij \mid i, j=1, 2, \cdots, n, i \neq j\}$. The problem is to find an admissible decision $(\Xi, U(\cdot)) \in \Omega = (\Xi, \Theta)$ that minimizes $J(i, X, s, \Xi, U(\cdot))$ which is subject to Equation (1).

When the aging of the system is not taken into consideration, i. e. , the jump rate $q_{\alpha\beta}(\cdot)$ is independent of $h(t)$, and is a stochastic constant, as in [4], it can be shown that the optimal control policy can be obtained by solving the HJB(Hamilton-Jacobi-Bellman) equation.

2.3 Simplified model for the system

It is very difficult to get analytic solutions of the HJB, and Sethi and Zhang reached significant conclusions in the direction via viscosity solution by assuming the rate of change in machine states approaches infinity. For vital setup cost and duration, once the equipment is set up to produce any type of product, the process will last for a while until its optimal inventory is reached. For any fixed $T>0$, the number of setups in $[0, T]$ is finite. Moreover, it is meaningful to consider the finite horizon in practice. According to the nature of the production system, a new finite horizon is denoted instead of infinite horizon to find the optimal production of the system, and a satisfying solution can be conveniently achieved. Without losing generality, let $s=0$, and P_i denote the initial product being produced. Over the finite horizon $[0, T(\alpha)]$ the objective function can be written as the following

$$\begin{aligned} J(i, X, 0, \Xi, U(\cdot), h) &= E\Big(\int_0^{T(\alpha)} e^{-\rho t} G(X(t), U(t)) dt + \sum_{l=0}^{k} e^{-\rho \tau_l} K_{i_l i_{l+1}}\Big) = \\ & E\Big(\int_0^{T_1(\alpha)} e^{-\rho t} G(X(t), U(t)) dt + \int_{T_1(\alpha)}^{T_1(\alpha)+\theta_1} e^{-\rho t} G(X(t), 0) dt + e^{-\rho T_1(\alpha)} K_1\Big) + \cdots + \\ & E\Big(\int_{\sum_{i=0}^{k-1}\theta_i + \sum_{i=0}^{k-1} T_i(\alpha)}^{\sum_{i=0}^{k-1}\theta_i + \sum_{i=0}^{k} T_i(\alpha)} e^{-\rho t} G(X(t), U(t)) dt + \int_{\sum_{i=0}^{k-1}\theta_i + \sum_{i=0}^{k} T_i(\alpha)}^{\sum_{i=0}^{k}\theta_i + \sum_{i=0}^{k} T_i(\alpha)} e^{-\rho t} G(X(t), 0) dt + e^{-\rho (\sum_{i=0}^{k-1}\theta_i + \sum_{i=0}^{k} T_i(\alpha))} K_i \Big) \end{aligned} \tag{7}$$

where $T_0 = K_0 = \theta_0 = 0$, and $T(\alpha)$ denotes the terminating time when the whole production process ends, and $T_i(\alpha)$ denotes the terminating time when the ith type production process is over, and θ_i denotes the setup time when the $i+1$ production process begins for $i=1, 2, \cdots, k$. Obviously, $T(\alpha)$ is a stochastic variable, and the definition of $T(\alpha)$ is different from the upper horizon of integral t_f in some lectures. And the conclusions in [5] cannot be used directly in this condition. It is obvious that $T(\alpha) \to \infty$ as $k \to \infty$, and the problem is the original problem. According to the above, what is discussed in the following is how to get satisfactory production planning based on hierarchical flow control policy over finite horizon (i. e. , k is a fixed constant).

3 Main results

In this section, the new framework of a hierarchical flow rate control model is discussed. The hierarchy is composed of two levels: the upper level——the static planning level——where the static optimal production of the system is decided without considering its unreliability and setup is treated as a typical controllable event. In the lower level——the dynamic operational level, where real-time processing is made to meet the production expectation with consideration of random breakdown, and repairs are subject to the age of the system.

3.1 The static planning level

Without considering the dynamic properties of the system Equation (7) can be simplified as the following.

$$J(i,X,0,\Xi,U(\cdot)) = \int_0^T e^{-\rho t}G(X(t),U(t))dt + \sum_{l=0}^{k} e^{-\rho\tau_l}K_{i_l i_{l+1}} =$$
$$\int_0^{T_1} e^{-\rho t}G(X(t),U(t))dt + \int_{T_1}^{T_1+\theta_1} e^{-\rho t}G(X(t),0)dt + e^{-\rho T_1}K_1 +$$
$$\int_{T_1+\theta_1}^{T_1+\theta_1+T_2} e^{-\rho t}G(X(t),U(t))dt + \cdots + \int_{\sum_{i=0}^{k-1}\theta_i+\sum_{i=0}^{k-1}T_i}^{\sum_{i=0}^{k-1}\theta_i+\sum_{i=0}^{k}T_i} e^{-\rho t}G(X(t),U(t))dt +$$
$$\int_{\sum_{i=0}^{k-1}\theta_i+\sum_{i=0}^{k}T_i}^{\sum_{i=0}^{k}\theta_i+\sum_{i=0}^{k}T_i} e^{-\rho t}G(X(t),0)dt + e^{-\rho(\sum_{i=0}^{k-1}\theta_i+\sum_{i=0}^{k}T_i)}K_i \tag{8}$$

Since T_i responds to the inventory $X(T_i)$ ($T_i(\alpha)$ is replaced by T_i without considering stochastic dynamic properties of the system), T_i as a new state variable is introduced. Let $N_i=[T_1, T_2, \cdots, T_i]$, $i=1, 2, \cdots, k$, $T_i\in R^+$. Then N_i denotes the production time series before the $i+1$th setup.

Let the optimal decision of Equation(8) be $V_{k-i}[j, X(i), N_i]$ when the initial state is $(j, X(i), N_i)$. Then a Bellman equation can be gotten by dynamic programming:

$$V_{k-i}[j,X(i),N_i] = \min_{u_j(i)}\{J(j,X(i),N_i) + V_{k-(i+1)}[l,X(i+1),N_{i+1}]\} \tag{9}$$

Theorem 1. Let $(\Xi^*, U^*(\cdot))=(\{(\tau_0^*, i_0 i_1), (\tau_1^*, i_1 i_2), \cdots\}, U^*(\cdot))\in\Omega$ denote an optimal control. Then there exists a constant $\eta>0$ such that $\tau_{l+1}^*\geqq\tau_l^*+\theta_{i_l i_{l+1}}+\dfrac{\eta}{1+|X(\tau_l^*)|^{K_g}}$, $l=0, 1, 2, \cdots$.

Proof. The result can be directly deduced from Lemma 4.2 by using $V_i[\cdot,\cdot]$ instead of $J_i(\cdot,\cdot)$ in [4]. □

From Theorem 1, it can be seen that in any optimal policy, there is always some nonzero time for producing the intended product after the completion of each setup, i.e., $T_i(\alpha)>\Delta>0$.

The solution of Equation (9) is the optimal production of the system over the finite horizon $[0,T]$ when unreliability is not considered. And the inventory $x_i(T_1)$ of the initial product being produced is its optimal inventory over $[0,T]$ when $J(\cdot,\cdot)$ reaches the minimum, and the other variable T_1 is its optimal production duration according to the initial condition $X(0)$. Both $x_i(T_1)$ and T_1 will be conveyed to the lower level as expected values.

3.2 The dynamic operational level

Since the optimal setup times, production rate and the optimal inventory have been gotten by the static planning level, and the optimal production duration of the initial product being produced is also determined, this paragraph is focused on how to make the real-time system catch up with the expected value when considering the unreliability and the aging of the system. For using receding algorithm, only given type of product is discussed.

Let x_i^2 and u_i^2 denote the surplus and the production rate of production P_i at the level, respectively. And at this level x_i^2 satisfies the following equation:

$$x_i^2(t) = \int_0^t u_i^2(s)\mathrm{d}s - \int_0^t u_i^1(s)\mathrm{d}s \text{ or } \frac{\mathrm{d}x_i^2}{\mathrm{d}t} = u_i^2 - u_i^1 \tag{10}$$

In the level we focus on the sub-problem that is to seek $u_i^2(t,\alpha)$ to minimize

$$JS(s,x_i^2,u_i^2(\cdot),\alpha,h(\cdot)) = E\left(\int_s^{t_f} g(x_i^2(t))\mathrm{d}t \mid x_i^2(s) = x_{is},\zeta(s) = \alpha\right) \quad \text{s. t. (1), (6)} \tag{11}$$

where s is the initial time of this production of P_i and t_f is the terminal time of this production process. Define a strictly convex function g such that $g(0)=0$; $g(x)\geqslant 0\ \forall x$; and $\lim_{\|x\|\to\infty} g(x)=\infty$; the initial condition is $x^2(s)$and $\zeta(s)$. Let $\Psi(s, x, \alpha, h(\cdot))$ for $x\in R^1$, $\alpha\in E$, $s\geqslant 0$, denote the value function of Equation (11), i. e.,

$$\Psi(s, x, \alpha, h(\cdot)) = \inf_{u(\cdot)\in\Theta} JS(s, x, u(\cdot)\alpha, h(\cdot)) \tag{12}$$

Based on [3], the following hold:

i) Given $\zeta(s)=\alpha$, the probability that there are no jumps of $\zeta(s)$ in the interval $[s, t]$ is

$$\exp\left(\int_s^t q_{\alpha\alpha}(h(\sigma))\mathrm{d}\sigma\right) \tag{13}$$

ii) Given $\zeta(s)=\alpha$, the probability density that the first jump of $\zeta(s)$ after time s is from α to β and occurs at time t is given by

$$q_{\alpha\beta}(h(t)) \exp\left(\int_s^t q_{\alpha\alpha}(h(\sigma))\mathrm{d}\sigma\right) \tag{14}$$

Theorem 2. The function $\Psi(s, x, \alpha, h(\cdot))$ satisfies the integral equation

$$\Psi(s, x, \alpha, h(\cdot)) = \int_s^{t_f} g(x(t))\mathrm{d}t \exp\left(\int_s^{t_f} q_{\alpha\alpha}(h(t))\mathrm{d}t\right) + \sum_{\beta\neq\alpha}\int_s^{t_f} q_{\alpha\beta}(h(\sigma)) \exp\left(\int_s^{\sigma} q_{\alpha\alpha}(h(t))\mathrm{d}t\right)\Psi(\sigma,X(\sigma),\beta,h(\sigma))\mathrm{d}\sigma \tag{15}$$

Proof. Let τ denote the time of the first jump of $\zeta(\cdot)$ after the initial time s. The formula for iterated conditional expectations implies

$$\begin{aligned} JS(s,x,u(\cdot),\alpha,h(\cdot)) &= \mathrm{E}\left(\int_s^{t_f} g(x(t))\mathrm{d}t \mid x(s) = x_s,\zeta(s) = \alpha,h(s) = h_s\right) = \\ &\mathrm{E}\left(\left(\int_s^{\tau} g(x(t))\mathrm{d}t \mid x(s) = x_s,\zeta(s) = \alpha,h(s) = h_s\right) + \right. \\ &\left.\left(\int_\tau^{t_f} g(x(t))\mathrm{d}t \mid x(\tau) = x_\tau,\zeta(\tau) = \beta,h(\tau) = h_\tau\right)\right) \end{aligned} \tag{16}$$

i) When

$$s\leqslant\tau\leqslant t_f, \Psi(s,x(s), u(\cdot),\alpha,h(s)) = \Psi(\tau, x(\tau), u(\cdot), \beta, h(\tau)) \tag{17}$$

ii) When

$$\tau\geqslant t_f, \Psi(s,x(s),u(\cdot), \alpha,h(s)) = \int_s^{t_f} g(x(t))\mathrm{d}t \mid x(s) = x_s,\zeta(\cdot) = \alpha,h(s) = h_s \tag{18}$$

Form Equation (16)and using Equations (17), (18), (13) and (14),Equation (15) follows. □

Notice that $\Psi(s, x(s), u(\cdot)\alpha, h(s))$ is piecewise continuously differentiable in t. Differentiate the value function with respect to s by using Equation (15):

$$\frac{\mathrm{d}\Psi(s,x,\alpha,h(\cdot))}{\mathrm{d}s} = \Psi_s(s,x,\alpha,h(\cdot)) + \Psi_x(s,x,\alpha,h(\cdot))\,\dot{x}(s,\alpha) + \Psi_h(s,x,\alpha,h(\cdot))\,\dot{h}(\cdot)$$

$$= -g(x(s))\exp\left(\int_s^{t_f} q_{\alpha\alpha}(h(t))\mathrm{d}t\right) - \int_s^{t_f} g(x(t_f))\mathrm{d}t \exp\left(\int_s^{t_f} q_{\alpha\alpha}(h(t))\mathrm{d}t\right) q_{\alpha\alpha}(h(s)) - \sum_{\beta\neq\alpha}[q_{\alpha\beta}(h(s))\Psi(s,x(s),\beta,h(s))] \tag{19}$$

$$\sum_{\beta\neq\alpha}\int_s^{t_f} q_{\alpha\beta}(h(\sigma))\exp\left(\int_s^{\sigma} q_{\alpha\alpha}(h(t))\mathrm{d}t\right) q_{\alpha\alpha}(h(s))\Psi(\sigma,X(\sigma),\beta,h(\sigma))\mathrm{d}\sigma = -g(x(s))\exp\left(\int_s^{t_f} q_{\alpha\alpha}(h(t))\mathrm{d}t\right) - \sum_{\beta}[q_{\alpha\beta}(h(s))\Psi(s,x(s),\beta,h(s))] \tag{20}$$

Let $s=t$, and the following theorem can be gotten from Equation (20).

Theorem 3. The value function $\Psi(s, x, \alpha, h(\cdot))$ satisfies the partial differential equations of the system:

$$\Psi_t(s,x,\alpha,h(\cdot)) + \Psi_x(s,x,\alpha,h(\cdot))F_i(\cdot) + \Psi_h(s,x,\alpha,h(\cdot))f(\cdot) + \sum_{\beta} q_{\alpha\beta}(h(t))\Psi(s,x,\beta,h(\cdot)) + g(x(s))\exp\left(\int_s^{t_f} q_{\alpha\alpha}(h(t))\mathrm{d}t\right) = 0 \tag{21}$$

The boundary condition is $\Psi(s, x,\alpha, h(\cdot)) = \int_0^{t_f} g(x(t_f))\mathrm{d}t$.

Based on the above, Theorem 4 gives necessary and sufficient dynamic programming optimality conditions for the production control problem.

Theorem 4. A necessary and sufficient condition for a control $u_i(t)\in\Theta$ to be optimum is that for each $\alpha\in E$ its performance function $\Psi(i,x_i^2,\alpha,h(\cdot)) = \inf_{u_i^2(\cdot)\in\Theta} JS(i,x_i^2,u_i^2(\cdot),\alpha,h(\cdot))$ satisfies the following Hamilton-Jacobi- Bellman equation

$$\min_{u_i^2(\cdot)\in\Theta}\{\Psi_t(0,x_i^2,\alpha,h(\cdot)) + \Psi_{x_i^2}(0,x_i^2,\alpha,h(\cdot))(u_i^2 - proj(u_i^1,2) + \Psi_h(0,x_i^2,\alpha,h(\cdot))f(\cdot) + \sum_{\beta} q_{\alpha\beta}(h(t))\Psi(0,x_i^2,\beta,h(\cdot)) + g(x(s))\exp\left(\int_0^{t_f} q_{\alpha\alpha}(h(t))\mathrm{d}t\right) = 0 \tag{22}$$

where $Proj(u_i^1, 2)$ is the projection of u_i^1 into the space of u_i^2, and $proj(u^1,2)=E_1(u^2)=u^1$, i. e., the expected value is gotten by the upper level.

Proof. The proof is a straightforward modification of the proof in [6]. □

Remark. Theorem 4 extends the results in [3] in which the age function is linear, and the theorem gives the necessary and sufficient condition when the optimal control policy exists according to general conditions. [2] gives the hedging point control policy when the relations between the rate of breakdown and the flow rate are nonlinear, but the optimality of the policy is not shown.

3.3 Optimization on receding horizon

The algorithm is iterated to the upper level automatically when the time T_i is reached, which guarantees the receding algorithm is carried out online in real-time. The $u_i^2(t,\alpha)$ and $x_i(T_i)$ are set values to the operating equipment. When the given process of one type of products is over, the real-time inventory $X(T_i)$ is recorded as an initial condition instead of the old data for the next scheduling. Since inventory control is considered, if the optimal inventory is not gotten, then the process will last until the inventory emerges.

4 Simulation of examples

The performance of the hierarchical policy is shown with examples including following specifications: $n=2$, and $\rho=0.9$, $z_1=z_2=0.4$, $\alpha\in E=\{0,1\}$. The other parameter is shown in Table 1 (q_{10} is the rate of jump from the operational state to breakdown state and q_{01} the reverse when the rates are independent of the aging of the system). It is assumed that f is linear. For various initial conditions $X(0)$, the optimal production controls are listed in Table 2. Only optimal production durations of the initial product being produced are listed. Simulation shows that the optimal production control policy is of region switching structure and of hedging point policy.

Table 1 Parameters of the system

θ_{12}	θ_{21}	K_{12}	K_{21}	q_{10}	q_{01}
0.65	0.75	1.25	1.15	0.1	0.2

Table 2 Results of simulation

Ex.	$x_1(0)$	$x_2(0)$	C_1^+	C_1^-	C_2^+	C_2^-	T_1	min $J(\cdot,\cdot)$	T_1^0	min $J^0(\cdot,\cdot)$
1	−2.5	−2.0	0.5	3.0	0.6	3.0	1.40	14.2745	1.65	14.1297
2	−1.5	0.0	0.5	3.0	1.0	3.0	3.20	4.5565	2.95	4.4251
3	−2.5	1.5	0.5	3.0	1.0	3.0	5.40	7.5554	5.20	7.3909
4	0.0	0.0	1.0	· 3.0	1.0	3.0	0.90	1.7482	0.85	1.7738
5	2.0	2.5	1.0	3.0	1.0	3.0	0.0	5.1151	0.0	5.1182

The trend of the objective function $J(\cdot,\cdot)$ changing with T_1 is illustrated in Fig. 1(a) as T_i is optimal, $i=2, 3, \cdots, k$. And T_1 is the optimal production duration when $J(\cdot,\cdot)$ is its minimum in Fig. 1(a). Five curves in Fig. 1(a) agree with those five examples in Table 2. The simulation results show that different initial conditions respond to different optimal production durations of the products. In Examples 1, 2, and 4 (the sold lines in Fig. 1(a)), since product P_2 is not sufficient, sometimes even deficient, the policy shortens the optimal production duration of P_1, which is different from Example 3 (the dotted line). In Example 3, P_2 is sufficient, which prolongs the optimal production duration of P_1, but each product is sufficient in Example 5 (the dashed line), which makes the system produce nothing. The results also agree with the hedging point policy and the results in [2].

For a finite time, the hierarchical policy not only keeps the system run at the least cost but makes the production perfectly meet the demand. Moreover the policy makes the production satisfy the customers in sum and balances all types of the products, keeping the inventory in a low level.

T_1^0, min$J^0(\cdot,\cdot)$ are the results without considering the factor of the system aging in Table 2. Compared with the results above, it is shown that the optimal production duration when considering the aging of the system is longer than that when not considering the aging of the system. The equipment is unreliable, and it can produce nothing under breakdown. Consequently, when it is up, its production rate must be accelerated. However, there is a constrain to the production capacity of the system, which prolongs the production duration to ensure the optimal inventory. This agrees with the fact. To Example 2, the trend of $J(\cdot,\cdot)$ changing with T_1 considering the aging of the system (the solid line) or not (the dotted line) is illustrated in Fig. 1(b).

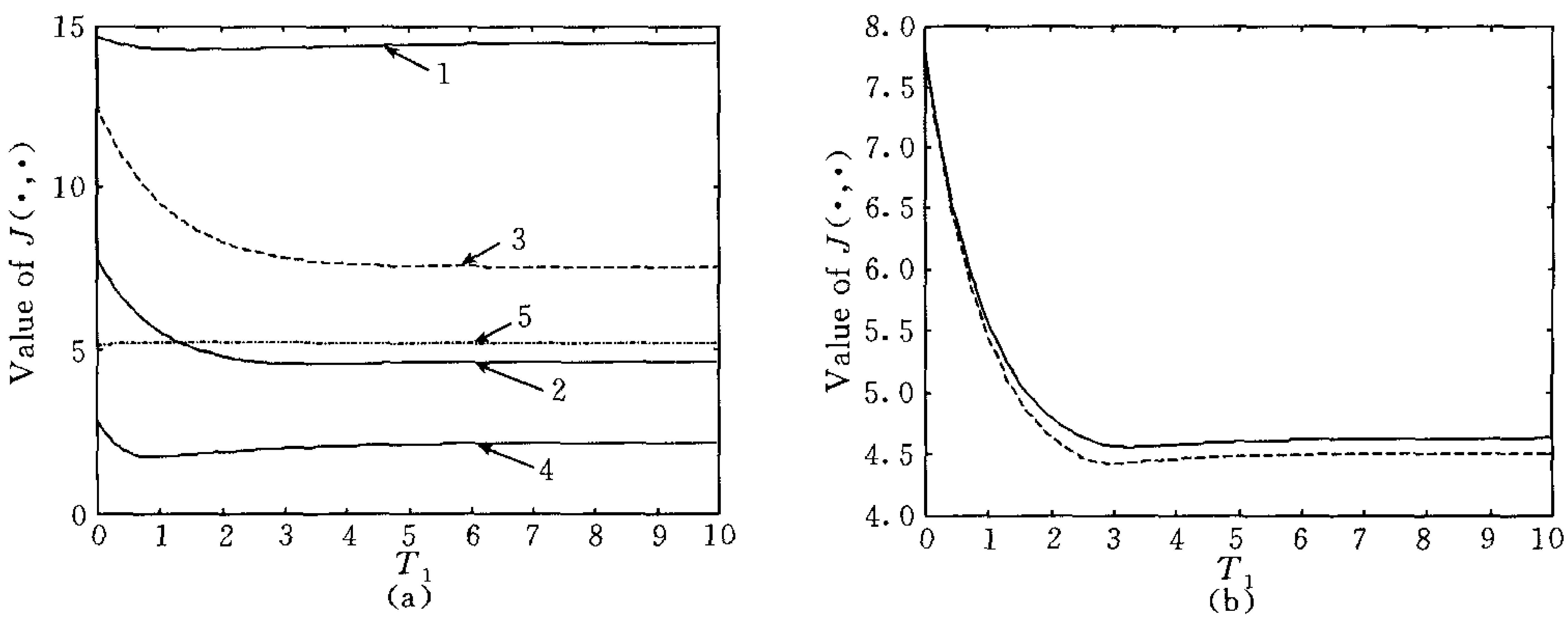

Fig. 1 Tendency of value $J(\cdot,\cdot)$ to T_1

5 Conclusions

In the paper, the proposed policy decreases the complexities of the original problem, i. e. , reduces the stochastic optimal production control problem of multi-dimension vector to the determinist optimal production control problem, and approximates to the stochastic problem by sliding in one dimension, which makes the receding algorithm feasible, more accurate and real-time. The decomposition and definition are feasible and meaningful in practice. The age function of the equipment makes the objective close to the reality. However, the optimized solution is not globally optimal, but an approximate solution. And the receding control policy can decrease the drawback. Our future work is how to get the optimal policy when products are perishable or may become obsolete and when systems have storage-space competition.

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SONG Chun-Yue Postdoctor of Zhejiang University. His research areas are the optimal control of production systems, the predictive control, modeling and optimization of hybrid systems.

WANG Hui Professor of Zhejiang University. Her research areas are modeling, control and optimization of industry processes.

LI Ping Professor. Received his Ph. D. degree from Zhejiang University in 1988. His research interests include complex industrial process modeling, intelligent control and integrated industrial control and management systems.

考虑工业设备老化时的递阶最优生产控制策略

宋春跃[1] 王 慧[2] 李 平[1]

[1](浙江大学工业控制技术研究所 杭州 310027)

[2](浙江大学系统工程研究所 杭州 310027)

(E-mail: cysong@iipc.zju.edu.cn)

摘 要 就切换并不太频繁的不可靠生产系统的最优生产控制进行研究，且系统的故障率与其老化程度有关. 以寻求最优的生产控制使其折扣率目标泛函取最小值. 根据系统自身特性，对目标泛函的积分上限给予新的含义，引入新的状态变量对问题进行分解，建立了考虑 setup 时间及成本的递阶流率控制结构框架，在最优生产时间标准下给出考虑设备故障情况下有限时域内的动态规划时最优控制率存在的充分必要条件. 仿真结果表明，该生产策略更易于工程实现.

关键词 递阶流率控制，最优生产，动态规划，设备老化

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