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# Controller Design and Stability Analysis for Fuzzy Singularly Perturbed Systems[1)]

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**Abstract** This paper defines a fuzzy singularly perturbed system by extending the ordinary Takagi-Sugeno fuzzy model. Stability conditions for the fuzzy singularly perturbed systems for all small enough values of a singular perturbation parameter are derived and represented in terms of a set of matrix inequalities. By the proposed two-stage procedure, the stable parallel distributed compensation (PDC) feedback gains and a common Lyapunov function can be found. The outcome of the stabilization problem is recast into linear matrix inequalities(LMIs) and bilinear matrix inequalities (BMIs) in each stage respectively. The resulting BMIs can be effectively solved by the proposed iterative linear matrix inequality approach. Furthermore, numerical examples and simulation results are given to verify the effectiveness of the algorithms proposed above.

**Key words** Fuzzy systems, linear matrix inequalities, parallel distributed compensation, singular perturbation

## 1 Introduction

Fuzzy models based fuzzy control system design methods, especially the parallel distributed compensation(PDC)[1] approach, have appeared in the fuzzy control fields for several decades. These methods are conceptually simple and straightforward. Linear feedback control techniques can be utilized as in the case of the feedback stabilization.

Compared to the rapid development of fuzzy control and neural control for nonlinear systems, there are relatively few in the literature focusing on the intelligent control for nonlinear singularly perturbed systems[2]. [3] proposed a multilayer neural network approach for stabilization via integral manifold theory. [4] proposed a T-S fuzzy singularly perturbed controller, but no any analytic results were given, and the stability was not investigated. The intelligent control problem for nonlinear singularly perturbed remains an open problem. On the other hand, utilizing linear matrix inequalities[5] to analyze and synthesize singularly perturbed systems has become a very interesting field[6,7].

In this paper, a new type of T-S fuzzy model is defined, of which consequent parts are represented by dynamic linear singularly perturbed models. This new fuzzy model, termed as fuzzy singularly perturbed systems, can be used to approximate nonlinear singularly perturbed systems on a compact set. Stability of this system is analyzed and the stabilization approach is proposed based on a set of matrix inequalities. Since the resulting matrix inequalities are not jointly affirm to the unknown variables, an iterative algorithm is utilized, where during any step, only some linear matrix inequalities need to solve. Finally, the convergence of the iterative algorithm can be guaranteed.

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## 2 Description of fuzzy singularly perturbed systems

The ordinary T-S fuzzy model has been investigated by a lot of researchers[8]. Here, we consider the following fuzzy singularly perturbed model to represent a nonlinear singularly perturbed system, which includes both local analytic linear singularly perturbed models and fuzzy membership functions. The $i$th rule of the fuzzy singularly perturbed model is of the following form:

Plant Rule $i$:

IF $y_1(t)$ is $F_{i1}$ and … and $y_g(t)$ is $F_{ig}$

THEN

$$\begin{cases} \dot{\boldsymbol{x}} = A_{11}^i \boldsymbol{x} + A_{12}^i \boldsymbol{z} + B_1^i \boldsymbol{u} \\ \varepsilon \dot{\boldsymbol{z}} = A_{21}^i \boldsymbol{x} + A_{22}^i \boldsymbol{z} + B_2^i \boldsymbol{u} \end{cases} \quad i = 1, 2, \cdots, r \tag{1}$$

where $F_{ij}$ $(j=1,2,\cdots,g)$ are fuzzy sets, $\boldsymbol{x}(t) \in R^n, \boldsymbol{z}(t) \in R^m, \boldsymbol{u}(t) \in R^p, A_{11}^i, A_{12}^i, A_{21}^i, A_{22}^i$, $B_1^i, B_2^i$ are matrices with approriate dimensions. $y_1(t), \cdots, y_g(t)$ are some measurable system variables.

Given a pair $[\boldsymbol{x}(t), \boldsymbol{z}(t); \boldsymbol{u}(t)]$, by using a standard fuzzy inference method——that is, using a singleton fuzzifier, product fuzzy inference and weighted average defuzzifier——the final state of the fuzzy system is inferred as follows:

$$E_\varepsilon \cdot \begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \sum_{i=1}^{r} \mu_i [\boldsymbol{y}(t)] \left\{ A^i \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + B^i \boldsymbol{u} \right\} \tag{2}$$

where
$$\mu_i(\boldsymbol{y}(t)) = \frac{w_i(\boldsymbol{y}(t))}{\sum_{i=1}^{r} w_i(\boldsymbol{y}(t))}, w_i(\boldsymbol{y}(t)) = \prod_{j=1}^{g} F_{ij}(y_j(t)),$$

$$E_\varepsilon = \begin{bmatrix} I_{n\times n} & \\ & \varepsilon \cdot I_{m\times m} \end{bmatrix}, \quad A^i = \begin{bmatrix} A_{11}^i & A_{12}^i \\ A_{21}^i & A_{22}^i \end{bmatrix}, \quad B^i = \begin{bmatrix} B_1^i \\ B_2^i \end{bmatrix}$$

For the convenience of notation, let $\mu_i = \mu_i(\boldsymbol{y}(t))$, $\boldsymbol{\theta}^{\mathrm{T}} = [\boldsymbol{x}^{\mathrm{T}}, \boldsymbol{z}^{\mathrm{T}}]^{\mathrm{T}}$

**Assumption 1.** For all $s$ with $\mathrm{Re}[s] \geqslant 0$, rank $\begin{bmatrix} sI_n - A_{11}^i & -A_{12}^i & B_1^i \\ -A_{21}^i & -A_{22}^i & B_2^i \end{bmatrix} = n+m$.

**Assumption 2.** $(A_{22}^i, B_2^i)$ is stabilizable.

**Theorem 1.** The fuzzy system (2) is asymptotically stable if there exists a common matrix $P_\varepsilon$ such that

$$\begin{aligned} &E_\varepsilon P_\varepsilon = P_\varepsilon^{\mathrm{T}} E_\varepsilon > 0 \\ &(A^i)^{\mathrm{T}} P_\varepsilon + (P_\varepsilon)^{\mathrm{T}} A^i < 0 \quad (i = 1, 2, \cdots, r) \end{aligned} \tag{3}$$

**Proof.** Select the Lyapunov function as $V(\boldsymbol{\theta}) = \boldsymbol{\theta}^{\mathrm{T}} E_\varepsilon P_\varepsilon \boldsymbol{\theta}$. Then the derivative of $V(\boldsymbol{\theta})$ with respect to $t$ is

$$\frac{\mathrm{d}}{\mathrm{d}t} V(\boldsymbol{\theta}) = \sum_{i=1}^{r} \mu_i \cdot \boldsymbol{\theta}^{\mathrm{T}} \cdot [(A^i)^{\mathrm{T}} P_\varepsilon + (P_\varepsilon)^{\mathrm{T}} A^i] \cdot \boldsymbol{\theta}$$

From(3) we can get $\frac{\mathrm{d}}{\mathrm{d}t} V(\boldsymbol{\theta}) < 0$, so system (2) is asymptotically stable. □

## 3 The design of fuzzy controller

### 3.1 Fuzzy controller

The parallel distributed compensation (PDC) controller is as follows.

Controller Rule $i$:

IF $y_1(t)$ is $F_{i1}$ and … and $y_g(t)$ is $F_{ig}$

THEN

$$\boldsymbol{u} = K_1^i \boldsymbol{x}(t) + K_2^i \boldsymbol{z}(t) \tag{4}$$

Then the resulting closed-loop system is

$$E_\varepsilon \cdot \begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j \left\{ \begin{bmatrix} A_{11}^i + B_1^i K_1^j & A_{12}^i + B_1^i K_2^j \\ A_{21}^i + B_2^i K_1^j & A_{22}^i + B_2^i K_2^j \end{bmatrix} \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \right\} \tag{5}$$

**Theorem 2.** If there exists common matrices $P_{11}, P_{21}, P_{22}$, and $K_1^i, K_2^i$ with compatible dimensions such that

$$P_{11} = P_{11}^{\mathrm{T}} > 0, \quad P_{22} = P_{22}^{\mathrm{T}} > 0$$
$$\Pi_{ij} + \Pi_{ji} < 0, \quad i \leqslant j \tag{6}$$

where

$$\Pi_{ij} = \begin{bmatrix} (A_{11}^i + B_1^i K_1^j)^{\mathrm{T}} P_{11} + P_{11}(A_{11}^i + B_1^i K_1^j) + (A_{21}^i + B_2^i K_1^j)^{\mathrm{T}} P_{21} + P_{21}^{\mathrm{T}}(A_{21}^i + B_2^i K_1^j) & (A_{21}^i + B_2^i K_1^j)^{\mathrm{T}} P_{22} + P_{11}(A_{12}^i + B_1^i K_2^j) + P_{21}^{\mathrm{T}}(A_{22}^i + B_2^i K_2^j) \\ * & (A_{22}^i + B_2^i K_2^j)^{\mathrm{T}} P_{22} + P_{22}(A_{22}^i + B_2^i K_2^j) \end{bmatrix}$$

then $\exists \varepsilon^* > 0$, $\forall \varepsilon \in (0, \varepsilon^*]$, the closed-loop systems (5) is asymptotically stable.

**Proof.** Let $P_\varepsilon = \begin{bmatrix} P_{11} & \varepsilon P_{21}^{\mathrm{T}} \\ P_{21} & P_{22} \end{bmatrix}$. Then $E_\varepsilon P_\varepsilon = \begin{bmatrix} P_{11} & \varepsilon P_{21}^{\mathrm{T}} \\ \varepsilon P_{21} & \varepsilon P_{22} \end{bmatrix}$, since $P_{11} > 0, P_{22} > 0$, $\exists \varepsilon_0^* > 0$, $\forall \varepsilon \in (0, \varepsilon_0^*]$, $E_\varepsilon P_\varepsilon = P_\varepsilon^{\mathrm{T}} E_\varepsilon > 0$. Select the Lyapunov function as: $V = \boldsymbol{\theta}^{\mathrm{T}} E_\varepsilon P_\varepsilon \boldsymbol{\theta}$. Then the derivative of it with respect to $t$ is

$$\frac{\mathrm{d}}{\mathrm{d}t}V(\boldsymbol{\theta}) = \dot{\boldsymbol{\theta}}^{\mathrm{T}} E_\varepsilon P_\varepsilon \boldsymbol{\theta} + \boldsymbol{\theta}^{\mathrm{T}} P_\varepsilon^{\mathrm{T}} E_\varepsilon \dot{\boldsymbol{\theta}} = [\dot{x}^{\mathrm{T}} \quad \varepsilon \dot{z}^{\mathrm{T}}] P_\varepsilon \begin{bmatrix} x \\ z \end{bmatrix} + [x^{\mathrm{T}} \quad z^{\mathrm{T}}] P_\varepsilon^{\mathrm{T}} \begin{bmatrix} \dot{x} \\ \varepsilon \dot{z} \end{bmatrix} =$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j \cdot \left\{ [x^{\mathrm{T}} \quad z^{\mathrm{T}}] \cdot \Pi_{ij}^\varepsilon \cdot \begin{bmatrix} x \\ z \end{bmatrix} \right\} = \boldsymbol{\theta}^{\mathrm{T}} \cdot \left\{ \sum_{i=1}^{r} \mu_i^2 \cdot \Pi_{ii}^\varepsilon + \sum_{i,j=1,\ i<j}^{r} \mu_i\mu_j \cdot (\Pi_{ij}^\varepsilon + \Pi_{ji}^\varepsilon) \right\} \cdot \boldsymbol{\theta}$$

where

$$\Pi_{ij}^\varepsilon = \begin{bmatrix} (A_{11}^i + B_1^i K_1^j)^{\mathrm{T}} P_{11} + P_{11}(A_{11}^i + B_1^i K_1^j) + (A_{21}^i + B_2^i K_1^j)^{\mathrm{T}} P_{21} + P_{21}^{\mathrm{T}}(A_{21}^i + B_2^i K_1^j) & (A_{21}^i + B_2^i K_1^j)^{\mathrm{T}} P_{22} + P_{11}(A_{12}^i + B_1^i K_2^j) + P_{21}^{\mathrm{T}}(A_{22}^i + B_2^i K_2^j) + \varepsilon (A_{11}^i + B_1^i K_1^j)^{\mathrm{T}} P_{21}^{\mathrm{T}} \\ * & (A_{22}^i + B_2^i K_2^j)^{\mathrm{T}} P_{22} + P_{22}(A_{22}^i + B_2^i K_2^j) + \varepsilon (A_{12}^i + B_1^i K_2^j)^{\mathrm{T}} P_{21}^{\mathrm{T}} + \varepsilon P_{21}(A_{22}^i + B_2^i K_2^j) \end{bmatrix}$$

It is obvious that $\Pi_{ij}^\varepsilon = \Pi_{ij} + \begin{bmatrix} 0 & O(\varepsilon) \\ O(\varepsilon) & O(\varepsilon) \end{bmatrix} = \Pi_{ij} + O(\varepsilon)$. Since $\Pi_{ij} < 0$, we have $\exists \varepsilon_{ij}^* > 0$, $\forall \varepsilon \in (0, \varepsilon_{ij}^*]$, such that $\Pi_{ij}^\varepsilon < 0$. Let $\varepsilon^* = \min\{\min_{i,j}(\varepsilon_{ij}^*), \varepsilon_0^*\}$. Then for $\varepsilon \in (0, \varepsilon^*]$, $\dot{V} < 0$, so the close-loop system remains asymptotically stable. □.

### 3.2 The two-stage design procedure

Note that (6) implies that

$$(A_{22}^i + B_2^i K_2^j)^{\mathrm{T}} P_{22} + P_{22}(A_{22}^i + B_2^i K_2^j) + (A_{22}^j + B_2^j K_2^i)^{\mathrm{T}} P_{22} + P_{22}(A_{22}^j + B_2^j K_2^i) < 0 \tag{7}$$

The main idea of the two-stage approach is to obtain $P_{22}, K_2^i$ from (7) firstly, then $P_{11}, P_{21}$ and $K_1^i$ can be obtained from (6) where $P_{22}, K_2^i$ are known. During the first stage, i.e., to obtain $P_{22}$ and $K_2^i$, the traditional techniques such as in [1] can be utilized to transform (7) into standard LMI. The second stage can be formulated as follows.

Though $P_{22}$ and $K_2^i$ have been determined, (6) is a set of BMI with unknown variables $P_{11}, P_{21}$ and $K_1^i$, and an iterative algorithm will be proposed to solve it.

For notation convenience, let $\Pi_{ij} + \Pi_{ji} = \begin{bmatrix} \Sigma_{ij} & \Delta_{ij} \\ * & \Xi_{ij} \end{bmatrix}$

where

$$\Sigma_{ij} = (A_{11}^i + B_1^i K_1^j)^{\mathrm{T}} P_{11} + P_{11}(A_{11}^i + B_1^i K_1^j) + (A_{21}^i + B_2^i K_1^j)^{\mathrm{T}} P_{21} + P_{21}^{\mathrm{T}}(A_{21}^i + B_2^i K_1^j) + (A_{11}^j + B_1^j K_1^i)^{\mathrm{T}} P_{11} + P_{11}(A_{11}^j + B_1^j K_1^i) + (A_{21}^j + B_2^j K_1^i)^{\mathrm{T}} P_{21} + P_{21}^{\mathrm{T}}(A_{21}^j + B_2^j K_1^i)$$

$$\Delta_{ij} = (A_{21}^i + B_2^i K_1^j)^{\mathrm{T}} P_{22} + P_{11}(A_{12}^i + B_1^i K_2^j) + P_{21}^{\mathrm{T}}(A_{22}^i + B_2^i K_2^j) + (A_{21}^j + B_2^j K_1^i)^{\mathrm{T}} P_{22} + P_{11}(A_{12}^j + B_1^j K_2^i) + P_{21}^{\mathrm{T}}(A_{22}^j + B_2^j K_2^i)$$

$\Xi_{ij} = (A_{22}^i + B_2^i K_2^j)^T P_{22} + P_{22}(A_{22}^i + B_2^i K_2^j) + (A_{22}^j + B_2^j K_2^i)^T P_{22} + P_{22}(A_{22}^j + B_2^j K_2^i)$

It can be shown that $\Delta_{ij}$ is linear to the unknown variables, and $\Xi_{ij}$ is a known term. The following theorem will form the basis of the iterative algorithm.

**Theorem 3.** There exist matrices $P_{11}, P_{21}$ and $K_1^i$ with compatible dimensions satisfying the matrix inequalities(6) if and only if there exist matrices $P_{110}$, $P_{210}$, $K_{10}^i$ and $P_{11}, P_{21}$, $K_1^i$ with compatible dimensions satisfying:

$$P_{11} = P_{11}^T > 0, \quad P_{110} = P_{110}^T > 0$$

$$\begin{bmatrix} \Omega_{ij} & P_{11}B_1^i + (K_1^j)^T & P_{11}B_1^j + (K_1^i)^T & P_{21}^T B_2^i + (K_1^j)^T & P_{21}^T B_2^j + (K_1^i)^T & \Delta_{ij} \\ & -I & & & & \\ & & -I & & & \\ & & & -I & & \\ & & & & -I & \\ & & & & & \Xi_{ij} \end{bmatrix} < 0 \quad (8)$$

where

$$\begin{aligned}\Omega_{ij} = {} & (A_{11}^i)^T P_{11} + P_{11}A_{11}^i + (A_{21}^i)^T P_{21} + P_{21}^T A_{21}^i + (A_{11}^j)^T P_{11} + P_{11}A_{11}^j + (A_{21}^j)^T P_{21} + \\ & P_{21}^T A_{21}^j - P_{11}B_1^i(B_1^i)^T P_{110} - P_{110}B_1^i(B_1^i)^T P_{11} - P_{11}B_1^j(B_1^j)^T P_{110} - P_{110}B_1^j(B_1^j)^T P_{11} - \\ & 2(K_1^i)^T K_{10}^i - 2(K_{10}^i)^T(K_1^i) - P_{21}B_2^i(B_2^i)^T P_{210} - P_{210}^T B_2^i(B_2^i)^T P_{21} - P_{21}B_2^j(B_2^j)^T P_{210} - \\ & P_{210}^T B_2^j(B_2^j)^T P_{21} - 2(K_1^j)^T K_{10}^j - 2(K_{10}^j)^T(K_1^j) + P_{110}B_1^i(B_1^i)^T P_{110} + P_{110}B_1^j(B_1^j)^T P_{110} + \\ & P_{210}^T B_2^i(B_2^i)^T P_{210} + P_{210}^T B_2^j(B_2^j)^T P_{210} + 2(K_{10}^i)^T(K_{10}^i) + 2(K_{10}^j)^T(K_{10}^j)\end{aligned}$$

**Proof**(Sufficiency). From Schur Complements, (8) is equivalent to $\begin{bmatrix} X_{ij} & \Delta_{ij} \\ * & \Xi_{ij} \end{bmatrix} < 0$

where

$$\begin{aligned}X_{ij} = {} & \Omega_{ij} + (P_{11}B_1^i + (K_1^j)^T)(P_{11}B_1^i + (K_1^j)^T)^T + (P_{11}B_1^j + (K_1^i)^T)(P_{11}B_1^j + (K_1^i)^T)^T + \\ & (P_{21}^T B_2^i + (K_1^j)^T)(P_{21}^T B_2^i + (K_1^j)^T)^T + (P_{21}^T B_2^j + (K_1^i)^T)(P_{21}^T B_2^j + (K_1^i)^T)^T = \\ & \Sigma_{ij} + (P_{11} - P_{110})B_1^i(B_1^i)^T(P_{11} - P_{110}) + (P_{11} - P_{110})B_1^j(B_1^j)^T(P_{11} - P_{110}) + \\ & (P_{21} - P_{210})^T B_2^i(B_2^i)^T(P_{21} - P_{210}) + (P_{21} - P_{210})^T B_2^j(B_2^j)^T(P_{21} - P_{210}) + \\ & 2(K_1^i - K_{10}^i)^T(K_1^i - K_{10}^i) + 2(K_1^j - K_{10}^j)^T(K_1^j - K_{10}^j)\end{aligned}$$

So,

$$\begin{aligned}& \Sigma_{ij} + (P_{11} - P_{110})B_1^i(B_1^i)^T(P_{11} - P_{110}) + (P_{11} - P_{110})B_1^j(B_1^j)^T(P_{11} - P_{110}) + \\ & (P_{21} - P_{210})^T B_2^i(B_2^i)^T(P_{21} - P_{210}) + (P_{21} - P_{210})^T B_2^j(B_2^j)^T(P_{21} - P_{210}) + \\ & 2(K_1^i - K_{10}^i)^T(K_1^i - K_{10}^i) + 2(K_1^j - K_{10}^j)^T(K_1^j - K_{10}^j) + \Delta_{ij}(\Xi_{ij})^{-1}(\Delta_{ij})^T < 0\end{aligned}$$

Then, $\Sigma_{ij} + \Delta_{ij}(\Xi_{ij})^{-1}(\Delta_{ij})^T < 0$, since $\Xi_{ij} < 0$, that is, $\Pi_{ij} + \Pi_{ji} = \begin{bmatrix} \Sigma_{ij} & \Delta_{ij} \\ * & \Xi_{ij} \end{bmatrix} < 0$(Necessity)

From $\Pi_{ij} + \Pi_{ji} = \begin{bmatrix} \Sigma_{ij} & \Delta_{ij} \\ * & \Xi_{ij} \end{bmatrix} < 0$, we have $\exists \mu_1 > 0, \mu_2 > 0, \mu_3 > 0$ such that $\begin{bmatrix} \Sigma_{ij} & \Delta_{ij} \\ * & \Xi_{ij} \end{bmatrix} +$ $(\mu_1 + \mu_2 + \mu_3) \cdot I < 0$, i. e., $\Sigma_{ij} + \Delta_{ij}(\Xi_{ij})^{-1}(\Delta_{ij})^T + (\mu_1 + \mu_2 + \mu_3) \cdot I < 0$

Select $\Gamma_1$ and $\Gamma_2$ such that $\Gamma_1(\Gamma_1)^T = \max\limits_{i,j}\{B_1^i(B_1^i)^T + B_1^j(B_1^j)^T\}$ and $\Gamma_2(\Gamma_2)^T = \max\limits_{i,j}\{B_2^i(B_2^i)^T + B_2^j(B_2^j)^T\}$

Let $P_{110} = P_{11} - \sqrt{\mu_1} \cdot [\Gamma_1(\Gamma_1)^T]^{-1/2}$, $P_{210} = P_{21} - \sqrt{\mu_2} \cdot [(\Gamma_2)^T \Gamma_2]^{-1}(\Gamma_2)^T$, and $K_{10}^i = K_1^i - \frac{1}{\sqrt{2}}\sqrt{\mu_3} I$

Then, $(P_{11} - P_{110}) \cdot [\Gamma_1(\Gamma_1)^T] \cdot (P_{11} - P_{110}) = \mu_1 \cdot [\Gamma_1(\Gamma_1)^T]^{-1/2}[\Gamma_1(\Gamma_1)^T][\Gamma_1(\Gamma_1)^T]^{-1/2} = \mu_1 \cdot I$

Similarly, $(P_{21} - P_{210})^T \cdot [\Gamma_2(\Gamma_2)^T] \cdot (P_{21} - P_{210}) = \mu_2 \cdot I$, $(K_1^i - K_{10}^i)^T(K_1^i - K_{10}^i) = \frac{1}{2}\mu_3 I$

Therefore,

$$\begin{aligned}& \Sigma_{ij} + (P_{11} - P_{110})B_1^i(B_1^i)^T(P_{11} - P_{110}) + (P_{11} - P_{110})B_1^j(B_1^j)^T(P_{11} - P_{110}) + \\ & (P_{21} - P_{210})^T B_2^i(B_2^i)^T(P_{21} - P_{210}) + (P_{21} - P_{210})^T B_2^j(B_2^j)^T(P_{21} - P_{210}) +\end{aligned}$$

$$(K_1^i - K_{10}^i)^{\mathrm{T}}(K_1^i - K_{10}^i) + (K_1^j - K_{10}^j)^{\mathrm{T}}(K_1^j - K_{10}^j) + \Delta_{ij}(\Xi_{ij})^{-1}(\Delta_{ij})^{\mathrm{T}} \leqslant$$
$$\Sigma_{ij} + (P_{11} - P_{110})\cdot[\Gamma_1(\Gamma_1)^{\mathrm{T}}]\cdot(P_{11} - P_{110}) + (P_{21} - P_{210})^{\mathrm{T}}\cdot[\Gamma_2(\Gamma_2)^{\mathrm{T}}]\cdot(P_{21} - P_{210}) +$$
$$(K_1^i - K_{10}^i)^{\mathrm{T}}(K_1^i - K_{10}^i) + (K_1^j - K_{10}^j)^{\mathrm{T}}(K_1^j - K_{10}^j) + \Delta_{ij}(\Xi_{ij})^{-1}(\Delta_{ij})^{\mathrm{T}} =$$
$$\Sigma_{ij} + \mu_1 I + \mu_2 I + \mu_3 I + \Delta_{ij}(\Xi_{ij})^{-1}(\Delta_{ij})^{\mathrm{T}} < 0$$

that is, $X_{ij} + \Delta_{ij}(\Xi_{ij})^{-1}(\Delta_{ij})^{\mathrm{T}} < 0$

Since $\Xi_{ij} < 0$, we can get $\begin{bmatrix} X_{ij} & \Delta_{ij} \\ * & \Xi_{ij} \end{bmatrix} < 0$ from Schur Complements. That completes the proof. □

Then the iterative algorithm can be described as.

Step 1. Initialization.

Select $Q > 0$, and solve the generalized Riccati equation(see [9] for the detailed algorithm)

$$A^{\mathrm{T}}P_0 + P_0^{\mathrm{T}}A - P_0^{\mathrm{T}}BB^{\mathrm{T}}P_0 + Q = 0, \quad E^{\mathrm{T}}P_0 = P_0^{\mathrm{T}}E$$

where
$$A = \frac{1}{r}\sum_{i=1}^{r}A^i \text{ and } B = \frac{1}{r}\sum_{i=1}^{r}B^i[10]$$

Then $P_{110}$ and $P_{210}$ can be obtained. $K_{10}^i$ can be arbitrarily selected, e. g., we can determine it as follows: select $K_0^i = -(B^i)^{\mathrm{T}}P_0$ where $P_0 = \begin{bmatrix} P_{110} & 0 \\ P_{210} & P_{220} \end{bmatrix}$, and let $K_{10}^i = K_0^i \cdot \begin{bmatrix} I_{n\times n} \\ 0_{m\times n} \end{bmatrix}$.

Step 2. Solve the following generalized eigenvalue problem (GEVP) in $P_{11}$, $P_{21}$, $K_1^i$ using the auxiliary variables $P_{110}$, $P_{210}$, $K_{10}^i$ determined in the previous step.

min $\alpha$

$$\text{s. t. } P_{11} = P_{11}^{\mathrm{T}} > 0 \tag{9}$$

$$\begin{bmatrix} \Omega_{ij} - \alpha P_{11} & P_{11}B_1^i + (K_1^j)^{\mathrm{T}} & P_{11}B_1^j + (K_1^i)^{\mathrm{T}} & P_{21}^{\mathrm{T}}B_2^i + (K_1^j)^{\mathrm{T}} & P_{21}^{\mathrm{T}}B_2^j + (K_1^i)^{\mathrm{T}} & \Delta_{ij} \\ & -I & & & & \\ & & -I & & & \\ & & & -I & & \\ & & & & -I & \\ & & & & & \Xi_{ij} \end{bmatrix} < 0$$

$$i \leqslant j, \quad i = 1, 2, \cdots, r, \quad j = 1, 2, \cdots, r \tag{10}$$

Assume the optimal value is $\alpha^t$.

Step 3. If $\alpha^t < 0$, then $P_{11}, P_{21}, K_1^i$ are feasible solutions and stop. Otherwise, next step.

Step 4. Solve the following optimal problem (OP) in $P_{11}$, $P_{21}$, $K_1^i$ using $\alpha^t$ determined in Step 2 and the auxiliary variables $P_{110}$, $P_{210}$, $K_{10}^i$.

min $trace(P_{11})$

s. t. (9), (10), where $\alpha$ is replaced by $\alpha^t$

Step 5. if $\| P_{11} - P_{110} \| > \delta$, where $\delta$ is a pre-determined tolerance, $\| \cdot \|$ is Frobenius norm, then set $P_{110} = P_{11}$, $P_{210} = P_{21}$, $K_{10}^i = K_1^i$, $t = t+1$, and go to Step 2, else, the problem may not be solved by this approach, Stop.

**Remark 1.** The term $\alpha P_{11}$ is involved in (10) to obtain the less conservative results. The LMI optimization in Step2 is a generalized eigenvalue problem (GEVP) and the algorithm can guarantee that the solution sequence $\{\alpha^t\}$ is a decreasing sequence[11].

**Remark 2.** The controller obtained by the above algorithm also solves the stabilization problem for the corresponding fuzzy descriptor systems $\begin{bmatrix} \dot{\boldsymbol{x}} \\ 0 \end{bmatrix} = \sum_{i=1}^{r}\mu_i[\boldsymbol{y}(t)]\left\{A^i\begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{z} \end{bmatrix} + B^i\boldsymbol{u}\right\}$ proposed in [14].

## 4 Simulation Examples

The considered plant is of two fuzzy rules with

$$A_{11}^1=\begin{bmatrix}-0.95 & -0.68\\ 1.478 & 0\end{bmatrix},\quad A_{12}^1=\begin{bmatrix}-0.92 & 0.11\\ 0 & 0\end{bmatrix},$$

$$A_{21}^1=\begin{bmatrix}0.2 & 0.4\\ 0.14 & 0.5\end{bmatrix},\quad A_{22}^1=\begin{bmatrix}0.68 & 0.428\\ -2.103 & -0.215\end{bmatrix},\quad B_1^1=\begin{bmatrix}0\\0\end{bmatrix},\quad B_2^1=\begin{bmatrix}0\\1\end{bmatrix}$$

$$A_{11}^2=\begin{bmatrix}-1.24 & 0.12\\ 0.511 & 0\end{bmatrix},\quad A_{12}^2=\begin{bmatrix}2.43 & -0.7\\ 0.5 & 1\end{bmatrix},$$

$$A_{21}^2=\begin{bmatrix}-2.5 & 1.73\\ -2.4 & 1.5\end{bmatrix},\quad A_{22}^2=\begin{bmatrix}-0.71 & 1.20\\ -1.111 & 0.325\end{bmatrix},\quad B_1^2=\begin{bmatrix}0\\1\end{bmatrix},\quad B_2^2=\begin{bmatrix}0\\1\end{bmatrix}$$

In the first stage, (7) can be solved by the LMI optimization tools to obtain

$$P_{22}=\begin{bmatrix}34.2040 & 8.2939\\ 8.2939 & 2.7635\end{bmatrix},\ K_2^1=[-12.6723\quad -3.6230],\ K_2^2=[-64.0851\quad -16.3892]$$

In the second stage, the algorithm finishes after 7 iterations, and the obtained feasible solution is:

$$\alpha=-0.0029,\ K_1^1=[-0.3544\quad -1.7789],\quad K_1^2=[5.2971\quad -5.4936]$$

$$P_{11}=\begin{bmatrix}1.6296 & 0.2118\\ 0.2118 & 0.5497\end{bmatrix},\quad P_{21}=\begin{bmatrix}-0.4094 & 2.2348\\ -1.1516 & 0.8994\end{bmatrix}$$

Fig. 1 shows the monotonic decrease of $\alpha^t$. $\alpha^t<0$ implies that the resulting local fuzzy controller $K_1^i$ and $K_2^i$ stabilizes the given fuzzy singularly perturbed plant for small enough $\varepsilon$.

In Fig. 2, the control response of the closed-loop system where $\varepsilon=0.1$ is monitored for an initial state [1, 0, 1, 0]. (The corresponding membership functions are defined as $\mu_1=\exp[-(x_1+x_2-1)^2]$, $\mu_2=1-\exp[-(x_1+x_2-1)^2]$).

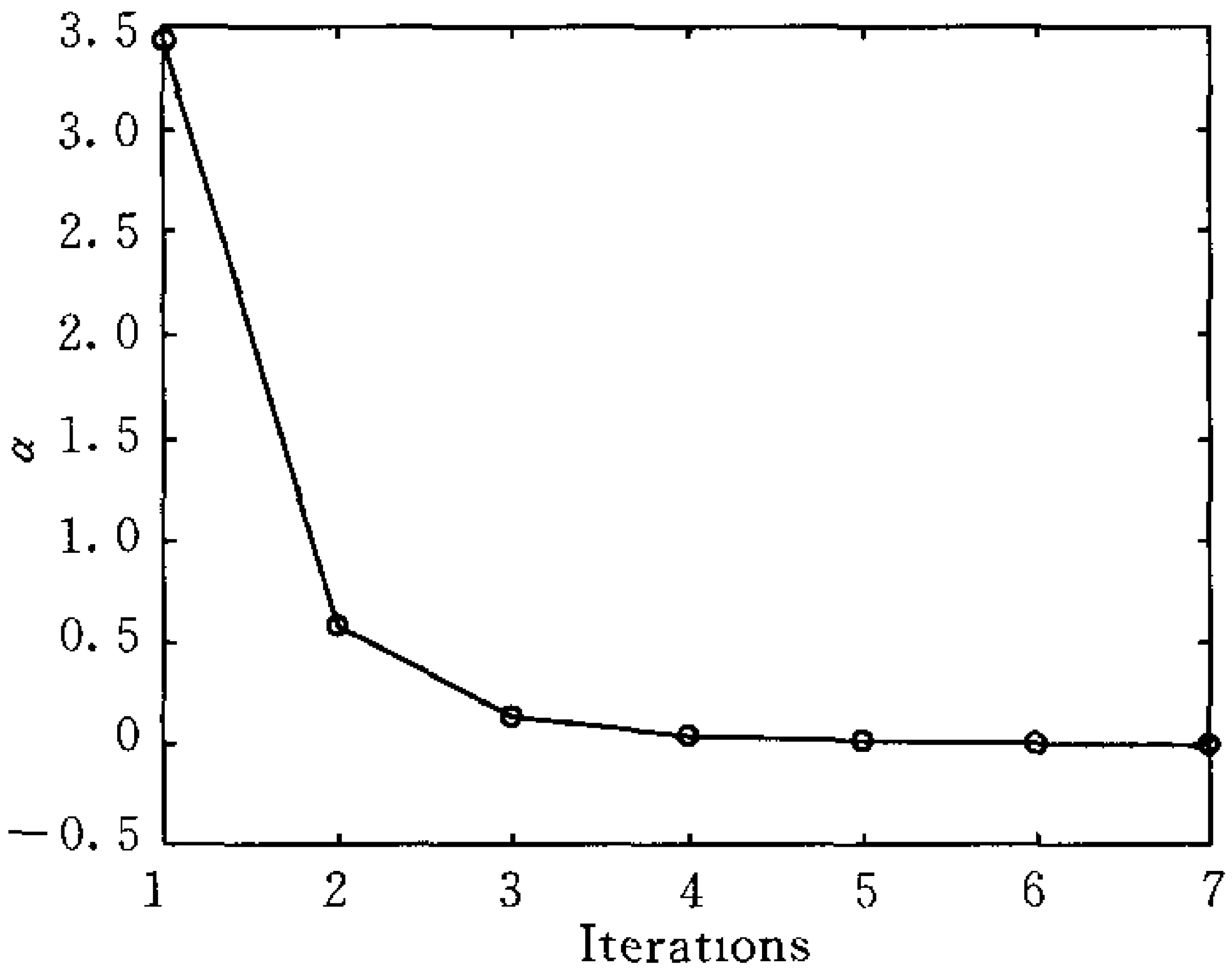


Fig. 1 $\{\alpha^t\}$ versus iteration

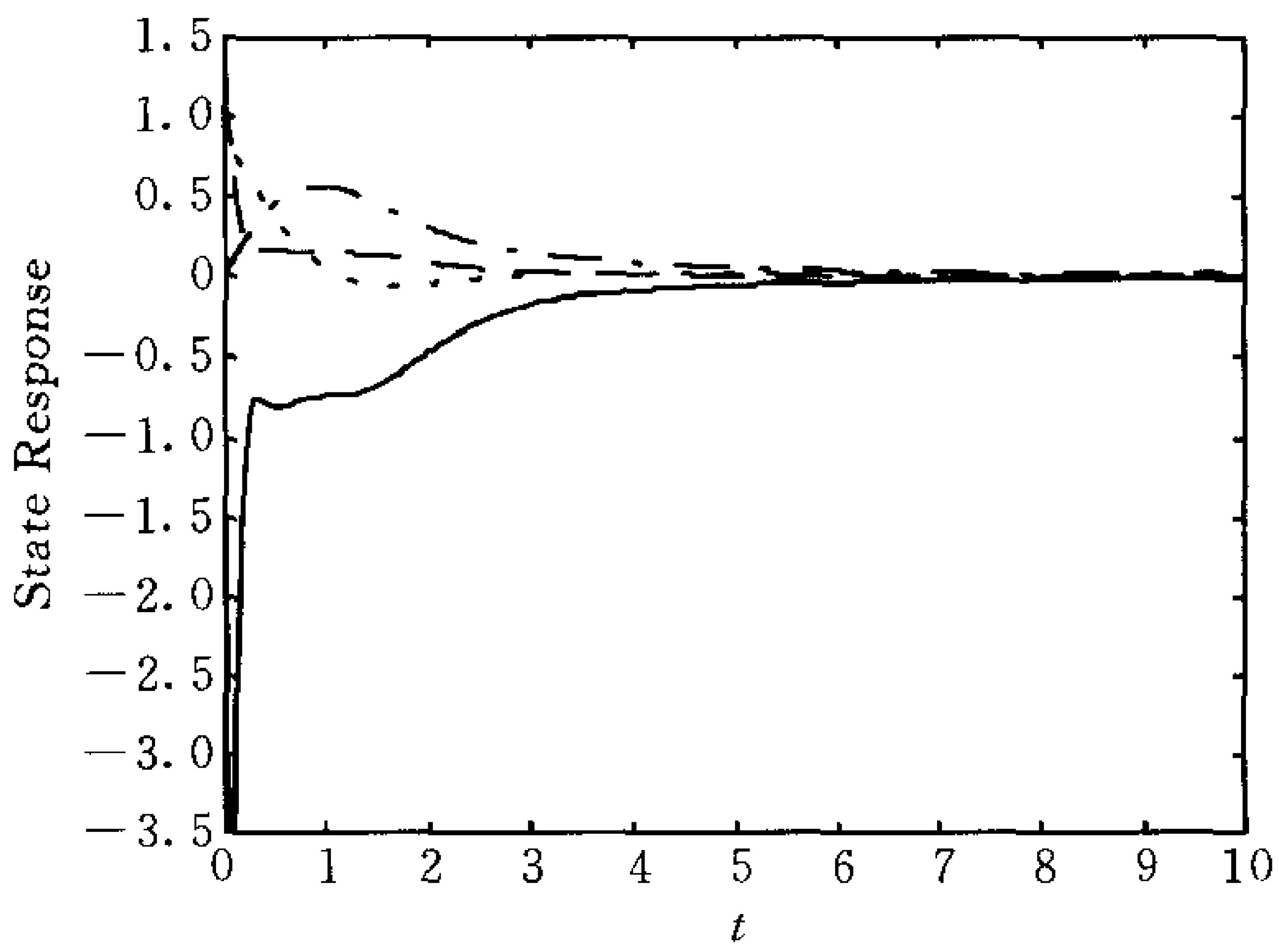


Fig. 2 The dynamics response

## 5 Conclusions

This paper defines a fuzzy singularly perturbed system by extending the ordinary T-S fuzzy model and the fuzzy descriptor systems. The stability condition is derived. The stabilization synthesis is reduced to a set of bilinear matrix inequalities, which can be effectively solved by the proposed iterative linear matrix inequalities approach. Finally, numerical examples are given to show the effectiveness of this approach. Moreover, both the standard and the nonstandard singularly perturbed systems can be dealt with using this approach. Future work is to research the dynamic output feedback controller and the multi-objective controller.

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# 模糊奇异摄动系统及其稳定性分析与综合

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**摘 要** 通过扩展常规 Takagi-Sugeno 模糊系统，定义了一类模糊奇异摄动系统，利用矩阵不等式表达出了在摄动参数足够小时的闭环稳定性. 镇定并行分布式补偿控制器增益和共同的 Lyapunov 函数可利用两步法得到，并可分别归结于一组线性矩阵不等式和双线性矩阵不等式，后者可以利用迭代线性矩阵不等式方法有效地求解. 文末给出了数值和仿真实例.

**关键词** 模糊系统，线性矩阵不等式，并行分布式补偿，奇异摄动
**中图分类号** TP273+.4