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# 参数不确定机械伺服系统的鲁棒非线性摩擦补偿控制[1)]

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**摘 要** 对含非线性摩擦环节的机械伺服系统,提出一种基于 Lyapunov 方法的鲁棒非线性控制方法,通过引入非线性增益和摩擦补偿项,来克服参数不确定性和补偿非线性摩擦,从而保证跟踪误差渐进收敛.对转台系统的实验研究,表明了该方法的有效性.

**关键词** 非线性控制, 鲁棒性, 伺服系统, 摩擦
**中图分类号** TP273

# Robust Nonlinear Friction Compensation of Mechanical Servo System with Time Variable Parameters

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**Abstract** A robust nonlinear control design is presented. By introducing the nonlinear gain and friction compensation term, the robust global convergence of the tracking error is proved using Lyapunov's direct method. Experiment results demonstrate the effectiveness of the proposed algorithm.

**Key words** Nonlinear control, robustness, servo system, friction

## 1 引言

对于高精度机械伺服系统,摩擦环节的存在是提高系统性能的障碍[1].在基于摩擦模型的补偿控制中,需要对摩擦环节建立数学模型[2,3],但在实践中,对摩擦建立精确的数学模型非常困难.本文提出一种基于 Lyapunov 直接法的非线性鲁棒控制方法,通过在控制中引

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入鲁棒补偿项，对摩擦进行补偿，同时采用非线性增益来克服系统参数的不确定性.

## 2　问题描述

对于机械伺服系统，采用如下微分方程描述

$$\frac{d^n\theta}{dt^n}+\alpha_{n-1}\frac{d^{n-1}\theta}{dt^{n-1}}+\cdots+\alpha_1\frac{d\theta}{dt}+\alpha_0\theta=\beta_0u-\beta_1f(\dot{\theta},u) \tag{1}$$

其中 $\theta$ 为系统输出转角，$u$ 为控制电压，且

$$0\leqslant\underline{\alpha_i}\leqslant\alpha_i\leqslant\overline{\alpha_i},\ i=0,1,\cdots,n-1 \tag{2}$$

$$0<\underline{\beta_0}\leqslant\beta_0\leqslant\overline{\beta_0},\quad 0<\underline{\beta_1}\leqslant\beta_1\leqslant\overline{\beta_1} \tag{3),(4}$$

式(1)中的 $f(\dot{\theta},u)$ 为非线性摩擦，表示为[4]

$$f(\dot{\theta},u)=\lambda(\dot{\theta})F_c(\dot{\theta})+[1-\lambda(\dot{\theta})]F_s(u,\dot{\theta}) \tag{5}$$

$$\lambda(\dot{\theta})=\begin{cases}1, & |\dot{\theta}|\geqslant D_V\\ 0, & |\dot{\theta}|<D_V\end{cases} \tag{6}$$

其中 $F_s(u,\dot{\theta})$ 和 $F_c(\dot{\theta})$ 为有界函数，且对于任意的 $u$，$\dot{\theta}$，有

$$|F_s(u,\dot{\theta})|\leqslant\overline{F_s},\quad |F_c(\dot{\theta})|\leqslant\overline{F_c} \tag{7}$$

对于系统(1)，引入如下渐进稳定参考模型

$$\frac{d^n\theta_m}{dt^n}+a_{n-1}\frac{d^{n-1}\theta_m}{dt^{n-1}}+\cdots+a_1\frac{d\theta_m}{dt}+a_0\theta_m=br \tag{8}$$

其中 $\theta_m$ 为模型输出，$r$ 为输入，$a_{n-1},a_{n-2},\cdots,a_0,b$ 为正实数. 定义误差信号 $e=\theta_m-\theta$，本文的问题在于寻求控制 $u$，使得对于任意初态，误差 $e$ 满足 $\lim\limits_{t\to\infty}e(t)=0$.

## 3　鲁棒非线性摩擦补偿控制

求出误差动态方程为

$$\frac{d^ne}{dt^n}+a_{n-1}\frac{d^{n-1}e}{dt^{n-1}}+\cdots+a_0e=br-\beta_0u+\beta_1f(\dot{\theta},u)+\sum_{i=0}^{n-1}(\alpha_i-a_i)\theta^{(i)} \tag{9}$$

定义向量 $\boldsymbol{\varepsilon}=[e,\dot{e},\cdots,e^{(n-1)}]^{\mathrm{T}}$，则有

$$\dot{\boldsymbol{\varepsilon}}=A\boldsymbol{\varepsilon}-\begin{bmatrix}0\\ \beta_0\end{bmatrix}u+\begin{bmatrix}0\\ \Delta\end{bmatrix} \tag{10}$$

其中 $A=\begin{bmatrix}0 & 1 & & 0\\ 0 & 0 & \ddots & 0\\ 0 & 0 & & 1\\ -a_0 & -a_1 & \cdots & -a_{n-1}\end{bmatrix}$，$\Delta=br+\beta_1f(\dot{\theta},u)+\sum\limits_{i=0}^{n-1}(\alpha_i-a_i)\theta^{(i)}$. 因为矩阵 $A$ 渐进稳定，则存在正定矩阵 $P$ 和 $Q$，使得

$$A^{\mathrm{T}}P+PA=-Q \tag{11}$$

定义信号 $e_1=[0,\ 0,\ \cdots,\ 1]P\boldsymbol{\varepsilon}$，取控制 $u$ 如下：

$$u = \sum_{i=0}^{n-1} k_i(e_1, \theta^{(i)})\theta^{(i)} + l(r)\ r + q(e_1, \dot{\theta}) \tag{12}$$

其中 $k_n(e_1, r)$和 $k_i(e_1, \theta^{(i)})$为增益系数,$q(e_1, \dot{\theta})$为摩擦环节的补偿项.

**定理 1.** 对于系统(1),采用式(12)控制,若取

$$l(r) = \begin{cases} b/\underline{\beta_0}, & e_1 r > 0 \\ b/\overline{\beta_0}, & e_1 r \leqslant 0 \end{cases} \tag{13}$$

$$k_i(e_1, \theta^{(i)}) = \begin{cases} \overline{\alpha_i}/\underline{\beta_0} - a_i/\overline{\beta_0}, & e_1\theta^{(i)} > 0, \\ \underline{\alpha_i}/\overline{\beta_0} - a_i/\underline{\beta_0}, & e_1\theta^{(i)} \leqslant 0, \end{cases} \quad i = 0, 1, \cdots, n-1 \tag{14}$$

$$q(e_1, \dot{\theta}) = \begin{cases} \overline{\beta_1}\ \overline{F_s}\, sgn(e_1)/\underline{\beta_0}, & |\dot{\theta}| < D_V \\ \overline{\beta_1}\ \overline{F_c}\, sgn(e_1)/\underline{\beta_0}, & |\dot{\theta}| \geqslant D_V \end{cases} \tag{15}$$

则对于任意的 $\alpha_i(i=0,1,\cdots,n-1)$, $f(\dot{\theta}, u)$及初始状态,误差 $e(t)$有界且渐进收敛于 0.

**证明.** 取 Lyapunov 函数

$$V(t) = \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{T}} P \boldsymbol{\varepsilon} \tag{16}$$

由式(9),(12)有

$$\dot{V} = \frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{T}}(A^{\mathrm{T}}P + PA)\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}^{\mathrm{T}} P \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} (\Delta - \beta_0 u) =$$

$$-\frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{T}} Q \boldsymbol{\varepsilon} + e_1 r(b - \beta_0 l) + \sum_{i=0}^{n-1}(\alpha_i - a_i - \beta_0 k_i) e_1 \theta^{(i)} + e_1[\beta_1 f(\dot{\theta}, u) - \beta_0 q].$$

当 $e_1 r > 0$ 时,$b - \beta_0 l = b\left(1 - \frac{\beta_0}{\underline{\beta_0}}\right) \leqslant 0$; 当 $e_1 r \leqslant 0$ 时,$b - \beta_0 l = b\left(1 - \frac{\beta_0}{\overline{\beta_0}}\right) \geqslant 0$. 从而有

$$e_1 r(b - \beta_0 l) \leqslant 0 \tag{17}$$

当 $e_1\theta^{(i)} > 0$ 时,$\alpha_i - a_i - \beta_0 k_i = a_i\left(\frac{\beta_0}{\overline{\beta_0}} - 1\right) + \alpha_i - \frac{\beta_0}{\underline{\beta_0}}\overline{\alpha_i} \leqslant 0$;同理可证,当 $e_1\theta^{(i)} \leqslant 0$ 时,$\alpha_i - a_i - \beta_0 k_i = a_i\left(\frac{\beta_0}{\underline{\beta_0}} - 1\right) + \alpha_i - \frac{\beta_0}{\overline{\beta_0}}\underline{\alpha_i} \geqslant 0$. 从而有

$$\sum_{i=0}^{n-1}(\alpha_i - a_i - \beta_0 k_i) e_1 \theta^{(i)} \leqslant 0 \tag{18}$$

由式(17),(18)有

$$\dot{V} \leqslant -\frac{1}{2}\boldsymbol{\varepsilon}^{\mathrm{T}} Q \boldsymbol{\varepsilon} + e_1[\beta_1 f(\dot{\theta}, u) - \beta_0 q] \tag{19}$$

由式(19),当 $e_1 > 0$ 时,

$$\beta_1 f(\dot{\theta}, u) - \beta_0 q = \begin{cases} \beta_1 F_s(u, \dot{\theta}) - \beta_0\, \overline{\beta_1}\ \overline{F_s}/\underline{\beta_0}\}, & |\dot{\theta}| < D_V \\ \beta_1 F_c(\dot{\theta}) - \beta_0\, \overline{\beta_1}\ \overline{F_c}/\underline{\beta_0}, & |\dot{\theta}| \geqslant D_V \end{cases} \leqslant 0 \tag{20}$$

同理可证，当 $e_1<0$ 时，$\beta_1 f(\dot{\theta},u)-\beta_0 q\geqslant 0$，则有

$$e_1[\beta_1 f(\dot{\theta},u)-\beta_0 q]\leqslant 0 \tag{21}$$

设 $\sigma_m(\cdot)$ 和 $\sigma_M(\cdot)$ 分别为矩阵的最大和最小奇异值，$\|\cdot\|$ 为 $\boldsymbol{\varepsilon}$ 范数，则有

$$\dot{V}\leqslant -\frac{\sigma_m(Q)}{2}\|\boldsymbol{\varepsilon}\|^2\leqslant -\frac{\sigma_m(Q)}{\sigma_M(P)}V \tag{22}$$

则 $V(t)\leqslant V(0)e^{-[\sigma_m(Q)/\sigma_M(P)]t}$，因为 $e^2(t)\leqslant\|\boldsymbol{\varepsilon}\|^2\leqslant\dfrac{2}{\sigma_m(P)}V(t)$，则

$$e^2(t)\leqslant\frac{2}{\sigma_m(P)}V(0)e^{-[\sigma_m(Q)/\sigma_M(P)]t} \tag{23}$$

由上式可知，对于任意的初始条件，误差 $e(t)$ 是有界的，且渐进收敛于 0.　　证毕.

对于采样系统，为避免响应出现抖振，用饱和函数 $sat(\cdot)$ 代替 $sgn(\cdot)$，即

$$sat(e)=\begin{cases}1, & e>\delta\\ 1/\delta, & |e|\leqslant\delta\\ -1, & e<-\delta\end{cases} \tag{24}$$

则对于系统(1)，当 $|e|\geqslant\delta$ 时，方程(22)成立. 由 LaSall 定理，存在有界集 $E=\{e\mid |e|\leqslant\delta\}$，对于任意的 $\alpha_i$，$f(\dot{\theta},u)$ 及初始条件，误差 $e(t)$ 有界，且存在 $T>0$，当 $t\geqslant T$，$e(t)$ 收敛于 $E$.

## 4　实验研究

某三轴转台，当负载为 30 kg 时，中框的频率特性如图 1 所示，其中实线为实测，由频率特性拟合，得到对象方程

$$\ddot{\theta}+15.0\dot{\theta}=150.0u-150.0f(\dot{\theta},u) \tag{25}$$

其频率特性如图中虚线所示. 取参考模型为

$$\ddot{\theta}_m+15.0\dot{\theta}_m+90.0\theta_m=90.0r \tag{26}$$

取 $\overline{F_s}=0.5$，$\overline{F_c}=0.2$，$D_v=0.3$，$\alpha_1\in[10,20]$，$\alpha_0\in[0,5]$，$\beta_0\in[130,180]$，$\beta_1\in[130,180]$，$Q=\begin{bmatrix}5.0 & 0\\ 0 & 5.0\end{bmatrix}$，$\delta=0.05$. 图 2 为转台中框跟踪 0.25 Hz 方波时的输出曲线. 可以看出，系

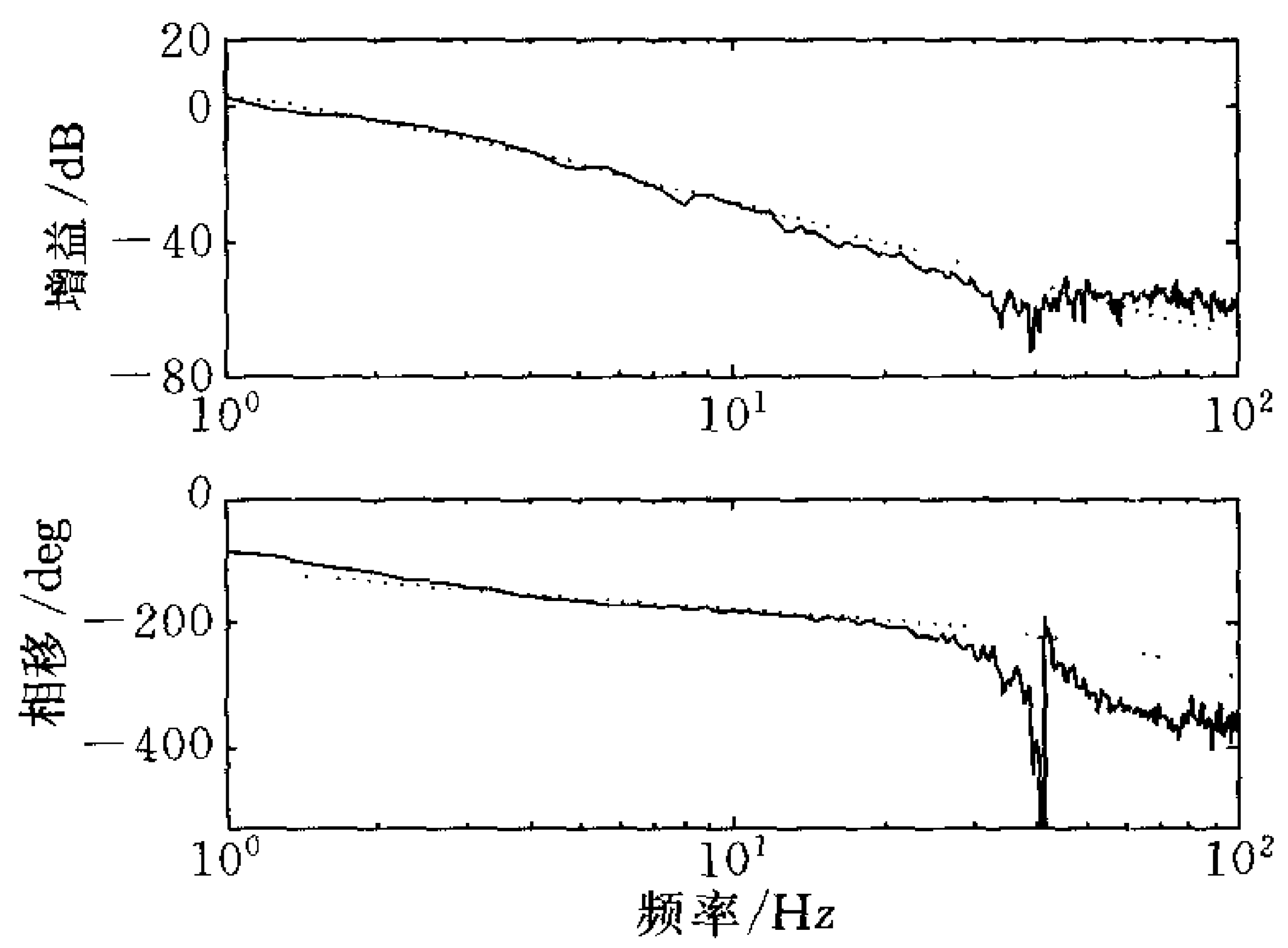


图 1　转台中框频率特性曲线
Fig. 1　Frequency response of middle frame

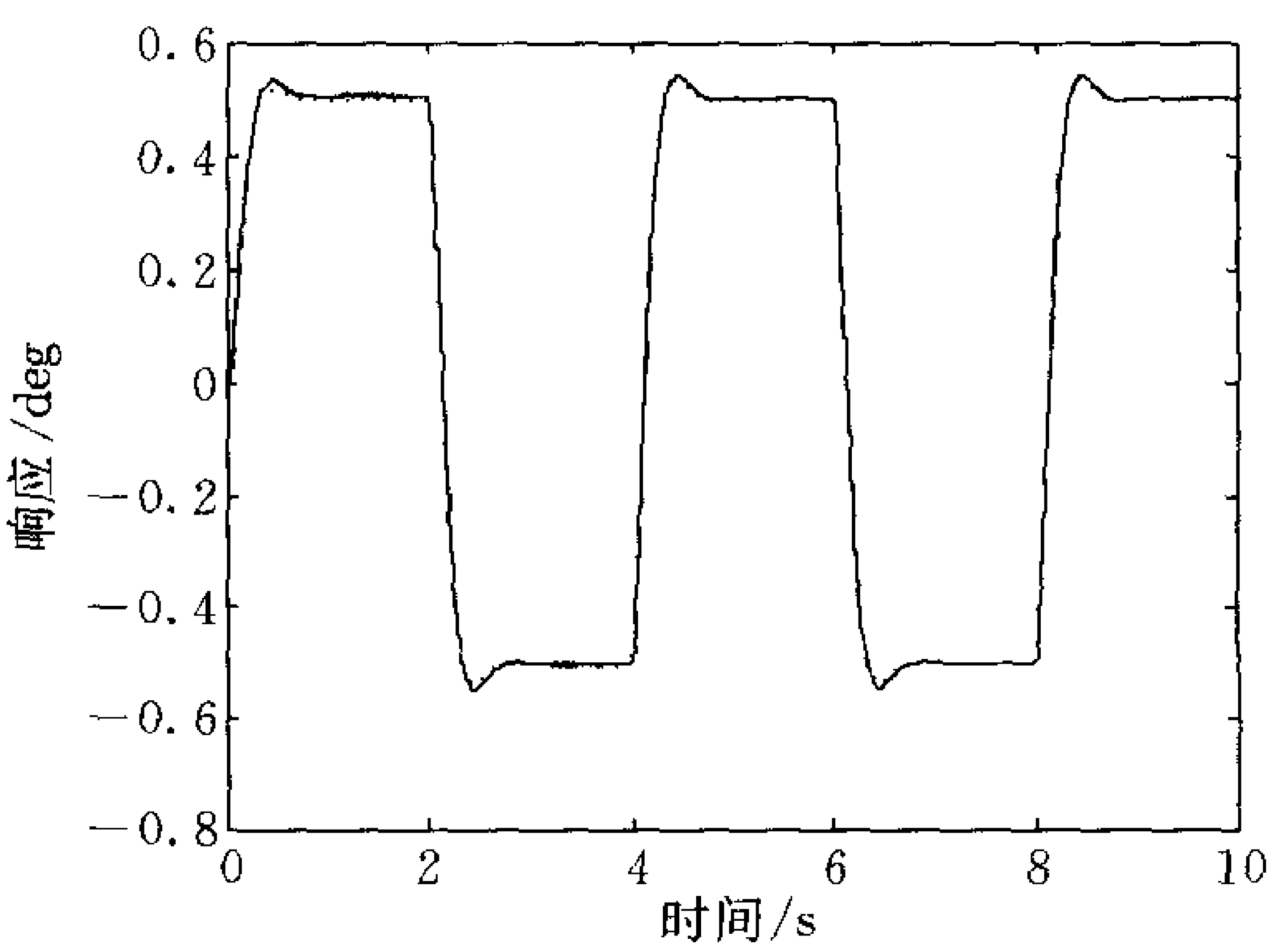


图 2　实验曲线
Fig. 2　Experimental results

统与模型之间的跟踪误差小,系统响应中没有死区,对摩擦有很强的抑制作用.

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