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Scheduling Problems under Linear Deterioration[1)]

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Abstract This paper considers the scheduling problem under linear deterioration. It is assumed that the deterioration function is a linear function. Optimal algorithms are presented respectively for single machine scheduling problems of minimizing the makespan, weighted sum of completion times, maximum lateness and maximum cost. For two-machine flow shop scheduling problem to minimize the makespan, it is proved that the optimal schedule can be obtained by Johnson's rule. If the processing times of operations are equal for each job, flow shop scheduling problems can be transformed into single machine scheduling problems.

Key words Scheduling, single machine, flow shop, linear deterioration

1 Introduction

In classical scheduling problems it is assumed that the processing times of jobs are constant and independent of the starting time. However, there are many situations where the processing time of the job depends on its starting time[1]. In this model, the processing time of a job can be described by a deterioration function. Browne and Yechiali[2] studied the single machine stochastic scheduling problem. Mosheiov[3] considered the single machine scheduling under simple linear deterioration. There are some solutions for the scheduling problems with arbitrary linear deterioration in [4,5]. Bachman and Janiak[6] proved that the single machine problem to minimize the maximum lateness with an arbitrary linear processing time is NP-complete. Yang and Chern[7] considered the two-machine flow shop problem.

This paper considers the scheduling problems under linear deterioration in which the actual processing time of a job is the product of its basic processing time and the deterioration function. It is the generalization case of the model in [3].

2 Single machine problems

The single machine problem can be described as follows.

There are n jobs $J_1, J_2, \cdots, J_n$. Associated with job J_j is a weight w_j and a due date d_j. The processing time of job J_j is $p_j(a+bt)$ if its starting time is t, where a and b are positive constants. The basic processing time of job J_j is p_j and deterioration function is $d(x)=a+bx$. $j=1,2,\cdots,n$. The problem is denoted as

$$1 \mid p_j(a+bt) \mid f(C)$$

where $f(C)=f(C_1, C_2, \cdots, C_n)$ is a non-decreasing function of completion time.

For a given schedule π, if the completion time of job J_j is C_j, the lateness of job J_j is $L_j=C_j-d_j$, $j=1,2,\cdots,n$. The makespan of schedule π is $C_{\max}=\max\limits_{1\leqslant j\leqslant n}\{C_j\}$, the weighted sum of completion times is $\sum w_jC_j=\sum\limits_{j=1}^{n} w_jC_j$, the maximum lateness is $L_{\max}=\max\limits_{1\leqslant j\leqslant n}\{L_j\}$, the maximum cost is $h_{\max}=\max\limits_{1\leqslant j\leqslant n}\{h_j(C_j)\}$, $h_j(C_j)$ is a non-decreasing function of C_j.

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2.1 The problem $1|p_j(a+bt)|C_{\max}$

Theorem 1. For the problem $1|p_j(a+bt)|C_{\max}$, the makespan is sequence independent. If the starting time of the first job is t, the makespan is

$$C_{\max}(t \mid J_1, J_2, \cdots, J_n) = t + (a+bt)\sum_{k=1}^{n} b^{k-1} C_n^k(p_1, p_2, \cdots, p_n)$$

where $C_n^k(p_1, p_2, \cdots, p_n)$ is sum of C_n^k items, each item is the product of k numbers of p_1, $p_2, \cdots, p_n$.

Proof(by induction). Without loss of generality we consider schedule $\pi = [J_1, J_2, \cdots, J_n]$.

$$C_1 = t + p_1(a+bt)$$

$$C_2 = C_1 + p_2(a+bC_1) = t + (a+bt)(\sum_{j=1}^{2} p_j + bp_1p_2) = t + (a+bt)\sum_{k=1}^{2} b^{k-1} C_2^k(p_1, p_2)$$

Suppose Theorem 1 holds for job J_j, i.e.,

$$C_j = t + (a+bt)\sum_{k=1}^{j} b^{k-1} C_j^k(p_1, p_2, \cdots, p_j)$$

Consider job J_{j+1}

$$C_{j+1} = C_j + p_{j+1}(a+bC_j) = t + (a+bt)\sum_{k=1}^{j} b^{k-1} C_j^k(p_1, p_2, \cdots, p_j) + p_{j+1}(a+bt) +$$
$$(a+bt)\sum_{k=1}^{j} p_{j+1} b^k C_j^k(p_1, p_2 \cdots, p_j) = t + (a+bt)[C_j^1(p_1, p_2, \cdots, p_j) +$$
$$p_{j+1} + \sum_{k=2}^{j} b^{k-1} C_j^k(p_1, p_2, \cdots, p_j) + \sum_{k=2}^{j+1} p_{j+1} b^{k-1} C_j^{k-1}(p_1, p_2 \cdots, p_j)] =$$
$$t + (a+bt)[C_j^1(p_1, p_2, \cdots, p_j) + p_{j+1} + \sum_{k=2}^{j} b^{k-1} C_j^k(p_1, p_2, \cdots, p_j) +$$
$$\sum_{k=2}^{j} p_{j+1} b^{k-1} C_j^{k-1}(p_1, p_2 \cdots, p_j) + p_{j+1} b^j C_j^j(p_1, p_2, \cdots, p_j)] =$$
$$t + (a+bt)\sum_{k=1}^{j+1} b^{k-1} C_{j+1}^k(p_1, p_2, \cdots, p_{j+1})$$

Hence, Theorem 1 holds for J_{j+1}. This completes the proof of Theorem 1. □

Corollary 1. $C_{\max}(t|J_1, J_2, \cdots, J_n) = C_{\max}(C_{\max}(t|J_1, \cdots, J_k)|J_{k+1}, \cdots, J_n)(1 \leqslant k \leqslant n)$.

Corollary 2. If $p' \leqslant p''$, then $C_{\max}(t|J_1, J_2, \cdots, J_k, J') \leqslant C_{\max}(t|J_1, J_2, \cdots, J_k, J'')$.

2.2 The problem $1|p_j(a+bt)|\sum w_jC_j$

Theorem 2. For the problem $1|p_j(a+bt)|\sum w_jC_j$, an optimal schedule can be obtained by arranging jobs in an order of non-decreasing $p_j/w_j(1+bp_j)$.

Proof(by contradiction). Suppose under an optimal schedule π, there are two adjacent jobs J_j and J_k, J_j followed by J_k, such that $p_j/w_j(1+bp_j) > p_k/w_k(1+bp_k)$.

Let the starting time of job J_j is t, and the weighted sum of completion times of J_j and J_k is

$$w_jC_j(\pi) + w_kC_k(\pi) = w_j[t + p_j(a+bt)] + w_k[t + (p_j + p_k + bp_jp_k)(a+bt)]$$

Perform an adjacent pair-wise interchange on jobs J_j and J_k, and call the new schedule $\bar{\pi}$. Under $\bar{\pi}$, the weighted sum of completion times of J_j and J_k is

$$w_jC_j(\bar{\pi}) + w_kC_k(\bar{\pi}) = w_k[t + p_k(a+bt)] + w_j[t + (p_j + p_k + bp_jp_k)(a+bt)]$$

If $p_j/w_j(1+bp_j) > p_k/w_k(1+bp_k)$, it is easily verified that

$$w_jC_j(\bar{\pi}) + w_kC_k(\bar{\pi}) < w_jC_j(\pi) + w_kC_k(\pi)$$

Since the weighted sum of completion times of other jobs is not affected by the interchange, then the weighted sum of completion times under $\bar{\pi}$ is strictly less than that under π. This contradicts the optimality of π.

2.3 The problem $1|p_j(a+bt)|h_{max}$

For the due date related model, we consider the general problem $1|p_j(a+bt)|h_{max}$[9].

From Theorem 1, for a set of jobs, the makespan is sequence independent. Let A denote the set of jobs to be scheduled, B denote the set of jobs already scheduled. $C_{max}(A)$ is the makespan of jobs in A.

Algorithm 1.

Step 1. Let $A=\{J_1, J_2, \cdots, J_n\}, B=\phi$

Step 2. Calculate $t=C_{max}(A)$. Find j^* such that

$$h_{j^*}(t) = \min\{h_j(t) \mid J_j \in A\}$$

Arrange job J_{j^*} in the last position.

Step 3. Set $A=A-\{J_{j^*}\}, B=B+\{J_{j^*}\}$. If $A=\phi$, stop; otherwise go to Step 2.

Theorem 3. Algorithm 1 generates an optimal schedule for the problem $1|p_j(a+bt)|h_{max}$.

Proof. Let π be an optimal schedule, $\bar{\pi}$ be the schedule of Algorithm 1. We will show that schedule π can be transformed into schedule $\bar{\pi}$ and the maximum cost does not increase. Without loss of generality we assume that the last job of π is $J_{j^{**}}$ and that of $\bar{\pi}$ is J_{j^*}. Let $\pi=[S_1, J_{j^*}, S_2, J_{j^{**}}]$, where $h_{j^{**}}(t) \geqslant h_{j^*}(t)$, S_1 and S_2 are the sets of the other $n-2$ jobs.

From π, by placing job J_{j^*} in the last position we can get a new schedule $\bar{\bar{\pi}}=[S_1, S_2, J_{j^{**}}, J_{j^*}]$. Since only the completion time of job J_{j^*} becomes larger and $h_{j^{**}}(t) \geqslant h_{j^*}(t)$, the maximum cost of $\bar{\bar{\pi}}$ can not be larger than that of π. Now $\bar{\bar{\pi}}$ and $\bar{\pi}$ have the same job in the last position. By a the similar method we can transform π into $\bar{\pi}$ and the maximum cost is not increased. □

The problem $1|p_j(a+bt)|L_{max}$ is a special case of the problem $1|p_j(a+bt)|h_{max}$, $h_j(C_j)=C_j-d_j$. At this condition, the schedule of Algorithm 1 satisfies the *EDD* (Earliest Due Date First) rule.

Theorem 4. For the problem $1|p_j(a+bt)|L_{max}$, an optimal schedule can be obtained by the *EDD* rule.

3 Flow shop problems

The *Flowshop* problems can be described as follows.

There are n jobs $J_1, J_2, \cdots, J_n$ to be processed successively on m machines M_1, M_2, $\cdots$, M_m in that order. Moreover, we assume that the same job order is chosen on each machine. Associated with job J_j is a weight w_j and a due date d_j. The operation of job J_j on M_i is denoted by T_{ij}. The processing time of operation T_{ij} is $p_{ij}(a+bt)(i=1,2,\cdots,m, j=1,2,\cdots,n)$ if its starting time is t. The problem is denoted as

$$Fm \mid p_j(a+bt) \mid f(C)$$

3.1 The problem $F2|p_j(a+bt)|C_{max}$

Johnson rule ($SPT(M_1)-LPT(M_2)$ rule)[8,9]

Partition the jobs into two sets with set A containing all jobs with $p_{1j}<p_{2j}$ and set B all the jobs with $p_{1j}>p_{2j}$. The jobs with $p_{1j}=p_{2j}$ may be in either set. The jobs in A go first in the order of non-decreasing p_{1j}, the jobs in B follow in the order of non-increasing p_{2j}.

Theorem 5. For the problem $F2|p_j(a+bt)|C_{max}$, an optimal schedule can be obtained by Johnson's rule.

Proof. Suppose an optimal π does not satisfy Johnson's rule, there must be two adjacent jobs J_j and J_k, J_j followed by J_k, which satisfy one of the following three conditions:

1) job J_j belongs to set B and J_k to set A;

2) jobs J_j and J_k belong to set A and $p_{1j}>p_{1k}$;

3) jobs J_j and J_k belong to set B and $p_{2j}<p_{2k}$.

In what follows we will show that under any of these three conditions the makespan is reduced after a pair-wise interchange of jobs J_j and J_k.

Let job J_l be followed by job J_j and let J_s follow job J_k. Perform an adjacent pair-wise interchange on jobs J_j and J_k, and call the new schedule $\bar{\pi}$. We denote the completion time of job J_j on machine M_i under $\pi(\bar{\pi})$ by $C_{ij}(\bar{C}_{ij})$. It is obvious that $C_{il}=\bar{C}_{il}(i=1,2)$. Interchanging jobs J_j and J_k clearly does not affect the starting time of job J_s on machine M_1 and it is $C_{\max}=(C_{il} \mid T_{1j}, T_{1k})$. Now consider the starting time of job J_s on machine M_2. Under π, the starting time of job J_s on machine M_2 is $\max\{C_{\max}(C_{1l} \mid T_{1j}, T_{1k}, T_{1s}), C_{2k}\}$, under $\bar{\pi}$, it is $\max\{C_{\max}(C_{1l} \mid T_{1j}, T_{1k}, T_{1s}), \bar{C}_{2j}\}$.

Under π

$$\begin{aligned} C_{2k} &= \max\{C_{\max}(C_{2j} \mid T_{2k}), C_{\max}(C_{1k} \mid T_{2k})\} = \\ &\max\{C_{\max}(C_{2l} \mid T_{2j}, T_{2k}), C_{\max}(C_{1j} \mid T_{2j}, T_{2k}), C_{\max}(C_{1k} \mid T_{2k})\} = \\ &\max\{C_{\max}(C_{2l} \mid T_{2j}, T_{2k}), C_{\max}(C_{1l} \mid T_{1j}, T_{2j}, T_{2k}), C_{\max}(C_{1l} \mid T_{1j}, T_{1k}, T_{2k})\}. \end{aligned}$$

Under $\bar{\pi}$

$$\begin{aligned} \bar{C}_{2j} &= \max\{C_{\max}(\bar{C}_{2k} \mid T_{2j}), C_{\max}(\bar{C}_{1j} \mid T_{2j})\} = \\ &\max\{C_{\max}(\bar{C}_{2l} \mid T_{2k}, T_{2j}), C_{\max}(\bar{C}_{1k} \mid T_{2k}, T_{2j}), C_{\max}(\bar{C}_{1j} \mid T_{2j})\} = \\ &\max\{C_{\max}(C_{2l} \mid T_{2k}, T_{2j}), C_{\max}(C_{1l} \mid T_{1k}, T_{2k}, T_{2j}), C_{\max}(C_{1l} \mid T_{1k}, T_{1j}, T_{2j})\}. \end{aligned}$$

From Theorem1, $C_{\max}(C_{2l} \mid T_{2k}, T_{2j}) = C_{\max}(C_{2l} \mid T_{2j}, T_{2k})$. Under condition (1), $p_{1j} \geqslant p_{2j}$, $p_{1k} \leqslant p_{2k}$. According to Theorem 1 and Corollary 2,

$$C_{\max}(C_{1l} \mid T_{1k}, T_{2k}, T_{2j}) \leqslant C_{\max}(C_{1l} \mid T_{1j}, T_{1k}, T_{2k})$$
$$C_{\max}(C_{1l} \mid T_{1k}, T_{1j}, T_{2j}) \leqslant C_{\max}(C_{1l} \mid T_{1j}, T_{2j}, T_{2k})$$

So, $\bar{C}_{2j} \leqslant C_{2k}$. Conditions (2) and (3) can be shown in a similar way as condition (1). □

3.2 The problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid f(C)$

For the classical problem $Fm \mid p_{ij}=p_j \mid f(C)$, the makespan is sequence independent. If the starting time of the first job is t, $p_l=\max\{p_j\}$, the makespan is

$$C_{\max}(\pi) = t + \sum_{j=1}^{n} p_j + (m-1)p_l$$

The above solution can be generalized to the problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid f(C)$. We consider each operation as a job, and the problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid f(C)$ is equivalent to the single problem $1 \mid p_j(a+bt) \mid f(C)$ with $n+m-1$ jobs. Job J_l is considered as m jobs ($p_l=\max\{p_j\}$). Hence, for the problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid f(C)$, the makespan is

$$C_{\max}(t \mid J_1, \cdots, J_l, \cdots, J_l, \cdots, J_n) = t + (a+bt) \sum_{k=1}^{n+m-1} b^{k-1} C_n^k(p_1, \cdots, p_l, \cdots, p_l, \cdots, p_n)$$

By the results of single machine problem we have the following solutions.

Theorem 6. For the problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid C_{\max}$, the makespan is sequence independent.

Theorem 7. For the problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid L_{\max}$, an optimal schedule can be obtained by the *EDD* rule.

Theorem 8. Algorithm 1 generates an optimal schedule for the problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid h_{\max}$(In step 1, set $A=\{J_1, J_2, \cdots, J_l, \cdots, J_l, \cdots, J_n\}$).

Not all results of single machine problems can be generalized to the problem $Fm \mid p_j(a+bt)$, $p_{ij}=p_j \mid f(C)$. It is easily verified that for the problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid \sum w_j C_j$, placing jobs in the order of non-decreasing $p_j/w_j(1+bp_j)$ does not necessarily minimize the weighted sum of completion times. Morever, if all jobs have equal weights, we have the following theorem.

Theorem 9. For the problem $Fm \mid p_j(a+bt), p_{ij}=p_j \mid \sum C_j$, an optimal schedule can be obtained by placing jobs in the order of non-decreasing p_j.

4 Conclusions

In this paper we consider scheduling problems with deteriorating jobs, the deterioration function being $d(x)=a+bx$. In this model, the actual processing time of a job is the product of its basic processing time and the deterioration function. For single machine problems and flow shop problems, various objective functions, including the makespan, the weighted sum of completion times, the maximum lateness and maximum cost functions are discussed, respectively.

References

1 Alidaee B, Womer N K. Scheduling with time dependent processing times: Review and extensions. *Journal of Operational Research Society*, 1999, **50** (5): 711~720

2 Browne S, Yechiali U. Scheduling deteriorating jobs on a single processor. *Operations Research*, 1990, **38**(3): 495~498

3 Mosheiov G. Scheduling jobs under simple linear deterioration. *Computers and Operations Research*, 1994, **21**(6): 653~659

4 Kunnathur A S, Gupta S K. Minimizing the makespan with late start penalties added to processing time in a single facility scheduling problem. *European Journal of Operational Research*, 1990, **47**(1): 56~64

5 Sundararaghavan P S, Kunnathur A S. Single machine scheduling with start time dependent processing times: Some solvable cases. *European Journal of Operational Research*, 1994, **78**(3): 394~403

6 Bachman A, Janiak A. Minimizing maximum lateness under linear deterioration. *European Journal of Operational Research*, 2000, **126**(4): 557~566

7 Yang D-L, Chern M-S. A generalized two-machine flow shop scheduling problem with processing time linearly dependent on job waiting- time. *Computers and Industrial Engineering*, 1999, **36**(3): 365~378

8 Johnson S M. Optimal two and three stage production schedules with setup times included. *Naval Research Logistics Quarterly*, 1954, **11**(1): 61~68

9 Pinedo M L. Scheduling: Theory, Algorithms and Systems. NJ: Prentice-Hall, Englewood Cliffs, 1995. 28~31

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具有线性恶化加工时间的调度问题

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摘 要 讨论了工件具有线性恶化加工时间的调度问题.在这类问题中,工件的恶化函数为线性函数.对单机调度问题中目标函数为极小化最大完工时间加权完工时间和,最大延误以及最大费用等问题分别给出了最优算法.对两台机器极小化最大完工时间的 Flowshop 问题,证明了利用 Johnson 规则可以得到最优调度.对于一般情况,如果同一工件的工序的加工时间均相等,则 Flowshop 问题可以转化为单机问题.

关键词 调度, 单机, Flowshop, 线性恶化

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