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Hybrid System Monitoring and Diagnosing Based on Particle Filter Algorithm[1)]

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Abstract State estimation of hybrid system, which consists of discrete state estimation and continuous state estimation, is a critical issue in hybrid system study. Utilizing the advantage of particle filter, which can estimate the discrete and continuous states simultaneously, we propose a new approach for monitoring and diagnosing of hybrid system. After giving the derivation of algorithm and design steps, we discuss the issues arising from algorithm implementation and propose an improved algorithm. The result of simulation shows the feasibility of particle filter in hybrid system monitoring and diagnosing, and it also demonstrates that the proposed improved algorithm can achieve better result.

Key words State estimation, hybrid spstem, particle filter, fault diagnosis

1 Introduction

With the technology development and ubiquitous application of computers in engineering, complicated systems have become prevalent in engineering environment today. Usually, complex systems comprise discrete and continuous states simultaneously and are called hybrid systems, in which two type states interact each other. It is far from satisfying the requirement to use only traditional analysis and synthesis approaches in analysis or design of such systems. So, the study of hybrid system theorem have attracted more and more attention since the end of last century. States estimation and diagnosis of hybrid systems have become an active research domain recently. In present stage of hybrid system fault diagnosis, traditional diagnosis techniques, e. g., model-based diagnosis approach etc, are indispensable means. Since hybrid system comprises discrete and continuous state variables simultaneously, its state estimation and diagnosis need some more powerful method than those traditional methods. Recently, particle filter has been used in states estimation and diagnosis of hybrid system since it can estimate discrete states and continuous state simultaneously. Motivated by this idea, we propose an approach to state estimation and diagnosis of hybrid system with particle filter. Although particle filter is effective in general state estimation, there exist some issues when it is applied to hybrid system fault diagnosis. We discuss these issues and propose a modified approach to make particle filters suitable for hybrid system fault diagnosis. Simulation results demonstrate that particle filter is a promising powerful means in state monitoring and fault diagnosis of hybrid system.

2 State monitoring and diagnosis of hybrid system

2.1 Hybrid system model and its state estimation

We call those complex dynamical systems as hybrid systems, in which interactive discrete state variables and continuous state variables coexist. Problem of state monitoring can be formulated as follows: given system model and measurement information, how to determine system states, including discrete and continuous states. State monitoring plays

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a critical role in engineering. When we treat some abrupt faults that may guickly change system operation mode as discrete states, fault diagnosis of hybrid system relates to state monitoring problem in some close way. So, we expect to apply those state estimation approaches to faults diagnosis. We do not discriminate between discrete state estimation and fault diagnosis in the sequel. State monitoring and diagnosis of hybrid system is a complicated and difficult task. Firstly, its model description is complex in general. Secondly, interactions between discrete state variables and continuous state variables make system dynamical characteristics more complicated. Finally but not the least, hybrid system is a high-autonomous large system in general. To cope with state estimation and diagnosis in such system, we need three diverse efforts as follows. The first factor is the state estimation technique, which ensures us to track the normal trajectory of hybrid system operation, i. e., given commands and measurements, to determine the operational mode of hybrid system and track continuous state variables simultaneously. The second issue is how to construct mode hypothesis set, in other words, the technique to determine fault candidates and their probability when predicted values differ from actual measurements. The third is how to determine the mode of the system , and this relates to traditional diagnosis technique closely. This paper concerns the first and third issues. The interesting point here is that we introduce a novel technique, i. e., particle filter algorithm, into hybrid system state monitoring and fault diagnosis.

2.2 Particle filter

The first particle filter algorithm was proposed by Gordon in 1993[1], in which SIR (sampling importance resampling) was proposed by Rubin[2]. Particle filter algorithm approximates states probability distribution by clusters of discrete samples (so-called particles) with corresponding weights. If every particle is regarded as an assumption of system state, we apply system stochastic model to those particles and get a new cluster particles, then compute the likelihood of every particle after measurements. The particle that predicts system performance better is endowed with a higher weight, since it is likely the state that the system should be. To prevent those particles with less weights to occupy probability density distribution, it is necessary to resample from the distribution and obtain new particle set to approximate the state distribution. Iteratively, resampling makes the approximation by those particles closer and closer to the true distribution. Compared with other filter technique (e. g. , Kalman filter), the advantage of particle filter is that it can express arbitrary distribution and follow system states naturally as continual measurements are acquired. Another merit of particle filter is that its computation burden depends on the number of particles and not on the model complexity. The three preconditions of particle filter implementation consist of: 1) Probability distribution of system states; 2) Stochastic system model; 3) Observation function O, which is used in likelihood computation.

3 Application of particle filter in hybrid system's diagnosis

3.1 System model and expected goal

One class of hybrid systems can be described by JMLM (Jump Markov Linear Model) equation

$$\begin{aligned} s_t &\sim P(s_t \mid s_{t-1}) \\ \boldsymbol{x}_t &= A(s_t)\boldsymbol{x}_{t-1} + B(s_t)\boldsymbol{u}_t + F(s_t)\boldsymbol{\eta}_t \\ \boldsymbol{y}_t &= C(s_t)\boldsymbol{x}_{t-1} + D(s_t)\boldsymbol{u}_t + G(s_t)\boldsymbol{\xi}_t \end{aligned} \tag{1}$$

where $\boldsymbol{y}_t \in \mathbb{R}^{n_y}$ denotes observations, $\boldsymbol{x}_t \in \mathbb{R}^{n_x}$ denotes unknown continuous states of the system, $\boldsymbol{u}_t \in \mathbb{R}^{n_u}$ is known control signals, $s_t \in \{1, \cdots, n_s\}$ denotes unknown discrete states including those abrupt faults we discussed above . It is assumed that the noise processes are ***i.i.d*** Gaussian, i.e., $\eta_t \sim \mathcal{N}(0, I)$ and $\xi_t \sim \mathcal{N}(0, I)$. So continuous states and output can be express with continuous density as the following:

$$p(\boldsymbol{x}_t \mid s_t, \boldsymbol{x}_{t-1}) = \mathcal{N}(A(s_t)\boldsymbol{x}_{t-1} + B(s_t)\boldsymbol{u}_t, F(s_t)F(s_t)^{\mathrm{T}})$$
$$p(\boldsymbol{y}_t \mid \boldsymbol{x}_t, s) = \mathcal{N}(C(s_t)\boldsymbol{x}_{t-1} + D(s_t)\boldsymbol{u}_t, G(s_t)G(s_t)^{\mathrm{T}})$$

where $(A, B, C, D, E, F, P(z_t|z_{t-1}))$ are system matrices with appropriate dimensions, depending on discrete state of operation s_t. It is assumed that $F(s_t)F(s_t)^{\mathrm{T}} > 0$ for any s_t. The initial states are $\boldsymbol{x}_0 \sim \mathcal{N}(\mu_0, \Sigma_0)$ and $s_0 \sim P(s_0)$. It is assumed that one can compute continuous system states accurately by linear Gaussian model, if discrete state s_t is given. In general, state estimation of hybrid system comprises estimation of continuous state and discrete state. The problem can be formulated as follows: how to determine discrete state s and continuous state $\boldsymbol{x}$ by system models when system output $\boldsymbol{y}_{1:t}$ are given, i.e., estimate discrete and continuous state probability distributions $p(s_{0:t}|y_{1:t})$ and $p(x_{0:t}|y_{1:t})$ (here $\boldsymbol{x}_{i:j}$ denotes all $\boldsymbol{x}$ in $[i, j]$).

In Bayesian monitoring, one usually derives $p(s_{0:t}|\boldsymbol{y}_{1:t})$ and $p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t})$ from the posteriori distribution $p(\boldsymbol{x}_{0:t}, s_{0:t}|\boldsymbol{y}_{1:t})$ by standard marginalization. When we only concern about the probability of state in present time, we can simplify them as $p(s_t|\boldsymbol{y}_{1:t})$ and $p(\boldsymbol{x}_t|\boldsymbol{y}_{1:t})$. In general, one can expect to compute posteriori density recursively, i.e.,

$$p(\boldsymbol{x}_{0:t}, s_{0:t} \mid \boldsymbol{y}_{1:t}) = p(\boldsymbol{x}_{0:t-1}, s_{0:t-1} \mid \boldsymbol{y}_{1:t-1}) \cdot \frac{p(\boldsymbol{y}_t \mid \boldsymbol{x}_t, s_t)\, p(\boldsymbol{x}_t, s_t \mid \boldsymbol{x}_{t-1}, s_{t-1})}{p(\boldsymbol{y}_t \mid \boldsymbol{y}_{t-1})}.$$

Apparently, it is an intractable problem. One should tackle it with some numeric approximation techniques, e.g., particle filter, etc.

3.2 Particle filter and its improvement

In particle filter algorithm, the posteriori distribution of probability is approximated by a set of weighted samples, i.e., particles $\{(\boldsymbol{x}_{0:t}^i, s_{0:t}^i), w_{0:t}^i\}_{i=1}^N$. One can approximate it as the following point-mass distribution $p_N(\mathrm{d}\boldsymbol{x}_{0:t}, s_{0:t}|\boldsymbol{y}_{1:t}) = \sum_{i=1}^{N} w_t^i \delta_{x_{0:t}^i, s_{0:t}^i}(\mathrm{d}\boldsymbol{x}_{0:t}, s_{1:t})$, where $\delta_{(\cdot)}(\cdot)$ is Dirac-delta function, and $\mathrm{d}\boldsymbol{x}$ denotes continuous distribution. Since $p(\boldsymbol{x}_{0:t}, s_{0:t}|\boldsymbol{y}_{1:t}) = p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t}, s_{0:t})p(s_{0:t}|\boldsymbol{y}_{1:t})$, it is possible to design a more effective algorithm. Here, density $p(\boldsymbol{x}_{0:t}|\boldsymbol{y}_{1:t}, s_{0:t})$ is Gaussian, and can be computed analytically if marginal density $p(s_{0:t}|\boldsymbol{y}_{1:t})$ is given. The last term in the above equation can be deduced recursively as

$$p(s_{0:t}| \boldsymbol{y}_{1:t}) = p(s_{0:t-1}| \boldsymbol{y}_{1:t-1}) \times \frac{p(\boldsymbol{y}_t \mid \boldsymbol{y}_{1:t-1}, s_{0:t})\, p(s_t \mid s_{t-1})}{p(\boldsymbol{y}_t \mid \boldsymbol{y}_{1:t-1})} \tag{2}$$

Assume that the posteriori distribution can be expressed by a weighted sample set $\{s_{0:t}^i, w_t^i\}_{i=1}^N$ as $p_N(s_{0:t}|\boldsymbol{y}_{1:t}) = \sum_{i=1}^{N} w_t^i \delta_{s_{0:t}^i}(s_{1:t})$. The posteriori distribution of $\boldsymbol{x}_{0:t}$ is a Gaussian mixture, and it can be computed effectively with a bank of Kalman filters as

$$p_N(\boldsymbol{x}_{0:t} \mid \boldsymbol{y}_{1:t}) = \int p(\boldsymbol{x}_{0:t} \mid s_{0:t}, \boldsymbol{y}_{1:t})\,\mathrm{d}p(s_{0:t}| \boldsymbol{y}_{1:t}) = \sum_{i=1}^{N} w_t^i p(\boldsymbol{x}_{0:t} \mid \boldsymbol{y}_{1:t}, s_{0:t}^i)$$

Generally, it is impossible to sample from real posteriori distribution P in implementation of particle filter. Alternatively, one can sample from the known distribution $\boldsymbol{Q}$, and multiply those samples from it by weight $\mathrm{Pr}_P(s)/\mathrm{Pr}_Q(s)$, i.e., the ratio of likelihood of samples from the two distributions. It is assumed that $p_N(\mathrm{d}\boldsymbol{x}_{0:t}, s_{0:t}|\boldsymbol{y}_{1:t})$ is replaced by $q(\mathrm{d}\hat{\boldsymbol{x}}_{0:t-1}^i, \hat{s}_{0:t-1}^i|\boldsymbol{y}_{1:t-1})$ in the sampling stage, where $\hat{x}_t$ denotes state estimation in time t. A new path $\{\hat{\boldsymbol{x}}_{0:t}^i, \hat{s}_{0:t}^i\}_{i=1}^N$ is obtained by integral $\int q(\mathrm{d}\hat{\boldsymbol{x}}_{0:t}, \hat{s}_{0:t}|\boldsymbol{x}_{0:t-1}, s_{0:t-1}, \boldsymbol{y}_{1:t})\,\mathrm{d}p(\boldsymbol{x}_{0:t-1}, s_{0:t-1}|\boldsymbol{y}_{1:t-1})$ from current path $\{\boldsymbol{x}_{0:t-1}^{(i)}, s_{0:t-1}^{(i)}\}_{i=1}^N$. For convenience of integral, we only concern the particle in time t and not the whole trajectory. As a result, one can use $q(\mathrm{d}\hat{\boldsymbol{x}}_t, \hat{s}_t|\boldsymbol{x}_{0:t-1}, s_{0:t-1}, \boldsymbol{y}_{1:t})\delta_{x_{0:t-1}, s_{0:t-1}}(\mathrm{d}\hat{\boldsymbol{x}}_{0:t-1}, \hat{s}_{0:t-1})$ to replace term $q(\mathrm{d}\hat{\boldsymbol{x}}_{0:t}, \hat{s}_{0:t}|\boldsymbol{x}_{0:t-1}, s_{0:t-1},$

$y_{1:t}$) in the above integral. All in all, particles gotten from $q(\cdot)$ must be multiplied by importance weight:

$$w_t = \frac{p(\mathrm{d}\hat{x}_{0:t}, \hat{s}_{0:t} \mid y_{1:t})}{q(\mathrm{d}\hat{x}_{0:t}, \hat{s}_{0:t} \mid y_{1:t})} = \frac{p(\mathrm{d}x_{0:t-1}, s_{0:t-1} \mid y_{1:t})}{p(\mathrm{d}x_{0:t-1}, s_{0:t-1} \mid y_{1:t-1})} \times \frac{p(\mathrm{d}\hat{x}_t, \hat{s}_t \mid x_{0:t-1} s_{0:t-1} y_{1:t})}{q(\mathrm{d}\hat{x}_t, \hat{s}_t \mid x_{0:t-1} s_{0:t-1} y_{1:t})} \propto \frac{p(y_t \mid \hat{x}_t, \hat{s}_t) p(\hat{x}_t, \hat{s}_t \mid x_{0:t-1}, s_{0:t-1}, y_{1:t})}{q(\hat{x}_t, \hat{s}_t \mid x_{0:t-1}, s_{0:t-1}, y_{1:t})}$$

Here, the prior $p(\hat{x}_t, \hat{s}_t | x_{0:t-1}, s_{0:t-1}, y_{1:t})$ is simplified as Markov density $p(\hat{x}_t, \hat{s}_t | x_{t-1}, s_{t-1}) = p(\hat{x}_t | x_{t-1}, s_{t-1}) p(\hat{s}_t | s_{t-1})$. Apparently, it is difficult to obtain the optimal suggestion distribution $q(\cdot \mid \cdot) = p(\mathrm{d}\hat{x}_t, \hat{s}_t | x_{0:t-1}, s_{0:t-1}, y_{1:t})$, so we use $q(\cdot \mid \cdot) = p(\mathrm{d}\hat{x}_t | x_{t-1}, s_{t-1}) p(\hat{s}_t | s_{t-1})$ instead. From the above formula , we can compute important weight by likelihood function, i. e., $w_t \propto p(y_t | \hat{x}_t, \hat{s}_t)$.

We obtain different particle filter algorithms when different function of optimal suggestion is adopted in important weight computation. One simplification of particle filter is introduced by Rao-Blackwell theorem [3~5], which is a well-known result of statistical probability theory[6,7]. With formula (2) and its deduction above, computation of $p(x_{0:t}, s_{0:t} | y_{1:t})$ is divided into two steps. One samples discrete state firstly, and then computes continuous states distribution using Kalman filter with corresponding model. Original problem turns into sampling from posterior $p(s_{0:t} | y_{1:t})$, and important weight is computed as:

$$w_t = \frac{p(s_{0:t} \mid y_{1:t})}{q(s_{0:t} \mid y_{1:t})} = \frac{p(s_{0:t-1} \mid y_{1:t})}{p(s_{0:t-1} \mid y_{1:t-1})} \frac{p(s_t \mid s_{0:t-1}, y_{1:t})}{q(s_t \mid s_{0:t-1}, y_{1:t})} \propto \frac{p(y_t \mid y_{1:t}, s_{0:t}) p(s_t \mid s_{0:t-1}, y_{1:t-1})}{q(s_t \mid s_{0:t-1}, y_{1:t})}$$

where the suggested distribution is chosen as $q(s_{0:t} | y_{1:t}) = q(s_t | s_{0:t-1}, y_{1:t}) p(s_{0:t-1} | y_{1:t-1})$. To avoid integral problem, we do not sample from past trajectory and use transition prior $q(s_t | s_{0:t-1}, y_{1:t}) = p(s_t | s_{0:t-1}, y_{1:t-1}) = p(s_t | s_{t-1})$.

Before computing important weight we consider the following issues. At first, we can compute output predictive density as $p(y_t | y_{1:t-1}, s_{1:t})$. On the other hand, one can notice that the optimal suggested distribution in last formula is $q(s_t | s_{0:t-1}, y_{1:t}) = p(s_t | s_{0:t-1}, y_{1:t})$, which satisfyies Bayesian rule $p(s_t | s_{0:t-1}, y_{1:t}) = p(y_t | y_{1:t-1}, s_{0:t}) p(s_t | s_{0:t-1}, y_{1:t-1}) / p(y_t | y_{1:t-1}, s_{0:t-1})$. As a result the important weight is simplified as $w_t \propto p(y_t | y_{1:t-1}, s_{0:t-1}) = \sum_{s_t=1}^{n_s} p(y_t | y_{1:t-1}, s_{0:t-1}, s_t) p(s_t | s_{t-1})$, where n_s is the number of discrete states. Convergence result of particle filter could be found in [4] and its references.

3.3 Implementation of particle filter algorithm

It is assumed that particle number is N. We implement weighted important sample in two stages by Rao-Blackwell theorem. Firstly we sample from discrete distribution $p(s_t | s_{t-1})$, and then we compute important weight of particle by continuous outputs predictive density $p(y_t | y_{1:t-1}, s_{1:t}) = \mathcal{N}(y_t | y_{t|t-1}, S_{1:t})$. Implementation steps of Rao-Blackwellised particle filter are given as follows:

Step 1 (Initialization). the aim is to produce initial values of particle filter stochastically. Sample N initial values $\{s_0^i\}_{i=1}^N$ from discrete state set $S = \{1, \cdots, n_s\}$ stochastically, and then continuous state initial values $\{x_0^i\}_{i=1}^N$, so we get a set $\{x_0^i, s_0^i\}_{i=1}^N$ with N particles.

Step 2 (Sample of discrete states). at time t, one samples N samples $\{\hat{s}_t^i\}_{i=1}^N$ from transition prior $\hat{s}_t \sim p(s_t | s_{t-1})$ as a prediction of discrete state.

Step 3 (Continuous prediction). predict system states and output with Kalman filter algorithm:

$$\begin{aligned} x_{\mathrm{pre}}^i(t) &= A(\hat{s}_t^i) x^i(t-1) + B(\hat{s}_t^i) u(t) \\ \Sigma_{\mathrm{pre}}^i &= A(\hat{s}_t^i) \Sigma^i A(\hat{s}_t^i)' + F(\hat{s}_t^i) F(\hat{s}_t^i)' \\ K^i &= C(\hat{s}_t^i) \Sigma_{pre}^i C(\hat{s}_t^i)' + G(\hat{s}_t^i) G(\hat{s}_t^i)'; \end{aligned}$$

$$y^i_{\text{pre}}(t) = C(\hat{s}^i_t)x^i_{pre}(t) + D(\hat{s}^i_t)u(t)$$

Step 4 (Computation of important weight). discrete state s^i is computed as follows: the variance between prediction and measurements $y(t)$ is given as $\varepsilon = y(t) - y^i_{\text{pre}}(t)$, and $w^i_t \propto p(y_t | y_{1:t-1}\hat{s}^i_t)$, then $w^i_t = \exp(-\frac{1}{2}\varepsilon'(K^i)^{-1}\varepsilon)/\sqrt{|K^i|}$. We get these normalized weights $w^i_t = w^i_t / \sum_{i=1}^{N} w^i_t$ at last.

Step 5 (Important sampling). according to high or low of important weight of particle, we decide to multiply or discard samples and form a particle set $\{s^i_t\}^N_{i=1} = \mathcal{F}(\{\hat{s}^i_t\}^N_{i=1})$ which is used as posterior in the next circle, where the algorithm $\mathcal{F}$ can be found in [4].

Step 6 (Update). update Kalman filter as follows and return to step 2:

$$x^i(t) = x^i_{\text{pre}}(t) + \Sigma^i_{\text{pre}}C(s^i_t)'(K^i)^{-1}(y(t) - y^i_{\text{pre}}(t))$$

$$\Sigma^i = \Sigma^i_{\text{pre}} - \Sigma^i_{\text{pre}}C(s^i_t)'(K^i)^{-1}C(s^i_t)\Sigma^i_{\text{pre}}$$

3.4 Issues arising from diagnosis based on particle filter

As aforementioned, particle filter describes system state probability distribution by a swarm of particles, which are endowed weights evaluated according to their predictive precision with measurements. In the important sampling, these weighted particles are multiplied or discarded to form the prior used in recurrence. Recursively, the approximation is closer and closer to the true state distribution. In particle filter algorithm, number of distinct discrete states in particles set (i. e., possible system mode) decreases and converges to some state gradually. So particle filter tracks the system mode effectively. In some sense, particle filters are suitable for state estimation of hybrid system as mentioned above.

Since we consider some abrupt faults as a discrete state in hybrid system as mentioned previously, particle filter can be applied to diagnosis of such faults in theoretical speaking. Unfortunately, some problems arise when particle filter is sued in diagnosis task. At first, the fault state is possibly detected only when it is included in the particle set. When fault is excluded from the particle set, the probability it becomes the last state of particle filter is zero. Since fault transition probability is very low, the probability fault state is sampled and included in particle set is very low. Consequently, it is difficult to be detected by particle filter algorithm in time. It is one result of phenomena so-called "Sample Impoverishment". On the other hand, one may think it is true that once a fault state is included in sample set, it will be replicated with more offspring in resample stage when fault occurred. But it is not true in practice. Due to transient dynamics characteristic arising from such abrupt fault, the filter predicts system output poorly, thus its weight is low and it is excluded from particle set[8].

One should cope with these issues before applying particle filter to fault diagnosis of hybrid system. Increasing particle number is the simplest solution. Its effect is equivalent to increase of the fault probability while keeping particle number fixed. Although it is infeasible in some restricted environment since its computation burden increases as particle number increases, it is a solution to this issue in some case. Another way is to ensure those important system modes (i. e., discrete states denoting faults) are included in particle set in resampling stage with some approaches, while the particle number is fixed.

3.5 Improvement scheme of particle filter

To cope with these issues arising when we apply particle filter to fault diagnosis discussed in last subsection, we improve Rao-Blackwellised particle filter. The basic idea here is to ensure those important states (i. e., fault states) to be included in the particle set in resample stage. If they explain following measurements well, they obtain higher weights, and are replicated in resample stage of next circle, otherwise they will disappear quickly. We propose the following two improved approaches to ensure those important states(i. e., all possible fault candidates) to be included in sampleset in some way.

i) To sample N particles from prior probability as the usual particle filter algorithm does, then increase the number of those discrete states with 0 samples in sample set to a small positive integer (e. g., 1) . The number of discrete state with the largest number decreases to a corresponding integer in the meantime.

ii) To assign every discrete state 1 sample firstly, then sample $N-n_s$ samples from prior probability as the usual particle filter algorithm.

Here we adopt the first method. Then we only need to modify the second stage in particle filter algorithm:

Step 2 (Sample of discrete states) at time t, one samples N samples $\{\hat{s}_t^i\}_{i=1}^N$ from transition prior $\hat{s}_t \sim p(s_t|s_{t-1})$ as a prediction of discrete state. Then check the members of the particle set. If the particle number of some discrete state is 0, then force it to be 1 and subtract 1 from the particle number of the state with maximum particles, thus we get a new particle set $\{\hat{s}_t^i\}_{i=1}^N$.

4 Simulation and its results analysis

To demonstrate the feasibility of algorithm proposed here, we consider the system as follows:

$$\begin{cases} \boldsymbol{x}(k) = A(s)\boldsymbol{x}(k-1) + B(s)u(k) + F(s)\eta(k) \\ \boldsymbol{y}(k) = C(s)\boldsymbol{x}(k) + D(s)u(k) + G(s)\xi(k) \end{cases}$$

where $n_x=3$, $n_s=3$ and $n_y=2$; system matrices are $A(1)=\begin{bmatrix} 0.4 & 0.15 & 0.1 \\ 0.15 & 0.6 & 0.15 \\ 0.1 & 0.1 & 0.7 \end{bmatrix}$, $A(2)=\begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.1 & 0.5 & 0.1 \\ 0.1 & 0.12 & 0.6 \end{bmatrix}$, $A(3)=\begin{bmatrix} 0.4 & 0.12 & 0.1 \\ 0.1 & 0.7 & 0.15 \\ 0.1 & 0.12 & 0.3 \end{bmatrix}$, $B(1)=B(2)=B(3)=\begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}$, $C(1)=C(2)=C(3)=\begin{bmatrix} 0.4 & 0.15 & 0.1 \\ 0.1 & 0.15 & 0.3 \end{bmatrix}$, $F=0.05I_{3\times 3}$, $D(1)=D(2)=D(3)=[0 \quad 0]^{\mathrm{T}}$, $G=0.05I_{2\times 2}$, noises are $\eta \sim N(0,I)$ and $\xi \sim N(0,I)$; System input is constant 1, i. e., $u(k)=1$; system mode evolves according to Markove rule, its transition matrix is $P=\begin{bmatrix} 0.998 & 0.001 & 0.001 \\ 0.001 & 0.998 & 0.001 \\ 0.001 & 0.001 & 0.998 \end{bmatrix}$. In the simulation example here, system mode changes from mode 3 to mode 1 and from 1 to 3 at $t=50$ and 150, respectively, see Fig. 1. We hope our results of the example can help us explain several aspects as follows: 1) Whether Rao-Blackwellization could improve particle filter; 2) Give some tips for choice of particle num-

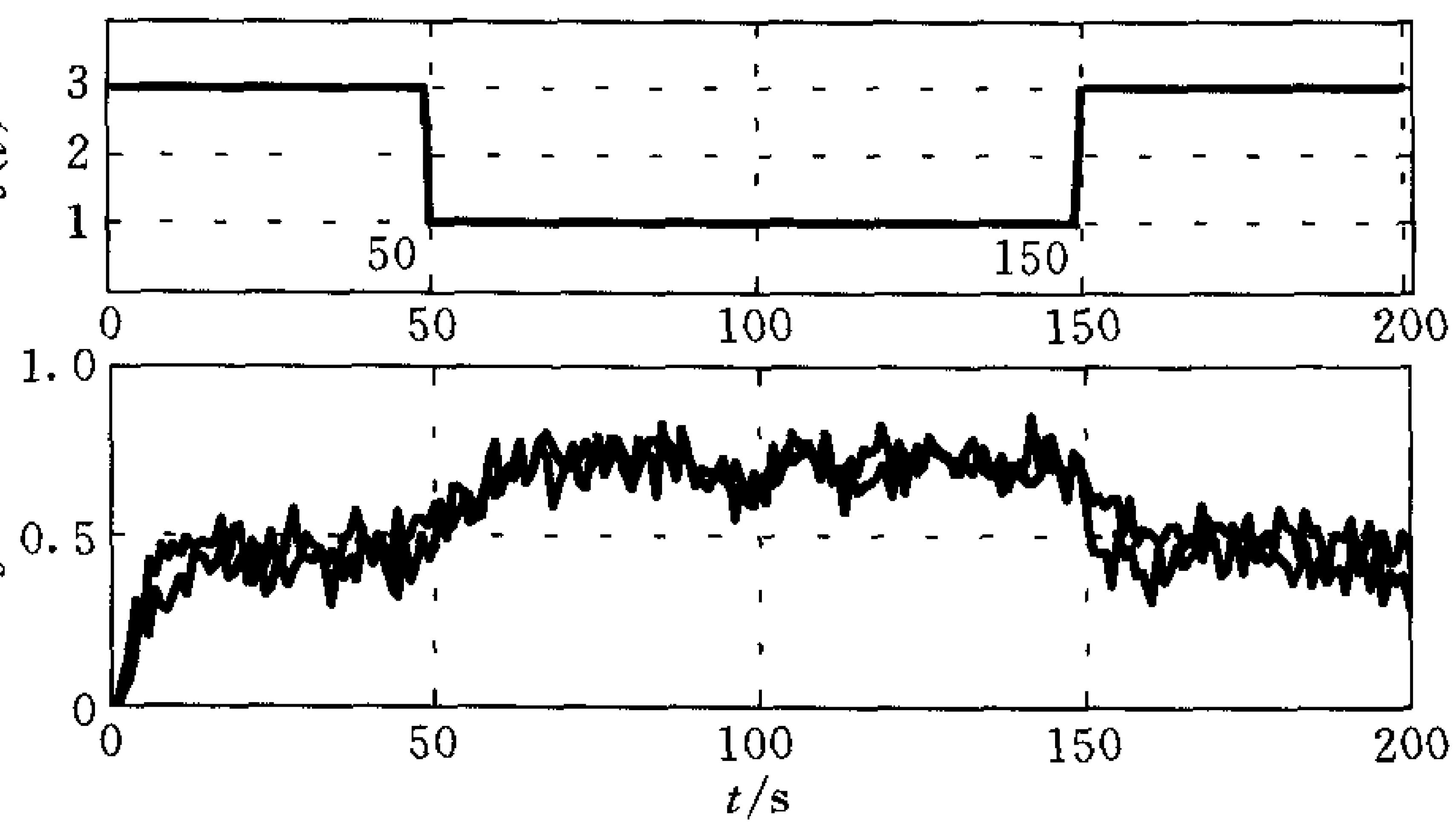

Fig. 1 Behaviors of system

ber; 3) Verify the effectiveness of the improved algorithm.

4.1 Comparison between general particle filter and Rao-Blackwellised particle filter

To compare RBPF (Rao-Blackwellised Particle filter) with PF (Particle filter), one can compare estimating error ratios and computation time along particle number increase from 50 to 500 by step size 50. Since particle filter algorithm is essentially stochastic, we repeat the experiment 30 times. The results, i. e., the mean estimating error ratio and computation time, are given in Fig. 2. It shows that the estimating error ratio of RBPF is lower than that of PF, but it consumes longer time. As a trade-off, we choose appropriate particle number as 100-200 in this example.

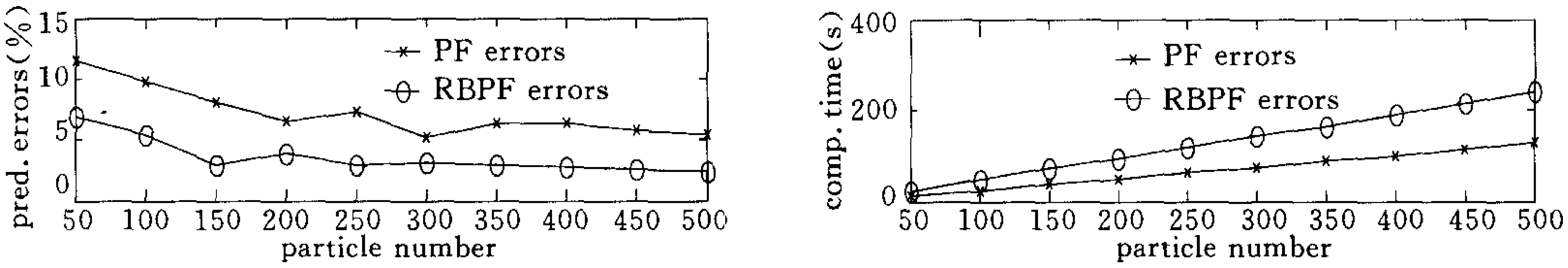

Fig. 2 Comparison of PF with RBPF

4.2 Improved effect of proposed scenario

The same experiment repeats with the improved particle filter proposed previously. Experiment result is presented in Fig. 3. Comparing it with Fig. 2, one can draw some conclusion about the improved approach. Firstly, one can obtain satisfied results when the particle number in algorithm takes a lower number than the original algorithm, especially in the case of Rao-Blackwellised particle filter algorithm. Improved algorithm consumed longer time than the generic particle filter. As compromise between error ratio and computation time, it is appropriate to choose sample number as 100 here. In the end, we demonstrate discrete state estimation results of the original algorithm and improved algorithm in Fig. 4 and Fig. 5, respectively. From two figures one can observe apparent effect of the improved scheme of particle filter. Obviously, improved algorithm proposed in this paper is feasible.

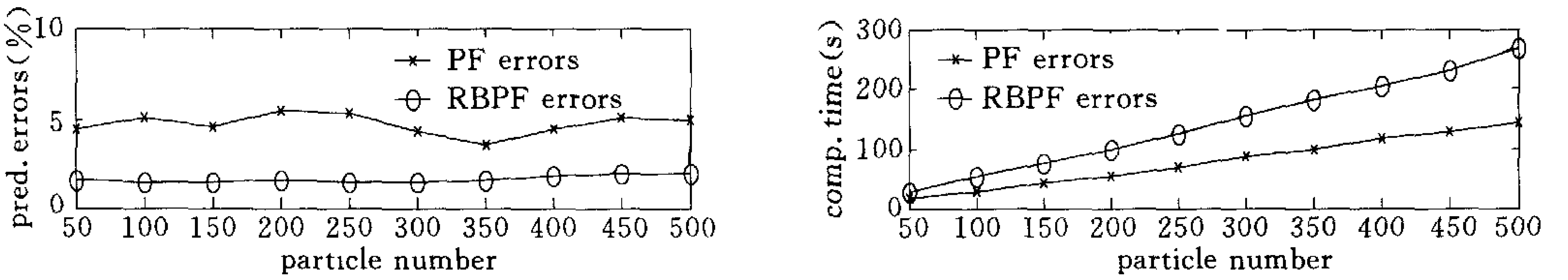

Fig. 3 Results with improved algorithm

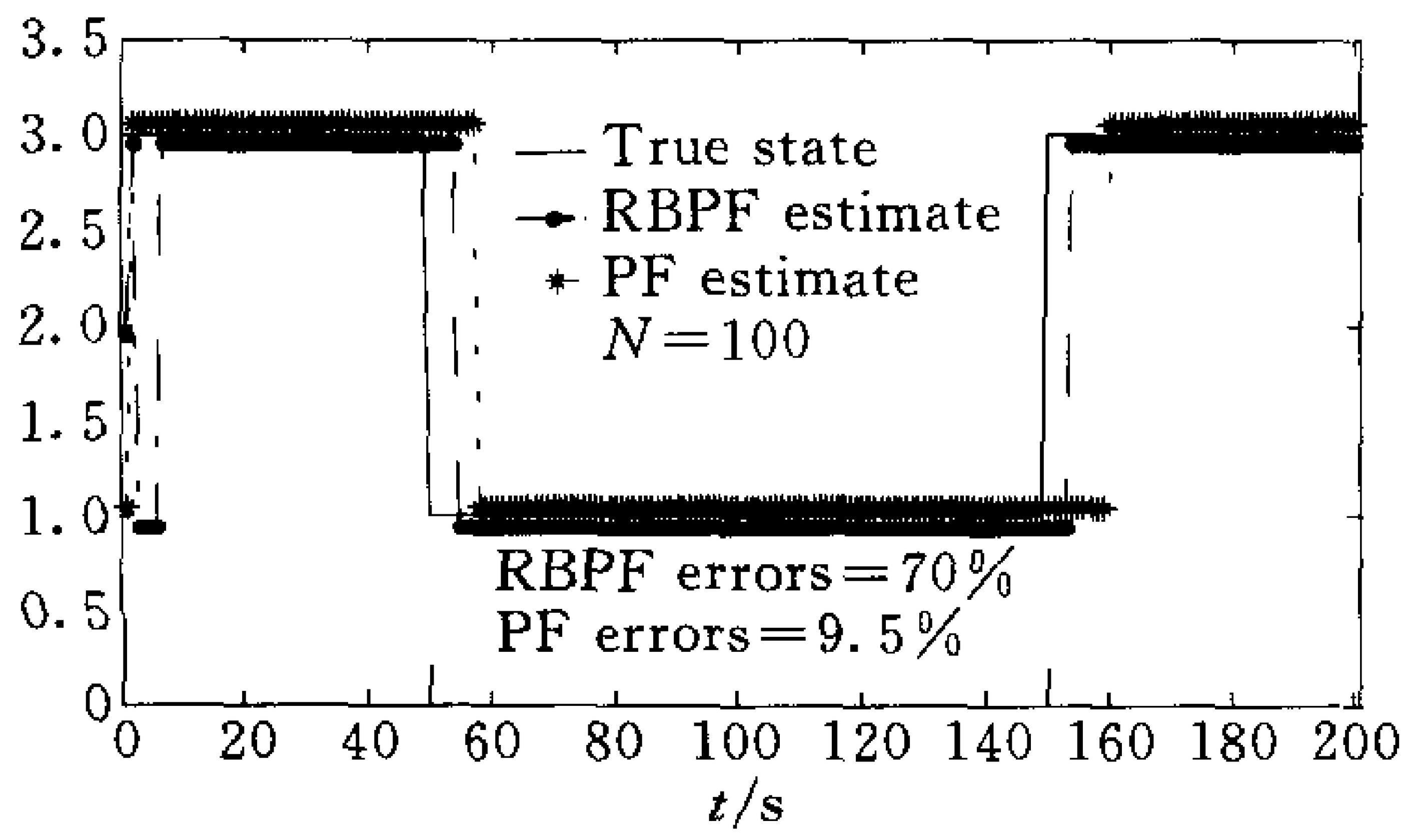

Fig. 4 Estimated results without improved algorithm

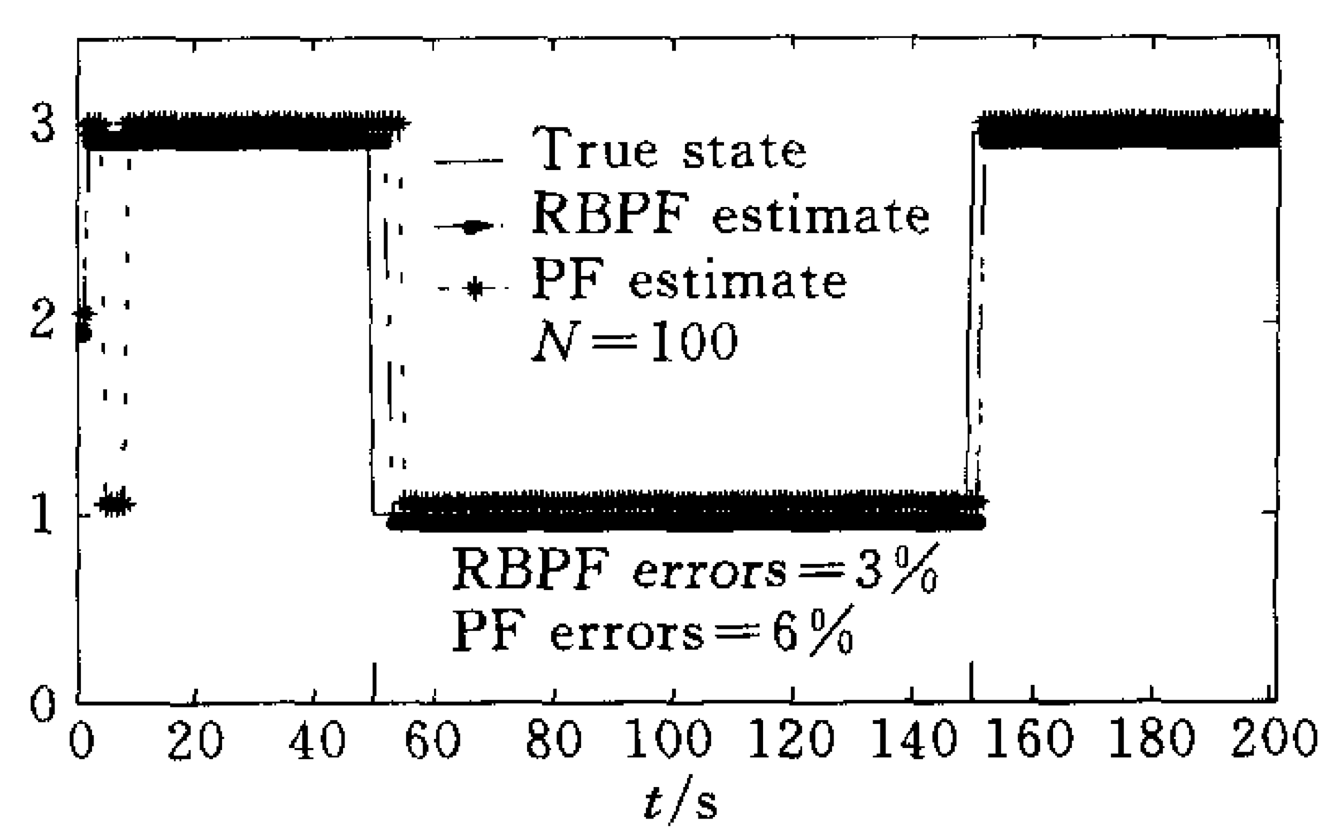

Fig. 5 Estimated results with improved algorithm

5 Conclusion

Taking the advantage of particle filter that can estimate continuous and discrete states simultaneously, we propose the state monitoring and diagnosis of hybrid system approach based on particle filter. It provides a new effective method for research on state monitoring and diagnosis of hybrid system. Simplification and deduction of particle filter are given at first. Then, we discuss issues for application of particle filter in hybrid system fault diagnosis, and propose improved algorithm and apply it to state estimation and diagnosis of hybrid system. Due to low probability of fault, it is possible that fault state is not included in particle set and it is impossible to be diagnosed. We propose the improved method to force all possible fault states to be included in particle set. The algorithm is suitable for state monitoring and diagnosis of hybrid system. Simulation results demonstrate that we can expect a better effect using the improved algorithm.

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基于粒子滤波算法的混合系统监测与诊断

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摘　要　利用粒子滤波算法具有同时估计连续状态和离散状态的特点，提出一种可用于混合系统状态监测与诊断的新方法.给出了该方法的理论推导和设计步骤，讨论了在诊断应用中粒子滤波器所遇到的问题，并给出了改善的措施.仿真结果证明用粒子滤波器对混合系统进行监测与诊断是可行的，所提的方法对估计结果有比较好的改善.

关键词　状态估计，混合系统，粒子滤波器，故障诊断

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