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Algorithm for Performance Sensitivity Estimation of Markov Discrete Event Dynamic System[1)]

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Abstract An efficient simulation algorithm for performance sensitivity estimation of Markov discrete event dynamic system is designed. This algorithm can give consistent and reliable results for both steady state and transient performance sensitivity estimation. Compared with other simulation based sensitivity estimation methods, the proposed algorithm is simple in mathematical description and is easy in program realization. The capability of this algorithm is illustrated with several examples. Furthermore, our research shows that performance sensitivity of Markov discrete event dynamic system is the sum of the estimation obtained from both smoothed perturbation analysis (SPA) and likelihood ratios method (LR), singly using any one of them usually can not give consistent and reliable result.

Key words Markov system, sensitivity evaluation, performance measure, simulation

1 Introduction

Performance sensitivity estimation plays an important role in the analysis, designing and optimization of discrete event dynamic system (DEDS). Due to the lack of effective and easy computable analytic model, simulation is the conventional approach, sometimes even the only means to analyze, evaluate and optimize such system[1]. Since Markov model is the most fundamental model for stochastic systems, efficient sensitivity estimation algorithm for Markov DEDS is very useful. Although there are efficient algorithms[2~4] for performance evaluation of such system, the estimation of performance sensitivity with respect to system parameters is still full of challenge. IPA[5] and LR[6] are two representative methods for sensitivity estimation of DEDS. Unfortunately, both of them have strong restriction and are only applicable to limited applications[5~14]. When applied to Markov system they usually give incorrect results[9~14]. To widen the application of IPA method, Gong[8] presented the idea of smoothed perturbation analysis (SPA), but it still has certain restriction. In recent years, performance sensitivity estimation of Markov system has achieved great breakthrough. Several algorithms that can give consistent results for sensitivity estimation of steady state Morkov system or discrete Markov chain were presented. Dai[9], Dai and Ho[10] put forward the SIPA method for sensitivity estimation of discrete Markov chain. Cao, Yuan and Lin[11] introduced the idea of perturbation realization for sensitivity estimation of discrete Markov chain. In [12,13], Cao and his colleague further developed the sensitivity estimation algorithm based on the potential theory for steady state Markov system. Dai[14] presented the PA method via coupling for steady state Markov system. The algorithm derived by Cao[12,13] is one of the most successful approaches for sensitivity estimation, some good application of Cao's algorithm can be found in [15~17].

Recently, most published sensitivity estimation work has focused on steady state system. In this paper, new algorithm for performance sensitivity estimation of Markov system is presented. This algorithm can be applied to steady state as well as transient per-

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formance sensitivity estimation, the later is often met in system reliability evaluation. The proposed algorithm is based on the Improved Standard Clock (ISC) method presented in [4], and has the advantages of simple mathematical description and easy program realization.

2 General description of the sensitivity estimation of DEDS

System performance measurement of DEDS is typically described as the following form [2~5]

$$J(\theta) = E[L(\theta,\boldsymbol{\omega})] \approx \frac{1}{N}\sum_{i=1}^{N} L(\theta,\omega_i) \tag{1}$$

where θ is a parameter of the system, $\boldsymbol{\omega}$ is a random vector defined on probability space, $L(\theta,\omega_i)$ is a performance sample extracted from ith sample path, N is the total number of simulation trials. According to the strong law of large number, the above estimation converges with probability 1 as $N\to\infty$.

$\partial J(\theta)/\partial\theta$ is called the system performance sensitivity with respect to θ, the simplest way to estimate sensitivity is via difference method:

$$\frac{\partial J(\theta)}{\partial\theta} = \frac{\partial E[L(\theta,\boldsymbol{\omega})]}{\partial\theta} \approx \frac{1}{N\Delta\theta}\Big[\sum_{i=1}^{N} L(\theta+\Delta\theta,\omega_i) - \sum_{i=1}^{N} L(\theta,\omega_i)\Big] \tag{2}$$

However, this approach is very inefficient. Cao[18] has proved that if perturbation sample path and nominal sample path adopt common random numbers technique, only when $N\cdot\Delta\theta\to\infty$, can Equation (2) convergence. While not employing common random numbers technique, it requires $N\cdot(\Delta\theta)^2\to\infty$. Furthermore, if $\boldsymbol{\theta}$ is a vector, each partial derivative component of $\partial J(\boldsymbol{\theta})/\partial\boldsymbol{\theta}$ needs to be estimated respectively, which is a time-consuming work.

Due to the above reason, algorithm with high efficiency for performance sensitivity estimation has drawn great attention. During the time of 1980s, a lot of algorithms such as IPA and LR were presented. The common characteristic of these algorithms is extracting the information for sensitivity estimation only from nominal sample path, rather than generating a perturbation sample path, therefore avoiding the inefficiency of difference method.

3 GSMP description of DEDS simulation

The estimation of system performance starts from sample path construction, which is customarily named as DEDS simulation. From the viewpoint of random process, the so-called DEDS simulation is to generate a sample path via Monte Carlo method. The usual way to simulate DEDS is with the event scheduling simulation scheme[1], which is based on the GSMP description of DEDS. The GSMP description of DEDS is a five-element group $\{X,E,f,\Gamma,G\}$, where X is the state set, E is the event set, and f is the state transfer function defined as

$$f: X\times E\to X \tag{3}$$

$\Gamma(x)\subseteq E$ is the feasible event set under state x, G is the event generating function set with $G_i(\cdot)$ representing the pdf of time interval of event i. Let system's initial state be x; for each feasible event $i\in\Gamma(x)$, let y_i be the residual trigger time. In DEDS simulation, $\{(i,y_i), i\in\Gamma(x)\}$ is called the future event table. Obviously, the first occurred event e' is

$$e' = \arg\min_{i\in\Gamma(x)}\{y_i\} \tag{4}$$

Now, move simulation clock t to the time of e' occurrence, and update the system as follows.

$$t' = t + y^* \tag{5}$$

$$x' = f(x,e') \tag{6}$$

$$y_i' = \begin{cases} y_i - y^*, & i\neq e' \text{ and } i\in\Gamma(x) \\ g_i, & i\in\Gamma(x') \text{ and } i\notin\Gamma(x) \end{cases} \tag{7}$$

where $y^* = \min_{i\in\Gamma(x)}\{y_i\}$ is the inter-event time span, g_i is the residual trigger time of new feasible event sampled from the pdf $G_i(\cdot)$. By repeating such steps, a sample path can be constructed. Notice that the state of a DEDS is piecewise constant and that it changes only when events occur. The sample path of DEDS can be simply represented by a sequence of triples $\{x_{k-1}, e_k, t_k\}$, $k=1,2,\cdots$, where t_k is the time of occurrence of kth event e_k. After the sample path is generated, the $L(\theta,\omega_i)$ can be extracted.

4 Improved standard clock algorithms for Markov DEDS

The Markov DEDS represents a special case of DEDS where all $G_i(\cdot)$are exponential distribution. A high efficient algorithm for performance evaluation of such system was developed in [4]. This algorithm, named as improved stand clock method (ISC), is the extension of Vakili's well-known stand clock method (SC)[2~4]. The ISC method only generates state transfer sequence $\{x_{k-1}, e_k\}$, $k=1,2,\cdots$ rather than the detail sample path as usual. ISC uses the conditional form for performance evaluation

$$J(\theta) = E[E[L(\theta,\omega) \mid Z]] \approx \frac{1}{N}\sum_{i=1}^{N} E[L(\theta,\omega) \mid Z = z_i] \tag{8}$$

where z is the state transfer sequence $\{x_{k-1}, e_k\}$, $k=1,2,\cdots$, in ith simulation trail, $E[L(\theta,\omega)\mid Z=z_i]$ is the mean performance measure under z_i, which can be calculated analytically in the case of Markov DEDS.

Suppose $G_i(t)=1-e^{-\lambda_i t}$, $i\in E$. Let the current state be x_{k-1}, and define

$$\Lambda(x_{k-1}) = \sum_{j\in\Gamma(x_{k-1})} \lambda_j \tag{9}$$

The evolution of the system has the following characteristics[2~4]:

1) The next event e_k is only state dependent and decoupled with time. The pdf of next event is

$$P\{e_k = j \mid x_{k-1}\} = \frac{\lambda_j}{\Lambda(x_{k-1})}, \quad j\in\Gamma(x_{k-1}) \tag{10}$$

2) The inter-event time y^* is also state dependent and obeys exponential distribution with parameter $\Lambda(x_{k-1})$.

$$F(y^* \mid x_{k-1}) = 1 - \exp[-\Lambda(x_{k-1})t] \tag{11}$$

Thus, if the initial state x_0 is specified, the corresponding state transfer sequence z can be easily and efficiently generated in terms of (10) and (6). After z is generated, $E[L(\theta,\omega)\mid Z=z_i]$ can be extracted on the basis of the above characteristics 2. In the case of integral style performance function, i. e.,

$$L(\theta,\omega_i) = \int h(x)\mathrm{d}t = \sum h(x_{k-1})y_k^* \tag{12}$$

it is easy to obtain

$$E[L(\theta,\omega) \mid Z = z_i] = \sum h(x_{k-1})/\Lambda(x_{k-1}) \tag{13}$$

where $h(x)$ is the state dependent weight function.

The algorithm of ISC presented in [4] has the capability of simultaneously generating a family of state transfer sequences with respect to different parameter set in one simulation run. Assuming that the system parameter set has M possible configurations, denoted by $\{\lambda_i^m, i\in E\}$, $m=1,2,\cdots,M$, the algorithm for simultaneous construction of M state transfer sequences is given as follows[4].

Algorithm

1) Generate random number $u \backsim U(0,1)$

for $m=1$ to M do {

2) On the basis of u, generate e_k^m according its pdf, formula (10)

3) Extract information for computation of $E[L(\theta,\omega)|Z=z]^m$

4) State updating, $x_k^m=f(x_{k-1}^m,e_k^m)$

}

5) Goto 1 until the end of simulation

6) Compute $E[L(\theta,\omega)|Z=z]^m$, $m=1,2,\cdots,M$

In N simulation runs, the system performance measurement with respect to M parameter set can be estimated simultaneously in terms of (8). From the above, we can see that ISC is tightly linked with conditional expectation variance reduction technique[1]. ISC method is more efficient than usual SC method because it batter utilizes system's prior information and dose not need to generate detail sample path.

5 ISC based parameter sensitivity estimation

Assume that the performance measure of interest is related to the dynamic process of system evolution from initial state to some specific state subsets. The steady state performance measure can be seen as the special case of above. Due to the ergodicity of steady state Markov process, system behavior only depends on the dynamic process in the regenerative cycle[1].

Let Ω be the set of all possible state transfer sequences obtained from simulation trial (or from regenerative cycle in steady state case). The system performance measurement can be expressed as

$$J(\theta) = E[E[L(\theta,\omega) \mid Z]]= \sum_{z\in\Omega}E[L(\theta,\omega) \mid Z = z]P(z) \tag{14}$$

Differentiating (14) with respect to θ, we obtain

$$\frac{\partial J(\theta)}{\partial\theta} = \sum_{z\in\Omega} \frac{\partial E[L(\theta,\omega) \mid Z = z]}{\partial\theta}P(z) + \sum_{z\in\Omega}E[L(\theta,\omega) \mid Z = z]\frac{\partial P(z)/\partial\theta}{P(z)}P(z) \tag{15}$$

By the theory of expectation of random function[19], we have

$$\frac{\partial J(\theta)}{\partial\theta} = E\left[\frac{\partial E[L(\theta,\omega) \mid Z]}{\partial\theta}\right]+E\left[E[L(\theta,\omega) \mid Z]\frac{\partial \ln P(Z)}{\partial\theta}\right] \tag{16}$$

The first item of (16) is actually the expression of SPA[8]. It reflects the sensitive of performance measure with respect to θ when the event sequence remains unchanged. The second item of (16) is nothing else but expression of LR method[6], which reflects the variation of performance measures with respect to event sequence changing caused by perturbation of θ. (16) indicates that performance sensitivity is the sum of that estimated by SPA and LR methods.

On the basis of (16) the unbiased estimation of $\partial J(\theta)/\partial\theta$ can be written as

$$\frac{\partial \hat{J}(\theta)}{\partial\theta} = \frac{1}{N}\sum_{i=1}^{N}\left\{\frac{\partial E[L(\theta,\omega) \mid Z = z_i]}{\partial\theta} + E[L(\theta,\omega) \mid Z = z_i]\frac{\partial \ln p(z_i)}{\partial\theta}\right\} \tag{17}$$

where in usual cases N is the total number of independent simulation trials, and z_i is the state transfer sequence in the ith simulation trial. In the cases of steady state, N is the total number of regenerative cycles in one simulation, and $z_i$is the state transfer sequence in ith epoch.

The estimation of $\partial J(\theta)/\partial\theta$ needs three statistical quantities: $E[L(\theta,\omega)|z_i]$, $\partial E[L(\theta,\omega)|z_i]/\partial\theta$ and $\partial \ln p(z_i)/\partial\theta$. Since $E[L(\theta,\omega)|z_i]$ can be acquired through analytic approach, so does $\partial E[L(\theta,\omega)|z_i]/\partial\theta$, i. e., in the case of integral style performance function, taking (13) into account, we have

$$\frac{\partial}{\partial\theta}E[L(\theta,\omega) \mid Z = z_i] =-\sum_{k=1}^{n}\frac{h(x_{k-1})}{[\Lambda(x_{k-1})]^2}\frac{\partial\Lambda(x_{k-1})}{\partial\theta} \tag{18}$$

where $z_i=[\{x_{k-1},e_k\}, k=1,2,\cdots,n]$, and n is the length of the sequence.

Thus, the applicable condition of (17) is that $\partial \ln p(z_i)/\partial\theta$ should be computed analytically. Fortunately, $p(z)$ can be expressed as the product of sequence state transition probability when characteristics 1 (see Section 4) is hold.

$$p(z) = p(e_1 \mid x_0)p(e_2 \mid x_1)\cdots p(e_n \mid x_{n-1}) \tag{19}$$

where $p(e_k|x_{k-1})=\lambda_{ek}/\Lambda(x_{k-1})$, $k=1,\cdots,n$.

Rewriting (19) in logarithm form and differentiating it with respect to θ yield

$$\frac{\partial \ln p(z_i)}{\partial\theta} = \sum_{k=1}^{n} \frac{\partial \ln p(e_k \mid x_{k-1})}{\partial\theta} = \sum_{k=1}^{n}\left[\frac{\partial\lambda_{e_k}/\partial\theta}{\lambda_{e_k}} - \frac{\partial\Lambda(x_{k-1})/\partial\theta}{\Lambda(x_{k-1})}\right] \tag{20}$$

Collecting and accumulating the three statistical quantities obtained from N simulation trials (or from N regenerative epochs in steady state case), the unbiased estimation of system performance sensitivity can be determined in terms of (17).

6 Simulation examples

In this section, M/M/1/K and M/M/1 queueing system are chosen as examples to verify the algorithm derived above, because such systems are widely used models and the theoretical analysis of them is perfect (see [20]).

Let the customer arrival rate be λ, the service rate be μ, and select the queue length as the state variable. The five-element group of M/M/1/K(M/M/1)system is defined as

$$X = \{0,1,2\cdots,\},\quad E = \{1,2\},\quad G = \{1-e^{-\lambda t},\ 1-e^{-\mu t}\} \tag{21}$$

$$\Gamma(x) = \begin{cases}\{1,2\}, & x>0\\ \{1\}, & x=0\end{cases},\quad f(x,e) = \begin{cases}x+1, & e=1\\ x-1, & e=2\end{cases} \tag{22}$$

where “1” denotes the event of customer arrival, and “2” denotes the event of customer leave.

6.1 Parameter sensitivity estimation of the mean crash time of M/M/1/K queue

The crash time T_c of M/M/1/K queue, which is a random variable, is defined as the first time when the queue $x(t)$ exceeds the capability queue length K under the given $x(0)$. $E[T_c]$, the mean crash time is an important measure reflecting the reliability of communication network[3,4].

In this experiment, the capability queue length and the initial state are set as $K=7$, $x(0)=0$ respectively, and the measurements of interest are $E[T_c]$, $\partial E[T_c]/\partial\lambda$ and $\partial E[T_c]/\partial\mu$.

First, construct the state transfer sequence according to the ISC method with simulation trial ending at queue length $x(t)$ exceeding K. Suppose that the state transfer sequence in the ith simulation is $z_i=[\{x_{k-1},e_k\},k=1,2,\cdots,n]$.

Next, extract three necessary statistical quantities form z_i. Using characteristics 2 in Section 4, the conditional expectation of T_c under z_i is given by

$$E[T_c \mid z_i] = \sum_{k=1}^{n} E[y_k^* \mid \Lambda(x_{k-1})] = \sum_{k=1}^{n} \frac{1}{\Lambda(x_{k-1})} \tag{23}$$

Differentiating (23), we have

$$\frac{\partial}{\partial\theta}E[T_c \mid z_i] = -\sum_{k=1}^{nz} \frac{\partial\Lambda(x_{k-1})/\partial\theta}{[\Lambda(x_{k-1})]^2},\quad \theta\in\{\lambda,\mu\} \tag{24}$$

By the definition of M/M/1/k queuing system, we have

$$\lambda_{e_k} = \begin{cases}\lambda, & e_k=1\\ \mu, & e_k=2\end{cases},\quad \Lambda(x_{k-1}) = \begin{cases}\lambda, & x_{k-1}=0\\ \lambda+\mu, & x_{k-1}>0\end{cases} \tag{25}$$

Thus, we get

$$\left[\frac{\partial\lambda_{e_k}}{\partial\lambda}\ \frac{\partial\lambda_{e_k}}{\partial\mu}\right] = \begin{cases}[1\quad 0], & e_k=1\\ [0\quad 1], & e_k=2\end{cases} \tag{26}$$

$$\left[\frac{\partial\Lambda(x_{k-1})}{\partial\lambda}\ \frac{\partial\Lambda(x_{k-1})}{\partial\mu}\right]=\begin{cases}[1\ \ 1], & x_{k-1}>0\\ [1\ \ 0], & x_{k-1}=0\end{cases} \tag{27}$$

Substituting (25)~(27) to (23), (24) and (20), we obtain the three necessary statistical quantities.

Finally, carry out N independent simulation trials, collect and accumulate the three statistical quantities in terms of (8) and (17), the unbiased estimation of $E[T_c]$, $\partial E[T_c]/\partial\lambda$ and $\partial E[T_c]/\partial\mu$ are obtained simultaneously.

Estimation of mean crash time and its parameter sensitivity under different λ,μ is shown in Table 1, which indicates that the maximal relative error is not greater than 2% for $E[T_c]$, and is less than 7% for both $\partial E[T_c]/\partial\lambda$ and $\partial E[T_c]/\partial\mu$. These results are obtained through 4000 simulation trials. Due to ISC's capability of generating multiple state transfer sequences in one simulation trial, all results in Table 1 can be estimated simultaneously.

Table 1 Mean crash time & its sensitivity estimation of M/M/1/7

$[\lambda,\mu]$	$[E[T_c], \partial E[T_c]/\partial\lambda, \partial E[T_c]/\partial\mu]$	
	Simulation results	Theoretical values
[0.60, 1]	[344.5, −2999., 1454.]	[345.8, −3182., 1563.]
[0.72, 1]	[137.9, −938.9, 538.2]	[135.3, −894.0, 508.4]
[0.84, 1]	[68.57, −321.8, 201.8]	[68.53, −331.8, 210.2]

The convergence histories of $\partial E[T_c]/\partial\theta, \theta\in\{\lambda,\mu\}$ using both ISC based algorithm and difference method are plotted in Fig. 1 and Fig. 2 respectively. The straight lines in both figures are theoretical values. In the case of difference method, the difference step is chosen as $\Delta\lambda=\Delta\mu=0.01$ (too small difference step should be avoided, see Section 2). The comparison shows that the convergence of ISC based algorithm is much faster than the difference method. Moreover, since ISC based algorithm obtains all measures of interest with only N trials while the difference method must rely on $3N$ trials for the same work, the actual computation burden of ISC based algorithm is far less than the difference method.

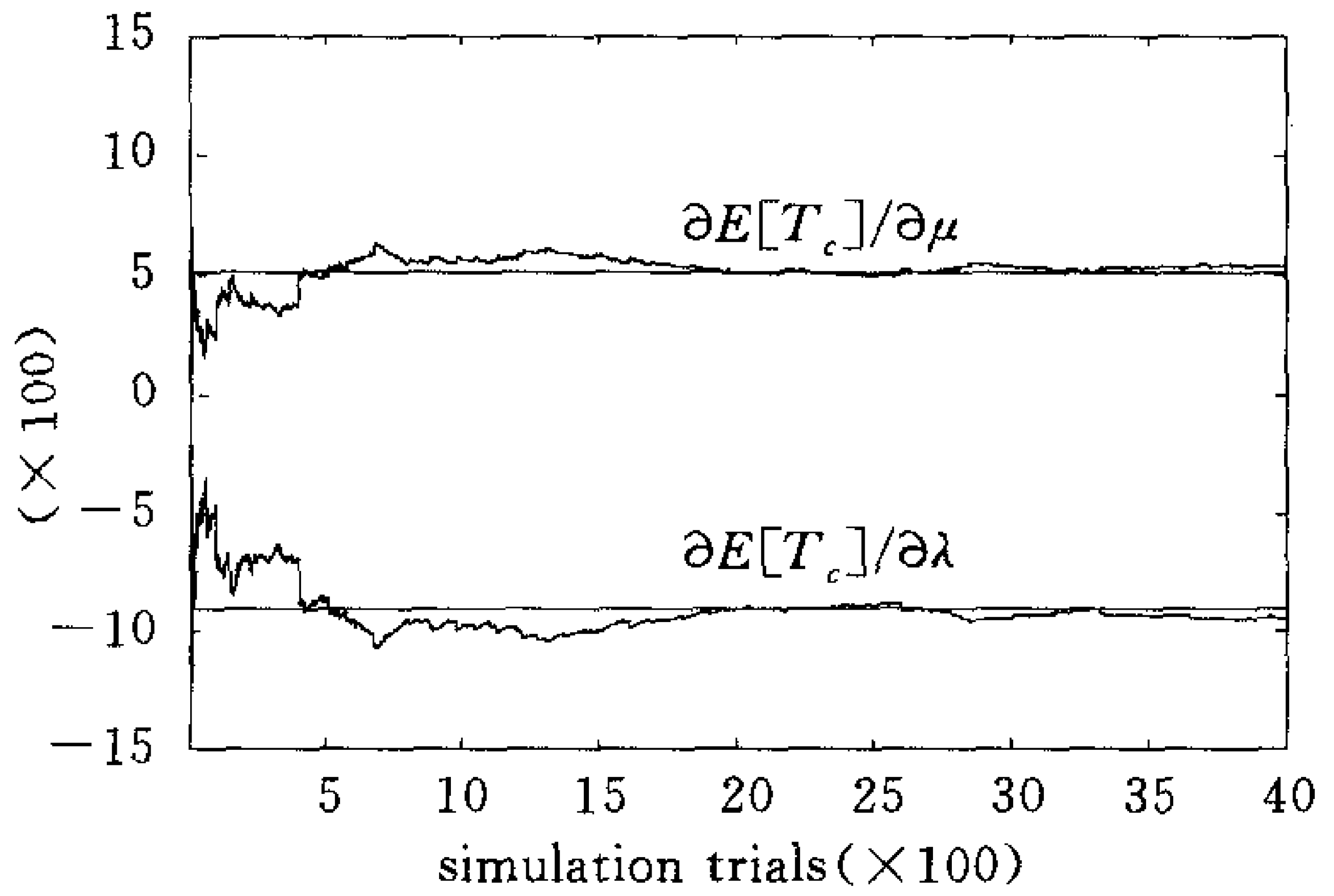

Fig. 1 Convergence history of ISC based algorithm

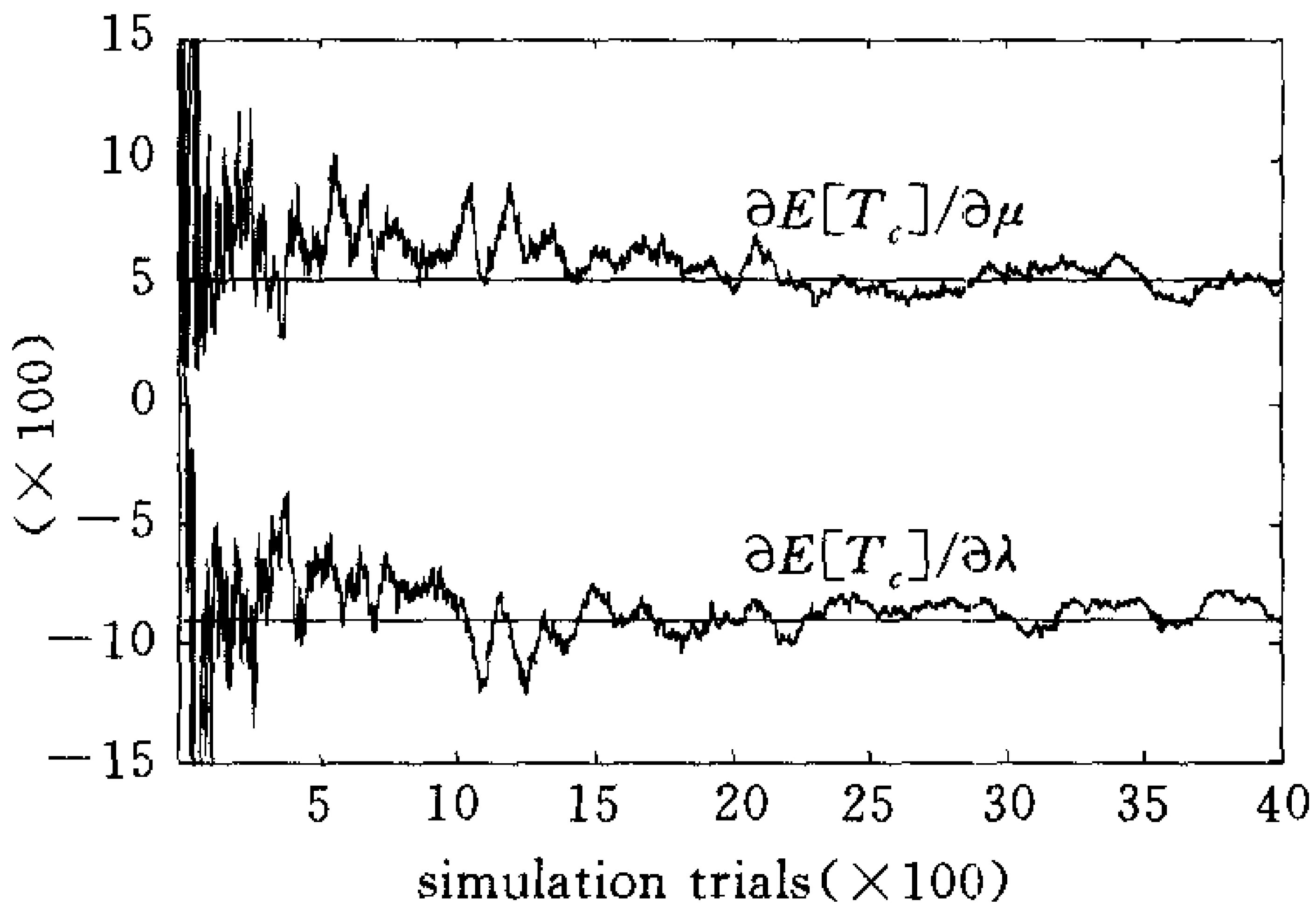

Fig. 2 Convergence history of difference method

6.2 Parameter sensitivity estimation of steady state mean length of M/M/1 queue

In this experiment, the interested performance measures are steady state mean queue length denoted by L, and $\partial L/\partial\theta, \theta\in\{\lambda,\mu\}$. By the regenerative theory[1], estimation of steady state performance measure of Markov system only needs one long-time simulation trial with enough regenerative cycles.

Set $x=0$ (the null queue) as the regenerative state and introduce the following random vector

$$\begin{bmatrix} L_c \\ C \end{bmatrix} = \begin{bmatrix} \text{"Integral of queue length in regenerative cycle"} \\ \text{"Time span of regenerative cycle"} \end{bmatrix} \tag{28}$$

According to the regenerative theory, we have

$$L = \frac{E[L_c]}{E[C]} \tag{29}$$

Thus

$$\frac{\partial L}{\partial \theta} = \frac{1}{E[C]^2}\left(\frac{\partial E[L_c]}{\partial \theta}E[C] - E[L_c]\frac{\partial E[C]}{\partial \theta}\right), \quad \theta \in \{\lambda, \mu\} \tag{30}$$

Obviously, to get $\partial L/\partial\theta$, $E[L_c]$, $\partial E[L_c]/\partial\theta$ and $E[C]$, $\partial E[C]/\partial\theta$ should be pre-estimated.

Let z be the state transfer sequence in regenerative cycles Apparently all possible z in regenerative cycles form a sample space. Rewrite $E[L_c]$, $E[C]$ in conditional expression

$$E[L_c] = E[E[L_c \mid Z]], \quad E[C] = E[E[C \mid Z]] \tag{31}$$

The followings is just like what we did in the above example. Perform a long-time simulation trial with N regenerative cycles. We denote the piece of state transfer sequence in ith epoch as $z_i = Z(\{x_{k-1}, e_k\}, k=1,\cdots,m), i \in \{1,\cdots,N\}$. By applying characteristics 2 in Section 4, the conditional expectations of L_c and C under z_i are given by

$$E[L_c \mid z_j] = \sum_{k=1}^{m} \frac{x_{k-1}}{\Lambda(x_{k-1})}, \quad E[C \mid z_j] = \sum_{k=1}^{m} \frac{1}{\Lambda(x_{k-1})} \tag{32}$$

Using the similar formulae in Section 6. 1 to extract the necessary statistical quantities: $\partial E[L_c \mid z_i]/\partial\theta$, $\partial E[C \mid z_i]/\partial\theta$ and $\partial \ln p(z_i)/\partial\theta$, where $\theta \in \{\lambda, \mu\}$, and feeding them to (8) and (17), we can obtain estimates of $E[L_c]$, $\partial E[L_c]/\partial\theta$ and $E[C]$, $\partial E[C]/\partial\theta$. Finally, estimations of L and $\partial L/\partial\theta, \theta \in \{\lambda, \mu\}$ can be determined in terms of (29) and (30).

Estimation of L, $\partial L/\partial\lambda$ and $\partial L/\partial\mu$ under different λ, μ are listed in Table 2. The results show maximal relative estimation errors not exceeding 8%. These results are obtained through one simulation trial with 4000 regenerative cycles.

Table 2 Steady state mean queue length & its sensitivity estimation of M/M/1

$[\lambda, \mu]$	$[L, \partial L/\partial\lambda, \partial L/\partial\mu]$	
	Simulation results	Theoretical values
[0.60, 1]	[1.50, 6.29, −3.77]	[1.50, 6.25, −3.75]
[0.70, 1]	[2.46, 11.9, −8.32]	[2.33, 11.1, −7.78]
[0.84, 1]	[5.66, 41.0, −34.5]	[5.25, 39.1, −32.8]

7 Conclusion

In this paper, we present an efficient algorithm for performance parameter sensitivity estimation of Markov DEDS. This algorithm can give consistent and reliable results for both steady state and transient performance sensitivity estimation. Another advantage of this algorithm is that aside from getting all partial derivatives of interest with respect to specific parameter set at the same time, it can simultaneously obtain a family of sensitivity estimations under different parameter sets. The third advantage of this algorithm is its relatively simple mathematical description and easy program realization. Moreover, an interesting and important conclusion of this paper is that the performance parameter sensitivity estimation of Markov system is the sum of those estimated by SPA method and LR method, and singly using any one of them usually can not give consistent and reliable result.

References

1 Law A M, Kelton W D. Simulation Modeling and Analysis. 3rd Edition. Beijing: Tsinghua Press, 2000

2 Vakili P. Using a standard clock technique for efficient simulation. *Operation Research Letters*, 1991, **10**(8): 445~

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3 Gassandras C G, Lee J I, Ho Y C. Efficient parametric analysis of performance measure for communication networks. *IEEE Journal on Selected Area in Communications*, 1990, **8**(9): 1709～1722
4 Zhou Jiang-Hua, Sun Guo-Ji, Guan Xiao-Hong. Performance evaluation of discrete event dynamic system using improved standard clock. *Information and Control*, 2002, **31**(5): 391～395 (in Chinese)
5 Ho Yu-Chi. Performance evaluation and perturbation analysis of discrete event dynamic system. *IEEE Transactions on Automatic Control*, 1987, **32** (7): 563～572
6 Reiman M I, Weiss A. Sensitivity analysis for simulation via likelihood ratios. *Operations Research*, 1989, **37**(5): 830～844
7 Glasserman P. Derivative estimate from simulation of continuous-time Markov chains. *Operations Research*, 1992, **40** (2): 292～308
8 Gong W B, HO Y C. Smoothed (conditional) perturbation analysis of discrete event dynamical systems. *IEEE Transactions on Automatic Control*, 1987, **32**(10): 858～866
9 Dai L. A consistent algorithm for derivative estimation of Markov chains. In: Proceedings of the 33rd IEEE Conference, Decision and Control, Vol. 2. Lake Buena Vista, FL, USA: IEEE, 1994. 1990～1995
10 Dai L. Ho Y C. Structural infinitesimal perturbation analysis (SIPA) for derivative estimation in discrete event systems. *IEEE Transactions on Automatic Control*, 1995, **40**(7): 1154～1166
11 Cao X R, Yuan X M, Qiu L. A single sample path-based performance sensitivity formula for Markov chains. *IEEE Transactions on Automatic Control*, 1996, **41**(12): 1814～1817
12 Cao X R, Chen H F. Perturbation realization, potentials, and sensitivity analysis of Markov processes. *IEEE Transactions on Automatic Control*, 1997, **42**(10): 1382～1393
13 Cao X R, Wan Y W. Algorithms for sensitivity analysis of Markov systems through potentials and perturbation realization. *IEEE Transactions on Control System Technology*, 1998, **6**(4): 482～494
14 Dai L. Perturbation analysis via coupling. *IEEE Transactions on Automatic Control*, 2000, **45**(4): 614～628
15 Yin Bao-Qun, Zhou Ya-Ping. Sensitivity formulas for performance in open queueing networks. *Journal of University of Science and Technology of China*, 1998, **28**(1): 27～32(in Chinese)
16 Yin Bao-Qun, Zhou Ya-Ping, Xi Hong-Sheng. Sensitivity analysis of performance with parameter-dependent performance function in closed queuing network. *Control Theory and Applications*, 2002, **19**(2): 311～312 (in Chinese)
17 Zhou Ya-Ping, Xi Hong-Sheng, Yin Bao-Qun. Optimality strategy of average cost based performance potentials for Markov control process. *Acta Automatica Sinica*, 2002, **28**(6): 904～910 (in Chinese)
18 Cao Xi-Ren. Convergence of parameter sensitivity estimates in a stochastic experiment. *IEEE Transactions on Automatic Control*, 1985, **30**(9): 845～853
19 Sheng Zhou, Xie Shi-Qian, Pan Cheng-Yi. Probability and Statistics. 2nd Edition. Beijing: High Education Press, 1990. 135～136 (in Chinese)
20 Leonard Kleinrock. Queueing System Volume 1: Theory. New York: John Wiley & Sons, Inc. 1975. 94～105

Appendix A: Algorithm for example in Section 6.1

For convince, we define

$$\hat{T}_c \equiv E[T_c \mid z], \quad Q \equiv \ln p(z), \quad \frac{\partial \tilde{T}_c}{\partial \theta} \equiv \frac{\partial \hat{T}_c}{\partial \theta} + \hat{T}_c \frac{\partial Q}{\partial \theta}$$

• Initialize

$$x = 0; \quad \Lambda(x) = \lambda; \quad \hat{T}_c = 0; \quad \frac{\partial \hat{T}_c}{\partial \lambda} = 0; \quad \frac{\partial \hat{T}_c}{\partial \mu} = 0; \quad \frac{\partial Q}{\partial \lambda} = 0; \quad \frac{\partial Q}{\partial \mu} = 0$$

• Procedure

1) Generating random number $u \backsim U(0,1)$

2) Generating next event

If $u \leqslant \lambda/\Lambda(x)$, $e=1$ //customer arrival event

Else $e=2$ //customer leave event

3) Calculating $\partial\Lambda(x)/\partial\lambda, \partial\Lambda(x)/\partial\mu$

If $x>0$ {

$\partial\Lambda(x)/\partial\lambda=1$; $\partial\Lambda(x)/\partial\mu=1$;

} Else {

$\partial\Lambda(x)/\partial\lambda=1$; $\partial\Lambda(x)/\partial\mu=0$;

}

4) $\hat{T}_c \leftarrow \hat{T}_c + 1/\Lambda(x)$

5) $\frac{\partial \hat{T}_c}{\partial \lambda} \leftarrow \frac{\partial \hat{T}_c}{\partial \lambda} - \frac{\partial\Lambda(x)/\partial\lambda}{[\Lambda(x)]^2}$, $\frac{\partial \hat{T}_c}{\partial \mu} \leftarrow \frac{\partial \hat{T}_c}{\partial \mu} - \frac{\partial\Lambda(x)/\partial\mu}{[\Lambda(x)]^2}$

6) Calculating $\lambda_e, \partial\lambda_e/\partial\lambda, \partial\lambda_e/\partial\mu$ in terms of event e

If $e=1$ {

$\lambda_e=\lambda$; $\partial\lambda_e/\partial\lambda=1$; $\partial\lambda_e/\partial\mu=0$;

} Else {

$\lambda_e=\mu$; $\partial\lambda_e/\partial\lambda=0$; $\partial\lambda_e/\partial\mu=1$;

}

7) $\frac{\partial Q}{\partial\lambda}\leftarrow\frac{\partial Q}{\partial\lambda}+\left[\frac{\partial\lambda_e}{\partial\lambda}\frac{1}{\lambda_e}-\frac{\partial\Lambda(x)}{\partial\lambda}\frac{1}{\partial\Lambda(x)}\right]$

$\frac{\partial Q}{\partial\mu}\leftarrow\frac{\partial Q}{\partial\mu}+\left[\frac{\partial\lambda_e}{\partial\mu}\frac{1}{\lambda_e}-\frac{\partial\Lambda(x)}{\partial\mu}\frac{1}{\partial\Lambda(x)}\right]$

8) State update

If $e=1$ {

$x\leftarrow x+1$;//customer arrival, queue length plus 1

} Else {

$x\leftarrow x-1$;//customer leave, queue length minus 1

}

If $x>0$ $\Lambda(x)=\lambda+\mu$

Else $\Lambda(x)=\lambda$

9) Check whether the simulation is finished?

If $x\leqslant K$ goto 1

else { //queue length exceeds K, simulation ends

$$\frac{\partial\hat{\mathrm{T}}_c}{\partial\lambda}=\frac{\partial\hat{\mathrm{T}}_c}{\partial\lambda}+\hat{\mathrm{T}}_c\frac{\partial Q}{\partial\lambda},\frac{\partial\hat{\mathrm{T}}_c}{\partial\mu}=\frac{\partial\hat{\mathrm{T}}_c}{\partial\mu}+\hat{\mathrm{T}}_c\frac{\partial Q}{\partial\mu}$$

}

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Markov 离散事件动态系统参数灵敏度估计算法

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摘 要 给出了马尔可夫离散事件系统参数灵敏度估计的高效率仿真算法.既可以用于稳态性能测度的参数灵敏度估计,也可用于终止型性能测度的灵敏度估计.和现有的仿真算法相比,其数学描述和仿真流程比较简洁,易于编程实现.给出的各种仿真算例均验证了该方法的适用性.此外,还指出了马尔可夫系统性能测度参数灵敏度的精确估计为光滑扰动分析(Smoothed Perturbatin Analysis)和似然比方法(Likelihood Ratios)得到的估计量之和,单独使用其中的任何一种均难以给出系统性能测度参数灵敏度的可靠、一致估计.

关键词 马尔可夫系统,灵敏度估计,性能测度,仿真

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