基于保 H_∞ 性能插值的变增益输出反馈控制[1)]

虞忠伟 陈辉堂 陈启军
(同济大学信息与控制工程系 上海 200092)
(E-mail: yuzhongwei7302@sina.com)

摘 要 针对线性变参数系统，基于保 H_∞ 性能插值设计一种无需变参数变化率反馈的变增益输出反馈控制器. 在将控制器设计转化为关于参数矩阵的 LMI 问题后，基于"H_∞ 性能覆盖"的概念给出了划分变参数集的充分条件，并将变参数集划分为若干充分小的子集，对各子集寻找满足要求的常数矩阵并利用插值来得到所要求的连续参数矩阵. 此控制器消除了变参数变化率反馈并通过对变参数变化率上界的限制降低了控制器设计的保守性. 实验结果验证了其有效性.

关键词 线性变参数系统，保 H_∞ 性能插值，变增益，参数线性矩阵不等式
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Gain Scheduling Output Feedback Control Base on H_∞ Performance Preserved Interpolation

YU Zhong-Wei CHEN Hui-Tang CHEN Qi-Jun
(*Department of Information and Control Engineering, Tongji University, Shanghai* 200092)
(E-mail: yuzhongwei7302@sina.com)

Abstract A gain scheduling output feedback controller without parameter-rate feedback is designed based on H_∞ performance preserved interpolation. After the controller design is translated into parameterized linear matrix inequalities about parameter matrixes, a sufficient condition is given to partition the parameter set based on H_∞ performance covering. The parameter set is parted into some subsets. After the constant matrixes are found for every subset, the continuous parameter matrixes are obtained using interpolation. The parameter-rate feedback is eliminated and the conservation of the designed controller is reduced via limiting the bound of the parameter-rate. Experiment results prove the effectiveness of the designed controller.

Key words Linear parameter varying system, H_∞ performance preserved interpolation, gain scheduling, parameterized linear matrix inequality(PLMI)

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1 引言

最近几年,对线性变参数(linear parameter varying)系统的变增益控制得到了深入地研究[1~5]. 文献[1]基于小增益理论设计了一种具有线性分式变换(linear fractional transformation)结构的LFT变增益控制器,但LFT描述的缺点是参数的变化允许为复数,当已知参数为实数时就引入了保守性. 文献[2,3]针对一类具有Polytopic结构的LPV系统,通过在整个变参数集内寻找一个单一的Lyapunov函数来保证LPV系统对所有可能轨迹都具有 H_∞ 性能,这在实际中比较困难,同时该方法对变参数变化率没有限制. 文献[4,5]虽然考虑了变参数变化率的有界性,但它只适合一类特殊的具有仿射形式的LPV系统,同时需利用变参数变化率反馈,这在实际中往往不可能. 上述方法的共同弱点是无法保证一定能找到满足要求的变增益控制器. 本文吸收了传统变增益控制器的框架,在"H_∞ 性能覆盖"基础上给出了划分变参数集的充分条件,并将变参数集分成若干充分小的子集,对各子集利用保 H_∞ 性能插值设计变增益控制器,在计算出各子集的插值以后得到保证系统在整个变参数集具有 H_∞ 性能的变参数变化率的上界. 虽然本文规定了变参数变化率上界,但在变增益控制器中消除了变参数变化率,所以无需变参数变化率反馈.

2 问题描述

考虑下列以 $\boldsymbol{\rho}(t)\in\Gamma\subset R^l$ 为变参数的线性变参数(LPV)系统(输出估计问题)

$$\begin{cases}\dot{\boldsymbol{x}}(t)=A(\boldsymbol{\rho}(t))\boldsymbol{x}(t)+B_1(\boldsymbol{\rho}(t))\boldsymbol{w}_1(t)+B_2(\boldsymbol{\rho}(t))\boldsymbol{u}(t)\\ \boldsymbol{z}_1(t)=C_{11}(\boldsymbol{\rho}(t))\boldsymbol{x}(t)\\ \boldsymbol{z}_2(t)=C_{12}(\boldsymbol{\rho}(t))\boldsymbol{x}(t)+\boldsymbol{u}(t)\\ \boldsymbol{y}(t)=C_2(\boldsymbol{\rho}(t))\boldsymbol{x}(t)+\boldsymbol{w}_2(t)\end{cases}\tag{1}$$

这里 $\boldsymbol{x}(t)\in R^n$, $\boldsymbol{u}(t)\in R^m$, $\boldsymbol{y}(t)\in R^r$ 分别表示系统的状态、控制和输出矢量;$\boldsymbol{z}_1(t)\in R^{m_1}$, $\boldsymbol{z}_2(t)\in R^{m_2}$ 为系统性能指标;$\boldsymbol{w}_1(t)\in R^p$ 为系统动态不确定、外部输入等引起的干扰;$\boldsymbol{w}_2(t)\in R^r$ 为系统测量噪声;参数矩阵 $A(\boldsymbol{\rho}(t)):\Gamma\to R^{n\times n}$, $B_1(\boldsymbol{\rho}(t)):\Gamma\to R^{n\times p}$, $B_2(\boldsymbol{\rho}(t)):\Gamma\to R^{n\times m}$, $C_{11}(\boldsymbol{\rho}(t)):\Gamma\to R^{m_1\times n}$, $C_{12}(\boldsymbol{\rho}(t)):\Gamma\to R^{m_2\times n}$, $C_2(\boldsymbol{\rho}(t)):\Gamma\to R^{r\times n}$. 设计全阶($n$ 阶)变增益输出反馈控制器

$$K(\boldsymbol{\rho}(t)):\begin{cases}\dot{\boldsymbol{x}}_K(t)=A_K(\boldsymbol{\rho}(t))\boldsymbol{x}_K(t)+B_K(\boldsymbol{\rho}(t))\boldsymbol{y}(t)\\ \boldsymbol{u}(t)=C_K(\boldsymbol{\rho}(t))\boldsymbol{x}_K(t)\end{cases}\tag{2}$$

定义 $\boldsymbol{x}_{cl}(t)=[\boldsymbol{x}(t)\quad \boldsymbol{x}_K(t)]^{\mathrm{T}}$, $\boldsymbol{w}(t)=[\boldsymbol{w}_1(t)\quad \boldsymbol{w}_2(t)]^{\mathrm{T}}$, $\boldsymbol{z}(t)=[\boldsymbol{z}_1(t)\quad \boldsymbol{z}_2(t)]^{\mathrm{T}}$,则闭环系统为

$$\begin{cases}\dot{\boldsymbol{x}}_{cl}(t)=A_{cl}(\boldsymbol{\rho}(t))\boldsymbol{x}_{cl}(t)+B_{cl}(\boldsymbol{\rho}(t))\boldsymbol{w}(t)\\ \boldsymbol{z}(t)=C_{cl}(\boldsymbol{\rho}(t))\boldsymbol{x}_{cl}(t)\end{cases}\tag{3}$$

其中 $A_{cl}(\boldsymbol{\rho}(t))=\begin{bmatrix}A(\boldsymbol{\rho}(t)) & B_2(\boldsymbol{\rho}(t))C_K(\boldsymbol{\rho}(t))\\ B_K(\boldsymbol{\rho}(t))C_2(\boldsymbol{\rho}(t)) & A_K(\boldsymbol{\rho}(t))\end{bmatrix}$, $B_{cl}(\boldsymbol{\rho}(t))=\begin{bmatrix}B_1(\boldsymbol{\rho}(t)) & 0\\ 0 & B_K(\boldsymbol{\rho}(t))\end{bmatrix}$, $C_{cl}(\boldsymbol{\rho}(t))=\begin{bmatrix}C_{11}(\boldsymbol{\rho}(t)) & 0\\ C_{12}(\boldsymbol{\rho}(t)) & C_K(\boldsymbol{\rho}(t))\end{bmatrix}$.

定义 1. 给定 $\gamma>0$,如果对任意干扰 $\boldsymbol{w}(t)$,闭环系统性能指标 $\boldsymbol{z}(t)$ 满足 $\int_0^\infty \boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)\mathrm{d}t<$

$\gamma^2\int_0^\infty \boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t)\mathrm{d}t$，那么闭环系统具有 H_∞ 性能 γ.

本文要解决的问题是设计变增益输出反馈控制器 $K(\boldsymbol{\rho}(t))$，使得对变参数集 Γ 内的任意 $\boldsymbol{\rho}(t)$，闭环系统均具有 H_∞ 性能 γ. 为方便起见，下文符号中省略了时间 t .

3　变增益控制器综合

定义 2. 对于闭环系统(3)，$\boldsymbol{\rho}\in\Gamma$，如果存在连续对称正定矩阵函数 $P(\boldsymbol{\rho}):\Gamma\to R^{2n\times 2n}$ 满足

$$A_{cl}^{\mathrm{T}}(\boldsymbol{\rho})P(\boldsymbol{\rho})+P(\boldsymbol{\rho})A_{cl}(\boldsymbol{\rho})+\frac{\mathrm{d}}{\mathrm{d}t}P(\boldsymbol{\rho})+\gamma^{-2}C_{cl}^{\mathrm{T}}(\boldsymbol{\rho})C_{cl}(\boldsymbol{\rho})+P(\boldsymbol{\rho})B_{cl}(\boldsymbol{\rho})B_{cl}^{\mathrm{T}}(\boldsymbol{\rho})P(\boldsymbol{\rho})<0 \tag{4}$$

则闭环系统具有 H_∞ 性能 γ.

在定义 2 的基础上给出变增益控制器综合的具体方法.

定理 1. 给定正定常数矩阵 Q_x，Q_y 和 $\gamma>0$，如果存在正定连续且可微的参数矩阵函数 $X(\boldsymbol{\rho})$，$Y(\boldsymbol{\rho})$，对所有 $\boldsymbol{\rho}\in\Gamma$ 都满足

$$\begin{bmatrix} S(\boldsymbol{\rho}) & Y(\boldsymbol{\rho})C_{11}^{\mathrm{T}}(\boldsymbol{\rho}) & \gamma^{-1}B_1(\boldsymbol{\rho}) \\ C_{11}(\boldsymbol{\rho})Y(\boldsymbol{\rho}) & -I & 0 \\ \gamma^{-1}B_1^{\mathrm{T}}(\boldsymbol{\rho}) & 0 & -I \end{bmatrix}<0 \tag{5}$$

$$\begin{bmatrix} R(\boldsymbol{\rho}) & X(\boldsymbol{\rho})B_1(\boldsymbol{\rho}) & \gamma^{-1}C_1^{\mathrm{T}}(\boldsymbol{\rho}) \\ B_1^{\mathrm{T}}(\boldsymbol{\rho})X(\boldsymbol{\rho}) & -I & 0 \\ \gamma^{-1}C_1(\boldsymbol{\rho}) & 0 & -I \end{bmatrix}<0,\quad \begin{bmatrix} X(\boldsymbol{\rho}) & \gamma^{-1}I \\ \gamma^{-1}I & Y(\boldsymbol{\rho}) \end{bmatrix}>0 \qquad (6),(7)$$

其中 $S(\boldsymbol{\rho})=[A(\boldsymbol{\rho})-B_2(\boldsymbol{\rho})C_{12}(\boldsymbol{\rho})]Y(\boldsymbol{\rho})+Y(\boldsymbol{\rho})[A(\boldsymbol{\rho})-B_2(\boldsymbol{\rho})C_{12}(\boldsymbol{\rho})]^{\mathrm{T}}-B_2(\boldsymbol{\rho})B_2^{\mathrm{T}}(\boldsymbol{\rho})+Q_y$，$R(\boldsymbol{\rho})=X(\boldsymbol{\rho})A(\boldsymbol{\rho})+A^{\mathrm{T}}(\boldsymbol{\rho})X(\boldsymbol{\rho})-C_2^{\mathrm{T}}(\boldsymbol{\rho})C_2(\boldsymbol{\rho})+Q_x$，$C_1(\boldsymbol{\rho})=\begin{bmatrix} C_{11}(\boldsymbol{\rho}) \\ C_{12}(\boldsymbol{\rho}) \end{bmatrix}$. 同时，对任意 $t\geqslant 0$，满足

$$\dot{X}(\boldsymbol{\rho})<Q_x,\ \dot{Y}(\boldsymbol{\rho})>-Q_y \tag{8}$$

那么，对任意可逆矩阵函数 $N(\boldsymbol{\rho}):\Gamma\to R^{n\times n}$，存在下列变增益控制器 $K(\boldsymbol{\rho})$

$F(\boldsymbol{\rho})=-[B_2^{\mathrm{T}}(\boldsymbol{\rho})Y^{-1}(\boldsymbol{\rho})+C_{12}(\boldsymbol{\rho})]$，$M(\boldsymbol{\rho})=[I-\gamma^2Y(\boldsymbol{\rho})X(\boldsymbol{\rho})]N^{-T}(\boldsymbol{\rho})$，$C_K(\boldsymbol{\rho})=\gamma^2F(\boldsymbol{\rho})Y(\boldsymbol{\rho})M^{-T}(\boldsymbol{\rho})$，$L(\boldsymbol{\rho})=-X^{-1}(\boldsymbol{\rho})C_2^{\mathrm{T}}(\boldsymbol{\rho})$，$B_K(\boldsymbol{\rho})=N^{-1}(\boldsymbol{\rho})X(\boldsymbol{\rho})L(\boldsymbol{\rho})$，

$$\begin{aligned} A_K(\boldsymbol{\rho})=&-N^{-1}(\boldsymbol{\rho})\{\gamma^2X(\boldsymbol{\rho})[A(\boldsymbol{\rho})+L(\boldsymbol{\rho})C_2(\boldsymbol{\rho})+B_2(\boldsymbol{\rho})F(\boldsymbol{\rho})]Y(\boldsymbol{\rho})+A^{\mathrm{T}}(\boldsymbol{\rho})+ \\ &[C_1^{\mathrm{T}}(\boldsymbol{\rho})C_1(\boldsymbol{\rho})+C_{12}^{\mathrm{T}}(\boldsymbol{\rho})F(\boldsymbol{\rho})]Y(\boldsymbol{\rho})+X(\boldsymbol{\rho})B_1(\boldsymbol{\rho})B_1^{\mathrm{T}}(\boldsymbol{\rho})+\gamma^2\dot{X}(\boldsymbol{\rho})Y(\boldsymbol{\rho})+ \\ &\dot{N}(\boldsymbol{\rho})M(\boldsymbol{\rho})\}M^{-\mathrm{T}}(\boldsymbol{\rho}) \end{aligned} \tag{9}$$

$K(\boldsymbol{\rho})=\begin{bmatrix} A_K(\boldsymbol{\rho}) & B_K(\boldsymbol{\rho}) \\ C_K(\boldsymbol{\rho}) & 0 \end{bmatrix}$，使得对所有 $\boldsymbol{\rho}\in\Gamma$，闭环系统具有 H_∞ 性能 γ.

证明. 定义

$$M(\boldsymbol{\rho})=[I-\gamma^2Y(\boldsymbol{\rho})X(\boldsymbol{\rho})]N^{-\mathrm{T}}(\boldsymbol{\rho}) \tag{10}$$

由 Schur 补引理[6]，式(7)等价为

$$X(\boldsymbol{\rho})-\gamma^{-2}Y(\boldsymbol{\rho})>0,\text{即 } Y^{-1}(\boldsymbol{\rho})-\gamma^2X(\boldsymbol{\rho})<0 \tag{11}$$

上式也表明$[I-\gamma^2X(\boldsymbol{\rho})Y(\boldsymbol{\rho})]Y^{-1}(\boldsymbol{\rho})$可逆，进而 $M(\boldsymbol{\rho})$可逆.

定义

$$P(\boldsymbol{\rho})=\begin{bmatrix} X(\boldsymbol{\rho}) & N(\boldsymbol{\rho}) \\ N^{\mathrm{T}}(\boldsymbol{\rho}) & -\gamma^2 N^{\mathrm{T}}(\boldsymbol{\rho})Y(\boldsymbol{\rho})M^{-\mathrm{T}}(\boldsymbol{\rho}) \end{bmatrix},\ P^{-1}(\boldsymbol{\rho})=\begin{bmatrix} \gamma^2 Y(\boldsymbol{\rho}) & M(\boldsymbol{\rho}) \\ M^{\mathrm{T}}(\boldsymbol{\rho}) & -N^{-1}(\boldsymbol{\rho})X(\boldsymbol{\rho})M(\boldsymbol{\rho}) \end{bmatrix},$$

$$H(\boldsymbol{\rho}) = A_{cl}^{\mathrm{T}}(\boldsymbol{\rho})P(\boldsymbol{\rho}) + P(\boldsymbol{\rho})A_{cl}(\boldsymbol{\rho}) + \dot{P}(\boldsymbol{\rho}) + \gamma^{-2}C_{cl}^{\mathrm{T}}(\boldsymbol{\rho})C_{cl}(\boldsymbol{\rho}) + P(\boldsymbol{\rho})B_{cl}(\boldsymbol{\rho})B_{cl}^{\mathrm{T}}(\boldsymbol{\rho})P(\boldsymbol{\rho}) \tag{12}$$

由定义 2,为了使闭环系统(3)具有 H_∞ 性能 γ,只需对所有 $\boldsymbol{\rho}\in\Gamma$,$P(\boldsymbol{\rho})>0$,同时 $H(\boldsymbol{\rho})<0$.

由 $P(\boldsymbol{\rho})P^{-1}(\boldsymbol{\rho})=I$,可得 $N(\boldsymbol{\rho})M^{\mathrm{T}}(\boldsymbol{\rho})=I-\gamma^2X(\boldsymbol{\rho})Y(\boldsymbol{\rho})$,因

$$X(\boldsymbol{\rho}) + N(\boldsymbol{\rho})[\gamma^2 N^{\mathrm{T}}(\boldsymbol{\rho})Y(\boldsymbol{\rho})M^{-\mathrm{T}}(\boldsymbol{\rho})]^{-1}N^{\mathrm{T}}(\boldsymbol{\rho}) = \gamma^{-2}Y^{-1}(\boldsymbol{\rho}) > 0 \tag{13}$$

同时 $-\gamma^2N^{\mathrm{T}}(\boldsymbol{\rho})Y(\boldsymbol{\rho})M^{-\mathrm{T}}(\boldsymbol{\rho})=-\gamma^2N^{\mathrm{T}}(\boldsymbol{\rho})[Y^{-1}(\boldsymbol{\rho})-\gamma^2X(\boldsymbol{\rho})]^{-1}N(\boldsymbol{\rho})$,由式(11)可知

$$-\gamma^2 N^{\mathrm{T}}(\boldsymbol{\rho})Y(\boldsymbol{\rho})M^{-\mathrm{T}}(\boldsymbol{\rho}) > 0 \tag{14}$$

所以由 Schur 补引理,式(13)和(14)等价于 $P(\boldsymbol{\rho})>0$.

为证明 $H(\boldsymbol{\rho})<0$,定义 $P_1(\boldsymbol{\rho})=\begin{bmatrix} \gamma^2Y(\boldsymbol{\rho}) & I \\ M^{\mathrm{T}}(\boldsymbol{\rho}) & 0\end{bmatrix}$,$P_2(\boldsymbol{\rho})=P(\boldsymbol{\rho})P_1(\boldsymbol{\rho})=\begin{bmatrix} I & X(\boldsymbol{\rho}) \\ 0 & N^{\mathrm{T}}(\boldsymbol{\rho})\end{bmatrix}$. 设

$$\overline{H}(\boldsymbol{\rho}) = P_1^{\mathrm{T}}(\boldsymbol{\rho})H(\boldsymbol{\rho})P_1(\boldsymbol{\rho}) \tag{15}$$

因 $P_1(\boldsymbol{\rho})$ 可逆,所以 $H(\boldsymbol{\rho})<0$ 的充要条件为 $\overline{H}(\boldsymbol{\rho})<0$. 将式(12)代入式(15),可得

$$\begin{aligned}\overline{H}(\boldsymbol{\rho}) =& P_1^{\mathrm{T}}(\boldsymbol{\rho})A_{cl}^{\mathrm{T}}(\boldsymbol{\rho})P_2(\boldsymbol{\rho}) + P_2^{\mathrm{T}}(\boldsymbol{\rho})A_{cl}(\boldsymbol{\rho})P_1(\boldsymbol{\rho}) + P_1^{\mathrm{T}}(\boldsymbol{\rho})\dot{P}(\boldsymbol{\rho})P_1(\boldsymbol{\rho}) + \\ &\gamma^{-2}P_1^{\mathrm{T}}(\boldsymbol{\rho})C_{cl}^{\mathrm{T}}(\boldsymbol{\rho})C_{cl}(\boldsymbol{\rho})P_1(\boldsymbol{\rho}) + P_2^{\mathrm{T}}(\boldsymbol{\rho})B_{cl}(\boldsymbol{\rho})B_{cl}^{\mathrm{T}}(\boldsymbol{\rho})P_2(\boldsymbol{\rho})\end{aligned} \tag{16}$$

定义 $\overline{H}(\boldsymbol{\rho})=\begin{bmatrix}\overline{H}_{11}(\boldsymbol{\rho}) & \overline{H}_{12}(\boldsymbol{\rho}) \\ \overline{H}_{12}^{\mathrm{T}}(\boldsymbol{\rho}) & \overline{H}_{22}(\boldsymbol{\rho})\end{bmatrix}$,式(16)代入 $P_1(\boldsymbol{\rho})$,$P_2(\boldsymbol{\rho})$,$A_{cl}(\boldsymbol{\rho})$,$B_{cl}(\boldsymbol{\rho})$,$C_{cl}(\boldsymbol{\rho})$ 并结合式(5)和(6),可得

$$\gamma^{-2}\overline{H}_{11}(\boldsymbol{\rho}) < -Q_y - \dot{Y}(\boldsymbol{\rho}), \quad \overline{H}_{22}(\boldsymbol{\rho}) < -Q_x + \dot{X}(\boldsymbol{\rho}) \tag{17),(18}$$

$$\begin{aligned}\overline{H}_{12}(\boldsymbol{\rho}) =& \gamma^2(\boldsymbol{\rho})X(\boldsymbol{\rho})[A(\boldsymbol{\rho})+L(\boldsymbol{\rho})C_2(\boldsymbol{\rho})+B_2(\boldsymbol{\rho})F(\boldsymbol{\rho})]Y(\boldsymbol{\rho})+N(\boldsymbol{\rho})A_K(\boldsymbol{\rho})M^{\mathrm{T}}(\boldsymbol{\rho})+A^{\mathrm{T}}(\boldsymbol{\rho})+ \\ &[C_1^{\mathrm{T}}(\boldsymbol{\rho})C_1(\boldsymbol{\rho}) + C_{12}^{\mathrm{T}}(\boldsymbol{\rho})F(\boldsymbol{\rho})]+X(\boldsymbol{\rho})B_1(\boldsymbol{\rho})B_1^{\mathrm{T}}(\boldsymbol{\rho})+\gamma^2\dot{X}(\boldsymbol{\rho})Y(\boldsymbol{\rho})+\dot{N}(\boldsymbol{\rho})M^{\mathrm{T}}(\boldsymbol{\rho}).\end{aligned}$$

当 $A_K(\boldsymbol{\rho})$ 取为式(9)时,$\overline{H}_{12}(\boldsymbol{\rho})=0$,所以结合式(17)和(18),$\overline{H}(\boldsymbol{\rho})<\begin{bmatrix}-\gamma^2[Q_y+\dot{Y}(\boldsymbol{\rho})] & 0 \\ 0 & -Q_x+\dot{X}(\boldsymbol{\rho})\end{bmatrix}$.

当式(8)成立时,$\overline{H}(\boldsymbol{\rho})<0$,即 $H(\boldsymbol{\rho})<0$,因此对所有 $\boldsymbol{\rho}\in\Gamma$,闭环系统具有 H_∞ 性能 γ. 证毕.

4 控制器中变参数变化率反馈的消除

推论 1. 定理 1 中,如果 $N(\boldsymbol{\rho})$ 满足

$$\frac{\partial N(\boldsymbol{\rho})}{\partial \rho_i} = -\gamma^2\frac{\partial X(\boldsymbol{\rho})}{\partial \rho_i}Y(\boldsymbol{\rho})[I-\gamma^2X(\boldsymbol{\rho})Y(\boldsymbol{\rho})]^{-1}N(\boldsymbol{\rho}) \quad (i=1,\cdots,l) \tag{19}$$

那么变增益控制器 $K(\boldsymbol{\rho})$ 中 $A_K(\boldsymbol{\rho})$ 重新构造为

$$\begin{aligned}A_K(\boldsymbol{\rho}) =& -N^{-1}(\boldsymbol{\rho})\{\gamma^2X(\boldsymbol{\rho})[A(\boldsymbol{\rho})+L(\boldsymbol{\rho})C_2(\boldsymbol{\rho})+B_2(\boldsymbol{\rho})F(\boldsymbol{\rho})]Y(\boldsymbol{\rho})+A^{\mathrm{T}}(\boldsymbol{\rho})+ \\ &[C_1^{\mathrm{T}}(\boldsymbol{\rho})C_1(\boldsymbol{\rho})+C_{12}^{\mathrm{T}}(\boldsymbol{\rho})F(\boldsymbol{\rho})]Y(\boldsymbol{\rho})+X(\boldsymbol{\rho})B_1(\boldsymbol{\rho})B_1^{\mathrm{T}}(\boldsymbol{\rho})\}M^{-\mathrm{T}}(\boldsymbol{\rho})\end{aligned} \tag{20}$$

后,控制器 $K(\boldsymbol{\rho})$ 同样可保证对所有 $\boldsymbol{\rho}\in\Gamma$,闭环系统具有 H_∞ 性能 γ.

证明. 定理 1 的证明中,设 $A_K(\boldsymbol{\rho})$ 取为式(20),那么 $\overline{H}_{12}(\boldsymbol{\rho})=\gamma^2\dot{X}(\boldsymbol{\rho})Y(\boldsymbol{\rho})+\dot{N}(\boldsymbol{\rho})M^{\mathrm{T}}(\boldsymbol{\rho})$,由式(10),$M^{-\mathrm{T}}(\boldsymbol{\rho})=[I-\gamma^2X(\boldsymbol{\rho})Y(\boldsymbol{\rho})]^{-1}N(\boldsymbol{\rho})$,上式代入式(19)以后,可得

$$\frac{\partial N(\boldsymbol{\rho})}{\partial \rho_i} = -\gamma^2\frac{\partial X(\boldsymbol{\rho})}{\partial \rho_i}Y(\boldsymbol{\rho})M^{-\mathrm{T}}(\boldsymbol{\rho}) \quad (i=1,\cdots,l),$$

则可推出$\frac{\partial N(\boldsymbol{\rho})}{\partial\rho_1}\dot{\rho}_1+\cdots+\frac{\partial N(\boldsymbol{\rho})}{\partial\rho_l}\dot{\rho}_l=-\gamma^2\left[\frac{\partial X(\boldsymbol{\rho})}{\partial\rho_1}\dot{\rho}_1+\cdots+\frac{\partial X(\boldsymbol{\rho})}{\partial\rho_l}\dot{\rho}_l\right]Y(\boldsymbol{\rho})M^{-\mathrm{T}}(\boldsymbol{\rho})$.

上式即为$\dot{N}(\boldsymbol{\rho})M^{\mathrm{T}}(\boldsymbol{\rho})=-\gamma^2\dot{X}(\boldsymbol{\rho})Y(\boldsymbol{\rho})$,所以$\bar{H}_{12}(\boldsymbol{\rho})=0$,则$\bar{H}(\boldsymbol{\rho})<\begin{bmatrix}-\gamma^2[Q_y+\dot{Y}(\boldsymbol{\rho})] & 0\\ 0 & -Q_x+\dot{X}(\boldsymbol{\rho})\end{bmatrix}$,由定理 1中式(8),$\bar{H}(\boldsymbol{\rho})<0$,即 $H(\boldsymbol{\rho})<0$,因此对所有 $\boldsymbol{\rho}\in\Gamma$,闭环系统具有 H_∞ 性能 γ. 证毕.

5 保 H_∞ 性能插值

定理 1 的关键是矩阵函数 $X(\boldsymbol{\rho})$,$Y(\boldsymbol{\rho})$的求取,文献[4,5]只适用于一类特殊的具有仿射形式的 LPV 对象,同时无法保证一定找得到满足要求的 $X(\boldsymbol{\rho})$和 $Y(\boldsymbol{\rho})$. 本文吸收了传统变增益的框架.

定义 3(H_∞ 性能覆盖). 假设对固定的 $\rho_i\in\Gamma(i=0,\cdots,m)$,分别存在常数矩阵 X_i,Y_i满足定理 1,设 U_i为包含 ρ_i的开邻域,并且对每一固定的 $\boldsymbol{\rho}\in U_i$, X_i,Y_i都能满足定理 1. 如果 $\Gamma\subset\bigcup_{i=0}^{m}U_i$,$U_i\cap U_{i+1}$非空集,那么这些矩阵集$\{X_i,Y_i,i=0,\cdots,m\}$满足 H_∞ 性能覆盖条件.

在 H_∞ 性能覆盖条件下,随着 $\boldsymbol{\rho}$ 在不同邻域 U_i的变化,通过矩阵 X_i,Y_i的切换可使系统获得 H_∞ 性能,但缺点是使控制器产生跳变. 本文利用插值来获得在整个 $\boldsymbol{\rho}\in\Gamma$ 内连续的矩阵函数 $X(\boldsymbol{\rho})$,$Y(\boldsymbol{\rho})$. 下面以标量变参数为例来说明这种插值技术,它很容易扩展为矢量变参数的情况.

定理 2. 对正常数 δ_x,δ_y,设 $Q_x=\delta_xI$,$Q_y=\delta_yI$,假设存在与 $\rho_0<\cdots<\rho_m\in\Gamma$ 相应的常数矩阵(X_0,Y_0),$\cdots$,(X_m,Y_m)满足 H_∞ 性能覆盖条件,即对包含 ρ_i的开邻域 U_i, X_i,Y_i都满足定理 1,那么存在区间$[r_{i-1},r_i]\subset U_{i-1}\cap U_i\cap[\rho_{i-1},\rho_i]$ $(i=1,\cdots,m)$和连续矩阵函数 $X(\boldsymbol{\rho})$,$Y(\boldsymbol{\rho})$

$$X(\boldsymbol{\rho})=\begin{cases}X_{i-1}, & \boldsymbol{\rho}\in[\rho_{i-1},r_{i-1}]\\ \dfrac{r_{i+1}-\boldsymbol{\rho}}{r_{i+1}-r_i}X_{i-1}+\dfrac{\boldsymbol{\rho}-r_i}{r_{i+1}-r_i}X_i, & \boldsymbol{\rho}\in(r_{i-1},r_i)\\ X_i, & \boldsymbol{\rho}\in[r_i,\rho_i]\end{cases}\tag{21}$$

$$Y(\boldsymbol{\rho})=\begin{cases}Y_{i-1}, & \boldsymbol{\rho}\in[\rho_{i-1},r_{i-1}]\\ \dfrac{r_{i+1}-\boldsymbol{\rho}}{r_{i+1}-r_i}Y_{i-1}+\dfrac{\boldsymbol{\rho}-r_i}{r_{i+1}-r_i}Y_i, & \boldsymbol{\rho}\in(r_{i-1},r_i)\\ Y_i, & \boldsymbol{\rho}\in[r_i,\rho_i]\end{cases}\tag{22}$$

当

$$|\dot{\boldsymbol{\rho}}|<\min_{i=1,\cdots,m}\left[\frac{(r_i-r_{i-1})\delta_x}{\|X_i-X_{i-1}\|},\frac{(r_i-r_{i-1})\delta_y}{\|Y_i-Y_{i-1}\|}\right]\tag{23}$$

时,闭环系统对所有 $\boldsymbol{\rho}\in\Gamma$ 具有 H_∞ 性能 γ.

证明. 由 H_∞ 性能覆盖条件,$\boldsymbol{\rho}\in[\rho_{i-1},r_i)\subset U_{i-1}$时 X_{i-1}和 Y_{i-1}满足式(5)～(7);$\boldsymbol{\rho}\in(r_{i-1},\rho_i]\subset U_i$时 X_i和 Y_i满足式(5)～(7). 所以对式(21)和(22)的连续矩阵函数 $X(\boldsymbol{\rho})$,$Y(\boldsymbol{\rho})$,$\boldsymbol{\rho}\in[\rho_{i-1},r_{i-1}]$时 $X(\boldsymbol{\rho})$和 $Y(\boldsymbol{\rho})$满足式(5)～(7);$\boldsymbol{\rho}\in[r_i,\rho_i]$时 $X(\boldsymbol{\rho})$和 $Y(\boldsymbol{\rho})$满足式(5)～(7);$\boldsymbol{\rho}\in(r_{i-1},r_i)$时,因此区域既属 U_{i-1}又属 U_i,所以(X_{i-1},Y_{i-1}),(X_i,Y_i)都满足式(5)～(7),则$\frac{r_{i+1}-\boldsymbol{\rho}}{r_{i+1}-r_i}X_{i-1}+\frac{\boldsymbol{\rho}-r_i}{r_{i+1}-r_i}X_i$,$\frac{r_{i+1}-\boldsymbol{\rho}}{r_{i+1}-r_i}Y_{i-1}+\frac{\boldsymbol{\rho}-r_i}{r_{i+1}-r_i}Y_i$也满足式(5)～(7). 进一步,可把连续 $X(\boldsymbol{\rho})$,$Y(\boldsymbol{\rho})$看作可微,因为总能找到与式(21)和(22)充分接近的光滑近似,使

得对$\boldsymbol{\rho}\in[\rho_{i-1},\rho_i]$，式(5)～(7)依然满足. 当 $\boldsymbol{\rho}\in[\rho_{i-1},\rho_i]$时，$\|\dot{X}(\boldsymbol{\rho})\|\leqslant\frac{\|X_i-X_{i-1}\|}{|r_i-r_{i-1}|}|\dot{\boldsymbol{\rho}}|$，$\|\dot{Y}(\boldsymbol{\rho})\|\leqslant\frac{\|Y_i-Y_{i-1}\|}{|r_i-r_{i-1}|}|\dot{\boldsymbol{\rho}}|$. 由式(23)，$\frac{\|X_i-X_{i-1}\|}{|r_i-r_{i-1}|}|\dot{\boldsymbol{\rho}}|<\delta_x$，$\frac{\|Y_i-Y_{i-1}\|}{|r_i-r_{i-1}|}|\dot{\boldsymbol{\rho}}|<\delta_y$，即 $\|\dot{X}(\boldsymbol{\rho})\|\leqslant\delta_x$，$\|\dot{Y}(\boldsymbol{\rho})\|\leqslant\delta_y$，所以定理 1 中式(8)也满足，因此闭环系统对所有 $\boldsymbol{\rho}\in\Gamma$ 具有 H_∞ 性能 γ. 证毕.

6 实验研究

实验对象为自行研制的平面两关节直接驱动机器人，其动力学方程为[7]

$$\begin{bmatrix} a & b\cos(\theta_2-\theta_1) \\ b\cos(\theta_2-\theta_1) & c \end{bmatrix}\cdot\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}+\begin{bmatrix}-b\dot{\theta}_2^2\sin(\theta_2-\theta_1)\\ b\dot{\theta}_1^2\sin(\theta_2-\theta_1)\end{bmatrix}=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} \tag{24}$$

其中 $a=5.6794\ \mathrm{kg\cdot m^2}$，$b=1.4730\ \mathrm{kg\cdot m^2}$，$c=1.7985\ \mathrm{kg\cdot m^2}$. 式(24)在平衡点 $X_e=(\theta_{1e}\quad\theta_{2e}\quad\dot{\theta}_{1e}\quad\dot{\theta}_{2e})^{\mathrm{T}}=(\theta_{1e}\quad\theta_{2e}\quad 0\quad 0)^{\mathrm{T}}$，$\boldsymbol{\tau}_e=(0\quad 0)^{\mathrm{T}}$. 经雅可比线性化为

$$\dot{\boldsymbol{x}}=A\boldsymbol{x}+B\boldsymbol{u} \tag{25}$$

其中 $A=\begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$，$B=\begin{bmatrix}0&0\\0&0\\ \frac{c}{ac-b^2\cos^2(\theta_{2e}-\theta_{1e})} & \frac{-b\cos(\theta_{2e}-\theta_{1e})}{ac-b^2\cos^2(\theta_{2e}-\theta_{1e})}\\ \frac{-b\cos(\theta_{2e}-\theta_{1e})}{ac-b^2\cos^2(\theta_{2e}-\theta_{1e})} & \frac{a}{ac-b^2\cos^2(\theta_{2e}-\theta_{1e})}\end{bmatrix}$，$\boldsymbol{x}=(\hat{\theta}_1\quad\hat{\theta}_2\quad\hat{\dot{\theta}}_1\quad\hat{\dot{\theta}}_2)^{\mathrm{T}}$，$\boldsymbol{u}=(\hat{\tau}_1\quad\hat{\tau}_2)^{\mathrm{T}}$，$\hat{\theta}_1=\theta_1-\theta_{1e}$，$\hat{\theta}_2=\theta_2-\theta_{2e}$，$\hat{\dot{\theta}}_1=\dot{\theta}_1$，$\hat{\dot{\theta}}_2=\dot{\theta}_2$，$\hat{\tau}_1=\tau_1$，$\hat{\tau}_2=\tau_2$. 实际中以 θ_1 和 θ_2 的测量值作为平衡点来对系统(24)进行线性化，设变参数 $\rho=\cos(\theta_2-\theta_1)$，则 $\boldsymbol{\rho}\in[-1,1]$. 设输出矢量 $\boldsymbol{y}(t)$为机器人关节位置，$\boldsymbol{w}_2(t)$为位置测量噪声，$\boldsymbol{w}_1(t)$为模型误差、动态不确定和外部干扰引起的等价扰动，式(25)可表示为与式(1)相同的形式，性能加权矩阵取为 $C_{11}(\boldsymbol{\rho}(t))=0_{1\times 4}$，$C_{12}(\boldsymbol{\rho}(t))=\begin{bmatrix}1&0&1&0\\0&1&0&1\end{bmatrix}$. 取 $\gamma=1$，如果限制 $X(\boldsymbol{\rho})$和 $Y(\boldsymbol{\rho})$在变参数空间内为常数矩阵，则定理 1 降为标准 LPV 综合问题，但该标准问题无解. 取 $\delta_x=0.015$，$\delta_y=0.85$，利用定理 2 设计无需变参数变化率反馈的变增益控制器$K(\boldsymbol{\rho})$. 实验中让机器人末端以 $0.50\ \mathrm{m\cdot s^{-1}}$ 的速度跟踪一直径为 0.50 m 的圆，同时实验了传统变增益 PID 控制，结果见图 1. 在传统变增益 PID 控制器下，关节 1 和 2 的最大误差都为 0.1°，同时存在明显的微振，而在本文的变增益控制器作用下，关节 1 和 2 的最大误差只有 0.04°和 0.03°，同时消除了

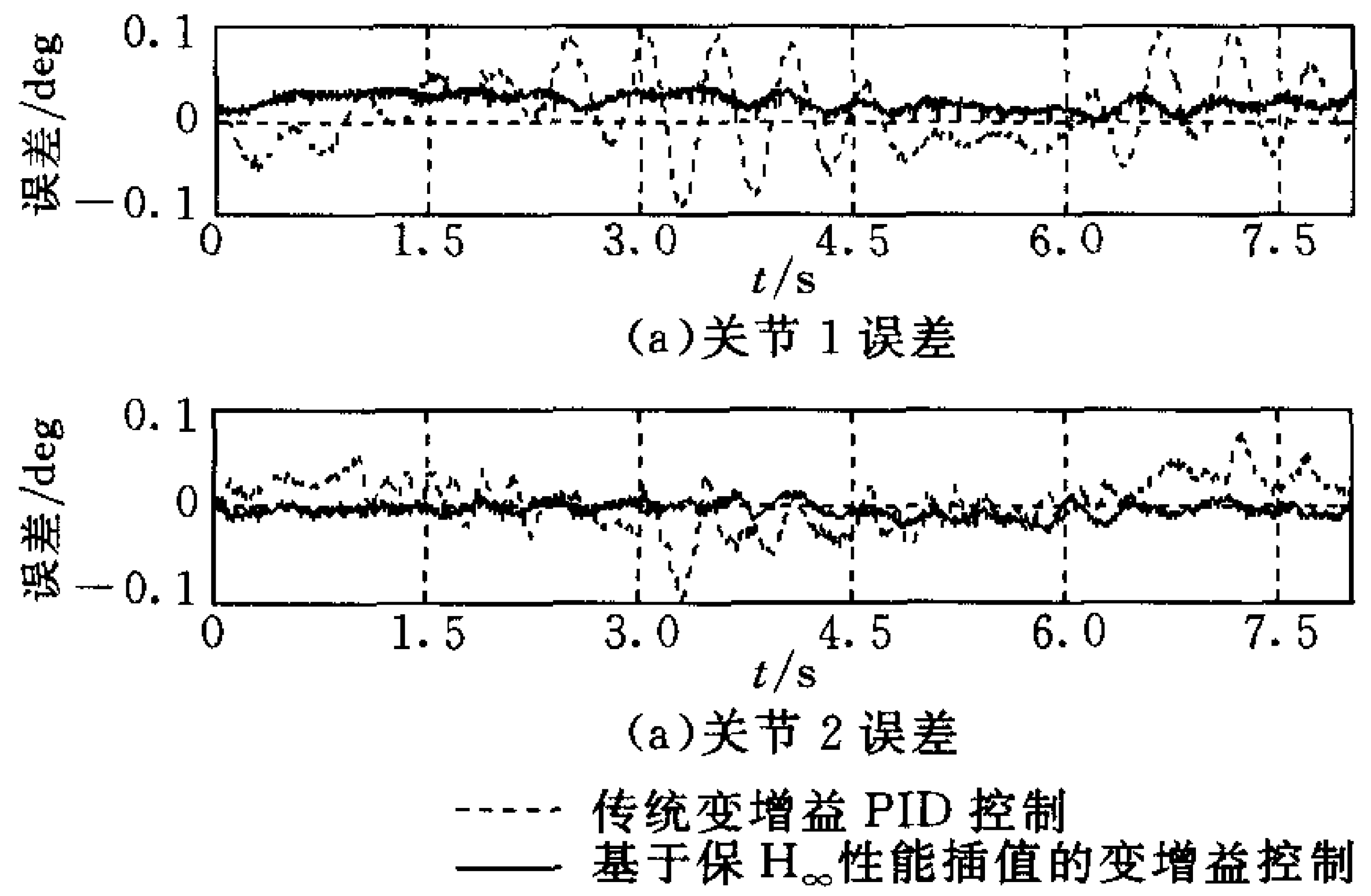

图 1 实验结果
Fig. 1 Experiment result

微振现象，取得了更好的控制效果.

7 结论

本文提出的方法是在保证变参数集内每一固定点均存在 LTI H_∞ 控制器的基础上利用插值设计的，所以克服了现有变增益 LPV 控制器综合无法保证一定能找得到满足要求的变增益控制器的缺点. 本文不仅从理论上证明了所提出的变增益控制器可使系统在整个变参数集均具有 H_∞ 性能，而且通过对两关节直接驱动机器人的实验验证了其有效性.

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虞忠伟 1999 年和 2002 年于同济大学电气工程系获硕士学位和博士学位. 主要研究方向为机器人变增益鲁棒控制，学习控制.

(**YU Zhong-Wei** Received his master and Ph. D. degree in Control Theory and Control Engineering from Shanghai Tongji University in 1999 and 2002 respectively. His research interests include gain-scheduled robust control and learning control for robot.)

陈辉堂 1953 年毕业于上海交通大学，1956 年研究生毕业于上海交通大学工业企业电气化专业，现为同济大学信息与控制工程系教授，博士生导师. 研究方向为机器人控制与智能控制.

(**CHEN Hui-Tang** He is currently professor in Tongji University. Graduated from Shanghai Jiaotong University in 1953 and postgraduated from Shanghai Jiaotong University in 1956. His research interests include robot control and intelligent control.)

陈启军 2000 年在同济大学获博士学位，现为同济大学信息与控制工程系教授。主要研究方向为机器人控制与智能控制.

(**CHEN Qi-Jun** He is currently the professor in the Department of Information and Control Engineering, Tongji University. Received the Ph. D. degree in Tongji University in 2002. His research interests include the robotic control and intelligent.)