第29卷 第5期 自 动 化 学 报 Vol. 29, No. 5
2003年9月 ACTA AUTOMATICA SINICA Sept., 2003

Fuzzy Indirect Adaptive Sliding Mode Tracking Control for a Class of Nonlinear Interconnected Systems[1)]

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Abstract Based on fuzzy logic approximation principle, a new fuzzy decentralized controller is designed for a class of nonlinear interconnected systems with unknown dynamics. Two cases are discussed, i. e. , whether or not the interconnected terms satisfy certain general restrict conditions. Different methods are adopted to counteract the affection of the interconnected terms to the whole system. The mode transformation function is used to realize the changing between the fuzzy indirect adaptive controller and fuzzy sliding mode controller, which keeps the state of the system changing in a close bounded set. By using Lyapunov method, it is proved that the close-loop system is stable and the tracking errors converge to a neighborhood of zero. The result of the emulation proves the validation of the designed controllers.

Key words Interconnected system, fuzzy indirect adaptive control, sliding mode control, tracking control

1 Introduction

In recent years, fuzzy control has been used successfully in complex control plants that are ill-defined or have no exact mathematical models[1~7]. Fuzzy adaptive control refers to fuzzy logic systems with adaptive learning algorithm. It is usually adopted to maintain certain system performance for controlled object with uncertainties or unknown factors in the parameters and structure. There are two cases: indirect one and direct one. The fuzzy indirect adaptive controller means that the fuzzy logic system in the controller is used to build models for the control objective but not as a controller itself[1]. Many existing references utilize the approximation property of the fuzzy logic system and the Lyapunov function syntheses approach to design globally stable adaptive controllers. However, some of them are limited to nonlinear systems with constant control gain; or the convergence of the tracking error depends on the assumption that the error is square integrable; or the control scheme depends on the fuzzy system to make the parameters matched completely, which may not be easy to check or realize. Su gave us a stable fuzzy indirect adaptive controller through sliding mode control and weakened the above constraints[2]. The fuzzy sliding mode control means that it mixes the fuzzy logic with the common sliding mode control. It utilizes the properties of the linguistic variables that fuzzy control itself possesses, adopts nonexact inference approach to substitute the discontinuous parts in the sliding mode to make the control signal smooth. Many researchers have made research on the fuzzy sliding mode control, but there is no exact mathematic expression, therefore, it is difficult to analyze the stability of close-loop system.

Based on Su[2], Tong[5] and Zhang[7], combining the fuzzy adaptive control algorithm, the fuzzy logic approximation method and the fuzzy sliding mode control, we study a class of nonlinear interconnected systems with more complex models in this paper, in

1) Supported in part by National Natural Science Foundation (79970114) and the National Key Project of P. R. China
Received April 20, 2001; in revised form August 23, 2002
收稿日期 2001-04-20; 收修改稿日期 2002-08-23

which not only the subsystems but also the interconnected terms have unknown functions. Obviously, for such high-dimension large-scale systems the decentralized controllers have outstanding advantages to realize the system stability.

2 System description and problem statement

Consider the following nonlinear unknown dynamic interconnected system:

$$\begin{aligned} \dot{\boldsymbol{x}}_i &= \boldsymbol{a}_i(\boldsymbol{x}_i) + \boldsymbol{b}_i(\boldsymbol{x}_i)[u_i + G_i(\bar{x})] \\ y_i &= h_i(\boldsymbol{x}_i) \end{aligned} \quad i = 1,2,\cdots,N \tag{1}$$

where $\boldsymbol{x}_i \in R^n, u_i, y_i \in R$ are the state vector, control input and measurable output of the ith subsystem $\sum_i$, respectively. $\boldsymbol{a}_i(\boldsymbol{x}_i)$ and $\boldsymbol{b}_i(\boldsymbol{x}_i)$ are the smooth vector fields on R^n, $h_i(\boldsymbol{x}_i)$ is the smooth function on R^n. They are all unknown dynamic of subsystem $\sum_i$. $G_i(\bar{x})$ is the interconnected term of the large-scale system and it is also an unknown smooth function on $R^{N\times n}$, and $\bar{x}=(\boldsymbol{x}_1,\cdots,\boldsymbol{x}_{i-1},\boldsymbol{x}_{i+1},\cdots,\boldsymbol{x}_N)$.

Definition 1. System (1) has uniform relative degree at $\boldsymbol{x}_0$, i. e., $r_1=r_2=\cdots=r_N=r$, if $r_1, r_2, \cdots, r_N$ satisfiy the following conditions:

(i) For all $1\leqslant i\leqslant N, k<r-1$ and all $\boldsymbol{x}_i$ in a neighborhood of $\boldsymbol{x}_0$, the following formula holds:

$$L_{b_i} L_{a_i}^k h_i(\boldsymbol{x}_i) = 0$$

(ii) $L_{b_i} L_{a_i}^{r-1} h_i(\boldsymbol{x}_i) \neq 0$

It is also said that system (1) has uniform strong relative degree at $\boldsymbol{x}_0$.

Assumption 1. Suppose system (1) has uniform strong relative degree for $\forall \boldsymbol{x}_i \in R^n$.

Then by definition 1, Assumption 1, and [4], there exists the diffemophism map $T_i(\boldsymbol{x}_i)=(\boldsymbol{\xi}_i, \eta_i)$ such that system (1) can be transformed into the following form:

$$\begin{aligned} \dot{\xi}_1^i &= \xi_2^i \\ \dot{\xi}_2^i &= \xi_3^i \\ &\vdots \cdots \vdots \\ \dot{\xi}_{r-1}^i &= \xi_r^i \\ \dot{\xi}_r^i &= f_i(\boldsymbol{\xi}_i, \eta_i) + g_i(\boldsymbol{\xi}_i, \eta_i)[u_i + G_i(\bar{x})] = f_i(\boldsymbol{x}_i) + g_i(\boldsymbol{x}_i)[u_i + G_i(\bar{x})] \\ \dot{\eta}_i &= q_i(\boldsymbol{\xi}_i, \eta_i) \\ y_i &= \xi_1^i \end{aligned} \tag{2}$$

where $f_i(\boldsymbol{x}_i)=L_{a_i}^r h_i(\boldsymbol{x}_i)=f_i(\boldsymbol{\xi}_i, \eta_i)$, $g_i(\boldsymbol{x}_i)=L_{b_i}L_{a_i}^{r-1}h_i(\boldsymbol{x}_i)=g_i(\boldsymbol{\xi}_i, \eta_i)$, $q_i(\boldsymbol{0}, \eta_i)$ is the zero dynamic system of system (1), $\boldsymbol{\xi}_i=(\xi_1^i, \cdots, \xi_r^i)$.

Assumption 2. System (2) is minimum phase, and for variable $\boldsymbol{\xi}_i$, the following Lipshcitz condition is always satisfied:

$$|q_i(\boldsymbol{\xi}_i, \eta_i) - q_i(\boldsymbol{0}, \eta_i)| \leqslant L|\boldsymbol{\xi}_i| \tag{3}$$

Remark 1. Under Assumption 2, if $\boldsymbol{\xi}_i$ is bounded, then η_i must be bounded [4].

Assumption 3. For the unknown functions $H_i(\boldsymbol{x}_i)=g_i^{-1}(\boldsymbol{x}_i)f(\boldsymbol{x}_i)$, $g_i(\boldsymbol{x}_i)$ and $G_i(\bar{x})$, suppose that there exist known functions $M_{1i}(\boldsymbol{x}_i), M_{2i}(\boldsymbol{x}_i), M_{3i}(\boldsymbol{x}_i)$ and γ_{ijk} such that

i) $|H_i(\boldsymbol{x}_i)| \leqslant M_{1i}(\boldsymbol{x}_i)$ ii) $|g_i^{-1}(\boldsymbol{x}_i)| \leqslant M_{2i}(\boldsymbol{x}_i)$

iii) $|(g_i^{-1}(\boldsymbol{x}_i))'| \leqslant |\Delta g_i^{-1}(\boldsymbol{x}_i)\dot{\boldsymbol{x}}_i| \leqslant M_{3i}(\boldsymbol{x}_i)\|\boldsymbol{x}_i\|$ iv) $G_i(\bar{x}) \leqslant \sum_{k=0}^{q}\sum_{j=1}^{N}\gamma_{ijk}\|\boldsymbol{x}_j\|^k$

For the given bounded reference output y_{im}, suppose $y_{im}, \dot{y}_{im}, \cdots, y_{im}^{(r)}$ are measurable and bounded. Let

$$X_{im} = (y_{im}, \dot{y}_{im}, \cdots, y_{im}^{(r-1)}), \quad X_{i1} = (y_i, \dot{y}_i, \cdots, y_i^{(r-1)})$$

Define the tracking errors as:

$$e_i(t) = y_i(t) - y_{im}(t)$$

The control objective: based on fuzzy logic system, design a state feedback fuzzy controller $u_i(\boldsymbol{x}_i|\theta_i)$ and the adaptive law of parameter θ_i such that:

1) All variables concerned with the close-loop system are uniformly bounded.

2) Under the restrictive condition iii), the tracking error $e_i(t)$ convergences to a small neighborhood of zero.

3 Designing of the fuzzy indirect adaptive controller(FIAC)

Define the sliding mode plane as:

$$s_i(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \eta_i\right)^{(r-1)} e_i \tag{4}$$

where η_i is a nonnegative constant.

If $f_i(\boldsymbol{x}_i)$, $g_i(\boldsymbol{x}_i)$ and $G_i(\bar{x})$ are known, then the controller can be designed as:

$$u_i^* = \frac{1}{g_i(\boldsymbol{x}_i)}[-f_i(\boldsymbol{x}_i) + \lambda_i s_i(t) + \dot{s}_i(t) + y_{im}^{(r)} + e_i^{(r)}] - G_i(\bar{x}) \quad \lambda_i > 0 \tag{5}$$

Substituting (5) into (2) yields

$$\dot{s}_i(t) + \lambda_i s_i(t) = 0$$

Thus, it can be deduced that $\lim_{t\to\infty} e_i = 0$. But in the case that $f_i(\boldsymbol{x}_i)$, $g_i(\boldsymbol{x}_i)$and $G_i(\bar{x})$ are unknown, u_i^* cannot be obtained. So the fuzzy indirect adaptive controller can be constructed by utilizing fuzzy logic system.

Suppose that for the unknown functions $H_i(\boldsymbol{x}_i)$, $g_i^{-1}(\boldsymbol{x}_i)$, there are the following fuzzy linguistic description rules (provided by experts):

$R_i^{(l)}$: If x_{i1} is $F_{i1}^{(l)}$ and x_{i2} is $F_{i2}^{(l)}$, and,$\cdots$, and x_{in} is $F_{in}^{(l)}$, then $H_i(\boldsymbol{x}_i)$ is $C_i^{(l)}$.

$R_i^{(s)}$: If x_{i1} is $B_{i1}^{(s)}$ and x_{i2} is $B_{i2}^{(s)}$ and,$\cdots$, and x_{in} is $B_{in}^{(s)}$, then $g_i^{-1}(\boldsymbol{x}_i)$ is $D_i^{(s)}$.

In the above $F_{ik}^{(l)}, B_{ik}^{(s)}, C_i^{(l)}, D_i^{(s)}$ $(k=\{1,\cdots,n\}, l=\{1,\cdots,p\}, s=\{1,\cdots,q\})$ are fuzzy sets on R. Their membership functions are Gaussian form. Then the following fuzzy logic system can be constructed:

$$H_i(\boldsymbol{x}_i \mid \alpha_i) = \sum_{l=1}^{p} \alpha_{i,l} P_{i,l}(\boldsymbol{x}_i) = \alpha_i^{\mathrm{T}} P_i(\boldsymbol{x}_i)$$

$$g_i^{-1}(\boldsymbol{x}_i \mid \beta_i) = \sum_{s=1}^{q} \beta_{i,s} Q_{i,s}(\boldsymbol{x}_i) = \beta_i^{\mathrm{T}} Q_i(\boldsymbol{x}_i) \tag{6}$$

Define the bounded close sets A_{id}, A_i as follows[2].

$$A_{id} = \{\boldsymbol{x}_i \mid \|\boldsymbol{x}_i - \boldsymbol{x}_{i0}\|_{p,\pi_i} \leqslant 1\}, \ A_i = \{\boldsymbol{x}_i \mid \|\boldsymbol{x}_i - \boldsymbol{x}_{i0}\|_{p,\pi_i} \leqslant 1 + \varphi_i\}$$

$$\|\boldsymbol{x}_i\|_{p,\pi_i} = \left\{\sum_{i=1}^{n}\left(\frac{|x_{i,n_i}|}{\pi_i}\right)^p\right\}^{1/p}$$

where $\{\pi_i\}_{i=1}^{n}$ is a group of strictly positive power weights. x_{i0} is a fixed point. φ_i represents the width of transition field. $\|\boldsymbol{x}_i\|_{p,\pi_i}$ is a kind of P-norm. Then in the close set A_i, by using omnipotent approximation theorem[1], for given $\varepsilon_{1i} \geqslant 0$, and $\varepsilon_{2i} \geqslant 0$, there exist the fuzzy logic systems with the form as (6) such that for arbitrary $x_i \in A_i$, the following inequalities hold:

$$|H_i(\boldsymbol{x}_i) - H_i(\boldsymbol{x}_i \mid \alpha_i)| \leqslant \varepsilon_{1i}$$

$$|g_i^{-1}(\boldsymbol{x}_i) - g_i^{-1}(\boldsymbol{x}_i \mid \beta_i)| \leqslant \varepsilon_{2i}$$

Based on [2] and [3], design the FIAC as:

$$u_i = -k_{d_i} s_{i\Delta}(t) - \frac{1}{2} M_{3i} \|\boldsymbol{x}_i\| s_{i\Delta} - \sum_{k=0}^{p}\sum_{j=1}^{N} \hat{r}_{ijk} \|\boldsymbol{x}_j\|^k - (1 - m_i(t)) u_{adi} + m_i(t) k_{1i}(s_i, t) u_{fsi} \tag{7}$$

where

$$u_{adi} = H_i(\boldsymbol{x}_i \mid \hat{\alpha}_i) + g_i^{-1}(\boldsymbol{x}_i \mid \hat{\beta}_i) a_{ri} + \hat{\varepsilon}_{1i} u_{fsi} + \hat{\varepsilon}_{2i} |a_{ri}| u_{fsi}$$

$$a_{ri} = -y_{im}^{(r)} + \sum_{k=1}^{r-1} \binom{r-1}{k} \eta_i^k e_i^{(r-k)} \tag{8}$$

$$s_{i\Delta}(t) = s_i - \psi_i \mathrm{sat}(s_i/\psi_i) \quad \psi_i > 0$$

where u_{adi} is the adaptive part. u_{fsi} is the fuzzy sliding mode controller, $k_{1i}(s_i, t) > 0$. $m_i(t)$ is a mode transformation function, and it realizes the changing between the adaptive control and fuzzy sliding mode control. The purpose of inducing the mode transformation function $m_i(t)$ is to keep the state in a bounded close set by using the designed fuzzy controllers. $0 \leqslant m_i(t) \leqslant 1$, and is defined as:

$$m_i(t) = \max\left\{0, \mathrm{sat}\left(\frac{\| \boldsymbol{x}_i - \boldsymbol{x}_{i0} \|_{p,\pi_i} - 1}{\varphi_i}\right)\right\} \tag{9}$$

where sat(•) is a saturation function (See [7]).

$s_{i\Delta}$ is the distance between the state and the border layer. $\eta_i > 0$ is a constant . $\psi_i \geqslant 0$ is the width of the border layer. $s_{i\Delta}$ possesses the following properties:

1) if $|s_i| > \psi_i$, then $|s_{i\Delta}| = |s_i| - \psi_i$, and $\dot{s}_{i\Delta} = \dot{s}_i$;

2) if $|s_i| \leqslant \psi_i$, then $\dot{s}_{i\Delta} = \dot{s}_i = 0$.

Based on the idea of Tong[3], the fuzzy sliding mode controller can be designed. The fuzzy linguistic variables s_i and u_{fsi}, the fuzzy membership function, and the exact expression can be found in [3]. Here we omit the details.

Let $\tilde{\alpha}_i = \hat{\alpha}_i - \alpha_i$, $\tilde{\beta}_i = \hat{\beta}_i - \beta_i$, and $\tilde{\boldsymbol{\gamma}}_{ijk} = \hat{\boldsymbol{\gamma}}_{ijk} - \boldsymbol{\gamma}_{ijk}$ be the parameter error vectors.

Choose the adaptive laws of the parameters and control gains as:

$$\dot{\hat{\alpha}}_i = \begin{cases} \eta_{1i}(1 - m_i(t)) P_i(\boldsymbol{x}_i) s_{i\Delta}, & \text{if } |\hat{\alpha}_i| < M_f, \text{or } |\hat{\alpha}_i| = M_f \text{ and } \hat{\alpha}_i^{\mathrm{T}} P_i(\boldsymbol{x}_i) s_{i\Delta} \leqslant 0 \\ P_1\{\eta_{1i}(1 - m_i(t)) P_i(\boldsymbol{x}_i) s_{i\Delta}\}, & \text{if } |\hat{\alpha}_i| = M_f \text{ and } \hat{\alpha}_i^{\mathrm{T}} P_i(\boldsymbol{x}_i) s_{i\Delta} > 0 \end{cases} \tag{10}$$

$$\dot{\hat{\beta}}_i = \begin{cases} \eta_{2i}(1 - m_i(t)) a_{ri} Q_i(\boldsymbol{x}_i) s_{i\Delta}, & \text{if } |\hat{\beta}_i| < M_g, \text{or } |\hat{\beta}_i| = M_g \text{ and } \hat{\beta}_i^{\mathrm{T}} Q_i(\boldsymbol{x}_i) a_{ri} s_{i\Delta} \leqslant 0 \\ P_2\{\eta_{2i}(1 - m_i(t)) a_{ri} Q_i(\boldsymbol{x}_i) s_{i\Delta}\}, & \text{if } |\hat{\beta}_i| = M_g \text{ and } \hat{\beta}_i^{\mathrm{T}} Q_i(\boldsymbol{x}_i) a_{ri} s_{i\Delta} > 0 \end{cases} \tag{11}$$

$$\dot{\hat{\varepsilon}}_{1i} = \eta_{3i}(1 - m_i(t)) |s_{i\Delta}| \tag{12}$$

$$\dot{\hat{\varepsilon}}_{2i} = \eta_{4i}(1 - m_i(t)) |s_{i\Delta} a_{ri}| \tag{13}$$

$$\dot{\hat{\gamma}}_{ijk} = \eta_{5i} |s_{i\Delta}(t)| \, \|\boldsymbol{x}_i\|^k \quad \hat{\gamma}_{ijk}(0) \geqslant 0 \tag{14}$$

where $\eta_{ki} > 0, (k = 1, \cdots, 5)$ is the learning ratio. M_f and M_g are the given positive constants.

$$P_1\{*\} = \eta_{1i}(1 - m_i(t)) s_{i\Delta} P_i(\boldsymbol{x}_i) - \eta_{1i}(1 - m_i(t)) s_{i\Delta} \frac{\hat{\alpha}_i \hat{\alpha}_i^{\mathrm{T}} P_i(\boldsymbol{x}_i)}{|\hat{\alpha}_i|^2} \tag{15}$$

$$P_2\{*\} = \eta_{2i}(1 - m_i(t)) s_{i\Delta} Q_i(\boldsymbol{x}_i) a_{ri} - \eta_{2i}(1 - m_i(t)) s_{i\Delta} a_{ri} \frac{\hat{\beta}_i \hat{\beta}_i^{\mathrm{T}} Q_i(\boldsymbol{x}_i)}{|\hat{\beta}_i|^2} \tag{16}$$

4 Analysis for system stability

Theorem 1. For system (1), under assumptions (1), (2), and (3), if one adopts the fuzzy indirect adaptive decentralized controller (7) and adaptive laws (10)~(16), and selects

$$k_{1i}(s_i, t) = M_{1i} + M_{2i} |a_{ri}|,$$

then the following properties hold:

1) $|\hat{\alpha}_i| \leqslant M_f$, $|\hat{\beta}_i| \leqslant M_g$, $X_{i1}, u_i \in L_\infty$

2) The tracking error $e_i(t)$ convergences to a neighborhood of zero.

Proof. Omitted

Remark 2. In the design of the FIAC, we fix the front fuzzy sets of the fuzzy logic system and only regulate the behind fuzzy sets. If we adopt the second case in [1], i. e., adjust the two kinds of fuzzy sets simultaneously, and use the same control scheme as above, we must utilize Taylor formula to approximate the fuzzy system $H_i(\boldsymbol{x}_i \mid \hat{\alpha}_i)$ and $g_i^{-1}(\boldsymbol{x}_i \mid \hat{\alpha}_i)$ as follows.

$$\begin{aligned} |H_i(\boldsymbol{x}_i \mid \hat{\alpha}_i) - H_i(\boldsymbol{x}_i \mid \alpha_i^*)| &\leqslant \tilde{\alpha}_i^{\mathrm{T}}\left[\frac{\partial H_i(\boldsymbol{x}_i \mid \hat{\alpha}_i)}{\partial \hat{\alpha}_i}\right] + O(|\tilde{\alpha}_i|^2) \\ |g_i^{-1}(\boldsymbol{x}_i \mid \hat{\beta}_i) - g_i^{-1}(\boldsymbol{x}_i \mid \beta_i^*)| &\leqslant \hat{\beta}_i^{\mathrm{T}}\left[\frac{\partial g_i^{-1}(\boldsymbol{x}_i \mid \hat{\beta}_i)}{\partial \hat{\beta}_i}\right] + O(|\tilde{\beta}_i|^2) \end{aligned} \tag{17}$$

Correspondingly, the adaptive laws of the parameters can be rewritten as:

$$\dot{\hat{\alpha}}_i = \begin{cases} \eta_{1i}(1 - m_i(t)) \dfrac{\partial \hat{H}_i}{\partial \hat{\alpha}_i} s_{i\Delta}, & \text{if } |\hat{\alpha}_i| < M_f, \text{or } |\hat{\alpha}_i| = M_f \text{ and } \hat{\alpha}_i^{\mathrm{T}} \dfrac{\partial \hat{H}_i}{\partial \hat{\alpha}_i} s_{i\Delta} \leqslant 0 \\ P_1\left\{\eta_{1i}(1 - m_i(t)) \dfrac{\partial \hat{H}_i}{\partial \hat{\alpha}_i} s_{i\Delta}\right\}, & \text{if } |\hat{\alpha}_i| = M_f \text{ and } \hat{\alpha}_i^{\mathrm{T}} \dfrac{\partial \hat{H}_i}{\partial \hat{\alpha}_i} s_{i\Delta} > 0 \end{cases} \tag{18}$$

$$\dot{\hat{\beta}}_i = \begin{cases} \eta_{2i}(1 - m_i(t)) a_{ri} \dfrac{\partial \hat{g}_i^{-1}}{\partial \hat{\beta}_i} s_{i\Delta}, & \text{if } |\hat{\beta}_i| < M_g, \text{of } |\hat{\beta}_i| = M_g \text{ and } \hat{\beta}_i^{\mathrm{T}} \dfrac{\partial \hat{g}_i^{-1}}{\partial \hat{\beta}_i} a_{ri} s_{i\Delta} \leqslant 0 \\ P_2\left\{\eta_{2i}(1 - m_i(t)) a_{ri} \dfrac{\partial \hat{g}_i^{-1}}{\partial \hat{\beta}_i} s_{i\Delta}\right\}, & \text{if } |\hat{\beta}_i| = M_g \text{ and } \hat{\beta}_i^{\mathrm{T}} \dfrac{\partial \hat{g}_i^{-1}}{\partial \hat{\beta}_i} a_{ri} s_{i\Delta} > 0 \end{cases} \tag{19}$$

If we ignore the high order terms, or adopt sliding mode control to attenuate the affection of them, the same result as the proof of theorem 1 can be obtained. $O(|\tilde{\alpha}_i|^2)$ and $O(|\tilde{\beta}_i|^2)$ are the high order terms.

$$\begin{aligned} \alpha_i^* &= \arg\min_{|\hat{\alpha}_i| \in M_f}[\sup_{x_i \in A_i} |H_i(\boldsymbol{x}_i \mid \hat{\alpha}_i) - H_i(\boldsymbol{x}_i)|] \\ \beta_i^* &= \arg\min_{|\hat{\beta}_i| \in M_g}[\sup_{x_i \in A_i} |g_i^{-1}(\boldsymbol{x}_i \mid \hat{\beta}_i) - g_i^{-1}(\boldsymbol{x}_i)|] \end{aligned} \tag{20}$$

Remark 3. In Theorem 1, it is discussed how to design decentralized fuzzy adaptive sliding mode controller while the interconnected terms satisfy condition (4) of Assumption 3. However, in the practice of large-scale systems, the interconnected terms are always not satisfying the above condition. Therefore, in the following we will discuss how to design controllers to get the objective while the interconnected terms are unknown, but bounded.

Thus, condition (4) in Assumption 3 is modified as:

$$G_i(\bar{x}) \leqslant M_{4i}(\bar{x})$$

For the unknown function $G_i(\bar{x})$, there is the following fuzzy linguistic description rules (provided by experts):

$R_i^{(c)}$: If x_{11} is $A_{i11}^{(c)}$ and x_{12} is $A_{i12}^{(c)}$, and $\cdots$, and x_{1n} is $A_{i1n}^{(c)}$,
and x_{21} is $A_{i21}^{(c)}$ and x_{22} is $A_{i22}^{(c)}$, and $\cdots$, and x_{2n} is $A_{i2n}^{(c)}$,
……
and x_{N1} is $A_{iN1}^{(c)}$ and x_{N2} is $A_{iN2}^{(c)}$, and $\cdots$, and x_{Nn} is $A_{iNn}^{(c)}$,
then $G_i(\bar{X})$ is $E_i^{(c)}$.

where $A_{ijk}^{(c)}, E_i^{(c)}$ $(i, j = \{1, \cdots, N\}, k = \{1, \cdots, n\}, c = \{1, \cdots, t\})$ are fuzzy sets on R, their membership functions are Gaussian form. Then the following fuzzy logic system can be constructed:

$$G_i(\bar{x} \mid \gamma_i) = \sum_{c=1}^{t} \gamma_{i,c} R_{i,c}(\bar{x}) = \gamma_i^{\mathrm{T}} R_i(\bar{x}) \tag{21}$$

Suppose the above formula satisfies

$$|G_i(\bar{x}) - G_i(\bar{x} \mid \gamma_i)| \leqslant \varepsilon_{3i}$$

Then, design the fuzzy indirect adaptive controller as:

$$u_i = -k_{d_i} s_{i\Delta}(t) - \frac{1}{2} M_{3i} \|\boldsymbol{x}_i\| s_{i\Delta} - (1 - m_i(t)) u_{adi} + m_i(t) k_{1i}(s_i, t) u_{fsi} \tag{22}$$

where

$$u_{adi} = H_i(\boldsymbol{x}_i \mid \hat{\alpha}_i) + g_i^{-1}(\boldsymbol{x}_i \mid \hat{\beta}_i) a_{ri} + G_i(\bar{x} \mid \hat{\gamma}_i) + \hat{\varepsilon}_{1i} u_{fsi} + \hat{\varepsilon}_{2i} |a_{ri}| u_{fsi} + \hat{\varepsilon}_{3i} u_{fsi}$$

The meaning of the other symbols are the same as the ones in theorem 1.

Add the followings to the adaptive laws:

$$\dot{\hat{\gamma}}_i = \begin{cases} \eta_{5_i}(1 - m_i(t)) R_i(\bar{x}) s_{i\Delta}, \\ \qquad \text{when } |\hat{\gamma}_i| < M_G, \text{or when } |\hat{\gamma}_i| = M_G \text{ and } \hat{\gamma}_i^{\mathrm{T}} R_i(\bar{x}) s_{i\Delta} \leqslant 0 \\ P_3\{\eta_{5_i}(1 - m_i(t)) R_i(\bar{x}) s_{i\Delta}\}, \quad \text{when } |\hat{\gamma}_i| = M_G \text{ and } \hat{\gamma}_i^{\mathrm{T}} R_i(\bar{x}) s_{i\Delta} > 0 \end{cases}$$

$$\dot{\hat{\varepsilon}}_{3i} = \eta_{6_i}(1 - m_i(t)) |s_{i\Delta}| \tag{23}$$

$$P_3\{*\} = \eta_{5_i}(1 - m_i(t)) s_{i\Delta} R_i(\bar{x}) - \eta_{5_i}(1 - m_i(t)) s_{i\Delta} \frac{\hat{\gamma}_i \hat{\gamma}_i^{\mathrm{T}} R_i(\bar{x})}{|\hat{\gamma}_i|^2}$$

It is easy to know that the designed controllers in Remark 3 are the centralized controllers.

5 Emulation

Example:

consider the following nonlinear interconnected system with two subsystems:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= (x_2^2 + 2x_1)\sin(x_2) + [1 + \exp(-x_1)][u_1 + 0.5\sin(3t)x_3] \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= (x_4^2 + 2x_3)\sin(x_4) + [1 + \exp(-x_3)][u_2 + 0.5\sin(3t)x_1] \end{aligned}$$

The given reference states are

$$x_{1m} = \sin\left(\frac{\pi}{2}t\right), \quad x_{2m} = \sin\left(\frac{\pi}{2}t\right)$$

Choose the fuzzy membership functions as:

$$A^1(x_i) = (1 + \exp(5 \times (x_i + 3)))^{-1}, \quad A^2(x_i) = \exp(-(x_i + 2)^2/2)$$

$$A^3(x_i) = \exp(-(x_i + 1)^2/2), \; A^4(x_i) = \exp(-(x_i)^2), \; A^5(x_i) = \exp(-(x_i - 1)^2/2)$$

$$A^6(x_i) = \exp(-(x_i - 2)^2/2), \quad A^7(x_i) = (1 + \exp(5 \times (x_i - 3)))^{-1}$$

Choose the parameters of the controllers as:

$k_{di} = 20$, $\varphi_i = 0.5$, $\lambda_i = 5$, $\eta_{1,i} = 0.6$, $\eta_{2,i} = 0.9$, $\eta_{3,i} = 0.4$, $\eta_{4,i} = 1$
$\eta_{5,i} = 0.01$, $M_{2,i} = 1$, $M_{3,i} = 1 (i = 1, 2)$, $M_{1,1} = x_2^2 + 2|x_1|$, $M_{1,2} = x_4^2 + 2|x_3|$

The initial values are chosen as follows.

$x_1(0) = 1.5$, $x_2(0) = 0$, $x_3(0) = 1.5$, $x_4(0) = 0$, $\hat{\varepsilon}_{1i}(0) = 1.5$, $\hat{\varepsilon}_{2i}(0) = 1$

$$\alpha_{11}(0) = (-0.1 \;\; -0.1 \;\; -0.1 \;\; 0 \;\; 0.1 \;\; 0.1 \;\; 0.1)^{\mathrm{T}}$$

$$\beta_{21}(0) = (0.1 \;\; 0.1 \;\; 0.1 \;\; 0 \;\; -0.1 \;\; -0.1 \;\; -0.1)^{\mathrm{T}}$$

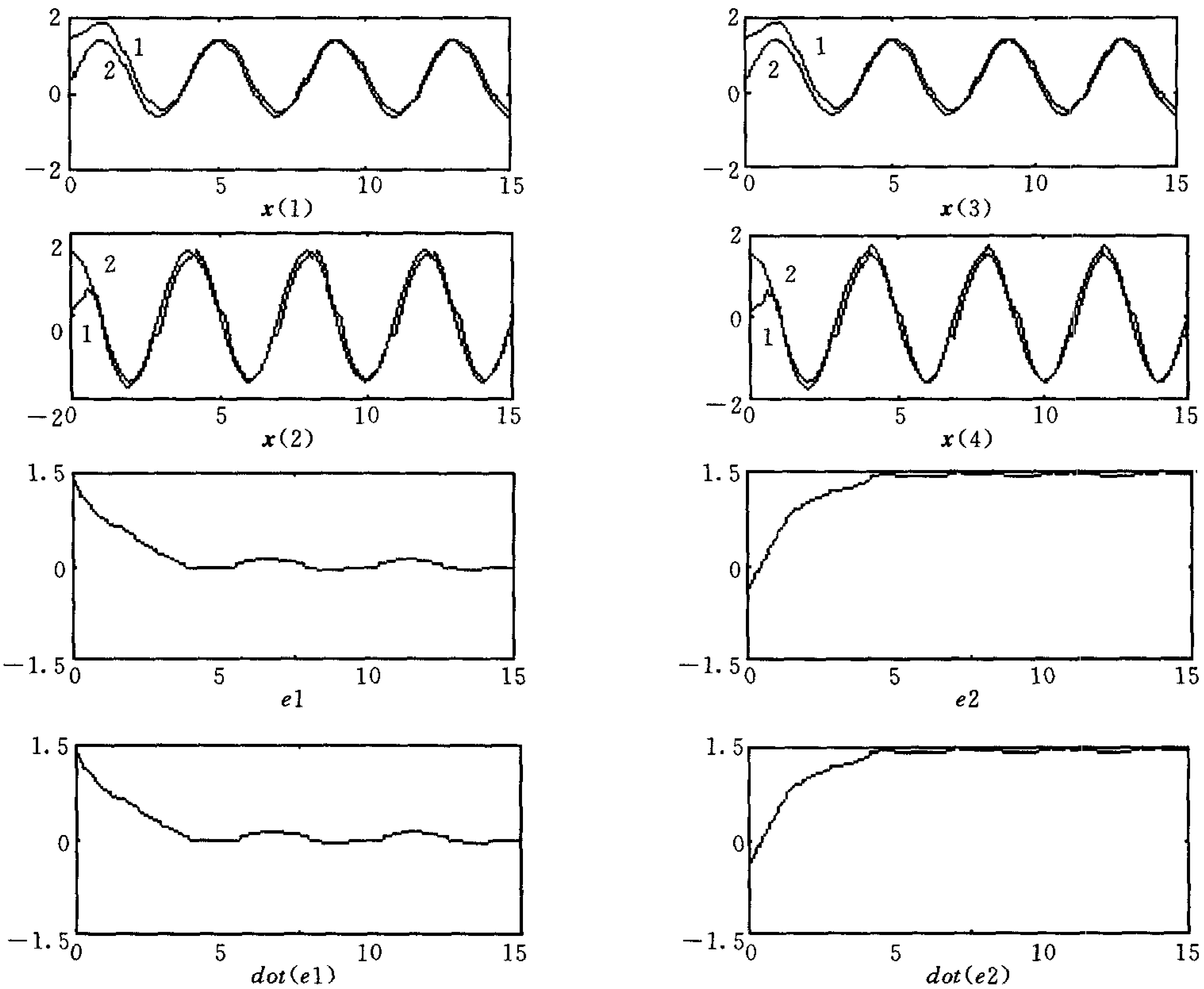
$$\alpha_{12}(0) = (-0.1 \;\; -0.1 \;\; -0.1 \;\; 0 \;\; 0.1 \;\; 0.1 \;\; 0.1)^{\mathrm{T}}$$

$$\beta_{22}(0) = (0.1 \;\; 0.1 \;\; 0.1 \;\; 0 \;\; -0.1 \;\; -0.1 \;\; -0.1)^{\mathrm{T}}$$

The emulation curves are shown in Fig. 1:

6 Conclusion

In this paper, a new scheme of fuzzy indirect adaptive control is proposed for the nonlinear interconnected systems with SISO subsystems. It combines the fuzzy adaptive control, fuzzy logic approximation and the fuzzy sliding mode control. The approach does not



(1 is the actual state, 2 is the reference state)

Fig. 1 Curves of the states and the errors

need the assumption that the tracking errors are square integrable. The sliding mode control compensates for the affection of the fuzzy approximation errors on the output tracking errors. It overcomes the defect that the controllers has no mathematical expressions such that it is difficult to analyze the stability of the systems. The scheme adopts sliding mode plane to concentrate the information about errors and error changing ratio. Using the system states information, the mode transformation functions are like the a "balance device" to make the decision between the two kinds of controllers (indirect adaptive controller and sliding mode controller) that which are more suitable to obtain the tracking objective. Of course, in this paper, only the fuzzy linguistic description information is utilized. Whether the fuzzy control rules can be utilized is our future work.

Acknowledgement. This paper had been inspired and guided by professor Tong Shaocheng (Liaoning Institute of Technology) before the manuscript was formed. The author makes sincerely thank him here.

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一类非线性互联系统的间接自适应模糊滑模跟踪控制

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摘 要 基于模糊逻辑逼近原理,针对非线性动态未知互联系统,设计一种新的模糊分散控制器.讨论了互联项满足或不满足一般性约束条件的两种情形,用不同方法来补偿其对大系统的影响.同时利用模式转换函数来实现间接自适应控制与模糊滑模控制之间的转换,使系统的状态在有界闭集内变化,并依据 Lyapunov 方法证明了闭环系统稳定,跟踪误差收敛到零的一个小邻域内.仿真结果证明所设计的模糊控制器的有效性.

关键词 互联系统,间接自适应模糊控制,滑模控制,跟踪控制

中图分类号 TP273