Using Linear Intersection for Node Location Computation in Wireless Sensor Networks[1)]

SHI Qin-Qin[1] HUO Hong[1] FANG Tao[1] LI De-Ren[1,2]

[1](*Institute of Image Processing and Pattern Recognition, Shanghai Jiaotong University, Shanghai* 200240)

[2](*State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing, Wuhan University, Wuhan, Hubei* 430072)

(E-mail: shiqq@sohu.com, honghuo@sjtu.edu.cn, tfang@sjtu.edu.cn)

Abstract Knowing the locations of nodes in wireless sensor networks (WSN) is essential for many applications. Nodes in a WSN can have multiple capabilities and exploiting one or more of the capabilities can help to solve the localization problem. In this paper, we assume that each node in a WSN has the capability of distance measurement and present a location computation technique called linear intersection for node localization. We also propose an applied localization model using linear intersection and do some concerned experiments to estimate the location computation algorithm.

Key words wirelell sensor network, localization, linear intersection, error

1 Introduction

Generally, a WSN is composed of a large number of sensor nodes that are deployed outdoor at random (*e.g.*, by dropping them from an aircraft) in a field[1]. It is essential to know the node location in many applications of WSN such as target tracking, event location reporting, geographic routing and directional querying. Determining the physical locations of sensor nodes after they have been deployed is known as the problem of localization[2].

GPS is the most popular localization system with relatively high accuracy, but it may not be realistic to equip each node in a WSN with GPS due to cost, form factor, energy consumption, and some other restrictive conditions[3]. So, several other localization schemes, classified into two categories in the gross, range-based and range-free[4], have been proposed to determine locations for randomly deployed sensor nodes and they can reduce or completely remove the dependence on GPS. Most of these schemes share a common feature: They use some special nodes called beacon nodes, which are assumed to know their own locations (*e.g.*, through GPS receivers or manual configuration), and other sensors discover their locations on the basis of the information provided by these beacon nodes[5]. The range-based schemes need exploiting node capability to directly measure either distances or angles for localization while the range-free schemes just estimate distances among nodes roughly or collect information of network connectivity only for localization, and so forth, without requiring additional hardware at nodes.

In this paper, we assume that the monitored area is planar, *i.e.* the elevation difference among the nodes can be ignored. We present a node location computation technique called linear intersection (LI) for quick localization in a localization scheme based on local distance measurement. LI is a routine method used for control point densification in surveying engineering. Here we apply it in localization for WSN and do some experiments to estimate its usability.

The rest of the paper is organized as follows. The next section gives an overview on current node location computation techniques used in localization schemes based on distance measurement. Section 3 presents linear intersection technique. Section 4 proposes a WSN localization model using LI. Section 5 estimates the algorithm of LI through some experiments. We summarize the paper in Section 6.

2 An overview on current node location computation techniques

In localization schemes based on distance measurement, each unknown node communicates with beacon nodes and gets enough known location and distance information for localization. The distance information needs exploiting the node′s own capability to measure. The distance measurement method can be RSSI, TOA, TDOA, and so on. Unknown nodes determine their locations locally using a certain

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location computation technique. Trilateration and multilateration are techniques common in use for the location computation in such localization schemes. We describe the computation principle of these two techniques with reference to [6].

2.1 Trilateration

Trilateration computes the intersection of three circles. As shown in Fig. 1 (a), A, B and C are three beacon nodes with known locations (x_A, y_A), (x_B, y_B), and (x_C, y_C), respectively, and D is an unknown node with assumed location (x, y). Let d_A, d_B, d_C be distances between D and A, B, C respectively and they can be expressed as

$$\begin{cases} \sqrt{(x-x_A)^2+(y-y_A)^2}=d_A \\ \sqrt{(x-x_B)^2+(y-y_B)^2}=d_B \\ \sqrt{(x-x_C)^2+(y-y_C)^2}=d_C \end{cases} \tag{1}$$

The location of D is deduced from equation system (1) and written in matrix format as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2(x_A-x_C) & 2(y_A-y_C) \\ 2(x_B-x_C) & 2(y_B-y_C) \end{bmatrix}^{-1} \begin{bmatrix} x_A^2-x_C^2+y_A^2-y_C^2+d_C^2-d_A^2 \\ x_B^2-x_C^2+y_B^2-y_C^2+d_C^2-d_B^2 \end{bmatrix} \tag{2}$$

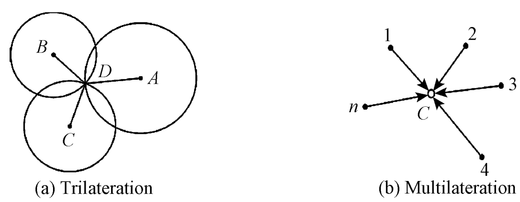


Fig. 1 Schematic diagram of trilateration and multilateration

2.2 Multilateration

In multilateration, usually more than three beacon nodes are needed to determine one unknown node′s location. As shown in Fig. 1 (b), $1, 2, 3, \cdots, n$ are all beacon nodes with locations $(x_1, y_1), (x_2, y_2), \cdots, (x_n, y_n)$, respectively, and the distances between unknown node C and each beacon node are $d_1, d_2, d_3, \cdots, d_n$, respectively. The location (x, y) of C can be deduced as follows. At first,

$$\begin{cases} (x_1-x)^2+(y_1-y)^2=d_1^2 \\ \quad\vdots \\ (x_n-x)^2+(y_n-y)^2=d_n^2 \end{cases} \tag{3}$$

Then, subtract the last equation from all the other $n-1$ equations in equation system (3), and the following equation system can be obtained.

$$\begin{cases} x_1^2-x_n^2-2(x_1-x_n)x+y_1^2-y_n^2-2(y_1-y_n)y=d_1^2-d_n^2 \\ \quad\vdots \\ x_{n-1}^2-x_n^2-2(x_{n-1}-x_n)x+y_{n-1}^2-y_n^2-2(y_{n-1}-y_n)y=d_{n-1}^2-d_n^2 \end{cases} \tag{4}$$

(4) can been expressed as $AX=b$, where

$$A=\begin{bmatrix} 2(x_1-x_n) & 2(y_1-y_n) \\ \vdots & \vdots \\ 2(x_{n-1}-x_n) & 2(y_{n-1}-y_n) \end{bmatrix}, \quad b=\begin{bmatrix} x_1^2-x_n^2+y_1^2-y_n^2+d_n^2-d_1^2 \\ \vdots \\ x_{n-1}^2-x_n^2+y_{n-1}^2-y_n^2+d_n^2-d_{n-1}^2 \end{bmatrix}, \quad X=\begin{bmatrix} x \\ \vdots \\ y \end{bmatrix}$$

At last, after using standard least mean square estimation, the location of C can be expressed as

$$\hat{X}=(A^{\mathrm{T}}A)^{-1}A^{\mathrm{T}}b \tag{5}$$

Compared with trilateration, multilateration can improve localization accuracy, but it involves higher computation overhead.

Remark. The formulas deduced above are directed at applications in 2-D space. Still they can be extended to 3-D space, but the computation will become more complicated due to the increasing number of necessary beacon nodes. In 2-D space, at least three beacon nodes are needed to determine one unknown node′s location both in trilateration and multilateration.

3 Linear intersection

3.1 Computation formula

LI is a flexible, effective and applied technique used in engineering surveying for control point′s densification. As shown in Fig. 2, A, B are two control points, and their coordinates are (x_A, y_A), (x_B, y_B) respectively in a surveying reference frame. P is the point whose location (x_P, y_P) is sought. Measure the distances between P and A, B as a, b, respectively, and then compute the coordinates (x_P, y_P). We introduce the computation formula of LI with reference to [7], and the computation procedure is simple and optimal.

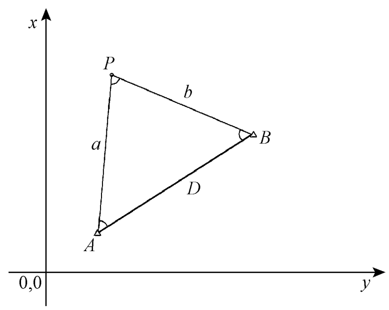


Fig. 2 Schematic diagram of linear intersection

At first, the distance between A and B can been computed according to their coordinates and expressed as $D = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$. Then, the computation formula of (x_P, y_P) can been expressed as

$$\begin{cases} x_P = x_A + L(x_B - x_A) + H(y_B - y_A) \\ y_P = y_A + L(y_B - y_A) - H(x_B - x_A) \end{cases} \tag{6}$$

where

$$\begin{cases} L = \dfrac{a^2 + D^2 - b^2}{2D^2} \\ H = \sqrt{\dfrac{a^2}{D^2} - L^2} \end{cases} \tag{7}$$

Obviously, above computation formula system is regular, simple and convenient. The formula system is highly fit for writing computer programs.

Remark. Above formula system is acceptable for the situation where the locations of P, A, and B accord with a counter-clockwise sequence $A \rightarrow B \rightarrow P$. This requirement must be paid attention to when using the computation formula to avoid the inapplicability.

3.2 Accuracy analysis

Since the distance measurements a and b contain error, the computed location of P contains error inevitably. We assume that the coordinates of control points are starting datum marks without error. Let the mean square errors of a and b be σ_a and σ_b, respectively, and the mean square error of point P′s location be σ_P. Three interior angles of ΔABP are named as $\angle A, \angle B$, and $\angle P$, respectively, according to the name of angle apex.

Rewrite (6) into complete differential equations as

$$\begin{cases} dx_P = (x_B - x_A)dL + (y_B - y_A)dH \\ dy_P = (y_B - y_A)dL + (x_A - x_B)dH \end{cases} \tag{8}$$

Rewrite (7) into complete differential equations as

$$\begin{cases} dL = \dfrac{a}{D^2}da - \dfrac{b}{D^2}db \\ dH = \dfrac{a(1-L)}{HD^2}da + \dfrac{bL}{HD^2}db \end{cases} \tag{9}$$

Substitute (9) into (8), and transfer the product into mean square error equation as

$$\sigma_p^2 = \left\{\left[\frac{a}{D}\right]^2 + \left[\frac{a(1-L)}{HD}\right]^2\right\}\sigma_a^2 + \left\{\left[\frac{b}{D}\right]^2 + \left[\frac{Lb}{HD}\right]^2\right\}\sigma_b^2 = \frac{a^2\sigma_a^2}{H^2D^2}[H^2+L^2-2L+1] + \frac{b^2\sigma_b^2}{H^2D^2}[H^2+L^2] \tag{10}$$

Since $H^2 + L^2 = \dfrac{a^2}{D^2}$, $\dfrac{b}{D} = \dfrac{\sin\angle A}{\sin\angle P}$ and $a\sin\angle A = HD$, substitute these equations into (10) and rewrite the product as

$$\sigma_P^2 = \frac{1}{\sin^2\angle P}[\sigma_a^2 + \sigma_b^2] \tag{11}$$

From (11), such a conclusion can be deduced that the location accuracy of point P is related with the distance measurement accuracy and the size of included angle $\angle P$. While the distance measurement accuracy has been determined, on condition that angle $\angle P$ equals to $90°$, the location accuracy of point P can get the highest degree. The location accuracy will fall while angle $\angle P$ is too small or too big.

4 A localization model using LI

4.1 Localization model

We assume that the sensor nodes are deployed randomly over a monitored area (2-D plane) and they are all unknown nodes with locations (x, y). As show in Fig. 3, two base stations 1 and 2 with known coordinates (x_1, y_1) and (x_2, y_2) respectively are placed beyond the boundary of the monitored area and all the unknown nodes are deployed on one side of the connecting line of the two base stations.

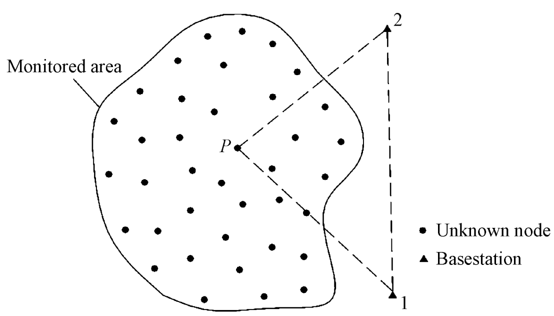


Fig. 3 Schematic diagram of the localization model

Each unknown node has limited resources (battery, CPU, *etc*), but each has range measurement capability (*e.g.* TOA, TDOA, or RSSI). Each base station has long-term power supplies and their beacon signals can reach all sensor nodes in the monitored area. The two base stations can be seen as beacon nodes in other localization schemes and they are the beacon nodes for all the unknown nodes in the monitored area. They will transmit beacon signals periodically to assist unknown nodes with location discovery. Each unknown node receives the beacon signals from base stations and measures the distances to them.

In location computation, the two base stations and each unknown node can be assumed as known control points and undetermined point in LI, respectively. After an unknown node has gotten the known location information of both base stations and two distance measures, it computes its own location locally using LI technique. The computation is executed on each unknown node in the WSN through application program designed on the basis of formula system provided in Section 3.

In our model, there are two obvious advantages. One is that each unknown node in the WSN computes its location independently with no need to translate all the information to an appointed place for location computation. The other is that each unknown node listens to two beacon signals passively during each beacon interval and is not required to transmit radio signal. These two characteristics both can reduce the communication overhead on nodes, in accord with the WSN design requirements.

4.2 Possible error source

The accuracy of node localization is influenced by several factors. As for our model, there are two major influencing factors, the error of local distance measure and the geometrical relationship between each unknown node and the two base stations.

Since the distance measurements contain error, the measure error affects the localization accuracy of the target node. Using different distance measurement techniques and using different communication signals may have big difference[8] in terms of measure error. Commonly, the higher measurement accuracy is at the cost of higher requirement for additional hardware at the sensor nodes. And different range measurement method may have different range of applications, so, the selection of distance measurement method should take the actual need into consideration. In practical applications, we can use (11) in Section 3 for node localization accuracy estimation.

In addition, according to the accuracy analysis in Section 3, while the distance measurement accuracy is determinate, the localization accuracy is still in relation to the graphic structure of each unknown node and the two base stations. So, a node with well graphic structure in the monitored area may have better localization accuracy than others′.

5 Algorithm estimation

5.1 General remark

To demonstrate the performance of the algorithm in our proposed location computation technique, we do some experiments. At first, we simulate the deployment of a WSN with AutoCAD. The deployment accords with that of the localization model we describe in Section 4, and the network is consisted of one hundred unknown nodes and two base stations. We collect all nodes′ coordinate data from the AutoCAD interface and record them as their real locations for the following computation and analysis. Then, we adopt the distance measurement error model introduced in [8] as our distance measurement error model and generate distance measurements for location computation algorithm simulation. LI algorithm is implemented using MATLAB 6.5.

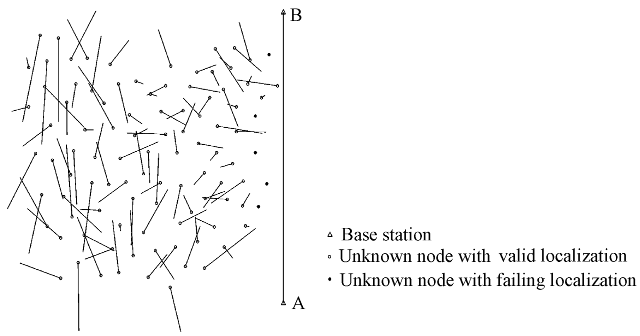
[8] introduces three distance error models of RSSI measurement, and the three models assume uniform distribution of the measurement error. If a node′s location is estimated in permissible error in the case of uniform distribution, then the localization system is expected to work in other distribution and real environment, because uniform distribution has a larger variation than other distributions such as normal distribution. The three error models are: 1) Measurement error is in proportion to the distance between the beacon node and the unknown node. A mean of the absolute value is 10% of the distance, for example, if distance between them is 20m, measurement error is given as a random value between -4.0m and 4.0m. 2) Measurement error is independent of distance and the mean of the absolute value is 1m, *i.e.* the measurement error is given as a random value between -2.0m and 2.0m. 3) This follows the upper boundaries of both the above two models.

In our experiments, the distances between base stations and unknown nodes are almost all longer than 10m, and using model (2) or using model (3) has little different influence on the localization. So, we analyze the localization error under model (1) and model (2) only.

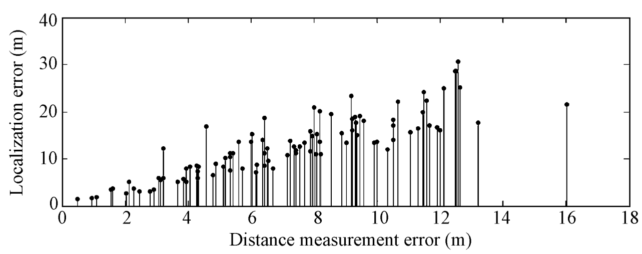
5.2 Error analysis

Under distance error model (1), we generate the distance measurements from all the unknown nodes to the two base stations and implement LI algorithm on each node. From the localization results, we find that there are three failing localization samples among total one hundred nodes. In the rest ninety-seven valid localization samples, the maximal localization error is 30.6m, the minimal localization error is 1.5m, and the average localization error is 12.8m. In order to investigate the relationship between affecting factors in theory and localization error, we diagrammatize localization error vector of each unknown node in Fig. 4(a), the relationship between "localization error" and "distance measurement error" of each unknown node in Fig. 4(b), and the relationship between $|\angle P - 90°|$ and "distance measurement error" of each unknown node in Fig. 4 (c), respectively. Here, what needs to be pointed out is that "distance measurement error" represents the mean value of the two distance measurement errors, "localization error" represents the location error of a node in 2-D space, and $\angle P$ is the included angle between the two sides connecting each unknown node and the two base stations, as has been introduced in Section 3.

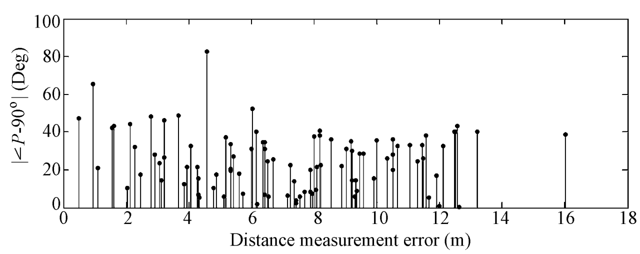
From Fig. 4(a), we can see the node deployment of the WSN, and the localization error vector of each unknown node. We can have a direct view of the relationship between the localization error and the geometric position of each unknown node. From Fig. 4(b), we can see the general tendency is that localization error is rising along with the accretion of the "distance measurement error", but there are some exceptional samples. Comparing Fig. 4 (b) with Fig. 4 (c), we can find the effect of included angle $\angle P$ on the localization error, and we can explain the cause of some exceptional samples not according with the general tendency in Fig. 4 (b). Integrating the analyses for these figures, we can deduce that the main affecting factor on node localization is distance measurement error, and that the size of included angle $\angle P$ still has effect on localization accuracy.



(a) Localization error vector of each unknown node



(b) Localization error $vs.$ distance measurement error

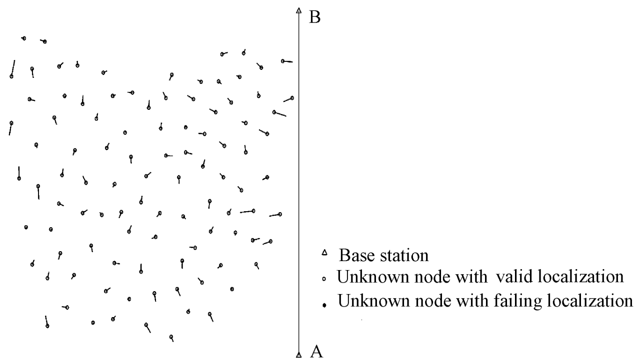


(c) $|\angle P - 90°|$ $vs.$ distance measurement error

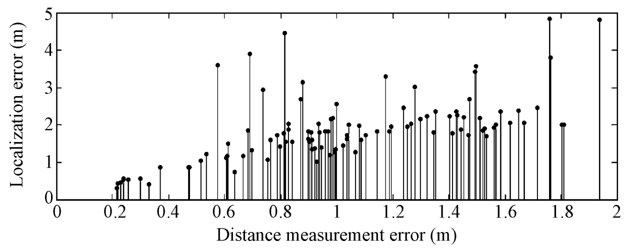
Fig. 4 Localization error relabed diagrams under distance error model (1)

We do the same experiment steps for distance error model (2) as for distance error model (1). From the localization results, we find that there is no failing localization sample. In the one hundred valid localization samples, the maximal localization error is 4.8m, the minimal localization error is 0.3m, and the average localization error is 1.9m. Fig. 5(a), Fig. 5 (b), and Fig. 5(c) have the same significance as the figures in Fig. 4, and they display almost the same rules as we have found under model (1). Of course, we get much better localization result under model (2) than under model (1) due to the better distance measurement error situation.

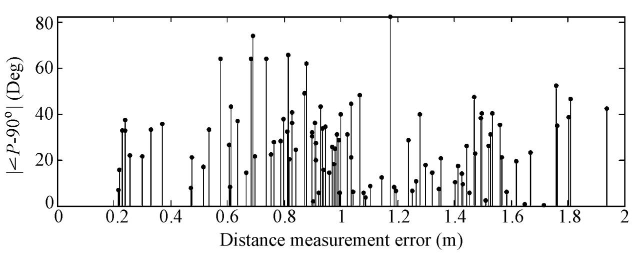
Integrating the above analyses, the accuracy of distance measurement is very important in localization schemes based on distance measurement. Using LI for node location computation, one basic requirement or a key problem is that the distance measurement accuracy should be ensured. The satisfaction for this requirement depends on the distance measurement techniques′ improvement for WSN.



(a) Localization error vector of each unknown node



(b) Localization error *vs.* distance measurement error



(c) $|\angle P - 90°|$ *vs.* distance measurement error

Fig. 5 Localization error related diagrams under distance error model (2)

6 Conclusion

In this paper, we present linear intersection technique for node location computation in localization schemes based on range measurement. Through describing an applied localization model and making some theoretical analyses, we illustrate the applying condition of this technique. We estimate the validity of the algorithm at the technique by concrete experiments. The conclusion is that the localization accuracy using the algorithm is mainly determined by the distance measurement accuracy. Being used in WSN localization, linear intersection technique has some advantages such as simple algorithm and small communication overhead, but some of it special requirements may limit its application. We hope that some concerned techniques of WSN localization such as distance measurement technique can get stepwise improvement and provide a satisfied accuracy in the future. Then, linear intersection should be considered as a pratical choice for WSN localization in 2-D space.

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SHI Qin-Qin Ph. D. candidate in the Institute of Image Processing and Pattern Recognition at Shanghai Jiaotong University. Her research interests include localization and routing technique for WSNs.

HUO Hong Lecturer in the Institute of Image Processing and Pattern Recognition at Shanghai Jiaotong University. Her research interests include information and image processing.

FANG Tao He is a professor in the Institute of Image Processing and Pattern Recognition at Shanghai Jiaotong University. Her research interests include information and image processing.

LI De-Ren Dual academician of both the Chinese Academy of Sciences and the Chinese Academy of Engineering. He is now the director of State Key Laboratory of Information Engineering in Surveying, Mapping and Remote Sensing at Wuhan University. His research interests include photogrammetry and remote sensing.