Construction of Control Lyapunov Functions for a Class of Nonlinear Systems

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Abstract The construction of control Lyapunov functions for a class of nonlinear systems is considered. We develop a method by which a control Lyapunov function for the feedback linearizable part can be constructed systematically via Lyapunov equation. Moreover, by a control Lyapunov function of the feedback linearizable part and a Lyapunov function of the zero dynamics, a control Lyapunov function for the overall nonlinear system is established.

Key words Nonlinear systems, control Lyapunov functions, semiglobal stabilization, zero dynamics.

1 Introduction

The seemingly obvious concept of a control Lyapunov function (CLF) introduced by Artstein[1] and Sontag[2] has made a tremendous impact on stabilization theory. It converts stability descriptions into tools for solving stabilization tasks. One way to stabilize a nonlinear system is to select a Lyapunov function $V(x)$ and then try to find a feedback control $u(x)$ that renders $\dot{V}(x, u(x))$ negative definite. With an arbitrary choice of $V(x)$ this attempt may fail, but if $V(x)$ is a CLF, there are many control laws that render $\dot{V}(x, u(x))$ negative definite, one of which is given by a formula due to Sontag[3]. The construction of a CLF is a hard problem, which has been solved for special classes of systems. For example, when the system is in the strict feedback form, CLFs can be constructed by backstepping[4]. For a linear system, we have obtained a universal formula to construct CLFs[5].

In this paper, the construction of control Lyapunov functions for a class of nonlinear systems is considered. For the feedback linearizable part, CLFs can be constructed by the method presented in [5]. Based on a CLF of the feedback linearizable part and a Lyapunov function of the zero dynamics, we present a method to obtain a CLF for the overall nonlinear system.

2 System description and preliminaries

Consider a nonlinear system described by

$$\dot{\boldsymbol{z}} = Q(\boldsymbol{z}, \boldsymbol{x}) \tag{1a}$$

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B[F(\boldsymbol{z}, \boldsymbol{x}) + G(\boldsymbol{z}, \boldsymbol{x})\boldsymbol{u}] \tag{1b}$$

$$\boldsymbol{y} = C\boldsymbol{x} \tag{1c}$$

where $\boldsymbol{x} \in R^r$, $\boldsymbol{z} \in R^{n-r}$ are the states, $\boldsymbol{u} \in R^m$ is the input, $\boldsymbol{y} \in R^l$ is the output. $Q(\boldsymbol{z}, \boldsymbol{x}) : R^n \to R^{n-r}$ is smooth. $f_i, g_{ij} : R^n \to R$, are assumed to be smooth with $f_i(0, 0) = 0, i = 1, 2, \cdots l$. $F(\boldsymbol{z}, \boldsymbol{x}) = \begin{bmatrix} f_1(\boldsymbol{z}, \boldsymbol{x}) & f_2(\boldsymbol{z}, \boldsymbol{x}) & \cdots & f_l(\boldsymbol{z}, \boldsymbol{x}) \end{bmatrix}^{\mathrm{T}}$, $G(\boldsymbol{z}, \boldsymbol{x}) = (g_{ij}(\boldsymbol{z}, \boldsymbol{x}))_{l\times m}$ and $\mathrm{rank}(G(\boldsymbol{z}, \boldsymbol{x})) = l$. $\{r_1, r_2, \cdots r_l\}$ is a vector relative degree of system (1), and $r_1 + r_2 + \cdots + r_l = r < n$. (1b) has the following cannonical form:

$$A = \mathrm{blockdiag}\{A_1, \cdots, A_l\}, \quad A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \mathrm{blockdiag}\{B_1, \cdots, B_l\}$$

$$\boldsymbol{B}_i = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix}^{\mathrm{T}}_{1\times r_i}, \qquad C = \mathrm{blockdiag}\{\boldsymbol{C}_1, \cdots, \boldsymbol{C}_l\}, \qquad \boldsymbol{C}_i = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}_{1\times r_i}.$$

From Isidori[6], if an affine nonlinear system

$$\begin{cases} \dot{\boldsymbol{x}} = f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} = h(\boldsymbol{x}) \end{cases} \tag{2}$$

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has relative degree $r < n$ for any $\boldsymbol{x} \in R^n$, and the distribution G =span$(g(\boldsymbol{x}))$ is involutive, then there exists a global diffeomorphism on R^n that transforms system (2) into system (1).

The dynamics of

$$\dot{\boldsymbol{z}} = Q(\boldsymbol{z}, 0) \tag{3}$$

is said to be the zero dynamics of system (1).

Assume M is an analytic $n-$dimensional manifold. Let $V : M \to R^+$ be a differential function. V is said to be positive definite on M if $V(\boldsymbol{x}) > 0$, $\boldsymbol{x} \in M - \{0\}$ and $V(\boldsymbol{0}) = 0$; V is said to be proper if $V(\boldsymbol{x}) \to \infty$ as $\|\boldsymbol{x}\| \to \infty$.

Definition 1. If there exists a differential, proper and positive definite function $V : M \to R^+$ such that

$$\inf_{\boldsymbol{u}}(L_{\boldsymbol{f}}V(\boldsymbol{x}) + L_g V(\boldsymbol{x})\boldsymbol{u}) < 0 \tag{4}$$

for each $\boldsymbol{x} \in M - \{0\}$, then $V(\boldsymbol{x})$ is said to be a control Lyapunov function (CLF) for system (2) on M.

Assumption 1. For system (3), there exists an open set $\Lambda \subset R^{n-r}$, a nonnegative real number $h > 1$, and a differential function $U : \Lambda \to R^+$ such that the set $\{\boldsymbol{z} : U(\boldsymbol{z}) \leqslant h+1\}$ is a compact subset of Λ, and we have

$$\dot{U}(\boldsymbol{z}) \leqslant -\phi_1(\boldsymbol{z}) \tag{5}$$

where $\phi_1(\boldsymbol{z})$ is continuous on Λ and positive definite on the set $\{\boldsymbol{z} : U(\boldsymbol{z}) \leqslant h+1\}$.

Lemma[7]**.** Let E be a compact set in a product space $R^m \times R^n$, and denote by E_z and E_x its respective projections (*i.e.*, $E \subset E_{\boldsymbol{z}} \times E_{\boldsymbol{x}}$). Let $\chi(\boldsymbol{z})$ be a continuous real function on E_z which is positive definite on the projection of the set $\{(\boldsymbol{z}, \boldsymbol{x}) : \boldsymbol{x} = 0\} \cap E$. Let $\psi(\boldsymbol{x})$ be a continuous real function on $E_{\boldsymbol{x}}$ which is positive definite on $E_{\boldsymbol{x}}/\{0\}$. Let $\xi(\boldsymbol{z}, \boldsymbol{x})$ be a continuous real function on E which satisfies $\xi(\boldsymbol{z}, \boldsymbol{x}) = 0$ for any $(\boldsymbol{z}, \boldsymbol{x}) \in \{(\boldsymbol{z}, \boldsymbol{x}) : \boldsymbol{x} = 0\} \cap E$. let κ be a function of class-K_∞. There exists a positive real number K_* such that for all $K \geqslant K_*$,

$$-\chi(\boldsymbol{z}) - \kappa(K)\psi(\boldsymbol{x}) + \xi(\boldsymbol{z}, \boldsymbol{x}) < 0, \forall(\boldsymbol{z}, \boldsymbol{x}) \in E \tag{6}$$

3 Main results

Consider system (1b). Divide A_i and B_i into their block forms as follows:

$$A_i = \begin{bmatrix} A_{i-1} & A_{i2} \\ 0 & 0 \end{bmatrix}, \text{ where } A_{i-1} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \boldsymbol{A}_{i2} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \boldsymbol{B}_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

Assume $\beta_{i1}, \beta_{i2}, \cdots, \beta_{i,r_{i-1}}$ are the coefficients of a Hurwitz polynomial

$$\lambda^{r_i-1} + \beta_{i,r_i-1}\lambda^{r_i-2} + \cdots + \beta_{i2}\lambda + \beta_{i1} \tag{7}$$

Let $p_{i3} > 0$, $\boldsymbol{P}_{i2} \in R^{r_i-1}$, $\boldsymbol{p}_{i3}^{-1}\boldsymbol{P}_{i2}^{\mathrm{T}} = \begin{bmatrix} \beta_{i1} & \beta_{i2} & \cdots & \beta_{i,r_i-1} \end{bmatrix}$. Then

$$A_{i-1} - \boldsymbol{A}_{i2}\boldsymbol{p}_{i3}^{-1}\boldsymbol{P}_{i2}^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\beta_{i1} & -\beta_{i2} & \cdots & -\beta_{i,r_i-1} \end{bmatrix}$$

is a Hurwitz matrix. Thus Lyapunov equation

$$S_{r_i-1}(A_{i-1} - \boldsymbol{A}_{i2}\boldsymbol{p}_{i3}^{-1}\boldsymbol{P}_{i2}^{\mathrm{T}}) + (A_{i-1} - \boldsymbol{A}_{i2}\boldsymbol{p}_{i3}^{-1}\boldsymbol{P}_{i2}^{\mathrm{T}})^{\mathrm{T}} S_{r_i-1} = -KF_i \tag{8}$$

has a unique positive definite solution S_{r_i-1} for an arbitrary positive definite matrix F_i and $K > 0$. Let $P_{r_i-1} = S_{r_i-1} + p_{i3}^{-1} P_{i2} P_{i2}^{\mathrm{T}}$. For $i = 1, 2 \cdots l$, then each P_{r_i-1} is positive definite.

Since $\det \begin{bmatrix} P_{r_i-1} & \boldsymbol{P}_{i2} \\ \boldsymbol{P}_{i2}^{\mathrm{T}} & p_{i3} \end{bmatrix} = p_{i3}\det[P_{r_i-1} - p_{i3}^{-1}\boldsymbol{P}_{i2}\boldsymbol{P}_{i2}^{\mathrm{T}}] = p_{i3}\det[S_{r_i-1}] > 0$, $P_i = \begin{bmatrix} P_{r_i-1} & \boldsymbol{P}_{i2} \\ \boldsymbol{P}_{i2}^{\mathrm{T}} & p_{i3} \end{bmatrix}$ is positive definite provided that P_{r_i-1} is positive definite.

Use block matrix to express x^{T}, that is,

$$\boldsymbol{x}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{x}_1^{\mathrm{T}} & \boldsymbol{x}_2^{\mathrm{T}} & \cdots & \boldsymbol{x}_l^{\mathrm{T}} \end{bmatrix}, \boldsymbol{x}_i^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{x}_{i,r_i-1}^{\mathrm{T}} & x_{i,r_i} \end{bmatrix}, \boldsymbol{x}_{i,r_i-1}^{\mathrm{T}} = \begin{bmatrix} x_{i1} & x_{i2} & \cdots & x_{i,r_i-1} \end{bmatrix}, i = 1, 2 \cdots l.$$

Denote $P = \text{blockdiag}\{P_1, \cdots, P_l\}$, and

$$V(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}} P \boldsymbol{x} \tag{9}$$

Let

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B\boldsymbol{v} \tag{10}$$

Theorem 1. $V(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}} P \boldsymbol{x}$ is a CLF for system (10) on R^r.

Theorem 2. $V(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}} P \boldsymbol{x}$ is a CLF for system (1b) on R^r.

Proofs of these Theorems are similar to that of Theorem 1 given in [5], so they are omitted.

In order to give Theorem 3, for any given $c > 0$, denote $S_1 = \{\boldsymbol{x} : V(\boldsymbol{x}) < c+1\} \times \{\boldsymbol{z} : U(\boldsymbol{z}) < h+1\}$. Define the function

$$W(\boldsymbol{z}, \boldsymbol{x}) = \frac{hU(\boldsymbol{z})}{h+1-U(\boldsymbol{z})} + \frac{cV(\boldsymbol{x})}{c+1-V(\boldsymbol{x})} \tag{11}$$

Then $W(\boldsymbol{z}, \boldsymbol{x}) : S_1 \to R^+$ is proper on S_1.

Theorem 3. If system (1) satisfies Assumption 1, then $W(\boldsymbol{z}, \boldsymbol{x}) : S_1 \to R^+$ is a CLF for system (1) on $S = \{(\boldsymbol{z}, \boldsymbol{x}) : W(\boldsymbol{z}, \boldsymbol{x}) \leqslant c^2 + h^2 + 1\}$.

Proof. Assume $W(\boldsymbol{z}, \boldsymbol{x}) \leqslant c^2 + h^2 + 1$. This implies

$$V(\boldsymbol{x}) \leqslant (c+1)\frac{c^2+h^2+1}{c^2+h^2+1+c}, U(\boldsymbol{z}) \leqslant (h+1)\frac{c^2+h^2+1}{c^2+h^2+1+h} \tag{12}$$

From (12), we get, when $W(\boldsymbol{z}, \boldsymbol{x}) \leqslant c^2 + h^2 + 1$,

$$\frac{c}{c+1} \leqslant \frac{c(c+1)}{(c+1-V)^2} \leqslant \frac{(c^2+h^2+1+c)^2}{c(c+1)} \tag{13}$$

$$\frac{h}{h+1} \leqslant \frac{h(h+1)}{(h+1-U)^2} \leqslant \frac{(c^2+h^2+1+h)^2}{h(h+1)} \tag{14}$$

By (12) and Assumption (1), the set S is compact. Also, from (12) the projections of S satisfy

$$S_{\boldsymbol{x}} \subset \{\boldsymbol{x} : V(\boldsymbol{x}) < c+1\}, S_{\boldsymbol{z}} \subset \{\boldsymbol{z} : U(\boldsymbol{z}) < h+1\} \tag{15}$$

Let $f(\boldsymbol{z}, \boldsymbol{x}) = \begin{bmatrix} Q(\boldsymbol{z}, \boldsymbol{x}) \\ A\boldsymbol{x} + BF(\boldsymbol{z}, \boldsymbol{x}) \end{bmatrix}, g(\boldsymbol{z}, \boldsymbol{x}) = \begin{bmatrix} 0 \\ BG(\boldsymbol{z}, \boldsymbol{x}) \end{bmatrix}$. Then we have

$$L_f W(\boldsymbol{z}, \boldsymbol{x}) = \frac{h(h+1)}{(h+1-U(\boldsymbol{z}))^2}\frac{\partial U}{\partial \boldsymbol{z}} Q(\boldsymbol{z}, \boldsymbol{x}) + \frac{c(c+1)}{(c+1-V(\boldsymbol{x}))^2}\frac{\partial V}{\partial x}(A\boldsymbol{x} + BF(\boldsymbol{z}, \boldsymbol{x})) \tag{16}$$

$$L_g W(\boldsymbol{z}, \boldsymbol{x}) = \frac{c(c+1)}{(c+1-V(\boldsymbol{x}))^2}\frac{\partial V}{\partial \boldsymbol{x}}(BG(\boldsymbol{z}, \boldsymbol{x})) \tag{17}$$

Let $\boldsymbol{X}_{r-l}^{\mathrm{T}} = \begin{bmatrix} \boldsymbol{X}_{1,r_1-1}^{\mathrm{T}} & \boldsymbol{X}_{2,r_2-1}^{\mathrm{T}} & \cdots & \boldsymbol{X}_{l,r_l-1}^{\mathrm{T}} \end{bmatrix}$, $F = \text{block diag} \begin{bmatrix} F_1 & F_2 & \cdots & F_l \end{bmatrix}$.

By Theorem 1, when $\dfrac{\partial V(\boldsymbol{x})}{\partial x} B = 0$, we have

$$\boldsymbol{x}^{\mathrm{T}}(PA + A^{\mathrm{T}}P)\boldsymbol{x} = -K\boldsymbol{X}_{r-l}^{\mathrm{T}} F \boldsymbol{X}_{r-l} \tag{18}$$

Since $\text{rank}(G(\boldsymbol{z}, \boldsymbol{x})) = l$, by (17) we have

$$L_g W(\boldsymbol{z}, \boldsymbol{x}) = 0 \Rightarrow \frac{\partial V}{\partial \boldsymbol{x}} B = 0 \tag{19}$$

By (14),(16), and (18), we get, when $L_g W(\boldsymbol{z}, \boldsymbol{x}) = 0, \boldsymbol{x} \neq 0$,

$$\begin{aligned} L_f W(\boldsymbol{z}, \boldsymbol{x}) =& \frac{h(h+1)}{(h+1-U(\boldsymbol{z}))^2}\frac{\partial U}{\partial \boldsymbol{z}} Q(\boldsymbol{z}, \boldsymbol{x}) + \frac{c(c+1)}{(c+1-V(\boldsymbol{x}))^2}\boldsymbol{x}^{\mathrm{T}}(PA + A^{\mathrm{T}}P)\boldsymbol{x} = \\ & \frac{h(h+1)}{(h+1-U(\boldsymbol{z}))^2}\frac{\partial U}{\partial \boldsymbol{z}} Q(\boldsymbol{z}, \boldsymbol{x}) - K\frac{c(c+1)}{(c+1-V(\boldsymbol{x}))^2}\boldsymbol{X}_{r-l}^{\mathrm{T}} F \boldsymbol{X}_{r-l} \end{aligned} \tag{20}$$

In view of (11), (12) and Assumption 1, then

$$L_fW(\boldsymbol{z},\boldsymbol{x}) \leqslant -\frac{Kc}{c+1}\boldsymbol{X}_{r-l}^{\mathrm{T}}F\boldsymbol{X}_{r-l} + \frac{(c^2+h^2+1+h)^2}{h(h+1)}|\frac{\partial U(\boldsymbol{z})}{\partial \boldsymbol{z}}(Q(\boldsymbol{z},\boldsymbol{x})-Q(\boldsymbol{z},\boldsymbol{0}))| - \frac{h(h+1)}{(h+1-U(\boldsymbol{z}))^2}\phi_1(\boldsymbol{z}) \tag{21}$$

Let us define

$$\chi(\boldsymbol{z}) = \frac{h(h+1)}{2(h+1-U(\boldsymbol{z}))^2}\phi_1(\boldsymbol{z}), \quad \psi(\boldsymbol{x}) = \frac{Kc}{2(c+1)}\boldsymbol{X}_{r-l}^{\mathrm{T}}F\boldsymbol{X}_{r-l}$$
$$\kappa = K, \quad \xi(\boldsymbol{z},\boldsymbol{x}) = \frac{(c^2+h^2+1+h)^2}{h(h+1)}|\frac{\partial U(\boldsymbol{z})}{\partial \boldsymbol{z}}(Q(\boldsymbol{z},\boldsymbol{x})-Q(\boldsymbol{z},\boldsymbol{0}))| \tag{22}$$

From Assumption 1, $\chi(\boldsymbol{z})$ is continuous on $S_{\boldsymbol{z}}$ and positive definite on the projection of the set $\{(\boldsymbol{z},\boldsymbol{x}) : \boldsymbol{x} = \boldsymbol{0}\} \cap S$. Since $x_{ir_i} = -\boldsymbol{X}_{i,r_i-1}^{\mathrm{T}}P_{i2}p_{i3}^{-1}, i = 1, 2\cdots l$, $\psi(\boldsymbol{x})$ is positive definite on $S_{\boldsymbol{x}}/\{0\}$. From (22), it follows that $\psi(\boldsymbol{x})$ is continuous on S_x, and $\xi(\boldsymbol{z},\boldsymbol{x}) = 0$, for any $(\boldsymbol{z},\boldsymbol{x}) \in \{(\boldsymbol{z},\boldsymbol{x}) : \boldsymbol{x} = \boldsymbol{0}\} \cap S$. This demonstrates that the conditions of Lemma are satisfied. It follows that there exists a positive real number K_* such that for all $K \geqslant K_*$,

$$\xi(\boldsymbol{z},\boldsymbol{x}) < \chi(\boldsymbol{z}) + K\psi(\boldsymbol{x}), \forall(\boldsymbol{z},\boldsymbol{x}) \in S \tag{23}$$

From (20)∼(22), we get, when $L_gW(\boldsymbol{z},\boldsymbol{x}) = 0, \boldsymbol{x} \neq 0$,

$$L_fW(\boldsymbol{z},\boldsymbol{x}) \leqslant -\frac{Kc}{2(c+1)}\boldsymbol{X}_{r-l}^{\mathrm{T}}F\boldsymbol{X}_{r-l} - \frac{h(h+1)}{2(h+1-U(\boldsymbol{z}))^2}\phi_1(\boldsymbol{z}) \tag{24}$$

Let $\phi(\boldsymbol{z},\boldsymbol{x}) = \dfrac{Kc}{2(c+1)}\boldsymbol{X}_{r-l}^{\mathrm{T}}F\boldsymbol{X}_{r-l} + \dfrac{h(h+1)}{2(h+1-U(\boldsymbol{z}))^2}\phi_1(\boldsymbol{z})$. From (24), when $L_gW(\boldsymbol{z},\boldsymbol{x}) = 0, \boldsymbol{x} \neq 0$, $L_{\boldsymbol{f}}W(\boldsymbol{z},\boldsymbol{x}) \leqslant -\phi(\boldsymbol{z},\boldsymbol{x})$. From (22) and (23), we have $\phi(\boldsymbol{z},\boldsymbol{x})$ is continuous on S_1, positive definite on S.

On the other hand, from Assumption 1, when $L_gW(\boldsymbol{z},\boldsymbol{x}) = 0, \boldsymbol{x} = 0, \boldsymbol{z} \neq \boldsymbol{0}$, $L_{\boldsymbol{f}}W(\boldsymbol{z},\boldsymbol{x}) = \dfrac{h(h+1)}{(h+1-U(\boldsymbol{z}))^2}\dfrac{\partial U}{\partial \boldsymbol{z}}Q(\boldsymbol{z},0) \leqslant -\dfrac{h}{h+1}\phi_1(\boldsymbol{z})$. In conclusion, $L_{\boldsymbol{f}}W(\boldsymbol{z},\boldsymbol{x}) < 0$, for $L_gW(\boldsymbol{z},\boldsymbol{x}) = 0, (\boldsymbol{z},\boldsymbol{x}) \neq 0$. Thus $W(\boldsymbol{z},\boldsymbol{x})$ is a CLF for system (1) on S.

4 Conclusion

The construction of control Lyapunov functions for a class of nonlinear systems is considered. We develop a method by which a control Lyapunov function for the feedback linearizable part can be constructed systematically *via* Lyapunov equation. Moreover, by a control Lyapunov function of the feedback linearizable part and a Lyapunov function of the zero dynamics, a control Lyapunov function for the overall nonlinear system is established.

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