The Reduced-order Design of Robust Adaptive Backstepping Controller[1)]

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Abstract For a class of systems with unmodeled dynamics, robust adaptive stabilization problem is considered in this paper. Firstly, by a series of coordinate changes, the original system is re-parameterized. Then, by introducing a reduced-order observer, an error system is obtained. Based on the system, a reduced-order adaptive backstepping controller design scheme is given. It is proved that all the signals in the adaptive control system are globally uniformly bounded, and the regulation error converges to zero asymptotically. Due to the order deduction of the controller, the design scheme in this paper has more practical values. A simulation example further demonstrates the efficiency of the control scheme.

Key words Robustness, reduced-order, backstepping, adaptive control, unmodeled dynamics

1 Introduction

Recently, more attention has been paid to backstepping design technique because of its systematic design method and the excellent transient performance of the closed-loop system[1]. Since the stability analysis of adaptive system in [1] was only limited to the ideal case, it is of practical and theoretical interest to study the design and performance of this kind of controller for systems in the presence of unmodeled dynamics. It is [2] that firstly studied this problem, however, the control law and the adaptive law were designed separately, and the implementation of the controller required a priori knowledge of unmodeled dynamics and disturbance. The work of [2] was improved by [3] and [4], with the adaptive law and control law being designed simultaneously. However, the introduction of a modification or a projection operator in the parameter adaptive laws resulted in more complicated controllers and might affect the system performance. To overcome the shortcomings an adaptive backstepping controller design based on K-filters was presented by [5] and it was proved that this kind of adaptive controller without modification still have certain robustness.

Under the same conditions, the asymptotic stabilization problem is further considered in this paper for uncertain systems with different unmodeled dynamics given by [5]. Compared with [5], our main work consists of the following aspects: 1) The original system is re-parameterized by introducing a novel coordinate change. 2) By using a reduced-order observer, and introducing an uncertain parameter in observer error, we obtain an error system based on which an adaptive backstepping controller design and its stability analysis are given. 3) The introduction of reduced-order observer and one dimension adaptive law can effectively reduce the dynamic order of the controller.

2 Problem formulation

Consider the following linear system with input and output unmodeled dynamics[5]

$$y=\frac{B(s)}{A(s)}(1+\mu_1\Delta_1(s))u+(1+\mu_2\Delta_2(s))y \tag{1}$$

where $A(s)=s^n+a_{n-1}s^{n-1}+\cdots+a_1s+a_0$, $B(s)=b_ms^m+\cdots+b_1s+b_0$, $a_i,b_j(i=0,\cdots,n-1,\ j=0,\cdots,m)$ are unknown parameters, $\Delta_k(s)(k=1,2)$ denote unmodeled dynamics, $\mu_k\geqslant 0$ denotes the amplitude of the unmodeled dynamics. Control objective: Design an adaptive backstepping controller to bound all signals in closed-loop system and to regulate the output asymptotically to zero. For system (1), the following assumptions are required.

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A1. $B(s)$ is Hurwitz polynomial, and the relative degree $\rho = n - m$ and the sign of b_m are known. For simplicity and without lose of generality, $b_m = 1$ is assumed.

A2. $\Delta_k(s)(k = 1, 2)$ is stable and strictly proper.

3 Controller design

The state-space realization of system (1) is presented as follows.

$$\dot{\boldsymbol{\chi}} = A_p\boldsymbol{\chi} + B\boldsymbol{u}, \quad \boldsymbol{y} = \boldsymbol{\chi}_1 + \boldsymbol{\aleph}$$
$$\dot{\boldsymbol{\xi}}_1 = A_1\boldsymbol{\xi}_1 + B_1\boldsymbol{\chi}_1, \quad \dot{\boldsymbol{\xi}}_2 = A_2\boldsymbol{\xi}_2 + B_2\boldsymbol{y}$$

where $A_p = \begin{pmatrix} & I_{n-1} \\ -a & \\ & 0_{1\times(n-1)} \end{pmatrix}$, $B = (0, \boldsymbol{b}^{\rm T})^{\rm T}, \boldsymbol{a} = (a_{n-1}, \cdots, a_0)^{\rm T}, \boldsymbol{b} = (1, b_{m-1}, \cdots, b_0)^{\rm T}, \boldsymbol{\aleph} = \mu_1\xi_{11} + \mu_2\xi_{21}, \boldsymbol{e}_1^{\rm T}(sI - A_1)^{-1}B_1 = \Delta_1(s), \boldsymbol{e}_1^{\rm T}(sI - A_2)^{-1}B_2 = \Delta_2(s)$. Introduce a similarity transformation[1,Chap.8]

$$\begin{pmatrix} \bar{\boldsymbol{\chi}}_\rho \\ \boldsymbol{\zeta} \end{pmatrix} = \begin{pmatrix} I_{\rho\times\rho} & 0_{\rho\times m} \\ & T_{m\times n} \end{pmatrix}\chi, \quad \boldsymbol{\zeta} \in R^m \tag{3}$$

where $\bar{\boldsymbol{\chi}}_\rho = (\chi_1, \cdots, \chi_\rho)^{\rm T}$, $T = (A_3^\rho e_1, \cdots, A_3 e_1, I_m)$, $A_3 = \begin{pmatrix} -b_{m-1} & \\ & I_{m-1} \\ \vdots & \\ & 0_{1\times(m-1)} \\ -b_0 & \end{pmatrix}$. Form (10.132) in [1], it is easy to verify that (3) satisfies the following property:

$$TB = 0, \quad TA = A_2T + TA^\rho Be_1^{\rm T} \tag{4}$$

where $A_p = \begin{pmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{pmatrix}$. From (2)~(4) and the definition of A, A_p we have

$$\dot{\boldsymbol{\zeta}} = A_3\boldsymbol{\zeta} + B_3\boldsymbol{\chi}_1 \tag{5}$$

where $B_3 = T(A^\rho B - a)$. By (3) and (5), $\bar{\chi}_\rho$-subsystem in (2) is rewritten as

$$\dot{\chi}_i = \chi_{i+1} - a_{n-i}\chi_1, \ i = 1, \cdots, \rho - 1, \qquad \dot{\chi}_\rho = u - a_m\chi_1 + \chi_{\rho+1} \tag{6}$$

From (3) and the definition of T, one gets $\chi_{\rho+1} = c_1\chi_1 + \cdots + c_\rho\chi_\rho + \zeta_1$, where $c_k = -e_1^{\rm T}A_3^{\rho-k+1}e_1$. Substituting this equality into (6) results in

$$\dot{\bar{\chi}}_\rho = \bar{A}\bar{\chi}_\rho + e_\rho u + e_\rho\zeta_1 \tag{7}$$

where $\bar{A} = \begin{pmatrix} -a_{n-1} & \\ \vdots & I_{\rho-1} \\ -a_{m+1} & c_2\cdots c_\rho \\ -a_m + c_1 & \end{pmatrix}$. Performing the following similarity transformation

$$\boldsymbol{x} = S\bar{\boldsymbol{\chi}}_\rho, \quad \boldsymbol{x} \in R^\rho \tag{8}$$

which satisfies $S\bar{A} = A_aS$, $Se_\rho = e_\rho$, where $S = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ -c_\rho & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -c_2 & \cdots & -c_\rho & 1 \end{pmatrix}$, $A_a = \begin{pmatrix} -\bar{a}_1 & \\ & I_{\rho-1} \\ \vdots & \\ & 0_{1\times(\rho-1)} \\ -\bar{a}_\rho & \end{pmatrix}$, $\bar{a}_1 = a_{n-1} - c_\rho$, $\bar{a}_i = a_{n-1} - c_\rho a_{n-i+1} - \cdots - c_{\rho-i+2}a_{n-1} - c_{\rho-i+1}(2 \leqslant i \leqslant \rho)$. Combining (7) with (8) gives

$$\dot{\boldsymbol{x}} = A_\alpha\boldsymbol{x} + e_\rho\boldsymbol{u} + e_\rho\boldsymbol{\zeta}_1 \tag{9}$$

From (8), $\chi_1 = x_1$ is known. Noting (9), system (2) equals

$$\dot{x}_i = x_{i+1} - \bar{a}_i x_1,\ i = 1, \cdots, \rho - 1, \quad \dot{x}_\rho = u - \bar{a}_\rho x_1 + \zeta_1, \quad y = x_1 + \aleph$$
$$\dot{\xi}_1 = A_1\xi_1 + B_1 x_1, \quad \dot{\xi}_2 = A_2\xi_2 + B_2 y, \quad \dot{\zeta} = A_3\zeta + B_3 x_1 \tag{10}$$

The following reduced-order observer is introduced

$$\dot{\hat{x}}_i = \hat{x}_{i+1} + k_{i+1} + k_{i+1}y - k_i(\hat{x}_1 + k_1 y),\ i = 1, \cdots, \rho - 2, \quad \dot{\hat{x}}_{\rho-1} = u - k_{\rho-1}(\hat{x}_1 + k_1 y) \tag{11}$$

where $k = (k_1, \cdots k_{\rho-1})^{\mathrm{T}}$ is chosen such that $A_0 = \begin{pmatrix} & I_{\rho-2} \\ -k & \\ & 0_{1\times(\rho-2)} \end{pmatrix}$ is stable. For $1 \leqslant i \leqslant \rho - 1$ the observer error $\varepsilon_i = (x_{i+1} - \hat{x}_i - k_i x_1)/p$, with $p = \max\{1, \sum\limits_{i=1}^{\rho-1}|q_{i1}|, \sum\limits_{i=1}^{\rho-1}|q_{i2}|, |P_3B_3|\}$, $q_{i1} = -\bar{a}_{i+1} + k_i\bar{a}_1$, $q_{i2} = -q_{i1} + k_i k_1 - k_{i+1}$, $k_\rho = 0$, satisfies

$$\dot{\varepsilon} = A_0\varepsilon + (\Delta + e_{\rho-1}\zeta_1)/p \tag{12}$$

where $\varepsilon = (\varepsilon_1, \cdots, \varepsilon_{\rho-1})^{\mathrm{T}}$, $\Delta = (\Delta_1, \cdots, \Delta_{\rho-1})^{\mathrm{T}}$, $\Delta_i = q_{i1}y + q_{i2}\aleph$. From (10) and the definition of observer error, the derivative of output is represented as

$$\dot{y} = \hat{x}_1 + p\varepsilon_1 + qx_1\dot{\aleph} \tag{13}$$

where $q = k_1 - \bar{a}_1$. Next we will develop the adaptive controller using backstepping techniques.

Step 1. Define

$$z_1 = y, \quad z_2 = \hat{x}_1 - \alpha_1 \tag{14}$$

From the stability of $A_i (i = 0, \cdots, 3)$, there exists P_i such that $P_iA_i + A_i^{\mathrm{T}}P_i = -I$. Let us consider the Lyapunov-like function candidate:

$$V_1 = \frac{1}{2}y_2 + \frac{1}{2r}\tilde{\theta}^2 + r_0\varepsilon^{\mathrm{T}}P_0\varepsilon + r_1\xi_1^{\mathrm{T}}P_1\xi_1 + r_2\xi_2^{\mathrm{T}}P_2\xi_2 + r_3\zeta^{\mathrm{T}}P_2\zeta \tag{15}$$

where $r, r_0, r_1, r_2, r_3 > 0$ will be chosen later, $\hat{\theta}$ is the estimate of $\theta = \max\{p^2, q^2\}$. From (14), the derivative of (15) satisfies

$$\begin{aligned}\dot{V}_1 =& z_1(z_2 + \alpha_1 + p\varepsilon_1 + qx_1 + \dot{\aleph}) - r_0|\varepsilon|^2 + \frac{2}{p}r_0\varepsilon^{\mathrm{T}}P_0(\Delta + e_{\rho-1}\zeta_1) - r_1|\xi_1|^2 + \\ & 2r_1\xi_1^{\mathrm{T}}P_1B_1x_1 - r_2|\xi_2|^2 + 2r_2\xi_2^{\mathrm{T}}P_2B_2y - r_3|\zeta|^2 + 2r_3\zeta^{\mathrm{T}}P_3B_3x_1 + r^{-1}\tilde{\theta}\dot{\hat{\theta}}\end{aligned} \tag{16}$$

With the choice of $r_0 = d_1d_2/12\|P_0\|^2$, $r_k = d_1/|P_kB_k|^2 (k = 1, 2, 3)$ for any parameters $d_1 > 0$, $\frac{1}{2} > d_2 > 0$ and by simple calculations we have

$$\begin{aligned}& z_1(p\varepsilon_1 + qx_1 + \dot{\aleph}) \leqslant \bar{\phi}_1\hat{\theta}z_1 + \phi_1 z_1 + \frac{r_0}{4\rho}|\varepsilon|^2 + d_0(y + \aleph^2 + \dot{\aleph}^2) - \bar{\phi}_1\tilde{\theta}z_1 \\ & \frac{2}{p}r_0\varepsilon^{\mathrm{T}}P_0(\Delta + e_{\rho-1}\zeta_1) \leqslant \frac{1}{4}r_0|\varepsilon|^2 + d_2r_3|\zeta|^2 + d_1d_2(z_1^2 + \aleph^2) \\ & 2r_1\xi_1^{\mathrm{T}}P_1B_1x_1 + 2r_2\xi_2^{\mathrm{T}}P_2B_2y + 2r_3\zeta^{\mathrm{T}}P_3B_3x_1 \leqslant \frac{1}{2}r_1|\xi_1|^2 + \frac{1}{2}r_2|\xi_2|^2 + \frac{1}{2}r_3|\zeta|^2 + 10d_1z_1^2 + 8d_1\aleph^2\end{aligned} \tag{17}$$

where $\phi_1 = z_1/4d_0$, $\bar{\phi}_1 = (\rho/r_0 + 1/2d_0)z_1$. Substituting (17) in (16) leads to

$$\begin{aligned}\dot{V}_1 =& -\frac{3}{4}r_0|\varepsilon|^2 - \frac{1}{2}r_1|\xi_1|^2 - \frac{1}{2}r_2|\xi_2|^2 - (\frac{1}{2} - d_2)r_3|\zeta|^2 + \frac{r_0}{4\rho}|\varepsilon|^2 + d_0(z_1^2 + \aleph^2 + \dot{\aleph}^2) + \\ & d_1(d_2 + 8)\aleph^2 + z_1(z_2 + \alpha_1 + d_1(d_2 + 10)z_1 + \bar{\phi}_1\hat{\theta} + \phi_1) - \bar{\phi}_1\tilde{\theta} + r^{-1}\tilde{\theta}\dot{\hat{\theta}}\end{aligned} \tag{18}$$

By the following tuning function and stabilizing function

$$\tau_1 = r\bar{\phi}_1 z_1, \quad \alpha_1(y, \hat{\theta}) = -c_1z_1 - d_0\rho z_1 - d_1(d_2 + 10)z_1 - \bar{\phi}_1\hat{\theta} - \phi_1 \tag{19}$$

and (18), we have

$$\dot{V}_1 = z_1 z_2 - c_1 z_1^2 - d_0\rho z_1^2 - \frac{3}{4}r_0|\varepsilon|^2 - \frac{1}{2}r_1|\xi_1|^2 - \frac{1}{2}r_2|\xi_2|^2 - (\frac{1}{2} - d_2)r_3|\zeta|^2 + \frac{r_0}{4\rho}|\varepsilon|^2 + d_0(z_1^2 + \aleph^2 + \dot{\aleph}^2) + d_1(d_2+8)\aleph^2 + r^{-1}\tilde{\theta}(\dot{\hat{\theta}} - \tau_1) \tag{20}$$

Step i (i= 2, $\cdots$, ρ), Define

$$z_{i+1} = \hat{x}_i - \alpha_i(y, \hat{x}_1, \cdots, \hat{x}_{i-1}, \hat{\theta}) \tag{21}$$

with $z_{\rho+1} = 0, \hat{x}_\rho = u$. For Lyapunov-like function $V_i = V_1 = \sum_{j=2}^{i}\frac{1}{2}z_j^2$, by the same argument as Step 1, with the following choice of tuning function and stabilizing function

$$\tau_i = \tau_{i-1} + r\bar{\phi}_i z_i, \quad \alpha_i = -z_{i-1} - c_i z_i - k_i z_1 + k_{i-1}k_1 z_1 + k_{i-1}\hat{x}_1 + \frac{\partial \alpha_{i-1}}{\partial y}\hat{x}_1 + \sum_{j=1}^{i-2}\frac{\partial\alpha_{i-1}}{\partial \hat{x}_j}\dot{\hat{x}}_j - \phi_i - \bar{\phi}_i\hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial\hat{\theta}}\tau_i + r\sum_{j=1}^{i-1}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}z_j\phi_i z_i \tag{22}$$

where $k_\rho = 0$, $\phi_i = z_i|\partial\alpha_{i-1}/\partial y|^2/4d_0$, $\bar{\phi}_1 = z_i(\rho/r_0 + 1/2d_0)|\partial\alpha_{i-1}/\partial y|^2/4d_0$, we can obtain

$$\dot{V}_i = z_i z_{i+1} - \sum_{j=1}^{i}c_j z_j^2 - d_0\rho z_1^2 - \frac{3}{4}r_0|\varepsilon|^2 - \frac{1}{2}r_1|\xi_1|^2 - \frac{1}{2}r_2|\xi_2|^2 - (\frac{1}{2} - d_2)r_3|\zeta|^2 + i(\frac{r_0}{4\rho}|\varepsilon|^2 + d_0(y + \aleph^2 + \dot{\aleph}^2)) + d_1(d_2+8)\aleph^2 + (r^{-1}\tilde{\theta} - \sum_{j=1}^{i}\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}z_j)(\dot{\hat{\theta}} - \tau_i) \tag{23}$$

Thus the control law and adaptive law are obtained as follows.

$$u = \alpha_\rho, \quad \dot{\hat{\theta}} = \tau_\rho \tag{24}$$

which gives

$$\dot{V}_\rho = -\sum_{j=1}^{\rho}c_j z_j^2 - \frac{1}{2}r_0|\varepsilon|^2 - \frac{1}{2}r_1|\xi_1|^2 - \frac{1}{2}r_2|\xi_2|^2 - (\frac{1}{2} - d_2)r_3|\zeta|^2 + d_0\rho(\aleph^2 + \dot{\aleph}^2) + d_1(d_2+8)\aleph^2 \tag{25}$$

Remark 1. The dynamic order of our controller is less $2(m+n)$ than that of [1, Chapter 10], which means that our control scheme possesses more practical values.

4 Main result

Our main result depends on a useful lemma[6] as follows.

Lemma 1. If $\Delta(s)$ is stable and strictly proper, then there exists a constant $\mu_0 > 0$ such that for all $\mu \in [0, \mu_0)$, $1/(1+\mu\Delta(s))$ is stable and strictly proper.

Theorem 1. For an adaptive feedback system consisting of (1), (11), and (24), if assumptions A1 and A2 hold, then there exists a constant $\bar{\mu} > 0$ such that for any $\mu_1, \mu_2 \in [0, \mu)$ and any initial condition $\chi(0)$, 1) all signals in the closed-loop system are globally uniformly bounded; 2) all signals except for parameter estimate tend to zero asymptotically.

Proof. Setting $\mu = \max\{\mu_1, \mu_2\}$, together with $x_1 = y - \aleph$, gives

$$\aleph^2 \leqslant 2\mu^2(|\xi_1|^2 + |\xi_2|^2), \ \dot{\aleph}^2 \leqslant (16|B_1|^2\mu^4 + 4\max\{\|A_1\|^2, \|A_2\|^2\}\mu^2)(|\xi_1|^2 + |\xi_2|^2) + (8|B_1|^2 + 4|B_2|^2)\mu^2 y^2 \tag{26}$$

Substituting (26) in (25) leads to

$$\dot{V}_\rho = -\sum_{j=1}^{\rho}c_j z_j^2 - \frac{1}{2}r_0|\varepsilon|^2 - \frac{1}{2}r_1|\xi_1|^2 - \frac{1}{2}r_2|\xi_2|^2 - (\frac{1}{2} - d_2)r_3|\zeta|^2 + (l_1\mu^4 + l_2\mu^2)(|\xi_1|^2 + |\xi_2|^2) + l_3\mu^2 z_1^2 \tag{27}$$

where $l_1 = 16\rho d_0|B_1|^2$, $l_2 = 2\rho d_0 + 2d_1(d_2+8) + 4\rho d_0 \max\{\|A_1\|^2, \|A_2\|^2\}$, $l_3 = 4\rho d_0(2|B_1|^2 + |B_2|^2)$. It is clear that there exists a parameter $\mu^* = \min\{\mu_1^*, \mu_2^*\}$, $\mu_1^* = (-l_2 + (l_2^2 + l_1 \min\{r_1, r_2\})^{1/2}/(2l_1))^{1/2}$, $\mu_2^* = (c_1/(2l_3))^{1/2}$, such that for any $\mu \in [0, \mu^*)$, $l_1\mu^4 + l_2\mu^2 \leqslant r_1/2$, $l_1\mu^4 + l_2\mu^2 \leqslant r_2/2$, $l_3\mu^2 \leqslant c_1/2$ hold. Substituting the above inequalities into (27) gives

$$\dot{V}_\rho \leqslant -\alpha|\varPsi|^2 \tag{28}$$

where $\alpha = \min\{c_1/2, c_2, \cdots, c_\rho, r_0/2, r_1/4, r_2/4, (1/2 - d_2)r_3\}$, $\varPsi = (z_1, \cdots, z_\rho, \varepsilon^{\mathrm{T}}, \xi_1^{\mathrm{T}}, \xi_2^{\mathrm{T}}, \zeta^{\mathrm{T}})^{\mathrm{T}}$. Thus $z_1, \cdots, z_\rho, \varepsilon, \xi_1, \xi_2, \zeta, \hat{\theta}$ are globally uniformly bounded. From (1) and (11), one gets

$$\hat{x}_1 = \frac{L(s)}{K(s)}y + \frac{1}{K(s)}\frac{A(s)}{B(s)}\frac{1-\mu_2\Delta_2(s)}{\mu_1\Delta_1(s)}y \tag{29}$$

where $K(s) = s^{\rho-1} + k_1 s^{\rho-2} + \cdots + k_\rho$, $L(s) = (k_2 - k_1^2)s^{\rho-2} + (k_3 - k_1k_2)s^{\rho-3} + \cdots + (k_{\rho-1} - k_1k_{\rho-2})s - k_1k_{\rho-1}$. From assumption A2 and Lemma 1, there exists a parameter $\mu_1 \in [0, \mu)$, $(1-\mu_2\Delta_2(s))/(1+\mu_1\Delta_1(s))$ such that for any, is stable and strictly proper also. Thanks to the stability of strict property of, and the global uniform boundedness of y and (29), there exists a parameter $\bar{\mu} = \min\{\mu_0, \mu^*\}$, such that for any $\mu \in [0, m\bar{u})$, $\hat{x}_1$ is globally uniformly bounded. From (19), it is known that $\tau_1, \alpha_1(y, \hat{\theta})$ are globally uniformly bounded also. From (21) and (22), one gets that $\alpha_2(y, \hat{x}_1, \hat{\theta}), \tau_2, \hat{x}_2$ are globally uniformly bounded. Recursively, the global uniform boundedness of $\hat{x}_i, \alpha_i (i = 3, \cdots, \rho-1)$, u is proved. From (3) and (8), it is obtained that x, χ are globally uniformly bounded, which concludes the first result in this theorem. From (28) and the definition of $\varPsi$, according to Lasalle-Yoshizaway Lemma[1], we have that $z_1, \cdots, z_\rho, \varepsilon, \xi_1, \xi_2, \zeta$ tend to zero asymptotically. By the same argument as above, the second result holds.

5 Simulation

For system (1), let $A(s) = s^2 + a_1 s + a_0$, $B(s) = 1$, $\Delta_1(s) = \Delta_2(s) = 0.5/(s+0.1)$. By using the coordinate change (14), from (19), (22) and (24) one gets $\dot{\hat{x}} = u - k(\hat{x} + ky)$, $\dot{\hat{\theta}} = \tau_2$, $u = \alpha_2$. Given $a_1 = -1$, $a_0 = 1$, $\chi = (0.4, -1)^{\mathrm{T}}$, choose design parameters $d_0 = 0.5$, $d_1 = 0.2$, $d_2 = 0.25$, $c_1 = 3.2$, $c_2 = 1$, $k = 2$, $r = 10^{-4}$, $\hat{x}(0) = -2$, $\hat{\theta}(0) = 0$. Using Matlab project, one gets $\bar{\mu} = 0.7304$. Fig. 1 shows the system responses of the closed-loop system with the choice of $\mu_1 = \mu_2 = 0.7$. The result of simulation verifies the effectiveness of our reduced-order controller design.

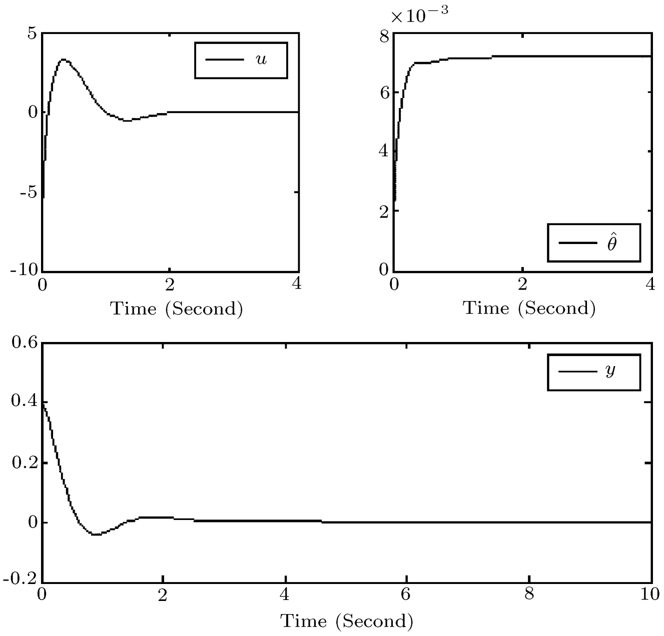

Fig.1 Responses of the closed-loop system

6 Conclusion

Compared with [5], by the designing reduced-order controller, the problem of robust adaptive stabilization is considered in this paper. It is proved that all signals in the closed-loop system are globally uniformly bounded, and the regulation error tends to zero asymptotically. The reduction of dynamic order of controller makes our controller design possess more practical values. To the authors′ knowledge this is a new result.

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