Cooperative Iterative Learning Control of Linear Multi-agent Systems with a Dynamic Leader under Directed Topologies

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Abstract This paper considers the cooperative tracking of linear multi-agent systems with a dynamic leader whose input information is unavailable to any followers. Cooperative iterative learning controllers, based on the relative state information of neighboring agents, are proposed for tracking the dynamic leader over directed communication topologies. Stability and convergence of the proposed controllers are established using Lyapunov-Krasovskii functionals. Furthermore, this result is extended to the output feedback case where only the output information of each agent can be obtained. A local observer is constructed to estimate the unmeasurable states. Then, cooperative iterative learning controllers, based on the relative observed states of neighboring agents, are devised. For both cases, it is shown that the multi-agent systems whose communication topologies contain a spanning tree can reach synchronization with the dynamic leader, and meanwhile identify the unknown input of the dynamic leader using distributed iterative learning laws. An illustrative example is provided to verify the proposed control schemes.

Key words Cooperative control, dynamic leader, linear multi-agent systems, iterative learning

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In recent years, compelling attention has been paid to cooperative control of multi-agent systems due to its potential application in numerous engineering areas including unmanned vehicles, manipulators, sensor networks, etc. Along this line of research, consensus as a fundamental control problem is of great importance due to that it is closely related to formation. The key of the consensus is to design a distributed control law based on local interactions with neighbors such that the agents agree on some value of interest[1−3]. Numerous results on the consensus control have been obtained; see [4−13] and the references therein. In general, these consensus protocols are falling neatly into two categories, namely, leaderless consensus and leader-follower consensus. In literature, the leader-follower consensus is also called as consensus tracking[10], distributed tracking[13], or cooperative tracking[12, 14].

During the past few years, cooperative tracking of multi-agent systems has been widely studied from different perspectives. In [8], a neighbor-based tracking controller together with a neighbor-based state observer is proposed for each agent to track a dynamic leader. In [9], consensus protocols are developed for first-order linear systems with a time-varying leader dynamics. In [10], robust consensus tracking controllers are proposed for second-order nonlinear systems for both undirected and directed topologies. In [11], consensus of multi-agent systems and synchronization of complex networks are unified in a framework. In [12], a framework for cooperative tracking of linear multi-agent systems is proposed, including state feedback, observer and output feedback. Note that the input is either assumed to be zero[12], or available to a fraction of followers[9−10], or known to all followers[8, 11], which may be restrictive in many circumstances.

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To deal with the unknown input of the leader, several methods can be applied[13−21]. In [14−18], neural networks are employed to compensate for the unknown input dynamics and unknown individual dynamics. In [19−20], the unknown input of leader can be rejected by a distributed output regulation approach. In [13, 21], the unknown input of leader can be handled using the sliding-mode control approach, which results in a discontinuous controller. Further, the undesirable chattering effect caused by the discontinuous controllers in the previous works[13, 21] can be avoided by the proposed continuous controllers in [22].

On the other hand, iterative learning method as an effective control strategy has been widely explored; see [23] and the references therein. The main advantage of iterative learning is using previous information to improve control performance. In [24], iterative learning adaptive law is developed to alleviate the constraint of identifying constant and slowly time-varying parameters. Since the input of the leader may be time-varying and changing very quickly, it is very attractive to apply it to the control of multi-agent systems. However, note that the iterative learning method proposed in [24] is for centralized identification of system parameters in single unknown system, and thus cannot be applied in this case.

Motivated by the above observations, we focus on the cooperative iterative learning control of linear multi-agent systems with a dynamic leader whose input is totally unknown to each agent. The communication topologies among the followers are assumed to be directed and containing a spanning tree. At first, cooperative iterative learning controllers, based on the relative state information of neighboring agents, are proposed. Lyapunov-Krasovskii functional is used to show the uniform ultimate boundedness of closed-loop network signals. Then, this result is extended to the output feedback case, and a local observer is constructed to estimate the unmeasurable states. Based on the relative observed states of neighboring agents, cooperative observer-based iterative learning controllers are devised. A key feature of the proposed controllers allows for tracking the dynamic leader over directed communication graphs without the knowledge of the input.

The comparisons with existing works are listed as follows. In contrast to the works[8−13], the input of the leader is not required to be zero, or available to any followers. Compared with the distributed tracking controllers developed in [13, 21], the proposed iterative learning controllers are able to capture the input of the leader, instead of capturing the upper bound of the input. Moreover, the proposed iterative learning control laws are easier to implement in digital processors due to the fact that algebraic equations are used for identification of the dynamics of leader. Unlike the cooperative tracking controllers developed for undirected graphs[13, 21−22], the communication topologies among agents considered here are directed. Finally, it is worth mentioning that the developed protocols are different from the iterative learning controllers proposed in [25−26] in the sense that the control laws are updated in a discrete time form without adaptive terms.

The paper is organized as follows. Section 1 introduces some preliminaries and states the problem formulation. Section 2 presents the state feedback design and stability analysis. Section 3 extends the preceding results to the output feedback case. Section 4 provides an example to illustrate the theoretical results. Section 5 concludes this article.

1 Preliminaries and problem formulation

1.1 Preliminaries

Consider a network of system consisting of N follower agents and one leader. If each follower is considered as a node, the neighbor relationship among the followers can be described by a graph $\mathcal{G}=\{\mathcal{V},\mathcal{E}\}$, where $\mathcal{V}=\{n_1,\cdots,n_N\}$ is a node set and $\mathcal{E}=\{(n_i,n_j)\in\mathcal{V}\times\mathcal{V}\}$ is an edge set with the element (n_i,n_j) that describes the communication from the node i to the node j. The neighbor set of the node i is denoted by $\mathcal{N}_i=\{j|(n_j,n_i)\in\mathcal{E}\}$. Define an adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbf{R}^{N\times N}$ with $a_{ij}=1$, if $(n_j,n_i)\in\mathcal{E}$ and $a_{ij}=0$, otherwise. Define an in-degree matrix as a diagonal matrix $\mathcal{D}=\mathrm{diag}\{d_i\}\in\mathbf{R}^{N\times N}$ with $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$ for node i. The Laplacian matrix associated with the graph $\mathcal{G}$ is defined as $L=\mathcal{D}-\mathcal{A}$. A directed path in the graph is an ordered sequence of nodes such that any two consecutive nodes in the sequence are an edge of the graph. A digraph has a spanning tree, if there is a node called as the root, such that there is a directed path from the root to every other node in the graph. Finally, define a leader adjacency matrix as $\mathcal{A}_0=\mathrm{diag}\{a_{10},\cdots,a_{N0}\}$, where $a_{i0}>0$ if and only if the ith agent has access to the leader information; otherwise, $a_{i0}=0$. Let $H=L+\mathcal{A}_0$.

Lemma 1[14]**.** Suppose the graph $\mathcal{G}$ has a directed spanning tree, and let the root agent has access to the leader. Define

$$\begin{aligned}&q=[q_1,\cdots,q_N]^{\mathrm{T}}=H^{-1}\mathbf{1}\\&T=\mathrm{diag}\{p_i\}=\mathrm{diag}\{1/q_i\},i=1,\cdots,N\\&G=TH+H^{\mathrm{T}}T\end{aligned}\tag{1}$$

Then, T and G are positive definite.

Throughout the paper, the Euclidean norm and trace are denoted by $||\cdot||$ and $\mathrm{tr}\{\cdot\}$, respectively. A diagonal matrix is represented by $\mathrm{diag}\{b_1,\cdots,b_N\}$ with b_i being the ith diagonal element. An identity matrix of dimension N is denoted by I_N. The Kronecker product is denoted by $\otimes$.

1.2 Problem formulation

Consider a class of multi-agent systems consisting of N followers and one leader. The dynamics of the ith follower is governed by a linear system

$$\left\{\begin{array}{l}\dot{x}_i(t)=Ax_i(t)+Bu_i(t)\\y_i(t)=Cx_i(t),i=1,\cdots,N\end{array}\right.\tag{2}$$

where $x_i=[x_{i1}(t),\cdots,x_{in}(t)]^{\mathrm{T}}\in\mathbf{R}^n$ is the system state; $u_i(t)\in\mathbf{R}^m$ is the control input; $y_i(t)\in\mathbf{R}^p$ is the output state; $A\in\mathbf{R}^{n\times n}$, $B\in\mathbf{R}^{n\times m}$, $C\in\mathbf{R}^{p\times n}$ are known matrices.

The leader is governed by a dynamic system

$$\dot{x}_0(t)=Ax_0(t)+Br(t)\tag{3}$$

where $x_0\in\mathbf{R}^n$ is the leader state. $r(t)\in\mathbf{R}^m$ is an unknown bounded input and may vary quickly in practice. For any initial conditions, assume that the solution x_0 exists for all $t\geq0$.

The control objective of this paper is to design a distributed control law u_i for each agent (2) to track the leader (3), i.e., $x_i(t)\to x_0(t)$ as $t\to\infty$.

Before designing the controllers, the following assumptions are needed.

Assumption 1. The pair (A,B) is stabilizable.

Assumption 2. The pair (A,C) is detectable.

Assumption 3. The time-varying input $r(t)$ is bounded by $\|r(t)\|\leq r_M$ with r_M being a positive constant.

2 State feedback

2.1 Controller design

In [12], it has been shown that synchronization to the leader (3) with $r(t)=0$ can be achieved under directed communication topologies. However, this synchronization controller may not work efficiently if the leader′s input is a time-varying trajectory. Motivated by the above observation, a distributed adaptive controller is proposed as follows

$$u_i(t)=cKe_i(t)+\nu_i(t)\tag{4}$$

where $c\in\mathbf{R}$ is a coupling gain to be specified later; $K\in\mathbf{R}^{m\times n}$ is a feedback matrix with

$$K=-B^{\mathrm{T}}P\tag{5}$$

where P is the unique positive definite solution to the following Riccati equation

$$A^{\mathrm{T}}P+PA+Q-PBB^{\mathrm{T}}P=0\tag{6}$$

where $Q\in\mathbf{R}^{n\times n}$ is positive definite; $e_i(t)$ is a local tracking error defined by

$$e_i(t)=\sum_{j=1}^{N}a_{ij}[x_i(t)-x_j(t)]+a_{i0}[x_i(t)-x_0(t)]\tag{7}$$

where a_{ij} are defined in Section 1.1. $\nu_i(t)$ is used to identify the unknown time-varying input $r(t)$ that is updated as

$$\nu_i(t)=\kappa_{i1}\nu_i(t-\tau)-\kappa_2p_i(d_i+a_{i0})B^{\mathrm{T}}Pe_i(t)\tag{8}$$

where $\tau\in\mathbf{R}$ is a positive updating interval. $\kappa_{i1}\in\mathbf{R}^{m\times m}$ and $\kappa_2\in\mathbf{R}$ satisfy $0<\kappa_{i1}^{\mathrm{T}}\kappa_{i1}<\alpha I_m$ with $0<\alpha<1$, and $\kappa_2>0$, respectively. p_i is defined in (1).

Define a global tracking error $\delta_i(t) = x_i(t) - x_0(t)$, whose time derivative along (2), (3) and (4) is given by

$$\dot{\delta}_i(t) = A\delta_i(t) + B[cKe_i(t) + \nu_i(t) - r(t)] \tag{9}$$

Define the input estimation error $\tilde{r}_i(t) = \nu_i(t) - r(t)$, and it follows that

$$\dot{\delta}_i(t) = A\delta_i(t) + B[cKe_i(t) + \tilde{r}_i(t)] \tag{10}$$

Let $\delta(t) = [\delta_1^{\rm T}(t), \cdots, \delta_N^{\rm T}(t)]^{\rm T}$, $\nu(t) = [\nu_1^{\rm T}(t), \cdots, \nu_N^{\rm T}(t)]^{\rm T}$, $\tilde{r}(t) = [\tilde{r}_1^{\rm T}(t), \cdots, \tilde{r}_N^{\rm T}(t)]^{\rm T}$, $e(t) = [e_1^{\rm T}(t), \cdots, e_N^{\rm T}(t)]^{\rm T}$. Then, the N subsystems of (10) resulting from the fact $e(t) = (H \otimes I_n)\delta(t)$ are

$$\dot{e}(t) = (I_N \otimes A + cH \otimes BK)e(t) + (H \otimes B)\tilde{r}(t) \tag{11}$$

2.2 Stability analysis

Theorem 1. Consider the multi-agent systems with the followers (2) and the dynamic leader (3) under Assumptions 1 and 3. The communication topologies among the followers are directed graphs containing a spanning tree, and let the root agent have access to the leader. Select the control law (4) with the adaptive law (8) and the coupling strength c satisfying

$$c \geq \frac{1}{\min_{i=1,\cdots,N}(q_i g_i)} \tag{12}$$

where g_i is the ith eigenvalue of Q. Then, the global tracking error $\delta_i(t)$ and the input estimation error $\tilde{r}_i(t)$ are uniformly ultimately bounded.

Proof. We first define

$$\begin{cases} \rho_i(t) = r(t) - \kappa_{i1} r(t-\tau) \\ \hat{\rho}_i(t) = \nu_i(t) - \kappa_{i1}\nu_i(t-\tau) \end{cases} \tag{13}$$

where $\|\rho_i(t)\| \leq \rho_{iM}$ with $\rho_{iM} = (1 + \|\kappa_{i1}\|)r_M$. The input estimation error $\tilde{r}_i$ can be written as

$$\tilde{r}_i(t) = \kappa_{i1}\tilde{r}_i(t-\tau) + \hat{\rho}_i(t) - \rho_i(t) \tag{14}$$

Let $\rho(t) = [\rho_1^{\rm T}(t), \cdots, \rho_N^{\rm T}(t)]^{\rm T}$, $\hat{\rho}(t) = [\hat{\rho}_1^{\rm T}(t), \cdots, \hat{\rho}_N^{\rm T}(t)]^{\rm T}$, $\kappa_1 = {\rm diag}\{\kappa_{11}, \cdots, \kappa_{N1}\}$, and then

$$\tilde{r}(t) = \kappa_1\tilde{r}(t-\tau) + \hat{\rho}(t) - \rho(t) \tag{15}$$

It follows that the error dynamics of $\delta(t)$ with (15) is

$$\begin{aligned}\dot{e}(t) = &(I_N \otimes A + cH \otimes BK)e(t) + (H \otimes B)[\kappa_1\tilde{r}(t-\tau) + \\ &\hat{\rho}(t) - \rho(t)]\end{aligned} \tag{16}$$

Consider the Lyapunov-Krasovskii functional

$$V_1 = e^{\rm T}(t)(T \otimes P)e(t) + \beta\left(\int_{t-\tau}^{t} \tilde{r}^{\rm T}(s)\tilde{r}(s){\rm d}s\right) \tag{17}$$

whose time derivative along (15) and (16) is given by

$$\begin{aligned}\dot{V}_1 = &e^{\rm T}(t)[T \otimes (PA + A^{\rm T}P) + cG \otimes PBK]e(t) + \\ &2e^{\rm T}(t)(TH \otimes PB)[\kappa_1\tilde{r}(t-\tau) + \hat{\rho}(t) - \rho(t)] + \\ &\beta[-\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) + \eta\tilde{r}^{\rm T}(t)\tilde{r}(t) - \\ &\tilde{r}^{\rm T}(t-\tau)\tilde{r}(t-\tau)]\end{aligned} \tag{18}$$

where $\eta = 1 + \mu$ and $\beta > 0$. Expanding $\tilde{r}^{\rm T}(t)\tilde{r}(t)$ from (15) gives

$$\begin{aligned}\tilde{r}^{\rm T}(t)\tilde{r}(t) = &\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\kappa_1\tilde{r}(t-\tau) + \rho^{\rm T}(t)\rho(t) + \\ &\hat{\rho}^{\rm T}(t)\hat{\rho}(t) - 2\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\rho(t) + \\ &2\hat{\rho}^{\rm T}(t)\kappa_1\tilde{r}(t-\tau) - 2\hat{\rho}^{\rm T}(t)\rho(t)\end{aligned}$$

which leads to

$$\begin{aligned}\dot{V}_1 = &e^{\rm T}(t)[T \otimes (PA + A^{\rm T}P) + cG \otimes PBK]e(t) + \\ &2e^{\rm T}(t)(T(\mathcal{A}_0 + \mathcal{D} - \mathcal{A}) \otimes PB)[\kappa_1\tilde{r}(t-\tau) + \hat{\rho}(t) - \rho(t)] + \\ &\beta[-\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) - \tilde{r}^{\rm T}(t-\tau)\tilde{r}(t-\tau) + \eta\rho^{\rm T}(t)\rho(t) + \\ &\eta\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\kappa_1\tilde{r}(t-\tau) + \eta\hat{\rho}^{\rm T}(t)\hat{\rho}(t) - \\ &2\eta\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\rho(t) - 2\eta\hat{\rho}^{\rm T}(t)\rho(t) + \\ &2\eta\hat{\rho}^{\rm T}(t)\kappa_1\tilde{r}(t-\tau)]\end{aligned} \tag{19}$$

Using (8) and (13) and letting $\kappa_2 = 1/(\beta\eta)$, it follows that

$$\begin{aligned}\dot{V}_1 = &e^{\rm T}(t)[T \otimes (PA + A^{\rm T}P) + cG \otimes PBK]e(t) - \\ &2e^{\rm T}(t)(T\mathcal{A} \otimes PB)\tilde{r}(t) - \beta\eta\hat{\rho}^{\rm T}(t)\hat{\rho}(t) + \\ &\beta[-\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) - \tilde{r}^{\rm T}(t-\tau)\tilde{r}(t-\tau) + \\ &\eta\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\kappa_1\tilde{r}(t-\tau) + \eta\rho^{\rm T}(t)\rho(t) - \\ &2\eta\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\rho(t)]\end{aligned} \tag{20}$$

From Young′s inequality, one has

$$\begin{aligned}-2\eta\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\rho(t) \leq &\gamma\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\kappa_1\tilde{r}(t-\tau) + \\ &\eta^2\rho^{\rm T}(t)\rho(t)/\gamma\end{aligned} \tag{21}$$

where $\gamma > 0$.

Substituting (21) into (20) yields

$$\begin{aligned}\dot{V}_1 \leq &e^{\rm T}(t)[T \otimes (PA + A^{\rm T}P) + cG \otimes PBK]e(t) - \\ &2e^{\rm T}(t)(T\mathcal{A} \otimes PB)\tilde{r}(t) - \beta\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) + \\ &\beta(\eta + \eta^2/\gamma)\rho^{\rm T}(t)\rho(t) - \beta[\tilde{r}^{\rm T}(t-\tau)[I_{Nm} - \\ &(\gamma + \eta)\kappa_1^{\rm T}\kappa_1]\tilde{r}(t-\tau)]\end{aligned} \tag{22}$$

where I_{Nm} represents an identity matrix of dimension $N \times m$.

By Lemma 1, we know G is positive definite. Then, let U be a unitary matrix such that $U^{\rm T}GU = {\rm diag}\{g_1, \cdots, g_N\}$. Introduce a state transformation $\epsilon(t) = (U^{\rm T} \otimes I_n)e(t)$ with $\epsilon = [\epsilon_1^{\rm T}, \cdots, \epsilon_N^{\rm T}]^{\rm T}$, and thus

$$\begin{aligned}\dot{V}_1 \leq &\sum_{i=1}^{N}\epsilon_i^{\rm T}[\xi_i(PA + A^{\rm T}P - cq_ig_iPBB^{\rm T}P)]\epsilon_i - \\ &2e^{\rm T}(t)(T\mathcal{A} \otimes PB)\tilde{r}(t) - \beta\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) + \\ &\beta(\eta + \eta^2/\gamma)\rho^{\rm T}(t)\rho(t) - \beta[\tilde{r}^{\rm T}(t-\tau)[I_{Nm} - \\ &(\gamma + \eta)\kappa_1^{\rm T}\kappa_1]\tilde{r}(t-\tau)]\end{aligned} \tag{23}$$

Recalling (6) and (12) and using the fact $\|\delta\| = \|\epsilon\|$, one has

$$\begin{aligned}\dot{V}_1 \leq &-\min_{i=1,\cdots,N}(\xi_i)\lambda_{\min}(Q)\|e(t)\|^2 + \\ &2\|e(t)\|\|T\mathcal{A}\|\|PB\|\|\tilde{r}(t)\| - \beta\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) - \\ &\beta\{\tilde{r}^{\rm T}(t-\tau)[I_{Nm} - (\gamma + \eta)\kappa_1^{\rm T}\kappa_1]\tilde{r}(t-\tau)\} + \\ &\beta(\eta + \eta^2/\gamma)\rho^{\rm T}(t)\rho(t)\end{aligned}$$

Letting $z_1(t) = [\|e(t)\|, \|\tilde{r}(t)\|, \|\tilde{r}(t-\tau)\|]^{\rm T}$, we have

$$\dot{V}_1 \leq -z_1^{\rm T}(t)S_1z_1(t) + \varpi_1 \tag{24}$$

where

$$\begin{cases} S_1 &= \begin{bmatrix} \min_{i=1,\cdots,N}(\xi_i)\lambda_{\min}(Q) & \zeta & 0 \\ \zeta & \beta\mu & 0 \\ 0 & 0 & \varsigma \end{bmatrix} \\ \varpi_1 &= \beta(\eta+\eta^2/\gamma)\rho_M^2 \end{cases}$$

with $\alpha = 1/(\gamma+\eta)$, $\zeta = \|T\mathcal{A}\|\|PB\|$, $\rho_M = \sqrt{\sum_{i=1}^N \rho_{iM}^2}$, $\varsigma = \beta\lambda_{\min}(I_{Nm} - \alpha^{-1}\kappa_1^{\rm T}\kappa_1)$. Select $\beta\mu > \zeta^2/(\min_{i=1,\cdots,N}(\xi_i)\lambda_{\min}(Q))$ such that S_1 is positive definite, and then define a compact set

$$\Omega_{z_1} = \{z_1(t)|\|z_1(t)\| \le \varepsilon_1\} \tag{25}$$

where $\varepsilon_1 = \sqrt{\varpi_1/\lambda_{\min}(S_1)}$. Note that $\|z_1(t)\| > \varepsilon_1$ renders $\dot{V}_1(t) < 0$. Therefore, the tracking error $\|e(t)\|$ and the estimate error $\|\tilde{r}(t)\|$ are uniformly ultimately bounded. Since $\|\delta(t)\| \le \|e(t)\|/\underline{\hbar}(H)$ where $\underline{\hbar}(H)$ denotes the minimal singular value of H[14], it follows that $\delta(t)$ is uniformly ultimately bounded, implying that $\delta_i(t)$ and $\tilde{r}(t)$ are bounded[27].

3 Output feedback

In the previous section, the proposed controller entails that all states of each agent can be obtained by each agent. However, some states cannot be available for the feedback control in many instances. Therefore, the control objective of this section is to develop a distributed control law $u_i(t)$ to track the leader under directed topologies, only using the output information of $y_i(t)$.

3.1 Controller design

At first, consider a local state observer

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_i(t) + F[y_i(t) - \hat{y}_i(t)] \\ \hat{y}_i(t) = C\hat{x}_i(t) \end{cases} \tag{26}$$

where $F \in \mathbf{R}^{n\times p}$ is a feedback gain matrix to be designed such that $A - FC$ is Hurwitz. Then, a distributed controller based on the observed states of neighboring agents is proposed as follows

$$u_i(t) = cK\hat{e}_i(t) + \nu_i(t) \tag{27}$$

where c, K are defined the same as (4); P is the unique positive definite solution to the Riccati equation (6); $\hat{e}_i(t)$ is defined as

$$\hat{e}_i(t) = \sum_{j=1}^N a_{ij}[\hat{x}_i(t) - \hat{x}_j(t)] + a_{i0}[\hat{x}_i(t) - x_0(t)] \tag{28}$$

$\nu_i(t)$ is updated by

$$\nu_i(t) = \kappa_{i1}\nu_i(t-\tau) - \kappa_2 p_i(d_i + a_{i0})B^{\rm T}P\hat{e}_i(t) \tag{29}$$

where κ_{i1}, κ_2, and τ are defined the same as (8).

Denote an estimated error $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$, whose dynamics can be written as

$$\dot{\tilde{x}}_i(t) = A_e\tilde{x}_i(t) \tag{30}$$

where $A_e = A - FC$. Let $\tilde{x} = [\tilde{x}_1^{\rm T}, \cdots, \tilde{x}_N^{\rm T}]^{\rm T}$, then the N subsystems of (30) is written as

$$\dot{\tilde{x}}(t) = (I_N \otimes A_e)\tilde{x}(t) \tag{31}$$

Define an estimated state tracking error $\tilde{\delta}_i(t) = \hat{x}_i(t) - x_0(t)$, whose time derivative along (3) and (27) is

$$\dot{\tilde{\delta}}_i(t) = A\tilde{\delta}_i(t) + cBK\hat{e}_i(t) + B[\nu_i(t) - r(t)] + FC\tilde{x}_i(t) \tag{32}$$

Let $\tilde{\delta}(t) = [\tilde{\delta}_1^{\rm T}(t), \cdots, \tilde{\delta}_N^{\rm T}(t)]^{\rm T}$, $\tilde{x}(t) = [\tilde{x}_1^{\rm T}(t), \cdots, \tilde{x}_N^{\rm T}(t)]^{\rm T}$, and then the dynamics of $\tilde{\delta}(t)$ can be expressed by

$$\begin{aligned}\dot{\hat{e}}(t) =&(I_N \otimes A + cH \otimes BK)\hat{e}(t) + \\ &(H \otimes FC)\tilde{x}(t) + (H \otimes B)\tilde{r}(t)\end{aligned} \tag{33}$$

3.2 Stability analysis

Theorem 2. Consider the multi-agent systems with the followers (2) and the leader (3) under Assumptions 1 ~ 3. The communication topologies among the followers are directed graphs containing a spanning tree, and let the root agent have access to the leader. Select the control law (27) with the coupling strength c satisfying (12) and the adaptive law (26), together with the state observer (29). Then, the estimated state tracking error $\tilde{\delta}_i(t)$, the global tracking error $\delta_i(t)$, and the input estimate error $\tilde{r}_i(t)$ are uniformly ultimately bounded.

Proof. Consider the following Lyapunov-Krasovskii functional

$$V_2 = \hat{e}^{\rm T}(T \otimes P)\hat{e} + \beta\left(\int_{t-\tau}^t \tilde{r}^{\rm T}(s)\tilde{r}(s){\rm d}s\right) \tag{34}$$

whose time derivative along (33) can be described by

$$\begin{aligned}\dot{V}_2 \le\ & \hat{e}^{\rm T}(t)[T \otimes (PA + A^{\rm T}P) + 2cG \otimes PBK]\hat{e}(t) - \\ & 2\hat{e}^{\rm T}(t)(T\mathcal{A} \otimes PB)\tilde{r}(t) + \beta[-\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) - \\ & \tilde{r}^{\rm T}(t-\tau)\tilde{r}(t-\tau) - 2\eta\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\rho(t) + \\ & \eta\tilde{r}^{\rm T}(t-\tau)\kappa_1^{\rm T}\kappa_1\tilde{r}(t-\tau) + \eta\rho^{\rm T}(t)\rho(t)] + \\ & 2\hat{e}^{\rm T}(t)(TH \otimes PFC)\tilde{x}(t)\end{aligned} \tag{35}$$

Using the state transformation $\epsilon = (U^{\rm T} \otimes I_n)\hat{e}$, one has

$$\begin{aligned}\dot{V}_2 \le & -\min(\xi_i)\lambda_{\min}(Q)\|\hat{e}(t)\|^2 - \beta\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) - \\ & 2\hat{e}^{\rm T}(t)(T\mathcal{A} \otimes PB)\tilde{r}(t) - \beta\{\tilde{r}^{\rm T}(t-\tau)[I_{Nm} - \\ & (\gamma+\eta)\kappa_1^{\rm T}\kappa_1]\tilde{r}(t-\tau)\} + \beta(\eta + \eta^2/\gamma)\rho^{\rm T}(t)\rho(t) + \\ & 2\hat{e}^{\rm T}(t)(TH \otimes PFC)\tilde{x}(t)\end{aligned} \tag{36}$$

Because A_e is Hurwitz, it follows that the system (31) is asymptotically stable, indicating that $\|\tilde{x}(t)\| \le \tilde{x}_M$ for all $t > 0$, where $\tilde{x}_M$ is a positive constant that depends on $\tilde{x}(0)$. Then

$$\begin{aligned}\dot{V}_2 \le & -\min_{i=1,\cdots,N}(\xi_i)\lambda_{\min}(Q)\|\hat{e}(t)\|^2 - \beta\mu\tilde{r}^{\rm T}(t)\tilde{r}(t) + \\ & 2\|\hat{e}(t)\|\|T\mathcal{A}\|\|PB\|\|\tilde{r}(t)\| - \beta\{\tilde{r}^{\rm T}(t-\tau)[I_{Nm} - \\ & (\gamma+\eta)\kappa_1^{\rm T}\kappa_1]\tilde{r}(t-\tau)\} + \beta(\eta+\eta^2/\gamma)\rho^{\rm T}(t)\rho(t) + \\ & 2\max_{i=1,\cdots,N}(\xi_i)\|\hat{e}(t)\|\|TH\|\|PFC\|\tilde{x}_M\end{aligned} \tag{37}$$

and using $\|\hat{e}(t)\| \leq \|\hat{e}(t)\|^2/\varrho + \varrho$ with $\varrho > 0$, it follows that

$$\begin{aligned}\dot{V}_2 \leq & -\min_{i=1,\cdots,N}(\xi_i)\lambda_{\min}(Q)\|\hat{e}(t)\|^2 - \beta\mu\tilde{r}^{\mathrm{T}}(t)\tilde{r}(t) + \\ & 2\|\hat{e}(t)\|\|T\mathcal{A}\|\|PB\|\|\tilde{r}(t)\| - \beta\{\tilde{r}^{\mathrm{T}}(t-\tau)[I_{Nm} - \\ & (\gamma+\eta)\kappa_1^{\mathrm{T}}\kappa_1]\tilde{r}(t-\tau)\} + \beta(\eta+\eta^2/\gamma)\rho^{\mathrm{T}}(t)\rho(t) + \\ & \max_{i=1,\cdots,N}(\xi_i)\|TH\|\|PFC\|\tilde{x}_M(\|\hat{e}(t)\|^2/\varrho + \varrho)\end{aligned} \tag{38}$$

Letting $z_2(t) = [\|\hat{e}(t)\|, \|\tilde{r}(t)\|, \|\tilde{r}(t-\tau)\|]^{\mathrm{T}}$, we have

$$\dot{V}_2 \leq -z_2^{\mathrm{T}}(t)S_2z_2(t) + \varepsilon_2 \tag{39}$$

where

$$\left\{\begin{aligned} S_2 &= \begin{bmatrix}\min_{i=1,\cdots,N}(\xi_i)\lambda_{\min}(Q) - d & \zeta & 0\\ \zeta & \beta\mu & 0 \\ 0 & 0 & \varsigma\end{bmatrix} \\ \varepsilon_2 &= \beta(\eta+\eta^2/\gamma)\rho_M^2 + \\ & \quad \varrho\max_{i=1,\cdots,N}(\xi_i)\|TH\|\|PFC\|\tilde{x}_M\end{aligned}\right.$$

with $d = \max_{i=1,\cdots,N}(\xi_i)\|TH\|\|PFC\|\tilde{x}_M/\varrho$. Choose $\beta\mu > \zeta^2/(\min_{i=1,\cdots,N}(\xi_i)\lambda_{\min}(Q) - d)$ such that S_2 is positive definite. Then, define a compact set

$$\Omega_{z_2} = \{z_2(t) | \|z_2(t)\| \leq \varepsilon_2\} \tag{40}$$

where $\varepsilon_2 = \sqrt{\varpi_2/\lambda_{\min}(S_2)}$. Note that $\|z_2(t)\| > \varepsilon_2$ renders $\dot{V}_2(t) < 0$. It follows that $\|\hat{e}(t)\|$, $\|\tilde{r}(t)\|$ and $\|\tilde{r}(t-\tau)\|$ are uniformly ultimately bounded, implying that $\|\tilde{\delta}\|$ is also bounded.

Noting that

$$\begin{aligned}\|\delta(t)\| = \|x(t) - x_0(t)\| = & \\ \|x(t) - \hat{x}(t) + \hat{x}(t) - x_0(t)\| \leq & \\ \|\tilde{x}(t)\| + \|\tilde{\delta}(t)\| & \end{aligned} \tag{41}$$

it follows that $\delta(t)$ is uniformly ultimately bounded, implying that $\tilde{\delta}_i(t)$, $\delta_i(t)$, and $\tilde{r}_i(t)$ are uniformly ultimately bounded. □

Remark 1. Cooperative tracking of linear multi-agent systems with a dynamic leader of a bounded unknown input is firstly considered in [13], and later extended to the output feedback case in [21]. Note that these controllers are developed for undirected graphs, while the focus of this paper is on the directed graphs, which take the undirected graphs as special cases. Moreover, the sliding-mode controller developed in [13] requires the upper bound of the control input, while the proposed controller does not require it to be known. Although the above restriction is relaxed in the second controller in [13] which tries to estimate the upper bound of the control input, the proposed controller aims to directly estimate the control input itself, which means that less control effort is needed when the actual control input is less than the upper bound.

4 An example

In this section, an illustrative example is given to verify the theoretical results.

Example. Synchronization of autopilots

Consider a network of autopilots whose dynamics can be described by a linear Nomoto model, which is given by

$$\left\{\begin{aligned}\dot{\psi}_i &= \omega_i \\ \dot{\omega}_i &= -\frac{1}{\mathcal{T}}\omega_i + \frac{\mathcal{K}}{\mathcal{T}}\delta_i\end{aligned}\right. \tag{42}$$

where ψ_i is the heading; ω_i is the yaw rate; δ_i is the actuator angle. The time constant $\mathcal{T}$ and gain $\mathcal{K}$ are given by

$$\mathcal{T} = \frac{I_{iz} - N_{i\dot{\omega}}}{-N_{i\omega}}, \mathcal{K} = -\frac{N_{i\delta}}{N_{i\omega}} \tag{43}$$

where I_{iz} is the moment of inertia in yaw axis; $N_{i\dot{\omega}}, N_{i\omega}, N_{i\delta}$ are hydrodynamic coefficients which can be estimated from trials in calm water. For more details on the linear Nomoto model, the readers are referred to [28]. In this example, $\mathcal{K}, \mathcal{T}$ are taken as $\mathcal{K} = 0.25, \mathcal{T} = 100$. Let $x_i = [\psi_i, \omega_i]^{\mathrm{T}}$ and choose $\delta_i = u_i - \psi_i - 396\omega_i$; then the model (42) can be rewritten as (2) with

$$A = \begin{bmatrix}0 & 1\\ -0.0025 & -0.1\end{bmatrix}, B = \begin{bmatrix}0\\ 0.0025\end{bmatrix} \tag{44}$$

Let the input be $r = 0.5\sin(0.1t)$ and none of the followers has access to the input of the leader. The information exchange topology is given by Fig. 1, with the node 0 being the leader. In simulation, the distributed controller given in Theorem 1 is adopted. Letting $Q = \mathrm{diag}(0.1, 0.1)$ and solving the Riccati equation (6) yields

$$P = \begin{bmatrix}1.9787 & 19.5235\\ 19.5235 & 382.3346\end{bmatrix}, K = \begin{bmatrix}-0.0488 & -0.9558\end{bmatrix} \tag{45}$$

and the other control parameters are chosen as $\kappa_{i1} = 0.998$, $\kappa_2 = 100$.

Simulation results are shown in Figs. 2 and 3. Fig. 2 shows that the state of each agent synchronizes with the leader. Fig. 3 indicates that unknown control input of the leader can be identified accurately.

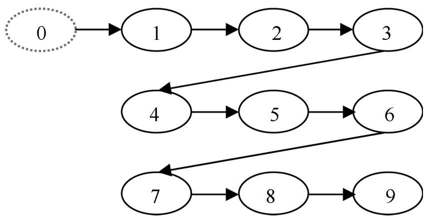

Fig. 1 Communication graph

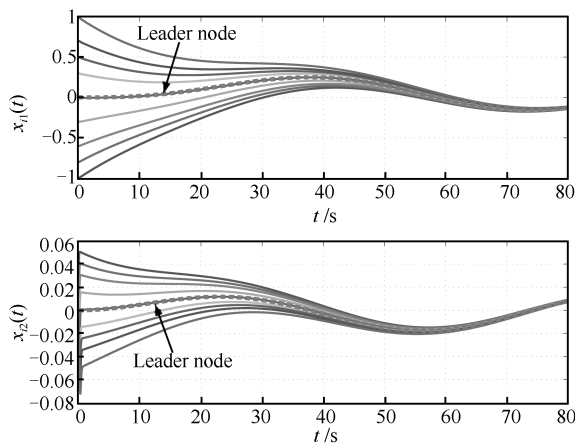

Fig. 2 Follower node states (solid line), leader node state (dotted line)

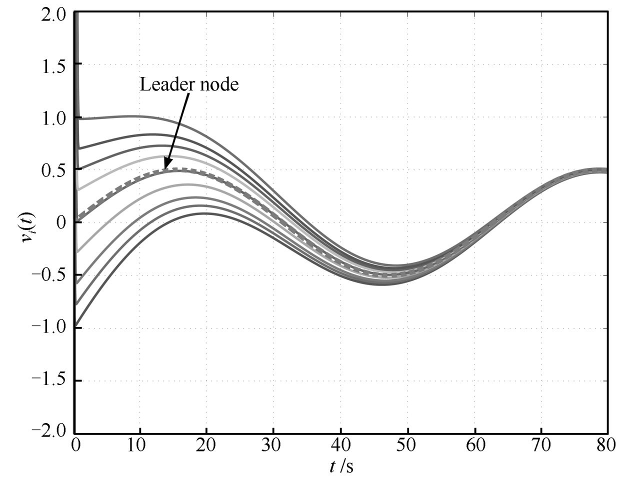

Fig. 3 Leader input estimation (solid line), leader input (dotted line)

In order to verify the performance of the proposed output feedback control, the controllers given in Theorem 2 are applied to the network. The feedback gain matrix F is designed as $F = [20, 99]^{\mathrm{T}}$ such that $\lambda(A_e) = \{-10, -10\}$.

Simulation results are shown in Figs. 4 and 5. Fig. 4 shows that the state of each agent synchronizes to the leader, despite the fact that only the output information ψ_i can be obtained. Fig. 5 demonstrates the state estimate errors approach zero.

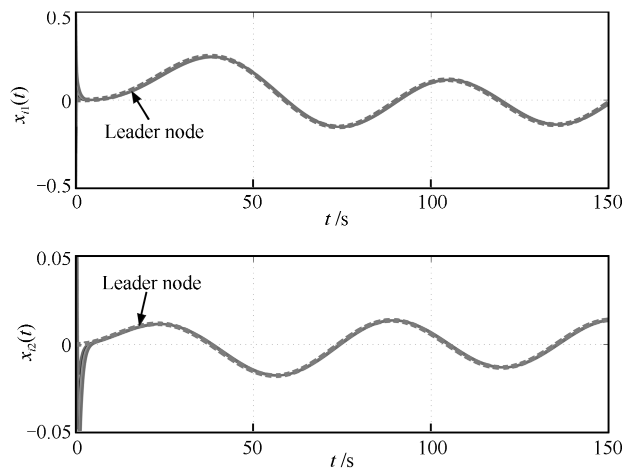

Fig. 4 Follower node states (solid line), leader node state (dotted line)

5 Conclusions

This paper has considered the cooperative tracking of linear multi-agent systems with a dynamic leader under directed communication topologies under the condition that the input of the leader is unavailable to any followers. At first, cooperative iterative learning controllers have been proposed based on the relative state information of the neighboring agents. Then, this result was further extended to the output feedback case where only partial states can be measured. Cooperative output feedback iterative learning controllers were devised based on a local observer. For both cases, the stability of the overall multi-agent systems has been established using Lyapunov-Krasovskii functionals. Compared with existing results, the main advantage of the proposed control designs allow for tracking the dynamic leader without the knowledge of the input, and meanwhile identifying the unknown input of the leader using distributed iterative learning laws. Moreover, the proposed iterative learning control laws are easier to implement in digital processors due to the fact that algebraic equations are used for identification of leader dynamics. An example has demonstrated the efficacy of the proposed approaches. Further works include an extension to the cooperative iterative learning control of nonlinear multi-agent systems.

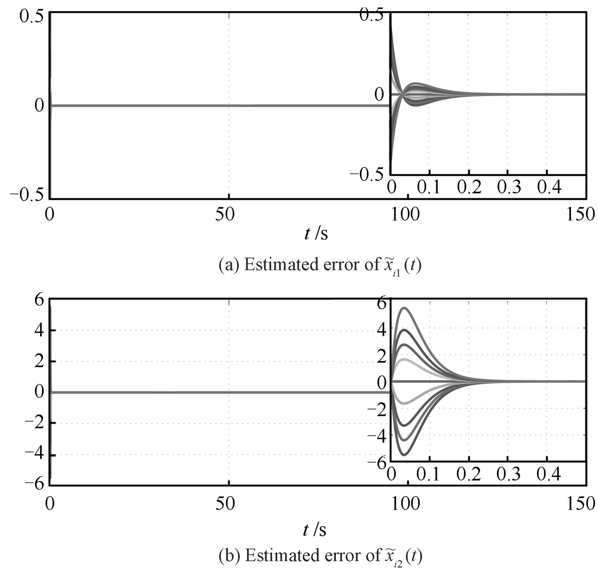

Fig. 5 Estimated errors of the states

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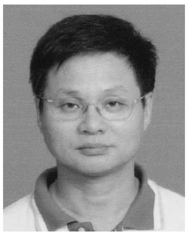

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