属性权重不确定条件下的区间直觉模糊多属性决策

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摘 要 在区间直觉模糊集 (Interval-valued intuitionistic fuzzy set, IVIFS) 的框架内, 重点研究了属性权重在一定约束条件下和属性权重完全未知的多属性群决策问题. 首先利用区间直觉模糊集成算子获得方案在属性上的综合区间直觉模糊决策矩阵, 进一步依据逼近理想解排序法 (Technique for order preference by similarity to an ideal solution, TOPSIS) 的思想计算候选方案和理想方案的加权距离, 最后确定方案排序. 其中针对属性权重在一定约束条件下的决策问题, 提出了基于区间直觉模糊集精确度函数的线性规划方法, 用以解决属性权重求解问题. 针对属性权重完全未知的决策问题, 首先定义了区间直觉模糊熵, 其次通过熵衡量每一属性所含的信息量来求解属性权重. 实验结果验证了决策方法的有效性和可行性.

关键词 不确定性, 区间直觉模糊集, 多属性决策, 线性规划, 熵

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Multi-attribute Decision Making with Uncertain Attribute Weight Information in the Framework of Interval-valued Intuitionistic Fuzzy Set

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Abstract In this paper, we investigate the multi-attribute group decision making problems with binding conditions of attribute weight information and unknown attribute weights in the framework of interval-valued intuitionistic fuzzy set (IVIFS). Firstly, a collective interval-valued intuitionistic fuzzy (IVIF) decision making matrix is determined by integrating all the decision making matrices derived from every decision makers. Secondly, we obtain the distance degree values between every alternative and the ideal-positive alternative depending on the technique for order preference by similarity to an ideal solution (TOPSIS) method. Finally, the ranking order of all the alternatives is determined through the obtained distance degree values. On one hand, a linear-programming method based on an accuracy function of IVIFS is proposed to calculate the attribute weights aiming at the decision making problem with binding attribute weight conditions. On the other hand, we propose a new definition of IVIF entropy, and choose attribute weights according to the information quantity of every alternative depending on IVIF entropy aiming at the decision making problem with completely unknown attribute weight information. The simulation shows the validity and feasibility of the proposed decision making methods.

Key words Uncertainty, interval-valued intuitionistic fuzzy set (IVIFS), multi-attribute decision making, linear-programming, entropy

自从 Zadeh[1] 于 1965 年引入模糊集理论以来, 一些专家学者在 Zadeh 工作的基础上, 相继引入众多的高阶模糊集[1−3] 来解决不确定性问题. 传统的模糊集因其不能完整地刻画所研究问题的全部信息受到一定的制约. Atanassov[2] 在 Zadeh 的模糊集的基础上, 引入了直觉模糊集, 直觉模糊集同时考虑了隶属度、非隶属度和未知度三方面的信息; Atanassov 等[3] 在直觉模糊集和区间模糊集的基础上, 进一步引入了区间直觉模糊集 (Interval-valued intuitionistic fuzzy set, IVIFS), 将隶属度、非隶属度以及未知度表示为 $[0,1]$ 区间的闭子区间, 有效地拓展了直觉模糊集处理不确定性信息的能力. 随着区间直觉模糊集的引入, 有关该理论的研究已受到国内外相关领域学者的极大关注[4−45], 并且被应用于决策、机器学习、模式识别、图像处理、医疗诊断和市场预测等诸多领域[7−42]. 随着区间直觉模糊集理论的深入研究, 直觉模糊集和区间直觉模糊集已经被成功应用于解决模糊多属性决策问题. 由于决策过程中知识的缺乏或者决策者的主观判断而导致的不确定的属性信息[7−9, 11−16, 18−42], 不仅包含方案在属性上的信任度和非信任度, 也包含了对属性的未知度. 因此针对决策过程中的这类不确定的属性信息, 可采用直觉模糊集或者区间直觉模糊集来刻画这种不确定性. 在直觉模糊集和区间直觉模糊集的框架内, 徐泽水[40]、Ye[41]、Nayagam 等[15] 和 Wang 等[18] 讨论了直觉模糊集和区间直觉模糊

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集的排序问题, 并给出了这些排序方法在多属性决策中的应用. 针对直觉模糊集和区间直觉模糊集集成问题, Xu 等[19, 21−22, 32] 引入了各类集成算子, 并讨论了这些算子在多属性决策中的应用. Chen 等[7]、Chen 等[9] 和 Xia 等[20] 提出各类直觉模糊熵、区间直觉模糊熵和交叉熵, 并用这些理论解决一些带有偏好的模糊多属性决策问题. Park 等[16] 和 Xu[38] 基于区间直觉模糊集的关联系数和距离测度提出几类决策方法. Li[13−14] 和 Xu 等[26] 基于数学规划的方法解决了属性权重部分已知的区间直觉模糊多属性决策问题. 目前的研究趋势表明, 如何解决属性权重部分已知和属性权重完全未知的区间直觉模糊多属性决策问题是当前研究的焦点和难点[13−14, 16, 18, 20−36].

本文重点研究了属性权重部分已知和属性权重完全未知的区间直觉模糊多属性决策问题, 并引入了区间直觉模糊熵的定义和相关的计算方法. 其中针对属性权重在一定约束条件下的决策问题, 提出了基于精确度函数的线性规划决策方法; 针对属性权重完全未知的决策问题, 提出了基于区间直觉模糊熵的决策方法. 最后通过相关的实验来验证方法的可行性和有效性.

1 预备知识

1.1 区间直觉模糊集及其相关运算法则

定义 1[3]**.** 设 X 为一非空集合, $\mathrm{Int}[0,1]$ 表示 $[0,1]$ 区间上所有闭子集的集合. 称

$$A=\{(x,\mu_A(x),\nu_A(x))|x\in X\} \tag{1}$$

为 X 上的区间直觉模糊集. 其中

$$\mu_A: X\to \mathrm{Int}[0,1],\ \ \nu_A: X\to \mathrm{Int}[0,1] \tag{2}$$

满足条件

$$0\le \sup(\mu_A(x))+\sup(\nu_A(x))\le 1,\ \forall x\in X \tag{3}$$

其中, $\mu_A(x)$ 和 $\nu_A(x)$ 分别表示 X 中元素属于 A 的隶属度和非隶属度. X 中元素属于 A 的不确定度或者未知度表示为

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x) \tag{4}$$

令 $\mu_{A_L}(x)=\inf(\mu_A(x)), \nu_{A_L}(x)=\inf(\nu_A(x))$, $\mu_{A_U}(x)=\sup(\mu_A(x))$, $\nu_{A_U}(x)=\sup(\nu_A(x))$, 则

$$\begin{aligned}A=\ &\{(x,[\mu_{A_L}(x),\mu_{A_U}(x)],\\ &[\nu_{A_L}(x),\nu_{A_U}(x)])|x\in X\}\end{aligned} \tag{5}$$

$$\pi_A(x)=[1-\mu_{A_U}(x)-\nu_{A_U}(x),1-\mu_{A_L}(x)-\nu_{A_L}(x)] \tag{6}$$

如果 $\mu_{A_L}(x)=\mu_{A_U}(x)$ 且 $\nu_{A_L}(x)=\nu_{A_U}(x)$, 那么区间直觉模糊集 A 就退化为一直觉模糊集.

设 $A=\{(x,\mu_A(x),\nu_A(x))|x\in X\}$ 和 $B=\{(x,\mu_B(x),\nu_B(x))|x\in X\}$ 是 X 上的区间直觉模糊集, 则有如下的运算法则[3, 4, 40]:

1) $A^c=\{(x,\nu_A(x),\mu_A(x))|x\in X\}$;

2) $A\preceq B$, 当且仅当 $\mu_{A_L}(x)\le\mu_{B_L}(x)$, $\mu_{A_U}(x)\le\mu_{B_U}(x)$, $\nu_{A_L}(x)\le\nu_{B_L}(x)$, $\nu_{A_U}(x)\le\nu_{B_U}(x)$.

设 $\alpha=([a_1,b_1],[c_1,d_1])$ 和 $\beta=([a_2,b_2],[c_2,d_2])$ 是两区间直觉模糊数, 则有如下运算法则[40]:

$$\alpha\otimes\beta=([a_1a_2,b_1b_2],[c_1+c_2-c_1c_2,d_1+d_2-d_1d_2]) \tag{7}$$

$$\alpha^\varepsilon=([a_1^\varepsilon,b_1^\varepsilon],[1-(1-c_1)^\varepsilon,1-(1-d_1)^\varepsilon])\ \varepsilon>0 \tag{8}$$

1.2 区间直觉模糊集的精确度函数

在模糊多属性决策中, 模糊数的排序是其中的一个关键点. 针对直觉模糊数和区间直觉模糊数的排序问题, Chen 等[8] 首先定义了得分函数, Hong 等[11] 在得分函数的基础上定义了精确度函数. Ye[41] 提出一种区间直觉模糊集的精确度函数. Xu 等[39−40] 将得分函数和精确度函数推广到区间直觉模糊集理论中. 在 Xu 工作的基础上, Nayagam 等[15] 提出一种新的精确度函数, 有效地克服了得分函数和 Ye 提出的精确度函数的缺陷. Wang 等[18] 进一步拓展了 Xu 的工作, 完全解决了区间直觉模糊数的排序问题. 针对属性一定约束条件的区间直觉模糊多属性决策问题, 利用文献 [18] 的排序规则构造决策矩阵的排序矩阵仅仅采用了得分函数, 因此本文采用 Nayagam 等引入的精确度函数构造优化模型.

定义 2[15]**.** 设 $\xi=([a,b],[c,d])$ 是一区间直觉模糊数, ξ 的精确度函数 h 为

$$h(\xi)=\frac{a+b-d(1-b)-c(1-a)}{2} \tag{9}$$

其中, $h(\xi)\in[-1,1]$.

1.3 区间直觉模糊熵

Zadeh 引入了模糊集, 并给出了模糊熵的定义, 模糊熵有效地刻画了模糊集的信息量[1]. 在 Zadeh 工作的基础上, Burillo 等[5] 提出了直觉模糊熵和区间模糊熵, 并分析了两者之间的关系. Chen 等[9] 归纳了各类直觉模糊熵, 并讨论了直觉模糊熵在多属性决策中的应用. 本文在 Burillo 等工作的基础上, 定义了区间直觉模糊熵.

令 IVIFS(X) 表示 X 上所有区间直觉模糊集的集合. 设 $A \in$ IVIFS(X), 引入如下算子:

$$F_p(A) = \{(x, \mu_{A_L}(x) + p\Delta\mu_A(x), \nu_{A_L}(x) + p\Delta\nu_A(x))|x \in X\} \tag{10}$$

其中, $\Delta\mu_A(x) = \mu_{A_U}(x) - \mu_{A_L}(x)$, $\Delta\nu_A(x) = \nu_{A_U}(x) - \nu_{A_L}(x)$, $p \in (0,1)$. 显然, $F_p(A)$ 是 X 上的直觉模糊集. 在此基础上, 定义 A 的区间直觉模糊熵 $I(A)$:

$$I(A) = 1 - (\bar{\mu}_A(x) + \bar{\nu}_A(x)) \times \sin\left(\frac{\bar{\mu}_A(x) + \bar{\nu}_A(x)}{2}\pi\right) \tag{11}$$

其中, $\bar{\mu}_A(x) = \mu_{A_L}(x) + p\Delta\mu_A(x)$, $\bar{\nu}_A(x) = \nu_{A_L}(x) + p\Delta\nu_A(x)$, $p \in (0,1)$.

区间直觉模糊熵 I 满足如下定理:

定理 1. 设 $A, B \in$ IVIFS(X), 则区间直觉模糊熵 I 满足如下 4 条性质:

1) $I(A) = 0$ 当且仅当 A 是模糊集;

2) $I(A) = 1$ 当且仅当 $\mu_A(x) = [0,0]$ 且 $\nu_A(x) = [0,0]$;

3) 对任意的 $A \in$ IVIFS(X), $I(A) = I(A^c)$;

4) 如果 $A \preceq B$, 那么 $I(A) \geq I(B)$.

证明. 1) $\Leftarrow$ 若 A 是模糊集, 即有 $\mu_{A_L}(x) = \mu_{A_U}(x)$, $\nu_{A_L}(x) = \nu_{A_U}(x)$, $\mu_{A_U}(x) + \nu_{A_U}(x) = 1$ 对 $\forall x \in X$, 依据式 (11), 可得 $I(A) = 0$.

$\Rightarrow I(A) = 0$ 等价于 $(\bar{\mu}_A(x) + \bar{\nu}_A(x))\sin(\frac{\bar{\mu}_A(x)+\bar{\nu}_A(x)}{2}\pi) = 1$ 对 $\forall p \in (0,1)$. 假设 A 不是模糊集, 即有 $\mu_{A_L}(x) + \nu_{A_L}(x) < 1$. 因为 $\mu_{A_U}(x) + \nu_{A_U}(x) \leq 1$, $\mu_{A_L}(x) + \nu_{A_L}(x) < 1$, 等价于 $\bar{\mu}_A(x) + \bar{\nu}_A(x) < 1$ 对 $\forall p \in (0,1)$. 显然可得 $(\bar{\mu}_A(x) + \bar{\nu}_A(x))\sin(\frac{\bar{\mu}_A(x)+\bar{\nu}_A(x)}{2}\pi) < 1$, 与条件 $(\bar{\mu}_A(x) + \bar{\nu}_A(x))\sin(\frac{\bar{\mu}_A(x)+\bar{\nu}_A(x)}{2}\pi) = 1$ 相矛盾, 故假设不成立. 因此若 $I(A) = 0$, 则 A 是模糊集.

2) $\Leftarrow$ 显然成立.

$\Rightarrow I(A) = 1$ 等价于 $(\bar{\mu}_A(x) + \bar{\nu}_A(x))\sin(\frac{\bar{\mu}_A(x)+\bar{\nu}_A(x)}{2}\pi) = 0$ 对 $\forall p \in (0,1)$. 假设 $\mu_{A_U}(x) + \nu_{A_U}(x) > 0$, 可得 $\bar{\mu}_A(x) + \bar{\nu}_A(x) > 0$ 对 $\forall p \in (0,1)$. 显然可得 $(\bar{\mu}_A(x) + \bar{\nu}_A(x))\sin(\frac{\bar{\mu}_A(x)+\bar{\nu}_A(x)}{2}\pi) > 0$, 与条件 $(\bar{\mu}_A(x) + \bar{\nu}_A(x))\sin(\frac{\bar{\mu}_A(x)+\bar{\nu}_A(x)}{2}\pi) = 0$ 相矛盾, 故假设不成立. 因此若 $\mu_{A_U}(x) + \nu_{A_U}(x) = 0$, 则有 $\mu_A(x) = [0,0]$ 及 $\nu_A(x) = [0,0]$.

3) 因为 $A^c = \{(x, \nu_A(x), \mu_A(x))|x \in X\}$, $I(A) = I(A^c)$ 显然成立.

4) 若 $A \preceq B$, 则 $\mu_{A_L}(x) \leq \mu_{B_L}(x)$, $\mu_{A_U}(x) \leq \mu_{B_U}(x)$, $\nu_{A_L}(x) \leq \nu_{B_L}(x)$, $\nu_{A_U}(x) \leq \nu_{B_U}(x)$, 可得 $(1-p)(\mu_{B_L}(x) - \mu_{A_L}(x)) + p(\mu_{B_U}(x) - \mu_{A_U}(x)) + (1-p)(\nu_{B_L}(x) - \nu_{A_L}(x)) + p(\nu_{B_U}(x) - \nu_{A_U}(x)) \geq 0$ 对 $\forall p \in [0,1]$, 进一步可得, $\mu_{A_L}(x) + p(\mu_{A_U}(x) - \mu_{A_L}(x)) + \nu_{A_L}(x) + p(\nu_{A_U}(x) - \nu_{A_L}(x)) \leq \mu_{B_L}(x) + p(\mu_{B_U}(x) - \mu_{B_L}(x)) + \nu_{B_L}(x) + p(\nu_{B_U}(x) - \nu_{B_L}(x))$, 即 $\bar{\mu}_A(x) + \bar{\nu}_A(x) \leq \bar{\mu}_B(x) + \bar{\nu}_B(x)$. 因为 $0 \leq \bar{\mu}_A(x) + \bar{\nu}_A(x) \leq \bar{\mu}_B(x) + \bar{\nu}_B(x) \leq 1$, 可得 $(\bar{\mu}_A(x) + \bar{\nu}_A(x))\sin(\frac{\bar{\mu}_A(x)+\bar{\nu}_A(x)}{2}\pi) \leq (\bar{\mu}_B(x) + \bar{\nu}_B(x))\sin(\frac{\bar{\mu}_B(x)+\bar{\nu}_B(x)}{2}\pi)$. 上式等价于 $I(A) \geq I(B)$. □

1.4 区间直觉模糊集成算子

本文采用区间直觉模糊混合几何 (Interval-valued intuitionistic fuzzy hybrid geometrical, IIFHG) 算子[21, 40], 定义如下:

定义 3[21, 40]**.** 令 Θ 表示全体区间直觉模糊数的集合, 设 $\alpha_i\,(i = 1,2,\cdots,n)$ 为一组 Θ 上的区间直觉模糊数, IIFHG : $\Theta^n \rightarrow \Theta$ 定义如下:

$$\text{IIFHG}_{\boldsymbol{\lambda},\,\boldsymbol{\omega}}(\alpha_1, \alpha_2, \cdots, \alpha_n) = \tilde{\alpha}_{\sigma(1)}^{\omega_1} \otimes \tilde{\alpha}_{\sigma(2)}^{\omega_2} \otimes \cdots \otimes \tilde{\alpha}_{\sigma(n)}^{\omega_n} \tag{12}$$

其中, $\boldsymbol{\omega} = [\omega_1, \omega_2, \cdots, \omega_n]^{\rm T}$ 是与函数 IIFHG 相关联的位置向量 (指数加权向量), $0 \leq \omega_i \leq 1\,(i = 1,2,\cdots,n)$ 且 $\sum_{i=1}^n \omega_i = 1$. $\tilde{\alpha}_{\sigma(i)}$ 是指加权的区间直觉模糊数 $\tilde{\alpha}_j\,(j = 1,2,\cdots,n)$ 中第 i 个最大的元素, 其中 $\tilde{\alpha}_j = \alpha_j^{n\lambda_j}\,(j = 1,2,\cdots,n)$, $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_n]^{\rm T}$ 是区间直觉模糊数 $\alpha_i\,(i = 1,2,\cdots,n)$ 的权重向量, $0 \leq \lambda_j \leq 1\,(j = 1,2,\cdots,n)$ 且 $\sum_{j=1}^n \lambda_j = 1$, n 表示平衡因子.

2 区间直觉模糊多属性群决策研究

对某一多属性群决策问题, 设 $D = \{d_1, d_2, \cdots, d_n\}$ 为决策者集合, 其中决策者的权重为 $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots, \lambda_n]^{\rm T}$, 满足 $\lambda_i \geq 0\,(i = 1,2,\cdots,n)$ 及 $\sum_{i=1}^n \lambda_i = 1$; $X = \{X_1, X_2, \cdots, X_k\}$ 为属性集; $A = \{A_1, A_2, \cdots, A_m\}$ 为方案集; 决策者 $d_i\,(i = 1,2,\cdots,n)$ 关于方案 A 在属性集 X 上的决策矩阵为

$$\bar{D}_i = \begin{bmatrix} \gamma_{11}^i & \gamma_{12}^i & \cdots & \gamma_{1k}^i \\ \gamma_{21}^i & \gamma_{22}^i & \cdots & \gamma_{2k}^i \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{m1}^i & \gamma_{m2}^i & \cdots & \gamma_{mk}^i \end{bmatrix} \tag{13}$$

其中, $\gamma_{jl}^i = ([a_{jl}^i, b_{jl}^i], [c_{jl}^i, d_{jl}^i])\,(i = 1,2,\cdots,n;\ j = 1,2,\cdots,m;\ l = 1,2,\cdots,k)$ 为区间直觉模糊数.

由于决策过程中存在主观或者客观的不确定性, 在实际决策问题中, 属性权重信息往往是部分已知

或者完全未知的[7, 12, 34−35, 40, 46−48]. 而在区间直觉模糊多属性决策问题中, 如何根据综合决策矩阵以及部分已知或者完全未知的属性权重信息获取属性权重成为目前研究的热点和难点. 接下来着重解决属性权重部分已知以及属性权重完全未知的区间直觉模糊多属性群决策问题.

2.1 属性权重信息部分已知的区间直觉模糊多属性群决策

Kim 等[12]、Xu 等[34−35] 以及 Wang 等[48] 指出在很多决策问题中, 由于决策者的差异, 属性权重信息不能完全确定, 其中的一类情形就是根据多个决策者的意见给出属性权重信息的约束条件. 针对带有属性权重约束条件的直觉模糊多属性决策问题, Xu 等[37, 40] 利用直觉模糊集的得分函数给出一种优化模型来选取权重. 在 Xu 等工作的基础上, 本节着重解决属性权重在一定约束条件下的区间直觉模糊多属性决策问题的权重求解.

设 $\bar{D} = [\gamma_{jl}]_{m\times k}$ 为综合决策矩阵, 为了便于描述, 令 $\gamma_{jl} = ([a_{jl}, b_{jl}], [c_{jl}, d_{jl}]), j = 1, 2, \cdots, m; l = 1, 2, \cdots, k$, 其中 γ_{jl} 表示区间直觉模糊数. 基于定义 2, 引入决策矩阵 $\bar{D}$ 的精确度矩阵, 可得定义如下:

定义 4. 设 $\bar{D}_{m\times k}$ 为综合决策矩阵. 称 $H = [h_{jl}]_{m\times k}$ 为 $\bar{D}_{m\times k}$ 的精确度矩阵, h_{jl} 满足

$$h_{jl} = \frac{a_{jl} + b_{jl} - d_{jl}(1 - b_{jl}) - c_{jl}(1 - a_{jl})}{2} \tag{14}$$

其中, $h_{jl}\,(j = 1, 2, \cdots, m; l = 1, 2, \cdots, k)$ 称 γ_{jl} 的精确度值.

根据精确度矩阵, 每个方案 $A_j\,(j = 1, 2, \cdots, m)$ 的综合精确值为

$$h_j(\boldsymbol{w}) = \sum_{l=1}^{k} w_l h_{jl} \tag{15}$$

其中, $\boldsymbol{w} = [w_1, w_2, \cdots, w_k]^{\rm T}$ 表示属性权重, 且 $\sum_{l=1}^{k} w_l = 1\,(w_l \geq 0)$. $h_j(\boldsymbol{w})$ 越大, 则表示方案 A_j 越好. 假设属性权重 $\boldsymbol{w}$ 的约束条件为 Λ. 如果仅仅考虑单个方案 $A_j\,(j = 1, 2, \cdots, m)$, 则属性向量 $\boldsymbol{w}$ 应使得 $h_j(\boldsymbol{w})$ 最大化, 可建立优化模型如下[37, 40]:

$$\begin{aligned} \max\ \ & h_j(\boldsymbol{w}) = \sum_{l=1}^{k} w_l h_{jl} \\ \text{s.t.}\ \ & \boldsymbol{w} \in \Lambda,\ w_l \geq 0\,(l = 1, 2, \cdots, k) \\ & \sum_{l=1}^{k} w_l = 1 \end{aligned}$$

依据此优化模型, 可得方案 $A_j\,(j = 1, 2, \cdots, m)$ 的最优权重

$$\boldsymbol{w}^{(j)} = [w_1^{(j)}, w_2^{(j)}, \cdots, w_k^{(j)}]^{\rm T} \tag{16}$$

但是在求解过程中, 需要对所有方案 $A = \{A_1, A_2, \cdots, A_m\}$ 进行综合考虑, 令 $W = [\boldsymbol{w}^{(1)}, \boldsymbol{w}^{(2)}, \cdots, \boldsymbol{w}^{(m)}]_{k\times m}$, 即

$$W = \begin{bmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(m)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(m)} \\ \vdots & \vdots & \ddots & \vdots \\ w_k^{(1)} & w_k^{(2)} & \cdots & w_k^{(m)} \end{bmatrix} \tag{17}$$

令 $\Gamma = (HW)^{\rm T}(HW)$, 得到 Γ 的归一化特征向量 $\boldsymbol{\rho} = [\rho_1, \rho_2, \cdots, \rho_m]^{\rm T}$. 最后计算属性权重

$$\boldsymbol{w} = W\boldsymbol{\rho} = \rho_1 w^{(1)} + \rho_2 w^{(2)} + \cdots + \rho_m w^{(m)} \tag{18}$$

满足 $\sum_{l=1}^{k} w_l = 1$ 及 $w_l \geq 0$.

2.2 属性权重完全未知的区间直觉模糊多属性决策

Wang 等[48] 对模糊多属性决策问题进行了系统的分析, Chen 等[7] 和 Xu 等[40] 讨论了属性权重完全未知的直觉模糊多属性决策问题, 并提出了基于熵的决策方法和投影决策方法. Chen 等[7] 分析了各类直觉模糊熵, 并利用熵求解直觉模糊多属性决策问题的属性权重. 在 Chen 等工作的基础上, 针对属性权重完全未知的区间直觉模糊多属性决策问题, 本节提出了基于区间直觉模糊熵的属性权重求解方法. 首先引入区间直觉模糊决策矩阵的熵矩阵.

定义 5. 设 $\bar{D}_{m\times k}$ 是一区间直觉模糊综合决策矩阵. $I = [I_{jl}]_{m\times k}$ 称之为 $\bar{D}_{m\times k}$ 的区间直觉模糊熵矩阵, $[I_{jl}]_{m\times k}$ 满足

$$I_{jl} = 1 - \Delta \sin\left(\frac{\Delta}{2}\pi\right) \tag{19}$$

其中, $\Delta = a_{jl} + p(b_{jl} - a_{jl}) + c_{jl} + p(d_{jl} - c_{jl})$, $p \in (0, 1)$.

接下来, 给出属性权重的求解过程:

步骤 1. 计算区间直觉模糊综合决策矩阵 $\bar{D}_{m\times k}$ 的熵矩阵 I:

$$I = \begin{bmatrix} I_{11} & I_{12} & \cdots & I_{1k} \\ I_{21} & I_{22} & \cdots & I_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ I_{m1} & I_{m2} & \cdots & I_{mk} \end{bmatrix} \tag{20}$$

步骤 2. 归一化熵矩阵 I:

$$I = \begin{bmatrix} \bar{I}_{11} & \bar{I}_{12} & \cdots & \bar{I}_{1k} \\ \bar{I}_{21} & \bar{I}_{22} & \cdots & \bar{I}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{I}_{m1} & \bar{I}_{m2} & \cdots & \bar{I}_{mk} \end{bmatrix} \tag{21}$$

其中, $\bar{I}_{jl}=\frac{I_{jl}}{\max\{I_{1l},I_{2l},\cdots,I_{ml}\}}$ $(j=1,2,\cdots,m;\ l=1,2,\cdots,k)$.

步骤 3. 计算属性权重 w_l $(l=1,2,\cdots,k)$:

$$w_l=\frac{1-\theta_l}{k-Z} \tag{22}$$

其中, $\theta_l=\sum_{t=1}^{m}\bar{I}_{tl}\,(l=1,2,\cdots,k)$, $Z=\sum_{e=1}^{k}\theta_e$.

2.3 区间直觉模糊群决策方法

本节基于 TOPSIS 的思想[36, 40], 提出区间直觉模糊群决策方法, 决策过程如下:

步骤 1. 利用 IIFHG 算子集成决策矩阵 $\bar{D}_i\,(i=1,2,\cdots,n)$, 获得综合决策矩阵 $\bar{D}=[\gamma_{jl}]_{m\times k}$, 其中 $\gamma_{jl}=([a_{jl},b_{jl}],[c_{jl},d_{jl}])\,(j=1,2,\cdots,m;\ l=1,2,\cdots,k)$.

步骤 2. 计算属性权重 $\boldsymbol{w}$.

步骤 3. 确定区间直觉模糊理想方案 $A^+=\{\alpha_1,\alpha_2,\cdots,\alpha_k\}$, 其中 $\alpha_l=([1,1],[0,0])\,(l=1,2,\cdots,k)$.

步骤 4. 计算候选方案 $A_j\,(j=1,2,\cdots,m)$ 和区间直觉模糊理想方案 A^+ 的加权距离:

$$d(A_j,A^+)=\Big[\frac{1}{4}\sum_{l=1}^{k}w_l((a_{jl}-1)^2+(b_{jl}-1)^2+(c_{jl}-0)^2+(d_{jl}-0)^2)\Big]^{\frac{1}{2}} \tag{23}$$

步骤 5. 依据距离测度值 $d(A_j,A^+)\,(j=1,2,\cdots,m)$, 对所有方案进行升序排列并确立最优方案.

显然, 距离 $d(A_j,A^+)$ 越小, 则方案 A_j 越接近区间直觉模糊理想方案 A^+.

3 实验与分析

本节采用两个区间直觉模糊环境下的多属性决策案例来验证本文提出的多属性决策方法.

例 1[16]**.** 在某一供应链选择问题中, 有 5 项考核指标 (属性): x_1 (价格指标)、x_2 (质量指标)、x_3 (服务指标)、x_4 (供应方指标) 和 x_5 (风险系数指标). 在这些指标下有 4 种备选方案 A_1, A_2, A_3, A_4. 为了决策的民主性和权威性, 包含 4 位决策专家, 记为 d_1, d_2, d_3, d_4, 4 位专家的权重 $\boldsymbol{\lambda}=[0.3,\ 0.2,\ 0.2,\ 0.2]^{\mathrm{T}}$. IIFHG 的位置权重向量为 $\boldsymbol{\omega}=[0.155,\ 0.345,\ 0.345,\ 0.155]^{\mathrm{T}}$. 4 位决策者根据各自的侧重点, 给出的综合属性约束条件为: $\Lambda=\{w_1\le 0.3, 0.2\le w_3\le 0.5, 0.1\le w_2\le 0.2, w_5\le 0.4, w_3-w_2\ge w_5-w_4, w_4\ge w_1, w_3-w_1\le 0.1, 0.1\le w_4\le 0.3\}$. 决策者 d_i $(i=1,2,3,4)$ 关于方案 A_j $(j=1,2,3,4)$ 在属性 x_l $(l=1,2,3,4,5)$ 上的决策矩阵 $\bar{D}_i\,(i=1,2,3,4)$ 见表 1 ∼ 表 4.

步骤 1. 利用 $\mathrm{IIFHG}_{\boldsymbol{\lambda},\boldsymbol{\omega}}$ 计算综合决策矩阵 $\bar{D}$, 见表 5 和表 6.

步骤 2. 计算属性权重, 过程如下. 计算决策矩阵 $\bar{D}$ 的精确度矩阵 H:

$$H=\begin{bmatrix}0.4316 & 0.1317 & 0.6105 & 0.4990 & -0.0901\\ 0.1920 & -0.0707 & 0.1230 & -0.1970 & 0.6504\\ 0.2185 & 0.6617 & 0.6045 & 0.4073 & 0.5049\\ 0.1071 & -0.4186 & -0.3716 & -0.0927 & -0.1670\end{bmatrix} \tag{24}$$

依据第 2.1 节中的属性权重求解算法, 可得

$$W=\begin{bmatrix}0.2667 & 0.1600 & 0.1000 & 0.3000\\ 0.1000 & 0.1000 & 0.2000 & 0.1000\\ 0.3667 & 0.2600 & 0.2000 & 0.2000\\ 0.2667 & 0.1600 & 0.2500 & 0.3000\\ 0.0000 & 0.3200 & 0.2500 & 0.1000\end{bmatrix} \tag{25}$$

$$\Gamma=\begin{bmatrix}0.4740 & 0.4038 & 0.4136 & 0.4155\\ 0.4038 & 0.4104 & 0.4033 & 0.3679\\ 0.4136 & 0.4033 & 0.4050 & 0.3720\\ 0.4155 & 0.3679 & 0.3720 & 0.3684\end{bmatrix} \tag{26}$$

进一步计算 Γ 的归一化特征向量 $\boldsymbol{\rho}$:

$$\boldsymbol{\rho}=[0.0003,\ 0.0023,\ 0.0291,\ 0.9683]^{\mathrm{T}} \tag{27}$$

最后得到属性权重 $\boldsymbol{w}$:

$$\boldsymbol{w}=[0.2939,\ 0.1029,\ 0.2002,\ 0.2982,\ 0.1048]^{\mathrm{T}} \tag{28}$$

步骤 3. 确定区间直觉模糊理想方案 $A^+=\{\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5\}$, 其中 $\alpha_l=([1,1],[0,0])\,(l=1,2,3,4,5)$.

步骤 4. 计算候选方案 $A_j\,(j=1,2,3,4)$ 和理想方案 A^+ 的距离测度, 可得 $d(A_1,A^+)=0.4420$, $d(A_2,A^+)=0.5486$, $d(A_3,A^+)=0.4041$, $d(A_4,A^+)=0.6432$.

步骤 5. 因为 $d(A_4,A^+)>d(A_2,A^+)>d(A_1,A^+)>d(A_3,A^+)$, 则所有方案排序为 $A_3\succ A_1\succ A_2\succ A_4$, 最佳方案为 A_3.

对例 1, 采用文献 [16] 的决策算法, 方案排序为 $A_3\succ A_1\succ A_2\succ A_4$, 最佳方案为 A_3. 结果说明了本文提出决策方法是有效的.

例 2[16]**.** 采用例 1 中的供应链选择问题, 假设属性权重信息完全未知, 决策过程如下:

表 1　区间直觉模糊决策矩阵 $\bar{D}_1$

Table 1　IVIF decision matrix $\bar{D}_1$

	x_1	x_2	x_3	x_4	x_5
A_1	([0.5, 0.6], [0.2, 0.3])	([0.3, 0.5], [0.4, 0.5])	([0.6, 0.7], [0.2, 0.3])	([0.5, 0.7], [0.1, 0.2])	([0.1, 0.4], [0.3, 0.5])
A_2	([0.3, 0.4], [0.4, 0.6])	([0.1, 0.3], [0.2, 0.4])	([0.3, 0.4], [0.4, 0.5])	([0.2, 0.4], [0.5, 0.6])	([0.7, 0.8], [0.1, 0.2])
A_3	([0.4, 0.5], [0.3, 0.5])	([0.7, 0.8], [0.1, 0.2])	([0.5, 0.8], [0.1, 0.2])	([0.4, 0.6], [0.2, 0.3])	([0.5, 0.6], [0.2, 0.3])
A_4	([0.3, 0.5], [0.4, 0.5])	([0.1, 0.2], [0.7, 0.8])	([0.1, 0.2], [0.5, 0.8])	([0.2, 0.3], [0.4, 0.6])	([0.2, 0.3], [0.5, 0.6])

表 2　区间直觉模糊决策矩阵 $\bar{D}_2$

Table 2　IVIF decision matrix $\bar{D}_2$

	x_1	x_2	x_3	x_4	x_5
A_1	([0.4, 0.5], [0.2, 0.4])	([0.3, 0.4], [0.4, 0.6])	([0.6, 0.7], [0.1, 0.2])	([0.5, 0.6], [0.1, 0.3])	([0.1, 0.3], [0.3, 0.5])
A_2	([0.3, 0.5], [0.4, 0.5])	([0.1, 0.3], [0.3, 0.7])	([0.3, 0.4], [0.4, 0.5])	([0.2, 0.3], [0.6, 0.7])	([0.6, 0.8], [0.1, 0.2])
A_3	([0.4, 0.6], [0.3, 0.4])	([0.6, 0.8], [0.1, 0.2])	([0.7, 0.8], [0.1, 0.2])	([0.4, 0.6], [0.3, 0.4])	([0.5, 0.6], [0.2, 0.4])
A_4	([0.3, 0.4], [0.4, 0.6])	([0.1, 0.2], [0.6, 0.8])	([0.1, 0.2], [0.7, 0.8])	([0.3, 0.4], [0.4, 0.6])	([0.2, 0.4], [0.5, 0.6])

表 3　区间直觉模糊决策矩阵 $\bar{D}_3$

Table 3　IVIF decision matrix $\bar{D}_3$

	x_1	x_2	x_3	x_4	x_5
A_1	([0.4, 0.7], [0.1, 0.2])	([0.3, 0.5], [0.3, 0.4])	([0.6, 0.7], [0.1, 0.2])	([0.5, 0.6], [0.1, 0.3])	([0.3, 0.5], [0.4, 0.5])
A_2	([0.4, 0.5], [0.2, 0.4])	([0.2, 0.4], [0.4, 0.5])	([0.4, 0.5], [0.3, 0.4])	([0.1, 0.2], [0.7, 0.8])	([0.6, 0.7], [0.2, 0.3])
A_3	([0.2, 0.4], [0.3, 0.4])	([0.6, 0.8], [0.1, 0.2])	([0.5, 0.7], [0.1, 0.3])	([0.5, 0.7], [0.2, 0.3])	([0.6, 0.8], [0.1, 0.2])
A_4	([0.3, 0.4], [0.2, 0.4])	([0.1, 0.2], [0.6, 0.8])	([0.1, 0.3], [0.5, 0.7])	([0.2, 0.3], [0.5, 0.7])	([0.1, 0.2], [0.6, 0.8])

表 4　区间直觉模糊决策矩阵 $\bar{D}_4$

Table 4　IVIF decision matrix $\bar{D}_4$

	x_1	x_2	x_3	x_4	x_5
A_1	([0.6, 0.7], [0.2, 0.3])	([0.3, 0.4], [0.3, 0.4])	([0.7, 0.8], [0.1, 0.2])	([0.5, 0.6], [0.1, 0.3])	([0.1, 0.2], [0.5, 0.7])
A_2	([0.4, 0.5], [0.4, 0.5])	([0.1, 0.2], [0.2, 0.3])	([0.3, 0.4], [0.5, 0.6])	([0.2, 0.3], [0.4, 0.6])	([0.6, 0.7], [0.1, 0.2])
A_3	([0.4, 0.5], [0.3, 0.4])	([0.6, 0.7], [0.1, 0.3])	([0.5, 0.8], [0.1, 0.2])	([0.4, 0.5], [0.2, 0.3])	([0.5, 0.6], [0.3, 0.4])
A_4	([0.3, 0.4], [0.4, 0.5])	([0.1, 0.3], [0.6, 0.7])	([0.1, 0.2], [0.5, 0.8])	([0.2, 0.3], [0.4, 0.5])	([0.3, 0.4], [0.5, 0.6])

表 5　综合区间直觉模糊决策矩阵 $\bar{D}$

Table 5　Collective IVIF decision matrix $\bar{D}$

	x_1	x_2	x_3
A_1	([0.4385, 0.6199], [0.1549, 0.2848])	([0.3000, 0.4573], [0.3404, 0.4710])	([0.6116, 0.7117], [0.1089, 0.2083])
A_2	([0.3502, 0.4797], [0.3114, 0.4681])	([0.1138, 0.3010], [0.2511, 0.4773])	([0.3379, 0.4387], [0.3872, 0.4887])
A_3	([0.3516, 0.4906], [0.2940, 0.4214])	([0.6395, 0.7711], [0.0980, 0.2263])	([0.5213, 0.7804], [0.0980, 0.2083])
A_4	([0.3000, 0.4170], [0.3114, 0.4887])	([0.1000, 0.2103], [0.6012, 0.7678])	([0.1000, 0.2366], [0.5577, 0.7569])

表 6　综合区间直觉模糊决策矩阵 $\bar{D}$

Table 6　Collective IVIF decision matrix $\bar{D}$

	x_4	x_5
A_1	([0.5000, 0.6395], [0.0980, 0.2567])	([0.1323, 0.3623], [0.3747, 0.5482])
A_2	([0.1758, 0.3134], [0.5305, 0.6496])	([0.6395, 0.7521], [0.1089, 0.2083])
A_3	([0.4387, 0.6252], [0.2263, 0.3262])	([0.5452, 0.6502], [0.1770, 0.3005])
A_4	([0.2103, 0.3109], [0.4050, 0.5613])	([0.1849, 0.3121], [0.5031, 0.6118])

步骤 1. 利用 $\text{IIFHG}_{\boldsymbol{\lambda},\,\boldsymbol{\omega}}$ 计算综合决策矩阵 $\bar{D}$, 见表 5 和表 6.

步骤 2. 计算属性权重. 令 $p = 0.5$, 计算综合区间直觉模糊决策矩阵 $\bar{D}$ 的熵矩阵 I:

$$I = \begin{bmatrix} 0.3084 & 0.2602 & 0.2122 & 0.3111 & 0.3641 \\ 0.2329 & 0.5530 & 0.2043 & 0.1933 & 0.1678 \\ 0.2677 & 0.1513 & 0.2338 & 0.2282 & 0.1910 \\ 0.2954 & 0.1868 & 0.2052 & 0.3157 & 0.2312 \end{bmatrix} \tag{29}$$

归一化熵矩阵, 可得

$$I = \begin{bmatrix} 1.0000 & 0.4705 & 0.9076 & 0.9854 & 1.0000 \\ 0.7552 & 1.0000 & 0.8738 & 0.6123 & 0.4609 \\ 0.8680 & 0.2736 & 1.0000 & 0.7228 & 0.5246 \\ 0.9578 & 0.3378 & 0.8777 & 1.0000 & 0.6350 \end{bmatrix} \tag{30}$$

基于归一化的熵矩阵 I, 可得属性权重

$$\boldsymbol{w} = [0.2515,\ 0.1054,\ 0.2591,\ 0.2261,\ 0.1579]^{\mathrm{T}} \tag{31}$$

步骤 3. 确定区间直觉模糊理想方案 $A^+ = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5\}$, 其中 $\alpha_l = ([1,1],[0,0])$ $(l = 1,2,3,4,5)$.

步骤 4. 计算候选方案 $A_j\,(j = 1,2,3,4)$ 和理想方案 A^+ 的距离测度, 可得 $d(A_1, A^+) = 0.4461$, $d(A_2, A^+) = 0.5307$, $d(A_3, A^+) = 0.3873$, $d(A_4, A^+) = 0.6588$.

步骤 5. 因为 $d(A_4, A^+) > d(A_2, A^+) > d(A_1, A^+) > d(A_3, A^+)$, 则所有方案排序为 $A_3 \succ A_1 \succ A_2 \succ A_4$, 最佳方案为 A_3.

对例 1, 采用文献 [40] 的投影决策算法, 方案排序为 $A_3 \succ A_1 \succ A_2 \succ A_4$, 最佳方案为 A_3. 结果说明了本文提出决策方法的有效性和可行性.

4　结论

本文首先引入了区间直觉模糊熵的定义及计算方法, 其次提出了基于精确度函数的线性规划决策方法和基于区间直觉模糊熵的决策方法, 有效地解决了属性权重为一定约束条件的区间直觉模糊多属性决策问题和属性权重完全未知的区间直觉模糊多属性决策问题. 最后通过不确定条件下的供应链选择问题验证了本文提出的决策方法的有效性和可行性.

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