Cooperative Tracking and Disturbance Suppression: A Classical Approach

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Abstract In this paper, the cooperative tracking problem of multiagent systems with a determinate disturbance input is analyzed. The communication topology of single-input-single-output (SISO) general linear node dynamics is directed and time invariant. Throughout this paper, the distributed control issue of multiagent systems is viewed and tackled as an output regulation problem and a distributed cooperative control law based on relative output measurements is proposed using classic pole assignment technique. Moreover, the notion of complex vector root locus (CVRL) is introduced, which is a generalization of classical root locus method, to analyze the stability of the control system.

Key words Multiagent systems, cooperative control, complex vector root locus (CVRL), graph theory

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Distributed control of multiagent systems has received significant attention due to its broad applications. Research areas include rendezvous problems[1−2], flock-ing[3−6], formation control[7−10], consensus problems[11−19], and so on. The mathematical models for agent dynamics include single-integrator model[11−13], double-integrator model[14−15], high-order-integrator model[16], and so on.

Great progress in recent years has been made and various distributed strategies that achieve agreements have been proposed in the field of multiagent distributed control. Sorensen et al.[7] considered tracking control for multiagent consensus with single or multiple leaders. The authors of [12] and [13] investigated the consensus problem for directed networks of agents with switching topology or time-delays. Fast consensus algorithms for multiagent systems with interconnected topology are studied in [15] and [19], to name a few.

A closely related issue on consensus problems of multiagent systems is the study of graph Laplacian. Graph Laplacian and its spectral properties[20−21] are important graph-related matrix that plays an important role in convergence analysis of consensus algorithms. The theory of nonnegative matrices is quite useful in understanding the links between graph theory and consensus reaching[9]. A brief introduction on graph Laplacian is also included in the next section.

The purpose of this paper is to provide a new viewpoint for tackling consensus-related problems. In our framework, consensus is viewed and handled as a tracking control problem (or the same as an output regulation problem). The benefit for this kind of perspective is that it allows us to tackle consensus (cooperative tracking) problem as well as disturbance suppression in much the same way. Secondly, most existing literature on consensus problems are focused

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on reaching agreement or synchronization of agent behaviors, but from the angle of control theory, by cooperative tracking we mean that each agent in the system has the capability of tracking its own desired trajectory with the help of the extra information provided by the neighboring agents. In our framework, this can be well posed and analyzed. Moreover, a wider set of models can be included in this framework besides some integrator-like dynamics. In this paper, the general single-input-single-output (SISO) linear dynamics in the frequency domain is further studied and the communication topology concerned throughout this paper is time-invariant.

We would like to mention that most existing consensus algorithms are based on state space models. However, this may not always be available for plants acquired from identification procedures, or some industrial plants such as electric motors. Besides, state variables are not always directly available either. So the control algorithms based on transfer function and output feedback studied in this paper is of much practical use.

Another contribution of this paper is that we introduced a generalized root locus method, so called complex vector root locus (CVRL), to analyze the stability of the proposed control law. The characteristic of CVRL essentially has no big difference with that of classic root locus method but a small modification, which will be further analyzed later on.

The rest of this paper is organized as follows: Section 1 introduces some basic notations including CVRL and some useful results of algebraic graph theory. Section 2 presents the framework to tackle the cooperative control problem using the internal model principle. Topology analysis and controller design issue are considered in Section 3. Section 4 concludes this paper.

1 Notion and preliminaries

1.1 Complex vector root locus

Root locus method is a branch of classic control theory with wide applications. The basic idea of this method is to identify the closed-loop poles of a negative feedback control system by varying the open-loop gain K from 0 to $+\infty$ with the open-loop transfer function characterized by

$$G(s) = \frac{K\prod\limits_{i=1}^{m}(s-z_i)}{\prod\limits_{i=1}^{n}(s-p_i)} \quad (n \geq m) \tag{1}$$

The CVRL technique studied in this paper is a generalized one in which $K = r\cdot \mathrm{e}^{\mathrm{j}\theta}$, z_i, $p_i \in \mathbf{C}$, where $r > 0$ and $\theta \in [0, 2\pi)$. The closed-loop poles move as r varies from 0 to $+\infty$ with θ being a constant.

Here we state 4 properties of CVRL:

Property 1. The number of the root locus branches is equal to the order of the closed-loop characteristic equation.

Property 2. The locus starts (when $r = 0$) at poles, and ends (when $r \to +\infty$) at the zeros. There are $n-m$ branch ends at $s = \infty$.

Property 3. If $n > m$ there are asymptotes of the root locus, and the starting point of the asymptotes is

$$\sigma = \frac{\sum\limits_{i=1}^{n} p_i - \sum\limits_{i=1}^{m} z_i}{n-m}$$

while the angle between the asymptotes and the positive real axis is

$$\varphi = \frac{2\kappa\pi + (\theta+\pi)}{n-m} \quad (\kappa = 0, \pm 1, \pm 2, \cdots)$$

Property 4. The angle of departure from a singular pole is

$$\theta_{pj} = 2\kappa\pi + (\pi+\theta) + \sum_{i=1}^{m}\angle(p_j - z_i) - \sum_{\substack{i=1,\\ i\neq j}}^{n}\angle(p_j - p_i)$$

and the angle of arrival at a singular zero is

$$\theta_{zj} = 2\kappa\pi + (\pi-\theta) - \sum_{\substack{i=1,\\ i\neq j}}^{m}\angle(z_j - z_i) + \sum_{i=1}^{n}\angle(z_j - p_i)$$

Proof. The proof of Properties 1∼3 are natural extensions of that of ordinary root locus from domain $\mathbf{R}$ to $\mathbf{C}$, which is available in many control theory tutorials, [22] for example. Property 4 is also easy to prove when the generalized angle condition

$$\sum_{i=1}^{m}\angle(s-z_i) - \sum_{i=1}^{n}\angle(s-p_i) = 2\kappa\pi + (\pi-\theta)$$

is applied to the proof in [22]. □

Remark 1. The starting point of CVRL asymptotes has the same form as that of ordinary root locus asymptotes. But generally it is not a real number any more as the former since the poles and zeros are complex. The angle between the asymptotes and the positive real axis rotates by $\theta/(n-m)$ counterclockwise compared with that of ordinary root locus.

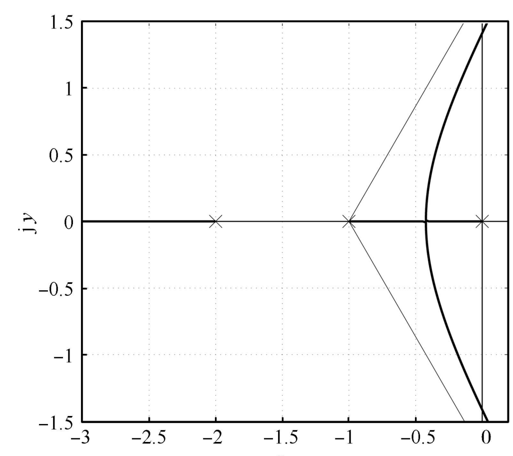
Remark 2. Compared with ordinary root locus, the angle of departure from a singular pole rotates by θ counterclockwise while the angle of arrival at a singular zero rotates clockwise also by θ.

Example 1. Draw the root locus of the system

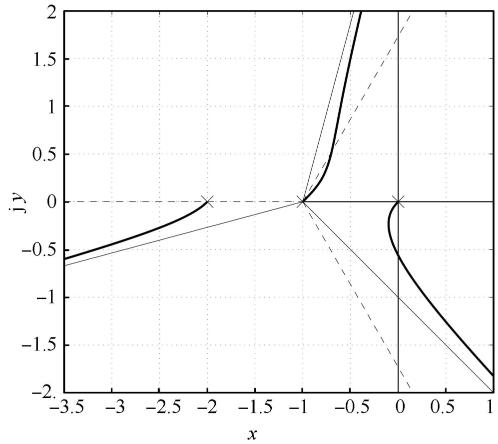
$$G_1(s) = \frac{r\cdot \mathrm{e}^{\mathrm{j}\theta}}{s(s+1)(s+2)}$$

where $r \in (0, +\infty)$, 1) $\theta = 0$, 2) $\theta = \pi/4$.

The root locus of 1) is shown in Fig. 1 (a) while 2) is in Fig. 1 (b). It is not hard to find that the asymptotes in 2) rotate by $\theta/(n-m) = \pi/12$ counterclockwise compared with 1) (the dashed line in Fig. 1 (b)) while the angles of departure from the poles rotate all by $\theta = \pi/4$ counterclockwise. The break-away point is gone.



(a) Root locus of 1)



(b) Root locus of 2)

Fig. 1 Root locus of $G_1(s)$

Example 2. Draw the root locus of the system

$$G_2(s) = \frac{r \cdot \mathrm{e}^{\mathrm{j}\frac{\pi}{4}}}{s(s+1)(s+j)}$$

where $r \in (0, +\infty)$.

According to Fig. 2, the starting point of the asymptotes equals $-(1+j)/3$ which is complex.

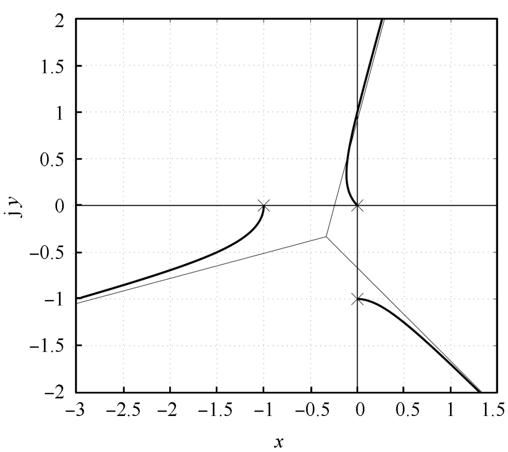


Fig. 2 Root locus of $G_2(s)$

1.2 Graph theory

A directed graph or digraph $G = (V, E)$ consists of a finite set V of vertices and a set $E \subseteq V \times V$ to be referred to as (directed) edges. We will assume that the digraph has no loops, that is, $(x, y) \in E$ implies $x \neq y$.

A (directed) walk in a digraph is a finite sequence of edges $(a_k, b_k), k = 1, 2, \cdots, r$ such that $b_k = a_{k+1}$ for $k = 1, 2, \cdots, r-1$. A walk with distinct vertices is called a (directed) path. A walk is called a (directed) circuit if in addition $b_r = a_1$. We say a digraph G is strongly connected if for any vertices $i, j \in V$, there exists a walk in G from i to j.

Definition 1 (Directed tree). A directed graph T is called a directed tree if it satisfies the following characteristics:

1) there is no circuit in T;

2) there exists a root vertex v such that any vertex in T has a path to v.

Definition 2 (Augmented graph). Suppose a digraph $G = (V, E)$ with a vertex set $V = \{1, 2, \cdots, r, \cdots, N\}$. A r-th order augmented graph of G is defined as $G^* = (V^*, E^*)$, where $V^* = V \cup \{0\}$ and $E^* = E \cup \{(1,0), (2,0), \cdots, (r,0)\}$.

The r-th order augmented graph of G has one additional vertex 0 in it with r directed edges $(1,0), (2,0), \cdots, (r,0)$ connected to 0.

The adjacency matrix $Q = [q_{ij}]$ of G is defined as:

$$q_{ij} = \begin{cases} 1, & \text{if } (i,j) \in E \\ 0, & \text{otherwise} \end{cases} \quad (i, j \in V) \tag{2}$$

The out-degree matrix of G is the diagonal matrix D with diagonal entries $d_{ii} = |\{j \in V | (i,j) \in E\}|$. The directed Laplacian of G is the matrix defined by[8]

$$L = D - Q \tag{3}$$

Lamma 1. Given a digraph G with L being its Laplacian, then 0 is an eigenvalue of L, and all the non-zero eigenvalues of L have positive real part[8].

Lamma 2. Given a digraph G, its Laplacian L has an eigenvalue with algebraic multiplicity one iff G has a rooted directed spanning tree[8].

2 Control law

2.1 Modeling

The dynamics of the agents are identical linear and single-input-single-output characterized by the transfer function

$$g(s) = \frac{n(s)}{d(s)} \quad (i = 1, 2, \cdots, N) \tag{4}$$

where $g(s)$ is a proper rational fraction with $n(s)$ and $d(s)$ coprime with each other.

We address the problem by making the following assumptions: 1) There is at least one agent in the system knowing about its reference input, and we call it (them) leader(s). 2) The other agents do not have the knowledge of their own reference input, nor can they get the absolute output of anyone else in the system, what they can get are the current output errors between their neighbors and themselves. Besides, the leaders can also have this ability. 3) Any agent knows about the desired bias between its neighbors and itself which can be predetermined and time-variant.

It is not hard to notice that the communication topology can be interpreted as a digraph G with each vertex representing an agent. N_i denotes the set containing the serial numbers of the i-th agent′s neighbors. So, $|N_i|$ is the i-th vertex′s out-degree. Directed edge (i, j) means that i can get the relative output error between agents i and j. In other words, j is a neighbor of i. The digraph shown in Fig. 3 is an example.

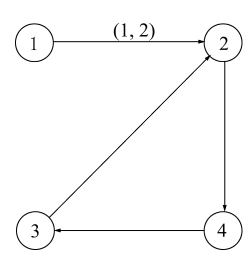


Fig. 3 Communication topology of the multiagent system

Without loss of generality, we assume that the leaders are numbered $1, 2, \cdots, r (1 \leqslant r \leq N)$. The reference input of agent i is $v_i(s) = n_{vi}(s)/d_{vi}(s)$ which is not available for anyone and the determinate disturb input is $w_i(s) = n_{wi}(s)/d_{wi}(s)$. The error between reference input and the actual output of agent i is denoted as $e_i = v_i - y_i$. The desired bias between i and j is denoted as $v_{ij} = v_i - v_j$ which is predetermined, while the actual error is $y_{ij} = y_i - y_j$.

Before dealing with the problem, we introduce the following truth, i.e., the internal model principle:

Theorem 1. Suppose that the controlled system $g(s) = n(s)/d(s)$ is a rational proper fraction with $n(s)$ and $d(s)$ coprime with each other.$\phi(s)$ is the least common factor of the unstable poles in $v(s) = n_v(s)/d_v(s)$ and $w(s) = n_w(s)/d_w(s)$ and has no common zeros with $g(s)$. Then, there exists a compensator $g_c(s) = n_c(s)/d_c(s)$ characterized by a rational fraction that makes the control system in Fig.4 have the capability of asymptotic tracking and disturbance suppression if and only if the following equation

$$d_f(s) = d_c(s)d(s)\phi(s) + n_c(s)n(s) = 0 \tag{5}$$

has all of its roots in the left part of the complex plane[23].

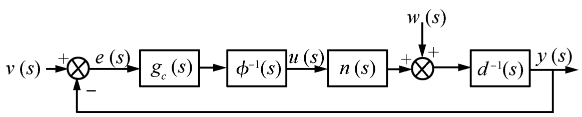


Fig. 4 Block diagram of asymptotic tracking

Proof. From Fig. 4, the following equation is yielded:

$$(v - y)\frac{n_c n}{d_c \phi} - dy + w = 0 \tag{6}$$

So, the output caused by disturbance w alone equals

$$y_w = \frac{d_c\phi}{d_c\phi + n_c n} w = \frac{d_c n_w}{d_c d\phi + n_c n} \cdot \frac{\phi}{d_w} \tag{7}$$

Similarly, the error caused by reference input v alone equals to

$$e = \frac{d_c d\phi}{d_c d\phi + n_c n} v = \frac{d_c d n_v}{d_c d\phi + n_c n} \cdot \frac{\phi}{d_v} \tag{8}$$

Since ϕ accurately offsets the unstable zeros in d_v and d_w, as long as the characteristic equation

$$d_f = d_c d\phi + n_c n = 0 \tag{9}$$

has all of its roots placed in the left part of the complex plane, both y_w and e are stable. So, (6) is stable. □

To begin with, design the input signal of $g_c(s)$ in each agent as follows:

$$\left\{\begin{array}{ll} \delta_i = e_i + \sum\limits_{j\in N_i}(y_{ji} - v_{ji}) & (1 \le i \le r) \\ \delta_i = \sum\limits_{j\in N_i}(y_{ji} - v_{ji}) & (r < i \le N) \end{array}\right. \tag{10}$$

Note that $\sum_{j\in N_i}(y_{ji} - v_{ji}) = \sum_{j\in N_i}(e_i - e_j)$. The following equations hold true:

$$\boldsymbol{e} = \boldsymbol{v} - \boldsymbol{y}; \quad \boldsymbol{\delta} = W\boldsymbol{e} \tag{11}$$

where $\boldsymbol{\delta} = [\delta_1, \delta_2, \cdots, \delta_N]^{\rm T}$, $\boldsymbol{e} = [e_1, e_2, \cdots, e_N]^{\rm T}$, $\boldsymbol{v} = [v_1, v_2, \cdots, v_N]^{\rm T}$, $\boldsymbol{y} = [y_1, y_2, \cdots, y_N]^{\rm T}$ and

$$W = L + E_r \quad \left(E_r = \left[\begin{array}{cc} I_r & 0 \\ 0 & 0 \end{array}\right]\right)$$

L is the Laplacian of G. Let $\boldsymbol{w} = [w_1, w_2, \cdots, w_N]^{\rm T}$, here we get the block diagram of the cooperative control system in Fig. 5:

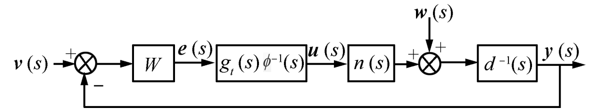


Fig. 5 Block diagram of cooperative tracking

2.2 Decoupling transformation

From Fig. 5, the following equation is yielded:

$$W(\boldsymbol{v} - \boldsymbol{y})\frac{n_c n}{d_c\phi} - d\boldsymbol{y} + \boldsymbol{w} = 0 \tag{12}$$

According to matrix theory, W can be similar to an upper triangular matrix as $U = P^{-1}WP$, where

$$U = \left[\begin{array}{cccc} \lambda_1 & * & \cdots & * \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & * \\ 0 & \cdots & 0 & \lambda_N \end{array}\right]$$

and λ_i is i-th eigenvalue of W, P is a unitary matrix. Thus a decoupling transformation is introduced. Let $\boldsymbol{v} = P\tilde{\boldsymbol{v}}$, $\boldsymbol{w} = P\tilde{\boldsymbol{w}}$, $\boldsymbol{y} = P\tilde{\boldsymbol{y}}$, and $\boldsymbol{e} = P\tilde{\boldsymbol{e}}$, v_i, w_i, y_i, and e_i be i-th entries of $\tilde{\boldsymbol{v}}, \tilde{\boldsymbol{w}}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{e}}$, respectively. We show the following to be true.

Theorem 2. The system in Fig. 5 is stable if and only if the following N systems are all stable

$$\lambda_i(\tilde{v}_i - \tilde{y}_i)\frac{n_c n}{d_c\phi} - d\tilde{y}_i + \tilde{w}_i = 0 \quad (i = 1, 2, \cdots, N) \tag{13}$$

Proof. The transformed equation of (12) is as follows:

$$\lambda_i(\tilde{v}_i - \tilde{y}_i)\frac{n_c n}{d_c\phi} - d\tilde{y}_i + \tilde{w}_i = f_i(\tilde{e}_{i+1}, \tilde{e}_{i+2}, \cdots, \tilde{e}_N) \tag{14}$$

where $f_N \equiv 0$. For $i \ne N$, $f_i \to 0$ if and only if $\tilde{e}_j \to 0\,(i < j \le N)$ simultaneously.

If (13) are all stable, firstly, N-th equation of (14) is stable and $f_{N-1} \to 0$; then $(N-1)$-th of (14) follows and $f_{N-2} \to 0$ and so on until $f_i \to 0\,(i = 1, 2, \cdots, N)$ so that (14) are all stable.

If (14) are all stable, that means $\tilde{e}_i \to 0$ for all $i = 1, 2, \cdots, N$ which is equivalent to $f_i \to 0\,(i = 1, 2, \cdots, N)$. So (13) are all stable.

Theorem 2 reveals the fact that in the cooperative manner, the goal is achieved if and only if one single internal model ϕ^{-1} and compensator $g_c(s)$ simultaneously solve the tracking problem of N independent systems:

$$g_i(s) = \frac{\lambda_i n(s)}{d(s)} \quad (i = 1, 2, \cdots, N) \tag{15}$$

with $\tilde{v}_i$ being the reference input and $\tilde{w}_i$ the disturb input whose denominators are the least common factor of the unstable poles of each v_i and $w_i\,(i = 1, 2, \cdots, N)$, respectively. The block diagram is shown in Fig. 6. □

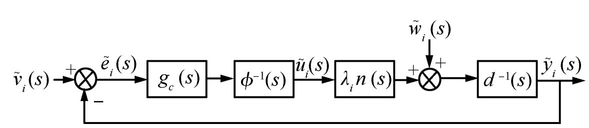


Fig. 6 Block diagram of the decoupled system

Corollary 1. The necessary and sufficient condition for the stability of the system in Fig. 5 as well as Fig. 6 is that the characteristic equations

$$d_{fi} = d_c d\phi + \lambda_i n_c n = 0 \tag{16}$$

have all of the roots in the left part of the complex plane.

Proof. This is a direct corollary of Theorem 1 and Theorem 2. □

3 Stability analysis

3.1 Leader selection

According to Corollary 1, it is obvious that W must have no zero eigenvalue unless $d(s)$ and $\phi(s)$ have no unstable factors, which is not allowed. Here, we first present the spectral properties of W.

Lamma 3. The spectrum of W has one less zero eigenvalue compared with L^*, which is the Laplacian of r-th order augmented graph of G, namely G^*.

Proof. L^* has the following form according to the definition of Laplacian:

$$L^* = \begin{array}{c} 0 \\ 1 \\ \vdots \\ r \\ r+1 \\ \vdots \\ N \end{array} \left[\begin{array}{cccc} 0 & 0 & \cdots 0 & 0 \\ -1 & & & \\ \vdots & & & \\ -1 & & W & \\ 0 & & & \\ \vdots & & & \\ 0 & & & \end{array}\right]$$

since $\det(sI - L^*) = s \cdot \det(sI - W)$, clearly W has one less zero eigenvalue compared with L^*. □

Lamma 4. W is nonsingular if and only if G^* has rooted direct spanning tree, where all the eigenvalues of W have positive real part.

Proof. With reference to Lemma 3, W is nonsingular iff 0 is an eigenvalue of L^* with algebraic multiplicity one. According to Lemma 2, this means G^* has a rooted direct spanning tree. And Lemma 1 reveals that all the eigenvalues of W have positive real part. □

That G^* has a direct rooted spanning tree or not correlates closely to the selection of leaders. Let us divide G into M subgraphs $G_m\,(m = 1, 2, \cdots, M)$ such that $\bigcup_{m=1}^{M} G_m = G$. Make sure that each of them has a direct rooted spanning tree in it. Since isolated vertices can also be treated as a directed tree, this kind of division always exists but may not be unique.

Theorem 3. The necessary and sufficient condition for G^* to have a direct rooted spanning tree is that there exists a division of G such that for any subgraph G_m, its root of (one of) the spanning trees is the leader of the system.

Proof. To prove the sufficiency, there is a need to prove that in this case any vertex $v_0 \in G$ has a path to 0. Assume that $v_0 \in G_{m_0}$; then if v_0 is not the root of the spanning tree of G_{m_0}, namely v_{m_0}, it must have a path to v_{m_0}. Since v_{m_0} is a leader, directed edge $(v_{m_0}, 0)$ exists. Thus v_0 can get to 0 via v_{m_0}. Else if $v_0 = v_{m_0}$, then it can get to 0 directly.

If G^* has a direct rooted spanning tree and the leaders $1, 2, \cdots, r$ connect 0 directly, then any other vertices can get to one of the leaders along a certain path. The subgraph is divided like this: $G_i = \bigcup\{$the vertex that has a path to i and this path$\}$. Obviously, G_i has a spanning tree. □

The theorem above puts forward a criterion for selecting leaders. An intuitive example is given in Fig. 7 with the agents painted black representing the leader. Though the division of G may not be unique, there is a smallest value of M when the number of leaders reaches its minimum. If G owns a direct rooted spanning tree itself, then $M = 1$, which means only one leader is needed. If G is strongly connected, any agent in the system can be selected as a leader.

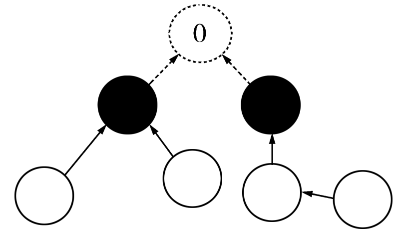


Fig. 7 Leader selection for communication topology

3.2 Controller design

The roots of (16) is the same as the closed-loop poles of the transfer function:

$$G_i(s) = \lambda_i \phi^{-1}(s) \cdot g_c(s) \cdot g(s) \tag{17}$$

which can be transformed into the following form:

$$G_i(s) = \frac{r \cdot \mathrm{e}^{\arg(\lambda_i)} \prod\limits_{j=1}^{m} (s - z_j)}{\prod\limits_{j=1}^{n} (s - p_j)} \quad (r > 0) \tag{18}$$

The difficulty increases for designing the open-loop gain as $n - m$, namely the relative order, gets higher. Here, we put forward the following conclusion for $n - m = 1$:

Theorem 4. The roots of (16), i.e., the closed-loop poles of $G_i(s)$, are strictly stable when r gets sufficiently large if the following two conditions are satisfied:

1) $n - m = 1$;

2) $G_i(s)$ has no zeros in the right half of the complex plane.

Proof. Based on Property 2, $p_j \to z_j\,(1 \le j \le m)$, and $p_j \to \infty\,(m < j \le n)$ as $r \to +\infty$. Since $n - m = 1$, only one branch goes to infinity, i.e., $p_n \to \infty$. Let φ_i be the angle between the asymptote and the positive direction of the real axis, $\varphi_i = \pi + \arg(\lambda_i)$ according to Property 3. Since λ_i has positive real part, $\arg(\lambda_i) \in (-\pi/2, \pi/2)$, we have $\varphi_i \in (\pi/2, 3\pi/2)$ and $p_n \to +\infty \cdot \mathrm{e}^{\mathrm{j}\varphi_i}$ which is stable as $r \to +\infty$. The other poles are also stable according to 2).

For $n - m > 1$, there is no ready solution to assure the stability of N distinct systems of (16) with a single parameter r adjustable. But there are still plenty of ways to achieve that by properly arranging the zeros and poles of $g_c(s)$.

Let us present a protocol for controller design based on the conclusions above:

1) Determine $\phi(s)$ according to the denominators of $v_i(s)$ and $w_i(s)$.

2) Select leaders of the system according to the criterion proposed in Theorem 3.

3) Disign $g_c(s)$ properly such that for $\lambda_i \in \mathbf{R}$, i.e., $\arg(\lambda_i) = 0$, which may be an imaginary one, $G_i(s)$ has a stable threshold as large as possible. During this step guidelines for root locus designing can be widely used.

4) Plot root locus with respect to the other eigenvalues to obtain the interval of r where all the systems are stable, test whether the designed controller meets the requirements or not.

5) If $g_c(s)$ meets the requirements, 5) is over, else return to 3) to get a better controller.

Maybe in the current framework there is a difficulty in accurately evaluating the transient performance since for an N-agent system, there are usually N eigenvalues to be considered but only one root locus can be designed (usually the ordinary root locus because of its simplicity), the rest ones are also determined afterwards. However, this drawback is partially caused by the property of the graph itself, in other words, the interaction mode between the agents has a substantial influence on the performance of the control algorithm. If the graph is also adjustable, say, it is an undirected graph where its Laplacian is symmetrical, the convergence rate can be analyzed and is related to the smallest nonzero eigenvalue of the (augmented) graph Laplacian known as the "algebraic connectivity" according to the existing literature[11]. This can also be figured out in our framework because in this case one ordinary root locus is enough to investigate the whole system since all the eigenvalues here are real, and the smallest nonzero eigenvalue is corresponding to the smallest open loop gain and a set of closed loop poles nearest to the imaginary axis.

Secondly, although accurate evaluation of the performance may not be available under general circumstances, there is still a possibility to improve it. It has been proved that the shape of CVRL is much the same as the corresponding ordinary root locus despite some differences in the direction of the asymptotes and departure (arrival) angles, so once the performance of the first root locus is improved, the rest will follow. □

Example 3. The communication topology is shown in Fig. 3. The eigenvalues of W are $\lambda_1 = 0.245, \lambda_2 = 2$ and $\lambda_{3,4} = 2.020 \cdot \mathrm{e}^{\pm \mathrm{j}0.378}$ when Agents 1 and 2 are chosen as leaders. The reference input are $v_i(t) = \sin t\,(i = 1, 2, 3, 4)$ with disturb input $w_{1,2}(t) = 1$ and $w_{3,4}(t) = t$. The dynamics of the agent is given by

$$g(s) = \frac{1}{(s+5)^2} \tag{19}$$

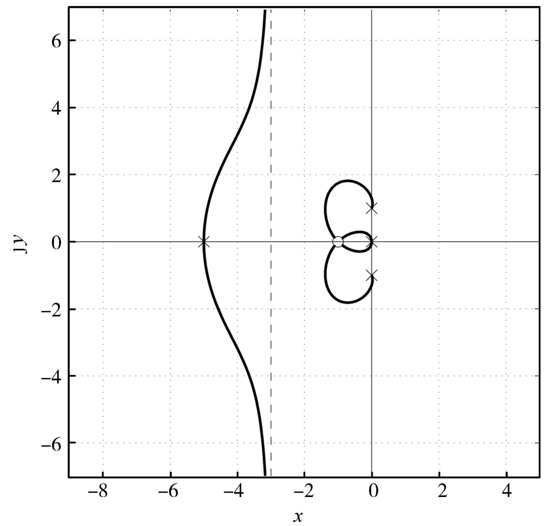
We see in this case that $\phi(s) = s^2(s^2+1)$. The controller is designed as

$$h_c(s) = \frac{g_c(s)}{\phi(s)} = \frac{K(s+1)^4}{s^2(s^2+1)}$$

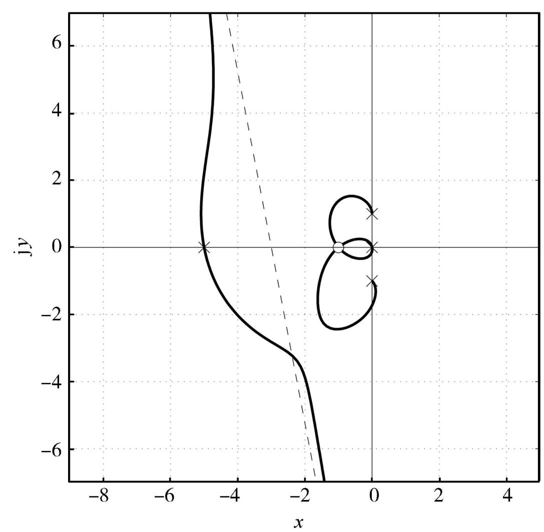
The root locus of $G_i(s)$ is shown in Fig. 8. Note that the root loci are symmetrical with each other with respect to the real axis when λ_i are mutually conjugate, so Fig. 8 (b) is of $\arg(\lambda_i) = 0.378$ alone. $G_{1,2}(s)$ is stable when $r \in (4, +\infty)$ while $G_{3,4}(s)$ is stable when $r \in (10, 240)$. Hence, the interval for stability is $(10, 240)$. The output result in Fig. 9 is for $K = 50$.

4 Conclusion

This paper deals with the cooperative tracking and disturbance suppression problem with single or multiple leaders using the internal model principle. We see that the control problem of a multiagent system consisting of N identical agents can be reduced into the stability problem of N distinct SISO systems modified by an eigenvalue of the augmented digraph. We show the necessary and sufficient conditions on the communication topology such that all the eigenvalues of W are nonzero. Simulation result shows that the CVRL technique is helpful and effective in stability analysis and pole assignment.



(a) Root locus for $G_{1,2}(s)$



(b) Root locus for $G_{3,4}(s)$

Fig. 8 Root locus of $G_i(s)$

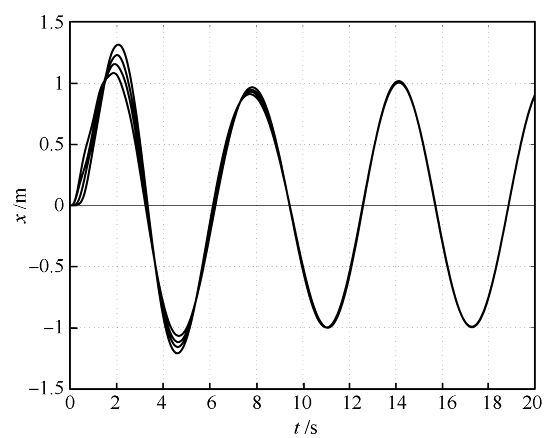


Fig. 9 Trajectory of the multiagent system

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