Fault-tolerant Control and Disturbance Attenuation of a Class of Nonlinear Systems with Actuator and Component Failures

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Abstract This paper investigates the stabilization problem of a class of nonlinear dynamic systems with actuator and component failures as well as external disturbances. New fault-tolerant control algorithms are derived without the need for analytically estimating bound on actuator failure variables, and thus the resultant control scheme has simpler structure and demands less online computations as compared with most existing methods. It is shown that with the proposed control, both actuator and subsystem/component failures can be accommodated, and the state dependent growth disturbances can be effectively attenuated. The algorithm is validated via a formative mathematical analysis based on a Lyapunov approach and numerical simulations in the presence of external disturbances, parametric uncertainties, as well as severe actuator/subsystem failures.

Key words Fault-tolerant control (FTC), actuator and subsystem failures, robust adaptive control, unbounded disturbances, nonlinear system

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Actuator and/or component failure could cause serious safety problem to engineering systems if no proper action is taken in time. Fault-tolerant control (FTC) has been counted as one of the most promising control technologies for maintaining specified safety performance of a system in the presence of unexpected faults. Various FTC methods have been proposed during the past decade (e.g., [1−15], to just name a few). Among the most existing FTC approaches, the one that does not rely on fault detection and diagnosis (FDD) is of particular interest in practice due to the fact that it does not demand timely and precise fault detection or diagnosis for implementation. Tang et al.[7] investigated a robust adaptive compensation method for linear time-invariant systems with actuator failures and applied it to rocket fairing structural-acoustic model. Although independent of FDD, this method needs to utilize an iterative algorithm to check the solvability of a linear matrix inequality (LMI). This is essentially an approximation process, which is quite time-consuming when multiple fault modes (patterns) are involved. Jin et al.[16] considered a similar problem in their recent work and proposed a robust adaptive FTC method to stabilize linear time-invariant systems with both actuator failures and bounded external disturbances. However, as noted in [17], their control scheme involves solving a failure-factor related Lyapunov equation (as with the one in Tang et al.[7]), which is non-trivial in control design. Fan et al.[17] have recently proposed a new FTC in which not only the state dependent growing external disturbances were addressed, but also the drawbacks associated with [7, 16] were completely circumvented.

This paper extends and improves the results in [17] from linear systems to nonlinear systems subject to external disturbances and actuator failures. The impact from subsystem/component malfunction is also considered. A control scheme that is FDD-independent is proposed, in which there is no need for explicit fault information in terms of its magnitude and time instance of the fault occurrence. The FTC control scheme developed herein does not need to solve a Lyapunov equation that contains uncertain and time-varying actuator failure variables, nor does it demand any explicit fault information for control design and implementation. The result presented here is inspired by the work of [16] and especially [17]. In fact, it can be viewed as the natural extension and improvement of the work by [16, 17].

1 FTC problem formulation

Consider the stabilization problem of the following nonlinear dynamic system under actuator and component faults as well as external disturbances.

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{N}(\boldsymbol{x},t_f)+B(\boldsymbol{u}_a(t)+\boldsymbol{d}(\boldsymbol{x},t)) \tag{1a}$$

with

$$\boldsymbol{N}(\boldsymbol{x},t_f)=A\boldsymbol{x}(t)+B[\boldsymbol{\xi}(\boldsymbol{x})+\mu(t-t_f)\boldsymbol{\eta}(\boldsymbol{x},t)] \tag{1b}$$

where $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the state, $\boldsymbol{u}_a(t)\in\mathbf{R}^m$ denotes the actual control input to the system, $\boldsymbol{d}(\boldsymbol{x},t)\in\mathbf{R}^m$ models external disturbances acting on the system, $\boldsymbol{\xi}(\boldsymbol{x})\in\mathbf{R}^m$ is a nonlinear and unknown vector function, $\mu(t-t_f)\boldsymbol{\eta}(\boldsymbol{x},t)$ denotes the uncertain effect due to subsystem/component failures occurring at the time instant $t\geq t_f$, A and B are known real constant matrices with appropriate dimensions.

When component and actuator failures (such as element outage, actuator loss of effectiveness, stuck, combination of all) occur, the actual control input $\boldsymbol{u}_a(t)$ and the designed control input $\boldsymbol{u}(t)$ function according to

$$\boldsymbol{u}_a(t)=\rho(t)\boldsymbol{u}(t)+\boldsymbol{E}(t) \tag{2}$$

where $\rho(t)$ =diag$\{\rho_i(t)\}$ is a diagonal matrix with $\rho_i(t)\in(0,1]\,(i=1,2,\cdots,m)$, being a time-varying scalar function called actuator efficiency factor[16], or "health indicator"[18], $\boldsymbol{E}(t)$ denotes a vector function corresponding to the portion of the control action produced by the actuator that is completely out of control. Note that $\boldsymbol{E}(t)$ might be immeasurable and time-varying.

Remark 1. It is noted that the system investigated here includes the one considered in [7, 16−17] as a special case. In fact, the system studied in [7] corresponds to the

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case that $\boldsymbol{d}(\boldsymbol{x},t)\equiv 0$, $\boldsymbol{\xi}(\boldsymbol{x})\equiv 0$ and $\mu(t-t_f)\boldsymbol{\eta}(\boldsymbol{x},t)\equiv 0$, the work of [16] considered the case that the disturbance $\boldsymbol{d}(\boldsymbol{x},t)$ is bounded by a constant and $\boldsymbol{E}(t)$ is constant, while the work of [18] assumed that the system is linear and $\mu(t-t_f)\boldsymbol{\eta}(\boldsymbol{x},t)\equiv 0$.

Remark 2. Regarding the disturbance, it is noteworthy that output disturbance and/or state disturbance are inevitable, thus a practical control scheme should is consider for such impact. In this work, the disturbance is treated as an overall impact on the system (unpredictable and immeasurable), and such impact grows depending on the state rather than being bounded by a constant. The lumped disturbances, together with the modeling uncertainties as well as actuator and subcomponent failures are compensated by the control scheme to be developed in the sequel.

For the system to admit a feasible FTC solution, the following assumptions are needed.

Assumption 1. All the states of the system are available at every instant.

Assumption 2. $(A,\ B)$ is controllable in that there exists a constant matrix K_0 such that the matrix $A-BK_0$ is Hurwitz matrix.

Assumption 3. The stuck-actuator fault is bounded by some unknown constant; the external disturbance, the nonlinear term, and the impact of the subsystem failure are piece-wise continuous and bounded by multiplication of some unknown nonnegative constants and known nonlinear functions, i.e., there exist some unknown constants $a_{_E}$, a_d, a_ξ and a_η as well as known functions $\psi_d(\boldsymbol{x})$, $\psi_\xi(\boldsymbol{x})$ and $\psi_\eta(\boldsymbol{x})$ such that

$$\|\boldsymbol{E}(t)\|\le a_{_E}<\infty$$

$$\|\boldsymbol{d}(\cdot)\|\le a_d\psi_d(\boldsymbol{x})$$

$$\|\boldsymbol{\xi}(\cdot)\|\le a_\xi\psi_\xi(\boldsymbol{x})$$

$$\|\mu(t-t_f)\boldsymbol{\eta}(\cdot)\|\le a_\eta\psi_\eta(\boldsymbol{x})$$

Remark 3. Since $(A,\ B)$ is controllable, one can choose K_0 properly such that $\bar{A}=A-BK_0$ is Hurwitz matrix in that for any given $Q=Q^{\rm T}>0$, there exists a symmetric and positive definite matrix P such that

$$-Q=\bar{A}^{\rm T}P+P\bar{A} \tag{3}$$

Since A and B are available and $\bar{A}$ can be specified as Hurwitz matrix by the designer, K_0 can be determined directly from $\bar{A}=A-BK_0$, and P can be readily solved from the Lyapunov equation (3) for a given $Q=Q^{\rm T}>0$.

2 Fault-tolerant control

2.1 Robust fault-tolerant control

In this section, a robust fault-tolerant control of the following form is proposed.

$$\boldsymbol{u}(t)=-(K_0+K(t))\boldsymbol{x} \tag{4a}$$

where K_0 is a constant matrix chosen such that $A-BK_0$ is Hurwitz matrix, and $K(t)$ is computed on-line by

$$K(t)=\frac{a}{\lambda_m}(1+\|K_0\boldsymbol{x}\|)\frac{B^{\rm T}P}{\|B^{\rm T}P\boldsymbol{x}\|} \tag{4b}$$

with

$$\begin{aligned}&0<\lambda_m\le\min\{\rho_1,\cdots,\rho_m\}\\&\psi(\boldsymbol{x})=\psi_d(\boldsymbol{x})+\psi_\xi(\boldsymbol{x})+\psi_\eta(\boldsymbol{x})\end{aligned} \tag{4c}$$

$$a=\max\{1,a_{_E},a_d,a_\xi,a_\eta\} \tag{4d}$$

Theorem 1. Consider the system described by (1) subject to actuator failures as defined in (2). Under Assumptions $1\sim 3$, if the robust FTC given in (4) is applied, the system is globally and asymptotically stable.

Proof. When the system is subject to the actuator failure as described in (2), its dynamic behavior is governed by

$$\dot{\boldsymbol{x}}=A\boldsymbol{x}+B[\rho(t)\boldsymbol{u}(t)+\boldsymbol{E}(t)+\boldsymbol{d}(\cdot)+\boldsymbol{\xi}(\cdot)+\mu(t-t_f)\boldsymbol{\eta}(\cdot)] \tag{5}$$

With the proposed control (4), the closed-loop dynamics becomes

$$\begin{aligned}\dot{\boldsymbol{x}}=\ &A\boldsymbol{x}+B[\rho(t)(-K_0\boldsymbol{x}-K(t)\boldsymbol{x})+\boldsymbol{E}(t)+\\&\boldsymbol{d}(\cdot)+\boldsymbol{\xi}(\cdot)+\mu(t-t_f)\boldsymbol{\eta}(\cdot)]=\\&(A-BK_0)\boldsymbol{x}+B[-\rho(t)K(t)\boldsymbol{x}+L(t)]=\\&\bar{A}\boldsymbol{x}+B[-\rho(t)K(t)\boldsymbol{x}+L(t)]\end{aligned} \tag{6}$$

where

$$L(\cdot)=(I-\rho)K_0\boldsymbol{x}+\boldsymbol{E}(t)+\boldsymbol{d}(\cdot)+\boldsymbol{\xi}(\cdot)+\mu(t-t_f)\boldsymbol{\eta}(\cdot)$$

By Assumption 3

$$\begin{aligned}\|L(\cdot)\|\le\ &\|K_0\boldsymbol{x}\|+\|\boldsymbol{E}(t)\|+\|\boldsymbol{d}(\cdot)+\boldsymbol{\xi}(\cdot)+\\&\mu(t-t_f)\boldsymbol{\eta}(\cdot)\|\le a(1+\|K_0\boldsymbol{x}\|+\psi(\boldsymbol{x})\|)\end{aligned}$$

where a and $\psi(\boldsymbol{x})$ are defined as in (4c). Therefore, it is true that

$$(B^{\rm T}P\boldsymbol{x})^{\rm T}L\le a(1+\|K_0\boldsymbol{x}\|+\psi(\boldsymbol{x}))\|B^{\rm T}P\boldsymbol{x}\| \tag{7}$$

Choose Lyapunov function candidate

$$V=\frac{1}{2}\boldsymbol{x}^{\rm T}P\boldsymbol{x}$$

By the Lyapunov equation (3), it is shown that

$$\begin{aligned}\dot{V}=&-\frac{1}{2}\boldsymbol{x}^{\rm T}Q\boldsymbol{x}+(B^{\rm T}P\boldsymbol{x})^{\rm T}L+\\&(B^{\rm T}P\boldsymbol{x})^{\rm T}\left[-\rho\frac{a}{\lambda_m}(1+\|K_0\boldsymbol{x}\|+\psi(\boldsymbol{x}))\frac{B^{\rm T}P\boldsymbol{x}}{\|B^{\rm T}P\boldsymbol{x}\|}\right]\le\\&-\frac{1}{2}\boldsymbol{x}^{\rm T}Q\boldsymbol{x}+a(1+\|K_0\boldsymbol{x}\|+\psi(\boldsymbol{x}))\|B^{\rm T}P\boldsymbol{x}\|-\\&\frac{a}{\lambda_m}\frac{(1+\|K_0\boldsymbol{x}\|+\psi(\boldsymbol{x}))}{\|B^{\rm T}P\boldsymbol{x}\|}(B^{\rm T}P\boldsymbol{x})^{\rm T}\rho(B^{\rm T}P\boldsymbol{x})\end{aligned} \tag{8}$$

Since $(B^{\rm T}P\boldsymbol{x})^{\rm T}\rho(B^{\rm T}P\boldsymbol{x})\ge\lambda_m\|B^{\rm T}P\boldsymbol{x}\|^2$, with certain calculation, the last two terms of (8) can be combined for cancellation therefore.

$$\dot{V}\le-\frac{1}{2}\boldsymbol{x}^{\rm T}Q\boldsymbol{x}\le 0$$

By Lyapunov stability theory, the system is globally and asymptotically stabilized by the proposed robust FTC. □

Remark 4. With certain information on the fault model and external disturbances, the two design parameters a and λ_m as defined in (4c) can be determined. Overall, the design and the implementation of the control scheme (4) is less involved as compared with the one in [7, 16], both are for linear systems. The next control scheme further simplifies the design procedure by avoiding the analytic determination of any design parameters.

2.2 Robust adaptive fault-tolerant control

The control scheme that is robust and adaptive is proposed as follows.

$$\boldsymbol{u}(t) = -(K_0 + \hat{K}(t))\boldsymbol{x} \tag{9a}$$

where $K_0 > 0$ is chosen such that $A - BK_0$ is Hurwitz matrix, and $\hat{K}(t)$ is on-line updated by

$$\hat{K}(t) = \frac{\hat{a}(t)\varphi(\boldsymbol{x})B^{\mathrm{T}}P}{\|B^{\mathrm{T}}P\boldsymbol{x}\|} \tag{9b}$$

with

$$\varphi(\boldsymbol{x}) = 1 + \|K_0\boldsymbol{x}\| + \psi(\boldsymbol{x}) \tag{9c}$$

and

$$\dot{\hat{a}}(t) = \gamma\varphi(\boldsymbol{x})\|B^{\mathrm{T}}P\boldsymbol{x}\|, \quad \gamma > 0 \tag{9d}$$

Theorem 2. For the system with external disturbances, subsystem failures and actuator failures as described by (1) ~ (2), let Assumptions 1 ~ 3 hold. If the robust adaptive FTC (9) is applied, the system is asymptotically stable.

Proof. Consider the following Lyapunov function candidate

$$V = \boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x} + \frac{1}{\lambda_m\gamma}(a - \hat{a}\lambda_m)^2 \tag{10}$$

where $\gamma > 0$ is a constant related to adaptation rate chosen by the designer and $\lambda_m > 0$ is constant as defined as before. Upon using the control scheme with the adaptive algorithm, it is not difficult to show that

$$\begin{aligned}\dot{V} = &\dot{\boldsymbol{x}}^{\mathrm{T}}P\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}P\dot{\boldsymbol{x}} + 2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1}) = \\ &[\bar{A}\boldsymbol{x} + B(-\rho\hat{K}\boldsymbol{x} + L)]^{\mathrm{T}}P\boldsymbol{x} + \\ &\boldsymbol{x}^{\mathrm{T}}P[\bar{A}\boldsymbol{x} + B(-\rho\hat{K}\boldsymbol{x} + L)] + 2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1}) = \\ &\boldsymbol{x}^{\mathrm{T}}(\bar{A}^{\mathrm{T}}P + P\bar{A})\boldsymbol{x} + 2\boldsymbol{x}^{\mathrm{T}}PB(-\rho\hat{K}\boldsymbol{x} + L) + \\ &2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1})\end{aligned}$$

By Lyapunov equation (3), it follows that

$$\begin{aligned}\dot{V} = &-\boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} + 2\boldsymbol{x}^{\mathrm{T}}PB\left\{\rho\left[-\frac{\hat{a}(t)\varphi(\boldsymbol{x})(B^{\mathrm{T}}P)}{\|B^{\mathrm{T}}P\boldsymbol{x}\|}\boldsymbol{x}\right] + L\right\} + \\ &2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1})\end{aligned} \tag{11}$$

In light of the definition of λ_m, it is true that $(B^{\mathrm{T}}P\boldsymbol{x})^{\mathrm{T}}\rho(B^{\mathrm{T}}P\boldsymbol{x}) \geq \lambda_m\|B^{\mathrm{T}}P\boldsymbol{x}\|^2$, thus the second term in (11) can be rewritten as

$$\begin{aligned}&2\boldsymbol{x}^{\mathrm{T}}PB\left\{\rho\left[-\frac{\hat{a}(t)\varphi(\boldsymbol{x})(B^{\mathrm{T}}P)}{\|B^{\mathrm{T}}P\boldsymbol{x}\|}\boldsymbol{x}\right] + L\right\} = \\ &-2\frac{\hat{a}(t)\varphi(\boldsymbol{x})}{\|B^{\mathrm{T}}P\boldsymbol{x}\|}(B^{\mathrm{T}}P\boldsymbol{x})^{\mathrm{T}}\rho(B^{\mathrm{T}}P\boldsymbol{x}) + 2(B^{\mathrm{T}}P\boldsymbol{x})^{\mathrm{T}}L \leq \\ &2(a - \lambda_m\hat{a})\varphi(\boldsymbol{x})\|B^{\mathrm{T}}P\boldsymbol{x}\|\end{aligned}$$

Thus, (11) becomes

$$\begin{aligned}\dot{V} \leq &-\boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} + 2(a - \lambda_m\hat{a})\varphi(\boldsymbol{x})\|B^{\mathrm{T}}P\boldsymbol{x}\| + \\ &2(a - \lambda_m\hat{a})(-\gamma^{-1}\dot{\hat{a}})\end{aligned} \tag{12}$$

Using the updating law (9d), one obtains from (12) that

$$\dot{V} \leq -\boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} \leq 0$$

Therefore, $\boldsymbol{x} \in L_2 \cap L_\infty$, and $\hat{a} \in L_\infty$, hence, $\boldsymbol{u}(t) \in L_\infty$ and $\dot{\boldsymbol{x}} \in L_\infty$. Namely, $\boldsymbol{x}$ is uniformly continuous. Then by Barbalat lemma[19], it is concluded that $\boldsymbol{x} \to 0$ as $t \to \infty$. □

The control block diagram of the control scheme and underlying system with actuator and component failures and external disturbances is depicted in Fig. 1. Clearly, one can easily build such control scheme with the information of A, B (thus K_0), and $\varphi(\boldsymbol{x})$, there is no need for any other analytic estimation on the uncertain faults and disturbances.

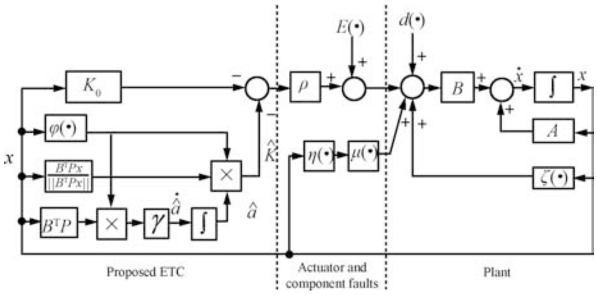

Fig. 1 The block diagram of the proposed FTC for systems with actuator/component failures

Remark 5. It is seen that the proposed control is independent of explicit information on faults and disturbances. As with most variable structure control methods, when the states get closer to zero, the control scheme might experience chattering, which can be easily avoided by replacing $\frac{z}{\|z\|}$ with $\frac{z}{\|z\| + \varepsilon}$, where ε is a small number, as commonly adopted in the literature. Also, to prevent the estimate $\hat{a}$ from drifting, (9d) can be modified to

$$\dot{\hat{a}}(t) = -\sigma\hat{a} + \gamma\frac{\varphi(\boldsymbol{x})^2\|B^{\mathrm{T}}P\boldsymbol{x}\|^2}{\varphi(\boldsymbol{x})\|B^{\mathrm{T}}P\boldsymbol{x}\| + \varepsilon}, \quad \sigma > 0, \quad \gamma > 0 \tag{13a}$$

In this case, we have the following ultimately uniformly bounded (UUB) stabilization result.

Remark 6. It is worth mentioning that none of the upper bounds on $\boldsymbol{E}(t)$, $\boldsymbol{d}(\boldsymbol{x}, t)$, $\boldsymbol{\xi}(\boldsymbol{x})$, and $\mu(t - t_f)\boldsymbol{\eta}(\boldsymbol{x}, t)$ need to be analytically estimated by the designer, and the algorithms automatically update them.

Theorem 3. Consider the nonlinear system (1). Let the Assumptions 1 ~ 3 hold. If the following robust adaptive control is applied

$$\boldsymbol{u}(t) = -(K_0 + \hat{K}(t))\boldsymbol{x} \tag{13b}$$

where K_0 is chosen such that $A - BK_0$ is Hurwitz, matrix and $\hat{K}(t)$ is generated by

$$\hat{K}(t) = \frac{\hat{a}(t)\varphi(\boldsymbol{x})^2B^{\mathrm{T}}P}{\|B^{\mathrm{T}}P\boldsymbol{x}\|\varphi(\boldsymbol{x}) + \varepsilon} \tag{13c}$$

and $\hat{a}$ is updated by (13a), then the system is ensured to UUB stable.

Proof. The result can be established by using the method similar to that as [18]. □

Remark 7. It is worth mentioning that the proposed fault-tolerant control does not involve parameters α and β as in [16], nor the iterative procedure in [7], thus no additional information on the fault model is required in control design and implementation.

3 Simulation verification

We conducted simulation on two numerical systems to test the effectiveness of the proposed method. The first one is a system with actuator failures, external disturbances, and uncertain nonlinearities. The second one is a nonlinear system with both failures from actuator and component.

3.1 Simulation 1

The nonlinear model considered is of the form (1) with the following system parameters

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & -2 \\ 0 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0.6 \\ 0.5 & 1 \end{bmatrix}$$

which is unstable but controllable. The system nonlinearities come from

$$\boldsymbol{\xi}(\boldsymbol{x}) = \begin{bmatrix} \omega_1 x_1 x_2 \sin(x_1 + x_2) \\ \omega_2 x_2 x_3 \cos(x_2 + x_3) \end{bmatrix}$$

with uncertain parameters $\omega_1 = 3\sin(t) \in [-3, 3]$, $\omega_2 = 6(1 + \sin(t)) \in [0, 12]$, and thus $\psi_\xi = |x_1 x_2| + |x_2 x_3|$. The state dependent disturbance simulated is of the form $\boldsymbol{d}(\boldsymbol{x}, t) = [0.14\sin(t)x_1^2, 0.12\cos(0.5t)x_3^2]^{\mathrm{T}}$ (thus $\psi_d = x_1^2 + x_3^2$), and the initial system states are $\boldsymbol{x}(0) = [1.4, -1, 2.1]^{\mathrm{T}}$. The uncontrollable portion of the actuator output is chosen as $\boldsymbol{E}(t) = [0.1\sin(0.3t), 0.15\cos(0.6t)]^{\mathrm{T}}$.

The actuator efficiency variables for each of the two control channels simulated are as follows

$$\rho_1(t) = \begin{cases} 1, & 0 \le t \le 5 \\ \dfrac{15 - t}{10}, & 5 < t \le 11 \\ 0.4, & 11 < t \le 14 \\ 0.7, & 14 < t \le 20 \\ 0.5, & t > 20 \end{cases}$$

$$\rho_2(t) = \begin{cases} 1, & 0 \le t \le 4 \\ 0.85 + 0.15\cos(t - 4), & 4 < t \le 10 \\ 0.5 + 0.1\sin(t - 10), & 10 < t \le 15 \\ 0.8 + 0.02(t - 20), & 15 < t \le 22 \\ 0.75, & t > 22 \end{cases}$$

as illustrated in Fig. 2.

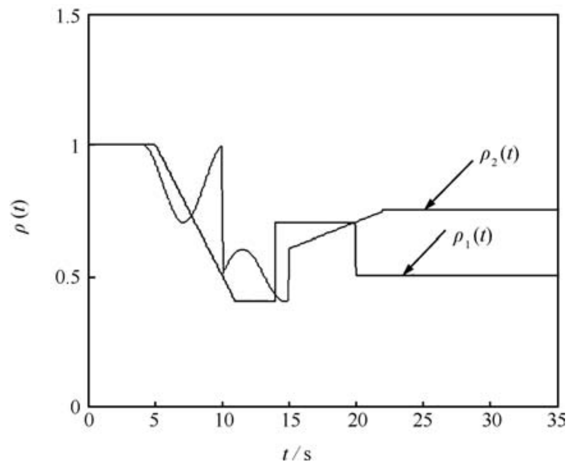

Fig. 2 Actuator failures from both control channels

It is noted that the actuator undergoes severe failures during the control process in that both channels lose effectiveness by over 50 % at some time, and the fault are time-varying. The FTC in (13) is used, where it is readily obtained that

$$\varphi(\cdot) = 1 + \|K_0 \boldsymbol{x}\| + |x_1 x_2| + |x_2 x_3| + x_1^2 + x_3^2$$

The simulation results presented in Fig. 3 correspond to $\hat{a}(0) = 0$, $\gamma = 5$, $\sigma = 0.02$, $\varepsilon = 0.01$, and

$$K_0 = \begin{bmatrix} -5.1429 & -6 & -6.8571 \\ 8.5714 & 10 & 6.4286 \end{bmatrix}$$

Fig. 4 is the estimated controller parameter $\hat{a}(t)$, which is updated automatically online. It is observed that with the proposed control scheme, good control performance is achieved and the result confirms the theoretical prediction.

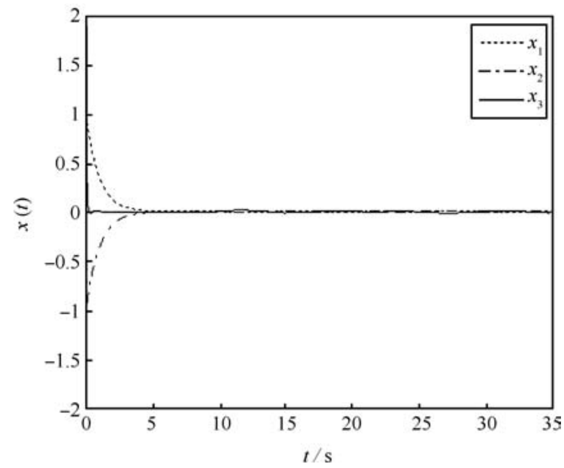

Fig. 3 Nonlinear system stabilization under actuator failures and state dependent growth external disturbances

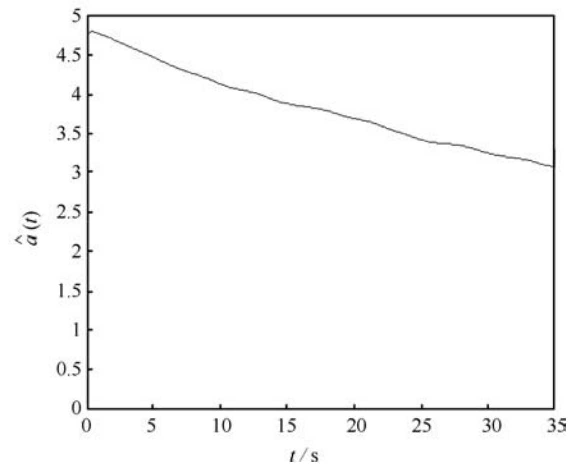

Fig. 4 Response curve of $\hat{a}(t)$

3.2 Simulation 2

The second simulation tests the effectiveness of the proposed fault-tolerant control scheme when applied to aircraft altitude stabilization[19], as shown in Fig. 5.

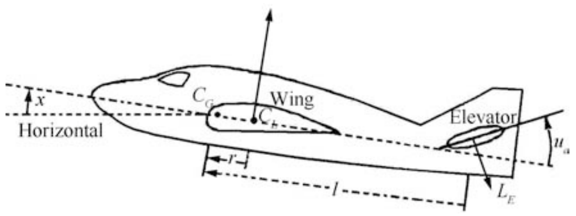

Fig. 5 Dynamic characteristics of an aircraft

The aircraft′s moment of inertia about the center of mass C_G is denoted by J, and r is the distance between the center of mass and the center of lift (positive r meaning that the center of mass is ahead of the center of lift). To control the vertical motion of the aircraft, one needs to adjust the elevator (a small surface located at the aircraft tail) by an angle u_a. This generates aerodynamic force L_E on the elevator, and thus a torque about C_G. This torque leads to a rotation of the aircraft about C_G, measured by an angle x. The sum of the lift forces applied on the aircraft wings and body is equivalent to a single lift force L_W, applied at the "center of lift" C_L. The lift force L_W applied on the wings is proportional to x, i.e., $L_W = C_{ZW}x$, where C_{ZW} is the aerodynamic coefficient. Similarly, L_E is proportional to the angle between the horizontal and the elevator, i.e., $L_E = C_{ZE}(u_a - x)$, and C_{ZE} is the aerodynamic coefficient. Furthermore, various aerodynamic forces create friction torques and aerodynamic drag of the form $k(\dot{x})\dot{x}$.

In summary, a simplified model of the aircraft vertical motion can be written into

$$J\ddot{x} + k(\dot{x})\dot{x} + (C_{ZE}l + C_{ZW}(x)r)x = C_{ZE}l(u_a + x\sin(t) + \mu(t-t_f)\eta(\cdot))$$

or

$$\ddot{x} = -\frac{k(\dot{x})}{J}\dot{x} - \frac{C_{ZE}l + C_{ZW}(x)r}{J}x + \frac{C_{ZE}l}{J}(u_a + x\sin(t) + \mu(t-t_f)\eta(\cdot))$$

which can be converted into the form of (1) with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \dfrac{C_{ZE}l}{J} \end{bmatrix}$$

and

$$\xi(x, \dot{x}) = \frac{J}{C_{ZE}l}\left(-\frac{k(\dot{x})}{J}\dot{x} - \frac{C_{ZE}l + C_{ZW}(x)r}{J}x\right)$$
$$d(x, t) = x\sin(t)$$

It is noted that in practice precise information of the aerodynamic coefficients and the aerodynamic drag is not available. To reflect this fact, we consider the case that uncertainties are involved in k and C_{ZE}, i.e.,

$$k(\dot{x}) = 4 + \dot{x}^2\cos(t), \quad C_{ZW}(x) = 5 + 0.02\sin(x), \quad r = 0.2$$

with $J = 1$, $C_{ZE} = 1$, $l = 3$. Assume that the actuator suffers from losing its effectiveness so that the elevator is partially adjustable in that $\boldsymbol{u}_a(t) = \rho(t)\boldsymbol{u}(t) + \boldsymbol{E}(t)$ with $\rho_1(t)$ denoting the degree of actuation failure, and $\boldsymbol{E}(t)$ being the uncontrollable portion of the elevator angle.

Also, we consider that a subsystem failure occurs at $t_f = 10$ (s) at which an additional term of the form $\eta(\cdot) = x^3\sin(t)$ is added to the system. Note that this is a nonlinear system with state dependent growth disturbance and subsystem failures, the control schemes developed in [18] are inapplicable. However, it can be easily dealt with by the proposed control scheme (9), where the control parameters can be chosen quite arbitrarily as

$$K_0 = [2, 3], \quad \hat{a}(0) = 0, \quad \gamma = 2, \quad \sigma = 0.013$$

and

$$\psi(x, \dot{x}) = 2|x| + |\dot{x}| + |\dot{x}^3| + |x^3|$$

The fault function used for simulation takes the form of

$$\mu(t - t_f) = \begin{cases} 1 - \mathrm{e}^{-\upsilon(t-t_f)}, & t \geq t_f \\ 0, & \text{otherwise} \end{cases}$$

where $\upsilon \in [0, \infty)$, as illustrated in Fig. 6. It is seen that $\upsilon = 0$ is the case where there is no subsystem fault and $\upsilon \to \infty$ corresponds to a jump fault.

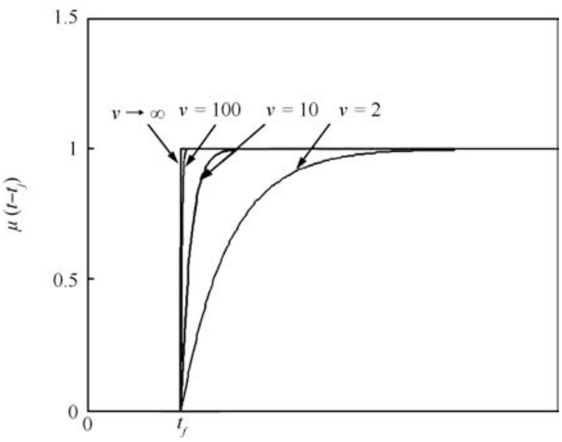

Fig. 6 Fault function profile

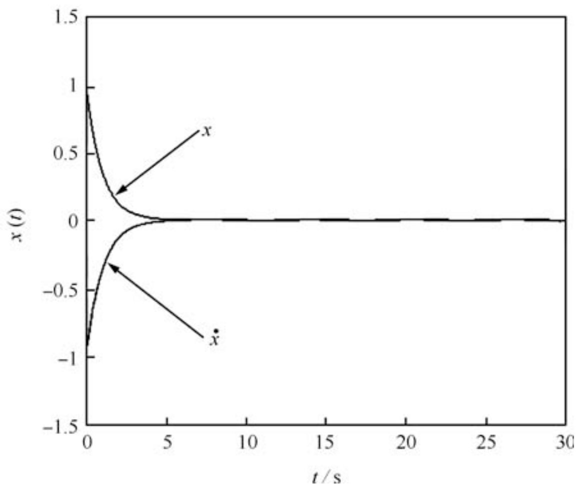

Fig. 7 Angle of attack stabilization of aircraft with actuator/subsystem faults and state dependent growth disturbances

The system response under the control of the proposed strategy (9) is shown in Fig. 7 and the control parameter $\hat{a}(t)$ is used is online updated as indicated in Fig. 8, where $\upsilon = 2$. One can observe that satisfactory control performance is achieved with the proposed control scheme in the presence of faults from actuator and subsystem as well as state dependent growth disturbance.

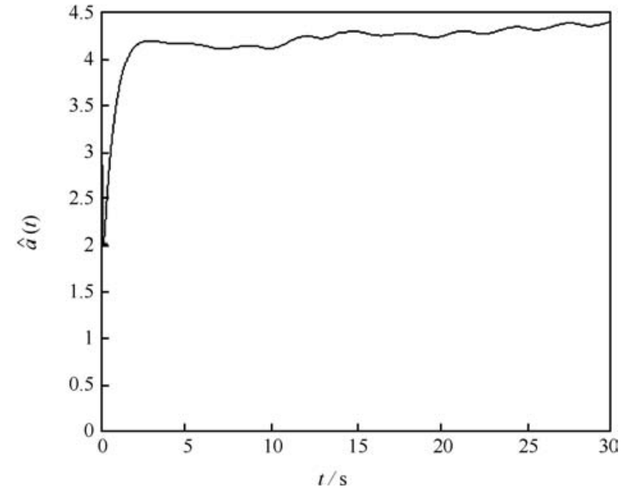

Fig. 8 Update of $\hat{a}(t)$

4 Conclusion

This paper presents a method for fault-tolerant control of a class of nonlinear systems with actuator and subsystem failures coupled with external disturbances. The result presented here is an extension and improvement of the work in [16, 17] in that it is more straightforward to design and implement the proposed control scheme, and the faults that can be coped with include both actuator and subsystem failures. The effectiveness of the developed approach is validated and confirmed by numerical simulations.

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