Robust Stabilization of Nonholonomic Chained Form Systems with Uncertainties

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Abstract A summary of recent developments concerning robust stabilization problems for the nonholonomic chained form systems with uncertainties is provided. Firstly, various models, main approaches, and results over past ten years for the uncertain chained form systems are presented. Then, several new exciting uncertain chained form models of special interest are proposed for the nonholonomic wheeled mobile robots. They are obtained by using the state and input transformations based on the visual servoing feedback. Finally, the novel robust regulation controllers are addressed for some new uncertain chained models by using two-steps technology, visual feedback, state-scaling and switching strategy. It is expected that this investigation will provide a good introduction about the development of robust stabilization for uncertain chained form systems.

Key words Nonholonomic, kinematic, mobile robot, uncertain chained system

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The term "nonholonomic system" has been widely accepted as "Lagrange system with linear constraints being non-integrable". There is an extensive list of research papers devoted to the study of stability of nonholonomic systems[1−17]. Mobile robots with velocities constraints are well-known nonholonomic nonlinear mechanical systems[18]. As explained and illustrated in [19−20] and references therein, many systems with nonholonomic constraints on velocities can be transformed, either locally or globally, to chained (form) systems by using coordinate and state-feedback transformations. Therefore, the models of nonholonomic wheeled mobile robots (NWMR) can be transformed into chained forms[11, 19−21]. Researchers have been attracted to search for an array of new control strategies around the important nonholonomic chained models[8−17]. However, in the control of nonholonomic systems, it is usually assumed that the robot states are available using sensor measurements. But in practice, some ideal conditions cannot be satisfied. There exist uncertainties in the nonholonomic systems[22−51] such as possible modeling errors, unknown parameters[22−27], external disturbances, uncertainties in the kinematic models, mechanical limitations and so on. For instance, there are unmodelled dynamics with small enough magnitude in the nonlinear chained systems in [29]. A dynamic nonholonomic mobile cart with unknown geometric and inertia parameters was considered in [40]. In this paper, we will mainly discuss the nonholonomic chained systems with uncertainties (uncertain chained systems).

For the stabilization of uncertain chained systems, many control models and strategies have been developed[28−51]. In [28], a global adaptive output feedback control strategy was presented for a class of nonholonomic systems in generalized chained form with drift nonlinearity and unknown virtual control parameters. Adaptive technology is an important method for the stabilization of the kinematic systems with uncertainties. The control design in [30] was applicable to solve the adaptive regulation of the parking problem of the robot in the presence of angle measurement errors. An interesting hybrid feedback algorithm based on Morse′s pioneering work in supervisory adaptive control was presented to globally asymptotically stabilize a wheeled mobile robot with parametric uncertainty[31]. In [32−35] and some references therein, adaptive global stabilization of nonholonomic systems with strong nonlinear drifts were investigated by using input-to-state scaling technology and adaptive state feedback control. Some new adaptive output feedback switching control strategies with time varying matrix observer, state estimator, or parametric estimator had been proposed[34, 37−38]. Exponential convergence is a desired performance for practical applications. The exponentially stabilizing and regulating problems have been investigated[40−43] for classes of uncertain chained systems in recent ten years. In [40], a smooth time-varying control scheme was developed, guaranteeing the convergence of the state variables to the desired set point exponentially despite unknown parameters. In [41], the globally exponentially converging robust dynamic output feedback law was proposed in perturbed chained form and with uncertain drift nonlinearity. This class of uncertain chained systems was motivated by the robust redesign of low-dimensional nonholonomic mechanical systems. A kind of exponential stabilization feedback control laws, or globally exponentially converging robust dynamic output feedback laws were obtained[42−43] by the use of switching algorithm and the input/state scaling. Finite time stabilization for an uncertain chained system was discussed in [44]. A novel switching control strategy[44] was proposed with the help of homogeneity, time-rescaling, and Lyapunov-based method.

Visual feedback[52−58] is also an important approach to improve the control performance of manipulators in the control procedure since it mimics the human sense of vision and allows operation on the basis of non-contact measurement and an unstructured environment. The adaptive tracking controller via visual servoing was developed for a mobile robot when the camera is onboard[53]. If a mobile robot with nonholonomic constraints is equipped with uncalibrated camera, we find that, uncertain kinematic systems in image plane can be obtained and they can also be transformed into chained systems with uncalibrated parameters. Therefore, several exciting new uncertain chained models for NWMR can be deduced by using coordinate and input transformations. They will be presented in Example 2 and Section 3. It is of interest to note that the models are different from the uncertain chained systems

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mentioned above. The uncertain nonlinearities do not satisfy triangularity conditions (see Subsection 2.2.1). In order to stabilize the uncertain chained systems, the equivalent uncertain system for type $(2,0)$ robot[18] was investigated at first[45−48]. Then, the robust regulation and tracking problems were solved for this mobile robot. Further, robust stabilizing controllers[49−50] are designed for uncertain chained systems of type $(1,1)$ and type $(1,2)$ robots[18] in particular case.

The main contribution of this paper is threefold. Firstly, we introduce the developments of robustly stabilizing classes of uncertain chained systems. The uncertain chained models, approaches, and main results over past ten years are presented. Secondly, based on visual servoing feedback, a series of new uncertain chained models and several general uncertain chained models are proposed by using the state and input transformations. They are different from those models studied in the past years. Finally, the novel time-varying controllers are presented to stabilize the uncertain chained systems by exploiting a new two-step technique or using switching scheme for type $(2,0)$, type $(1,1)$ and type $(1,2)$ robots.

The rest paper is organized as follows. In Section 1, two motivating practical examples are given for the uncertain chained models. Section 2 addresses a few important uncertain chained models, schemes and some results which have been proposed over past decade. In Section 3, the special new exciting nonholonomic uncertain chained models and several new general uncertain chained models are presented for NWMR. Section 4 addresses novel time-varying controllers to stabilize the uncertain chained systems for three types of robots. Conclusions and future work are presented in the last section.

1 Examples

In practice, there are many interesting motivating examples which stimulate researchers to expand them into broad classes and further investigated them. In this section, we introduce two nonholonomic kinematic examples with uncertainties.

1.1 Example 1

A simple tricycle-type mobile robot (see Fig. 1) with nonholonomic constraints on the linear velocity has often been used as a benchmark example in nonholonomic control systems design[2, 41]. This robot is called type $(2,0)$ robot. It has two fixed front wheels with one axis and a rear castor wheel which prevents the robot from tipping over as it moves on a plane. The nonholonomic constraint is defined by

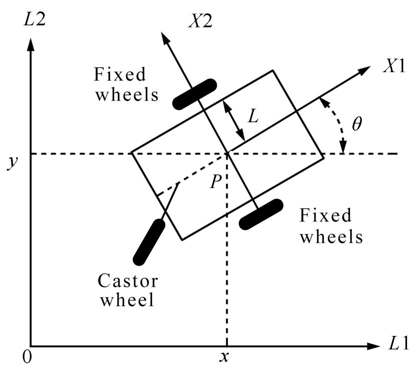

Fig. 1 Tricycle-type mobile robot

$$\dot{x}\sin\theta - \dot{y}\cos\theta = 0$$

where (x, y) denotes the position P of the center of mass, which is also located in the middle of the axis of the two front wheels, θ is the angle between the $L1$ axis and the $X1$ axis with a positive anticlockwise direction. Using this formula, the kinematics of the robot can be modeled by the following differential equations[2]

$$\left\{\begin{array}{rcl}\dot{x} &=& v\cos\theta\\ \dot{y} &=& v\sin\theta\\ \dot{\theta} &=& \omega\end{array}\right. \tag{1}$$

where v is the forward velocity while ω is the angular velocity of the robot.

For (1), by taking the following state and input transformation

$$\left\{\begin{array}{rcl}x_0 &=& x\\ x_1 &=& y\\ x_2 &=& \tan\theta\\ u_0 &=& v\cos\theta\\ u_1 &=& (\sec\theta)^2\omega\end{array}\right.$$

we obtain

$$\left\{\begin{array}{rcl}\dot{x}_0 &=& u_0\\ \dot{x}_1 &=& x_2u_0\\ \dot{x}_2 &=& u_1\end{array}\right. \tag{2}$$

System (2) is so-called canonical chained form with three-order and two control inputs. However, it only represents the modeling of the robot in the ideal case. In [31], Hespanha and his co-authors addressed the parking problem for a mobile robot of the unicycle type in the presence of parametric uncertainties.

$$\left\{\begin{array}{rcl}\dot{x} &=& p_1^*v\cos\theta\\ \dot{y} &=& p_1^*v\sin\theta\\ \dot{\theta} &=& p_2^*\omega\end{array}\right. \tag{3}$$

where p_1^* and p_2^* are (unknown) positive parameters determined by the radius of the front wheels and the distance between them. Taking the following change of coordinates and feedback

$$\left\{\begin{array}{rcl}x_0 &=& \theta\\ x_1 &=& x\sin\theta - y\cos\theta\\ x_2 &=& x\cos\theta + y\sin\theta\\ u_0 &=& \omega\\ u_1 &=& v\end{array}\right.$$

system (3) can be transformed into

$$\left\{\begin{array}{rcl}\dot{x}_0 &=& p_2^*u_0\\ \dot{x}_1 &=& p_2^*x_2u_0\\ \dot{x}_2 &=& p_1^*u_1 - p_2^*x_1u_0\end{array}\right. \tag{4}$$

comparing (2) with (4), the first terms on the right side of corresponding equations are identical except the unknown parameters p_1^* and p_2^*. So, system (4) is an uncertain chained system.

1.2 Example 2

The second interesting example is from [45, 53]. In Fig. 2, it is assumed that a pinhole camera is fixed to the ceiling. The mobile robot mentioned in Example 1 is under the camera. There are three coordinate frames, namely the inertial frame X-Y-Z, the camera frame x-y-z, and the image frame u-o_1-v. C is the intersection point of the optical axis of the camera with the X-Y plane. Its coordinate relative to the X-Y plane is (c_x, c_y). The coordinate of the original point of the camera frame with respect to the image frame is defined by (O_{c1}, O_{c2}). (x, y) is the coordinate of the mass center P of the robot with respect to the X-Y plane. Suppose that (x_m, y_m) is the coordinate of (x, y) relative to the image frame. The pinhole camera model yields

$$\begin{bmatrix} x_m \\ y_m \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} R \left[\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} c_x \\ c_y \end{bmatrix} \right] + \begin{bmatrix} O_{c1} \\ O_{c2} \end{bmatrix} \tag{5}$$

where α_1, α_2 are constants[53], which are dependent on the depth information, focal length, and scalar factors along the u axis and v axis, respectively.

$$R = \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix}$$

where θ_0 denotes the angle between the u axis and the X axis with a positive anticlockwise orientation.

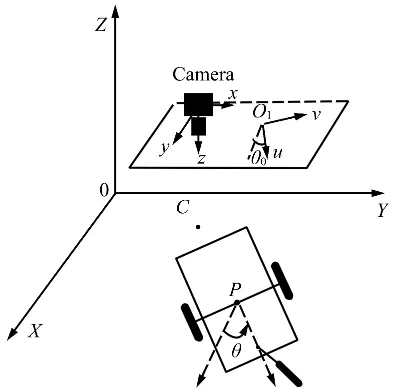

Fig. 2 Wheeled mobile robots with monocular camera

Then, we have

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \tag{6}$$

Considering the mobile robot described by (1) with monocular camera, the kinematic model in the image frame is

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \alpha_1 v \cos(\theta - \theta_0) \\ \alpha_2 v \sin(\theta - \theta_0) \\ \omega \end{bmatrix} \tag{7}$$

Taking the following state and input transformation

$$\begin{cases} x_0 = x_m \\ x_1 = y_m \\ x_2 = \tan\theta \\ u_0 = v\cos\theta \\ u_1 = (\sec\theta)^2\omega \end{cases}$$

we obtain the uncertain chained system

$$\begin{cases} \dot{x}_0 = (\alpha_1\cos\theta_0)u_0 + (\alpha_1\sin\theta_0)x_2u_0 \\ \dot{x}_1 = (\alpha_2\cos\theta_0)x_2u_0 - (\alpha_2\sin\theta_0)u_0 \\ \dot{x}_2 = u_1 \end{cases} \tag{8}$$

where α_1, α_2, and θ_0 are three unknown parameters (in practice, they are usually uncalibrated). By using the same methods, a series of new exciting uncertain chained models about NWMR can be obtained, which are presented in Section 3 in this paper.

2 Control models and methodologies for uncertain chained systems

As indicated in introduction, many researchers have been investigating the stabilization problems for uncertain chained systems. In this section, we will present the important classes of uncertain chained models and the main control methodologies about adaptive stabilization, exponential stabilization, and finite settling time stabilization developed in the past ten years.

2.1 Adaptive stabilization

In this subsection, the main results of adaptive state feedback control[31−35] and adaptive output feedback control[38] for different classes of uncertain chained systems are presented.

2.1.1 Adaptive state feedback stabilization

In 2002, the following perturbed canonical nonholonomic system was studied by Do et al.[32]

$$\begin{cases} \dot{x}_0 = u_0 + \varphi_0(x_0)^{\mathrm{T}}\theta \\ \dot{x}_i = u_0x_{i+1} + \varphi_i(x_0, u_0, x_1, \cdots, x_i)^{\mathrm{T}}\theta, \quad 1 \le i \le n-1 \\ \dot{x}_n = u_1 + \varphi_n(x_0, u_0, x)^{\mathrm{T}}\theta \end{cases} \tag{9}$$

where $\varphi_0(x_0) \in \mathbf{R}^p$, $\varphi_i(x_0, u_0, x_1, \cdots, x_i) \in \mathbf{R}^p$, $x = [x_1, \cdots, x_n]^{\mathrm{T}}$, and $\theta \in \mathbf{R}^p$ is a vector of unknown bounded constant parameters.

For (9), by using input-to-state scaling, backstepping technique, and switching scheme in two separate stages, the new adaptive controller was designed which made the uncertain chained system adaptive and globally stabilized.

In 2003, Ge et al.[34] investigated another class of uncertain chained system. A novel adaptive switching technique was employed to overcome the uncontrollability problem. Other researchers such as Wang et al.[35] and Gao et al.[36] also discussed different classes of uncertain chained system. By using parameter separation, state scaling, backstepping technique or switching strategy, adaptive asymptotic regulation of the closed-loop system was achieved.

2.1.2 Adaptive output feedback stabilization

The adaptive output feedback stabilization had been investigated for different uncertain chained systems in many papers[34, 37−38].

In [34], Ge et al. considered the output feedback model expressed by

$$\begin{cases} \dot{x}_0 = u_0 + c_0x_0 \\ \dot{x}_i = u_0x_{i+1} + \phi_i^{\mathrm{T}}(u_0, x_0, \bar{x}_i)\theta, \quad 1 \le i < n, \ n \ge 2 \\ \dot{x}_n = u_1 + \phi_n^{\mathrm{T}}(u_0, x_0, x)\theta \\ y = (x_0, x_1)^{\mathrm{T}} \end{cases} \tag{10}$$

where c_0 is known, $[x_0, x^{\mathrm{T}}] = [x_0, x_1, \cdots, x_n]$, $\bar{x}_i = [x_1, \cdots, x_i]^{\mathrm{T}} \in \mathbf{R}^i$, $\phi_i(u_0, x_0, \bar{x}_i) \in \mathbf{R}^l$, $1 \le i \le n$, $\theta \in \mathbf{R}^l$ is a vector of unknown bounded constant parameters.

For (10), it is assumed that for each i $(1 \le i \le n)$ there is a known smooth function vector $\bar{\phi}_i$ which is a function of u_0 and the available states x_0 and x_1 such that

$$\phi_i(u_0, x_0, \bar{x}_i) = x_1\bar{\phi}_i(u_0, x_0, x_1)$$

In [34], a filtered observer rather than the traditional linear observer was used to handle the technical problem due to the presence of unavailable states in the regressor matrix. The proposed control strategies can steer the system states converging to the origin globally while the estimated parameters remain bounded.

In 2009, a new nonholonomic system with strong nonlinear drifts was developed[38] by Zheng et al.. The model is with modeled nonlinear dynamics, unmodeled dynamics, and unknown parameters. A novel observer and estimator were introduced for states and parameter estimates, respectively. A constructive procedure of design for an output feedback adaptive controller was given by using the integrator backstepping approach which was based on the proposed observer and parameter estimator. Then, the system discussed was globally asymptotically stable.

In [39], Wang and her co-authors dealt with another class of uncertain chained system. The robust adaptive NN control laws were developed using state scaling and backstepping in two separate stages. The system states have been proved to converge to a small neighborhood of zero by appropriately choosing the design parameters while all other signals in the closed-loop have been guaranteed to be uniformly ultimately bounded.

2.2 Exponential stabilization

In 2000, motivated by Example 1, Jiang[41] proposed a perturbed version of the chained form and investigated exponential stabilization in three cases. They are global exponential regulation, global dynamic extension regulation, and output feedback regulation. In recent years, Xi et al.[42−43] also considered classes of uncertain chained systems and then investigated their global exponential stabilization and output feedback exponential stabilization problems. The main results are presented below.

2.2.1 Global exponential regulation (GER)

In [41], the discussed uncertain chained system is

$$\begin{cases} \dot{x}_0 &= d_0(t)u_0 + \phi_0^d(t, x_0) \\ \dot{x}_1 &= d_1(t)x_2u_0 + \phi_1^d(t, x_0, x, u_0) \\ &\vdots \\ \dot{x}_{n-2} &= d_{n-2}(t)x_{n-1}u_0 + \phi_{n-2}^d(t, x_0, x, u_0) \\ \dot{x}_{n-1} &= d_{n-1}(t)u + \phi_{n-1}^d(t, x_0, x, u_0) \end{cases} \tag{11}$$

where $x = (x_1, \cdots, x_{n-1}) \in \mathbf{R}^{n-1}$, the functions $d_i(\cdot)$ and $\phi_i^d(\cdot)$ $(i = 0, 1, \cdots, n-1)$ denote the possible modeling error and neglected dynamics, which include uncertain drift terms.

It is assumed that

$$\begin{aligned} &c_{i1} \leq d_i(t) \leq c_{i2}, \quad 0 \leq i \leq n-1, \ \forall\, t \geq 0 \\ &\left|\phi_0^d(t, x_0)\right| \leq c_{03}\,|x_0|\,, \quad c_{03} > 0 \\ &\left|\phi_i^d(t, x_0, x, u_0)\right| \leq |(x_1, \cdots, x_i)|\,\phi_i(x_0, x_1, \cdots, x_i, u_0) \end{aligned} \tag{12}$$

where ϕ_i $(i = 1, \cdots, n-1)$ are known smooth non-negative functions, and $(t, x_0, x, u_0) \in \mathbf{R}^+ \times \mathbf{R} \times \mathbf{R}^{n-1} \times \mathbf{R}$. It is worth noting that assumption (12) imposes the uncertain nonlinearities ϕ_i^d $(i = 1, \cdots, n-1)$ to satisfy the triangularity condition.

For (11), a discontinuous state scaling transformation was first introduced to obtain a scalar system and a simpler disturbed low dimensional system. Then, the well-known backstepping method was used to design a robust global exponential stabilizer for the transformed lower-size system. Finally a robust nonlinear state feedback law was proposed and the global exponential regulation problems were solved[41].

In [41], Jiang also discussed the following dynamic extension:

$$\begin{cases} \dot{u}_0 &= v_0 \\ \dot{x}_0 &= u_0 + c_0x_0 \\ \dot{x}_1 &= d_1(t)x_2u_0 + \phi_1^d(t, x_0, x, u_0) \\ &\vdots \\ \dot{x}_{n-2} &= d_{n-2}(t)x_{n-1}u_0 + \phi_{n-2}^d(t, x_0, x, u_0) \\ \dot{x}_{n-1} &= d_{n-1}(t)u + \phi_{n-1}^d(t, x_0, x, u_0) \\ \dot{u} &= d_n(t)v + \phi_n^d(t, x_0, x, u_0, u) \end{cases} \tag{13}$$

where $x = (x_1, \cdots, x_{n-1}) \in \mathbf{R}^{n-1}$, $d_i(\cdot)$ and $\phi_i^d(\cdot)$ $(i = 1, 2, \cdots, n)$ denote the possible modeling error and neglected dynamics. v_0 and v are considered as the (torque) control input, the constant c_0 is assumed to be known.

It is assumed that there exist a positive constant c_n and a smooth non-negative function ϕ_n such that

$$\begin{aligned} &c_n \leq d_n(t) \\ &\left|\phi_n^d(t, u_0, x_0, x, u)\right| \leq |(x, u)|\,\phi_n(u_0, x_0, x, u) \end{aligned} \tag{14}$$

Under assumptions (12) and (14), Jiang proposed the backstepping based controller and switching control strategy such that system (13) is GER.

In 2003, Xi et al. investigated another uncertain chained system[42]. A novel switching control strategy was proposed involving the use of input/state scaling and integrator backstepping. The new controllers can make the uncertain system Lyapunov stable and globally κ-exponentially convergent.

In 2005, Ma designed a new time-varying robust controller to yield global exponential convergence of cart′s position and orientation to the desired point[40].

2.2.2 Output-feedback regulation

In 2007, Xi et al. dealt with output feedback system with uncertain chained form[43] described by

$$\begin{cases} \dot{x}_0 &= u_0 + x_0\phi_0(t, x_0) \\ \dot{x}_1 &= x_2u_0 + \phi_1^d(t, x_0, x_1, u_0) \\ &\vdots \\ \dot{x}_{n-2} &= x_{n-1}u_0 + \phi_{n-2}^d(t, x_0, x_1, u_0) \\ \dot{x}_{n-1} &= u + \phi_{n-1}^d(t, x_0, x_1, u_0) \\ y &= (x_0, x_1)^{\mathrm{T}} \end{cases} \tag{15}$$

where $x = (x_1, \cdots, x_{n-1}) \in \mathbf{R}^{n-1}$, the functions $\phi_i^d(\cdot)$ denote the possible modeling error and neglected dynamics, and $\phi_0(t, x_0)$ is a known smooth function.

It is assumed that for every $1 \leq i \leq n-1$ and all $(t, x_0, x, u_0) \in \mathbf{R}^+ \times \mathbf{R} \times \mathbf{R}^{n-1} \times \mathbf{R}$, there are (known) smooth non-negative functions ϕ_i such that

$$\left|\phi_i^d(t, x_0, x_1, u_0)\right| \leq |x_1|\,\phi_i(t, y, u_0)$$

For (15), a systematic control design procedure to construct a robust nonlinear output feedback control law was presented. Furthermore, two special cases were considered which do not use the observer gain filter. By using a particular input-state scaling, the backstepping technique, the switching scheme and observer gain filter to design a dynamic output feedback controller, both robust global exponential regulation and Lyapunov stability with output feedback were obtained for a class of disturbed nonlinear chained systems (15).

In [41], Jiang also extended the state feedback results to the output-feedback case. By using the modified recursive design scheme, the dynamic output-feedback control law was proposed with suitable design parameters along with the switching control strategy. The uncertain chained system was globally exponentially regulated at the origin.

2.3 Finite time stabilization

In [44], Hong et al. investigated an uncertain chained system given by

$$\begin{cases} \dot{x}_0 &= q_0u_0 \\ \dot{x}_1 &= q_1x_2u_0 \\ &\vdots \\ \dot{x}_{n-1} &= q_{n-1}x_nu_0 \\ \dot{x}_n &= q_nu+\psi_n(x) \end{cases} \tag{16}$$

where $x=(x_1,\cdots,x_n)^{\rm T}\in\mathbf{R}^n$, $q_i>0$ $(i=0,\cdots,n)$ are uncertain parameters but they are located in known intervals (i.e., $0<q_i\in[q_i^{\min},q_i^{\max}]$).

It is assumed that $\psi_n(x)$ is an uncertain function satisfying

$$|\psi_n(x)|\le M\sum_{i=1}^n|x_i|,\quad M>0 \tag{17}$$

For (16), a novel control design procedure was proposed to construct a switching nonlinear control scheme that solved the problem of finite time convergence and Lyapunov stability for these nonholonomic systems, where a time-rescaling technique was employed in stabilizing these controlled systems within any given settling time. By using homogeneity, time-rescaling and Lyapunov function techniques, a finite-time stabilizing feedback law was designed to guarantee both Lyapunov stability and finite time convergence in any given settling time for the closed-loop system.

To sum up, many control strategies were developed to stabilize the uncertain chained systems. However, it is obviously seen that most of the unknown nonlinear functions of uncertain chained systems mentioned in this section satisfy triangularity conditions. Motivated by Example 2, a series of new exciting uncertain chained models for NWMR are obtained which do not satisfy the so-called triangularity conditions. They are presented in the next section.

3 New uncertain chained models of NWMR

In Example 2 of Section 1, the uncertain nonholonomic kinematic systems with visual feedback were considered for type $(2,0)$ robot. As indicated in [18], all nonholonomic mobile robots can be classified into four types called type $(2,0)$, type $(2,1)$, type $(1,1)$ and type $(1,2)$. A car towing a single trailer or towing n trailers is also a nonholonomic system. By using the same method as Example 2, we can obtain the following new uncertain chained models based on visual feedback and the state-input transformations.

3.1 Uncertain chained models of type (2, 0) robot

For the nonholonomic system of type $(2,0)$ robot, two different uncertain chained models can be obtained by using two different transformations.

3.1.1 Uncertain chained model 1

For type $(2,0)$ robot[18], the posture kinematic model is described by

$$\begin{cases} \dot{x} &= -v\sin\theta \\ \dot{y} &= v\cos\theta \\ \dot{\theta} &= \omega \end{cases} \tag{18}$$

where θ denotes the angle between the X axis and the heading direction of the robot with a clockwise direction.

Considering formula (6), we have

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\alpha_1v\sin(\theta-\theta_0) \\ \alpha_2v\cos(\theta-\theta_0) \\ \omega \end{bmatrix} \tag{19}$$

Taking the state-input transformation

$$\begin{cases} x_0 &= \theta \\ x_1 &= x_m\cos\theta+y_m\sin\theta \\ x_2 &= -x_m\sin\theta+y_m\cos\theta \\ u_0 &= \omega \\ u_1 &= v-x_1u_0 \end{cases}$$

we obtain the following system

$$\begin{cases} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= x_2u_0+\dfrac{1}{2}(x_1u_0+u_1)(\alpha_{12}s_0+\alpha_{21}s_\Theta) \\ \dot{x}_2 &= u_1+\dfrac{1}{2}(x_1u_0+u_1)(\alpha_{12}c_0+\alpha_{21}c_\Theta-2) \end{cases} \tag{20}$$

where

$$\begin{matrix} \alpha_{12}=\alpha_1+\alpha_2, & s_0=\sin\theta_0, & s_\Theta=\sin(2x_0-\theta_0) \\ \alpha_{21}=\alpha_2-\alpha_1, & c_0=\cos\theta_0, & c_\Theta=\cos(2x_0-\theta_0) \end{matrix} \tag{21}$$

(20) cannot be regarded as a special case of the models in Section 2, because the second term on the right side of the second equation is dependent on u_1. It does not satisfy the triangularity condition in (12). Therefore (20) is a new uncertain chained system.

3.1.2 Uncertain chained model 2

For system (19), by taking another state-input transformation

$$\begin{cases} x_0 &= y_m \\ x_1 &= -x_m \\ x_2 &= \tan\theta \\ u_0 &= v\cos\theta \\ u_1 &= (\sec\theta)^2\omega \end{cases}$$

one can obtain another uncertain chained model as follows

$$\begin{cases} \dot{x}_0 &= (\alpha_2\cos\theta_0)u_0+(\alpha_2\sin\theta_0)x_2u_0 \\ \dot{x}_1 &= (-\alpha_1\cos\theta_0)x_2u_0+(\alpha_1\sin\theta_0)u_0 \\ \dot{x}_2 &= u_1 \end{cases} \tag{22}$$

Expression (22) also cannot be regarded as a special case of the models discussed previously because the second term on the right side of the first equation is dependent on x_2. The triangularity condition is not satisfied.

3.2 Uncertain chained model of type (2, 1) robot

For type $(2,1)$ robot[18], the posture kinematic model is of the form

$$\begin{cases} \dot{x} &= -v_1\sin(\theta+\beta) \\ \dot{y} &= v_1\cos(\theta+\beta) \\ \dot{\theta} &= v_2 \\ \dot{\beta} &= v_3 \end{cases} \tag{23}$$

Taking the transformations $\varphi=\theta+\beta$ and $v=v_2+v_3$, we can convert (23) into (18). Except for another variables that can be controlled by an independent control input, there is nothing new. But it is a system with three inputs, and may be regarded as an expansion of (18).

3.3 Uncertain chained model of type (1, 1) robot

For type $(1,1)$ robot[18], its equation of the posture kinematic model can be expressed by

$$\left\{\begin{array}{rcl} \dot{x} &=& -Lv\sin\theta\sin\beta \\ \dot{y} &=& Lv\cos\theta\sin\beta \\ \dot{\theta} &=& v\cos\beta \\ \dot{\beta} &=& \omega \end{array}\right. \tag{24}$$

Considering formula (6), we have

$$\left[\begin{array}{c} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \\ \dot{\beta} \end{array}\right] = \left[\begin{array}{c} -Lv\alpha_1\sin\beta\sin(\theta-\theta_0) \\ Lv\alpha_2\sin\beta\cos(\theta-\theta_0) \\ v\cos\beta \\ \omega \end{array}\right]$$

Take the state-input transformation

$$\left\{\begin{array}{rcl} x_0 &=& \theta \\ x_1 &=& x_m\cos\theta + y_m\sin\theta \\ x_2 &=& -x_m\sin\theta + y_m\cos\theta \\ x_3 &=& L\tan\beta - x_m\cos\theta - y_m\sin\theta \\ u_0 &=& v\cos\beta \\ u_1 &=& L\omega\sec^2\beta - x_2 v\cos\beta \end{array}\right.$$

and note that

$$\sin\theta\cos(\theta-\theta_0) = \frac{1}{2}(s_0 + s_\Theta), \quad x_3 = L\tan\beta - x_1$$
$$\cos\theta\cos(\theta-\theta_0) = \frac{1}{2}(c_0 + c_\Theta), \quad Lv\sin\beta = (x_1+x_3)u_0$$

where s_0, c_0, s_Θ, c_Θ are defined in (21). The uncertain chained form system is obtained as follows

$$\left\{\begin{array}{rcl} \dot{x}_0 &=& u_0 \\ \dot{x}_1 &=& x_2u_0 + \dfrac{1}{2}(x_1+x_3)(\alpha_{12}s_0 + \alpha_{21}s_\Theta)u_0 \\ \dot{x}_2 &=& x_3u_0 + \dfrac{1}{2}(x_1+x_3)(\alpha_{12}c_0 + \alpha_{21}c_\Theta - 2)u_0 \\ \dot{x}_3 &=& u_1 - \dfrac{1}{2}(x_1+x_3)(\alpha_{12}s_0 + \alpha_{21}s_\Theta)u_0 \end{array}\right. \tag{25}$$

It is obviously seen that the second terms on the right side of the second and third equations of (25) are dependent on x_3, so the triangularity conditions cannot be satisfied. This means that (25) also cannot be regarded as a special case of the models in Section 2.

3.4 Uncertain chained model of type (1, 2) robot

The posture kinematic equation of type $(1,2)$ robot[18] can be given by

$$\left\{\begin{array}{rcl} \dot{x} &=& -Lv_1[\sin\beta_1\sin(\theta+\beta_2) + \sin\beta_2\sin(\theta+\beta_1)] \\ \dot{y} &=& Lv_1[\sin\beta_1\cos(\theta+\beta_2) + \sin\beta_2\cos(\theta+\beta_1)] \\ \dot{\theta} &=& v_1\sin(\beta_2-\beta_1) \\ \dot{\beta}_1 &=& v_2 \\ \dot{\beta}_2 &=& v_3 \end{array}\right. \tag{26}$$

where v_1, v_2, v_3 denote the virtual velocity of robot and two angular velocities of steering wheels respectively.

For system (26), take the state and input transformations as follows

$$\left\{\begin{array}{rcl} x_0 &=& \theta \\ x_1 &=& xc + ys \\ x_2 &=& -xs + yc - 2L\dfrac{s_1s_2}{s_{21}} \\ x_3 &=& xs - yc \\ x_4 &=& xc + ys - L\dfrac{s_{12}}{s_{21}} \end{array}\right.$$

$$\left\{\begin{array}{rcl} u_0 &=& v_1s_{21} \\ u_1 &=& -x_4v_1s_{21} - 2Lv_2\dfrac{s_2^2}{s_{21}^2} + 2Lv_3\dfrac{s_1^2}{s_{21}^2} \\ u_2 &=& x_2v_1s_{21} - Lv_2\dfrac{\sin(2\beta_2)}{s_{21}^2} + Lv_3\dfrac{\sin(2\beta_1)}{s_{21}^2} \end{array}\right.$$

where $s_{21} \neq 0$, and

$$\begin{array}{l} s = \sin\theta,\ s_1 = \sin\beta_1,\ s_{12} = \sin(\beta_1+\beta_2) \\ c = \cos\theta,\ s_2 = \sin\beta_2,\ s_{21} = \sin(\beta_2-\beta_1) \end{array} \tag{27}$$

The following system yields[20]

$$\left\{\begin{array}{rcl} \dot{x}_0 &=& u_0 \\ \dot{x}_1 &=& x_2u_0 \\ \dot{x}_2 &=& u_1 \\ \dot{x}_3 &=& x_4u_0 \\ \dot{x}_4 &=& u_2 \end{array}\right. \tag{28}$$

This is commonly called canonical chained form with two chains and three inputs. If (x, y) is measured using a camera with uncalibrated visual parameters, the uncertain chained form of type $(1,2)$ robot can be obtained by means of the same method employed for type $(1,1)$ robot.

For system (26), by using formula (6) we have

$$\left[\begin{array}{c} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \\ \dot{\beta}_1 \\ \dot{\beta}_2 \end{array}\right] = \left[\begin{array}{c} -\alpha_1Lv_1(s_1s_{\triangle 2} + s_2s_{\triangle 1}) \\ \alpha_2Lv_1(s_1c_{\triangle 2} + s_2c_{\triangle 1}) \\ v_1s_{21} \\ v_2 \\ v_3 \end{array}\right] \tag{29}$$

where s_1, s_2 and s_{21} are denoted in (27), and

$$s_{\triangle i} = \sin(\theta-\theta_0+\beta_i),\ c_{\triangle i} = \cos(\theta-\theta_0+\beta_i),\ i = 1,2$$

Taking the following state and input transformations

$$\left\{\begin{array}{rcl} x_0 &=& \theta \\ x_1 &=& x_mc + y_ms \\ x_2 &=& -x_ms + y_mc - 2L\dfrac{s_1s_2}{s_{21}} \\ x_3 &=& x_ms - y_mc \\ x_4 &=& x_mc + y_ms - L\dfrac{s_{12}}{s_{21}} \end{array}\right. \tag{30}$$

$$\left\{\begin{array}{rcl} u_0 &=& v_1s_{21} \\ u_1 &=& -x_4v_1s_{21} - 2Lv_2\dfrac{s_2^2}{s_{21}^2} + 2Lv_3\dfrac{s_1^2}{s_{21}^2} \\ u_2 &=& x_2v_1s_{21} - Lv_2\dfrac{\sin(2\beta_2)}{s_{21}^2} + Lv_3\dfrac{\sin(2\beta_1)}{s_{21}^2} \end{array}\right. \tag{31}$$

and noting that $\theta = x_0$,

$$cc_{\triangle 1} = \frac{1}{2}c_{10} + \frac{1}{2}c_{\Lambda 1},\ s_1s_{20} + s_2s_{10} = 2s_1s_2c_0 - s_0s_{12}$$
$$cc_{\triangle 2} = \frac{1}{2}c_{20} + \frac{1}{2}c_{\Lambda 2},\ s_1c_{20} + s_2c_{10} = 2s_1s_2s_0 + c_0s_{12}$$
$$sc_{\triangle 1} = -\frac{1}{2}s_{10} + \frac{1}{2}s_{\Lambda 1},\ s_1s_{\Lambda 2} + s_2s_{\Lambda 1} = 2s_1s_2c_\Theta + s_{12}s_\Theta$$
$$sc_{\triangle 2} = -\frac{1}{2}s_{20} + \frac{1}{2}s_{\Lambda 2},\ s_1c_{\Lambda 2} + s_2c_{\Lambda 1} = -2s_1s_2s_\Theta + s_{12}c_\Theta$$

where s_0, c_0, α_{12}, α_{21}, s_Θ, and c_Θ are defined in (21), and for $i = 1, 2$,

$$s_{i0} = \sin(\beta_i - \theta_0),\ \ s_{\Lambda i} = \sin(2\theta - \theta_0 + \beta_i)$$
$$c_{i0} = \cos(\beta_i - \theta_0),\ \ c_{\Lambda i} = \cos(2\theta - \theta_0 + \beta_i)$$

we have the following uncertain chained form[50]

$$\left\{\begin{array}{rcl}\dot{x}_0 &=& u_0\\ \dot{x}_1 &=& x_2u_0 + \dfrac{1}{2}(x_1 - x_4)(\alpha_{12}s_0 + \alpha_{21}s_\Theta)u_0 - \\ && \dfrac{1}{2}(x_2 + x_3)(2 - \alpha_{12}c_0 + \alpha_{21}c_\Theta)u_0\\ \dot{x}_2 &=& u_1 - \dfrac{1}{2}(x_2 + x_3)(\alpha_{12}s_0 - \alpha_{21}s_\Theta)u_0 - \\ && \dfrac{1}{2}(x_1 - x_4)(2 - \alpha_{12}c_0 - \alpha_{21}c_\Theta)u_0\\ \dot{x}_3 &=& x_4u_0 + \dfrac{1}{2}(x_2 + x_3)(\alpha_{12}s_0 - \alpha_{21}s_\Theta)u_0 + \\ && \dfrac{1}{2}(x_1 - x_4)(2 - \alpha_{12}c_0 - \alpha_{21}c_\Theta)u_0\\ \dot{x}_4 &=& u_2 + \dfrac{1}{2}(x_1 - x_4)(\alpha_{12}s_0 + \alpha_{21}s_\Theta)u_0 - \\ && \dfrac{1}{2}(x_2 + x_3)(2 - \alpha_{12}c_0 + \alpha_{21}c_\Theta)u_0\end{array}\right. \tag{32}$$

In contrast to the canonical chained model (28), three new parameters α_1, α_2, and θ_0 exist in (32) which are usually uncalibrated. So, model (32) is an uncertain chained form with two chains and three inputs. It is obvious that states x_2, x_3, and x_4 appear in the second and third terms on the right side of four equations in (32). The triangularity conditions cannot be satisfied. So, (32) is different from every class of the previous models in Section 2.

3.5 Uncertain chained model of a car with a single trailer

The model of kinematic motion of a car towing a single trailer[11] can be described by

$$\left\{\begin{array}{rcl}\dot{x}_c &=& v\cos\beta_0\\ \dot{y}_c &=& v\sin\beta_0\\ \dot{\phi} &=& \omega\\ \dot{\beta}_0 &=& \dfrac{1}{l}v\tan\phi\\ \dot{\beta}_1 &=& \dfrac{1}{d_1}v\sin(\beta_0 - \beta_1)\end{array}\right. \tag{33}$$

If the state and input transformations shown in [11] are used, two-input and five-order canonical chained system can be obtained. However, by using formula (6), we can convert system (33) into

$$\begin{bmatrix}\dot{x}_m\\ \dot{y}_m\\ \dot{\phi}\\ \dot{\beta}_0\\ \dot{\beta}_1\end{bmatrix} = \begin{bmatrix}\alpha_1 v\cos(\beta_0 - \theta_0)\\ \alpha_2 v\sin(\beta_0 - \theta_0)\\ \omega\\ \dfrac{1}{l}v\tan\phi\\ \dfrac{1}{d_1}v\sin(\beta_0 - \beta_1)\end{bmatrix} \tag{34}$$

Let

$$\begin{array}{lll}s_\phi = \sin\phi, & c_\phi = \cos\phi, & t_{\beta i} = \tan\beta_i\\ s_{\beta i} = \sin\beta_i, & c_{\beta i} = \cos\beta_i, & s_{01} = \sin(\beta_0 - \beta_1)\\ se_i = \sec\beta_i, & t_\phi = \tan\phi, & c_{01} = \cos(\beta_0 - \beta_1)\end{array}$$

where $i = 0, 1$, and then take the following state-input transformation

$$\left\{\begin{array}{rcl}x_0 &=& x_m\\ x_1 &=& y_m - d_1\log\left(\dfrac{1 + s_{\beta 1}}{c_{\beta 1}}\right)\\ x_2 &=& t_{\beta 1}\\ x_3 &=& \dfrac{1}{d_1}se_0s_{01}se_1^2\\ x_4 &=& \dfrac{1}{d_1 l}se_0^3se_1t_\phi + \dfrac{1}{d_1^2}se_0^2s_{01}^2s_{\beta 1}se_1^3 - \dfrac{1}{d_1^2}se_0s_{01}se_1^3\\ v &=& (se_0)u_0\\ \omega &=& \lambda_1(\phi, \beta_0, \beta_1)u_0 + \lambda_2(\phi, \beta_0, \beta_1)u_1\end{array}\right.$$

where

$$\begin{aligned}\lambda_1 =\ & -\frac{3}{l}se_0s_{\beta 0}s_\phi^2 - \left(\frac{1}{d_1} + \frac{2}{d_1}s_{\beta 0}se_1s_{01}\right)se_0s_{01}t_{\beta 1}s_\phi c_\phi - \\ & 2\left(\frac{1}{d_1}s_\phi c_\phi - \frac{l}{d_1^2}s_{01}c_\phi^2\right)s_{\beta 1}se_1^2s_{01}c_{01} - \\ & \frac{l}{d_1^2}(s_{01} + 3s_{01}t_{\beta 1}^2 - 3c_{\beta 0}t_{\beta 1}se_1)se_1c_\phi^2s_{01}^2 + \\ & \frac{1}{d_1}se_1^2c_{\beta 1}s_\phi c_\phi - \frac{1}{d_1^2}c_{\beta 0}se_1^2s_{01}c_{01}c_\phi^2\end{aligned}$$

$$\lambda_2 = ld_1c_{\beta 0}^3c_{\beta 1}c_\phi^2$$

We obtain the uncertain chained system of a car towing a single trailer as follows

$$\left\{\begin{array}{rcl}\dot{x}_0 &=& (\alpha_1c_0)u_0 + \alpha_1s_0(x_2 + \dfrac{d_1x_3}{\sqrt{1 + x_2^2}})u_0\\ \dot{x}_1 &=& x_2u_0 + [(\alpha_2c_0 - 1)(x_2 + \dfrac{d_1x_3}{\sqrt{1 + x_2^2}}) - (\alpha_2s_0)]u_0\\ \dot{x}_2 &=& x_3u_0\\ \dot{x}_3 &=& x_4u_0\\ \dot{x}_4 &=& u_1\end{array}\right. \tag{35}$$

It is obviously seen that the second terms on the right side of the first and second equations of (35) are functions with respect to states x_2 and x_3. Thus, the triangularity conditions are not satisfied.

3.6 Uncertain chained model of a car with *n* trailers

The kinematic model of a car with n trailers[19] was given by

$$\left\{\begin{array}{rcl}\dot{x} &=& v_n\cos\beta_n\\ \dot{y} &=& v_n\sin\beta_n\\ \dot{\beta}_n &=& \dfrac{1}{d_n}v_{n-1}\sin(\beta_{n-1} - \beta_n)\\ &\vdots&\\ \dot{\beta}_i &=& \dfrac{1}{d_i}v_{i-1}\sin(\beta_{i-1} - \beta_i)\\ &\vdots&\\ \dot{\beta}_1 &=& \dfrac{1}{d_1}v_0\sin(\beta_0 - \beta_1)\\ \dot{\beta}_0 &=& \omega\end{array}\right. \tag{36}$$

Take the coordinate and feedback transformations presented in [19]. We can change (36) into canonical chained form. However, by using (6), we have

$$\begin{bmatrix} \dot{x}_m \\ \dot{y}_m \end{bmatrix} = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} v_n\cos\beta_n \\ v_n\sin\beta_n \end{bmatrix}$$

Let $\beta_{n+1} = 0$ and denote

$$\begin{aligned}
&\underline{\beta}_i = [\beta_i, \cdots, \beta_n]^{\mathrm{T}} \\
&p_i(\underline{\beta}_i) = \prod_{j=i}^{n} \cos(\beta_j - \beta_{j+1}) \\
&\tau_i(\underline{\beta}_{i-1}) = \frac{\partial x_{i+1}}{\partial \underline{\beta}_i} \underline{f}_i(\underline{\beta}_{i-1}) \\
&c_i(\underline{\beta}_{i-1}) = p_{i-1}^2(\underline{\beta}_{i-1}) \prod_{j=i-1}^{n} d_j p_j(\underline{\beta}_j) \\
&f_i(\underline{\beta}_{i-1}) = \frac{1}{d_i} \frac{\tan(\beta_{i-1} - \beta_i)}{p_i(\underline{\beta}_i)} \\
&\underline{f}_i(\underline{\beta}_{i-1}) = [f_i(\underline{\beta}_{i-1}), \cdots, f_n(\underline{\beta}_{n-1})]^{\mathrm{T}}
\end{aligned}$$

where $i \in \{0, \cdots, n\}$. Then, by taking the following state-input transformation

$$\begin{cases}
x_0 &= x_m \\
x_1 &= y_m \\
x_2 &= \tan\beta_n \\
x_3 &= \dfrac{\tan(\beta_{n-1} - \beta_n)}{d_n \cos^3\beta_n} \\
x_4 &= \dfrac{\tan(\beta_{n-2} - \beta_{n-1})}{c_n(\underline{\beta}_{n-1})} + \tau_n(\underline{\beta}_{n-1}) \\
&\vdots \\
x_{n+2} &= \dfrac{\tan(\beta_0 - \beta_1)}{c_2(\underline{\beta}_1)} + \tau_2(\underline{\beta}_1) \\
u_0 &= p_0(\underline{\beta}_0) v_0 \\
u_1 &= \dfrac{1}{\cos^2(\beta_0 - \beta_1) c_2(\underline{\beta}_1)} \omega + \tau_1(\underline{\beta}_0) p_0(\underline{\beta}_0) v_0
\end{cases}$$

we obtain uncertain chained system as follows

$$\begin{cases}
\dot{x}_0 &= (\alpha_1\cos\theta_0) u_0 + (\alpha_1 \sin\theta_0) x_2 u_0 \\
\dot{x}_1 &= (\alpha_2\cos\theta_0) x_2 u_0 - (\alpha_2 \sin\theta_0) u_0 \\
\dot{x}_2 &= x_3 u_0 \\
&\vdots \\
\dot{x}_{n+1} &= x_{n+2} u_0 \\
\dot{x}_{n+2} &= u_1
\end{cases} \tag{37}$$

In (37), the second term on the right side of the first equation is dependent on x_2. The triangularity condition is not satisfied.

3.7 New general uncertain models of NWMR

In addition to the models (9) ~ (11), (13), (15), and (16), we will show several new general uncertain chained models deduced from the explicit NWMR models in Subsection 3.1 ~ 3.6.

3.7.1 General uncertain chained model 1

From systems (22) and (37), we can deduce the following general uncertain chained form

$$\begin{cases}
\dot{x}_0 &= d_0(t) u_0 + \phi_0^d(t, x_2) u_0 \\
\dot{x}_1 &= d_1(t) x_2 u_0 + \phi_1^d(t) u_0 \\
\dot{x}_2 &= x_3 u_0 \\
&\vdots \\
\dot{x}_{n-2} &= x_{n-1} u_0 \\
\dot{x}_{n-1} &= u_1
\end{cases} \tag{38}$$

where the functions $d_i(\cdot)$ and $\phi_i^d(\cdot)$ $(i = 0, 1)$ denote the possible modeling error, neglected dynamics, or unknown functions. It may be assumed that for every $i = 0, 1$ there are (known) positive c_{i1} and c_{i2} such that $c_{i1} \le d_i(t) \le c_{i2}$ for all $t \ge 0$. $\phi_0^d(t, x_2)$ is a linear function with respect to x_2, $\phi_1^d(t)$ is bounded for all $t \ge 0$.

3.7.2 General uncertain chained model 2

From systems (22), (35), and (37), we can deduce

$$\begin{cases}
\dot{x}_0 &= d_0(t) u_0 + \phi_0^d(t, x_2, x_3, \cdots x_{n-1}) u_0 \\
\dot{x}_1 &= d_1(t) x_2 u_0 + \phi_1^d(t, x_2, x_3, \cdots x_{n-1}) u_0 \\
\dot{x}_2 &= x_3 u_0 \\
&\vdots \\
\dot{x}_{n-2} &= x_{n-1} u_0 \\
\dot{x}_{n-1} &= u_1
\end{cases} \tag{39}$$

where the functions $d_i(\cdot)$ and $\phi_i^d(\cdot)$ $(i = 0, 1)$ denote the possible modeling error, neglected dynamics, or unknown functions. It may be assumed that for every $i = 0, 1$ there are (known) positive c_{i1} and c_{i2} such that $c_{i1} \le d_i(t) \le c_{i2}$ for all $t \ge 0$.

3.7.3 General uncertain chained model 3

From system (25), we have the following model

$$\begin{cases}
\dot{x}_0 &= u_0 \\
\dot{x}_i &= x_{i+1} u_0 + f_i(x) \phi_i^d(t, x_0) u_0, \\
&\qquad 1 \le i \le n-2 \\
\dot{x}_{n-1} &= u_1 + f_{n-1}(x) \phi_{n-1}^d(t, x_0) u_0
\end{cases} \tag{40}$$

where $x = (x_1, \cdots, x_{n-1})^{\mathrm{T}}$, the functions $\phi_i^d(\cdot)$ denote the possible modeling error, neglected dynamics or unknown functions, $f_i(x)$ are linear functions with respect to x $(i = 1, \cdots, n-1)$. It may be assumed that $\phi_i^d(\cdot)$ $(i = 1, \cdots, n-1)$ are bounded for all $t \ge 0$.

3.7.4 General uncertain chained model 4

Consider (32) and expand it to multi-input cases. We can obtained the uncertain chained model as follows

$$\begin{cases}
\dot{x}_0 &= u_0 \\
\dot{x}_i^1 &= x_{i+1}^1 u_0 + \sum\limits_{j=1}^{m} f_{ij}^1(x) \phi_{ij}^{d_1}(t, x_0) u_0, \\
&\qquad 1 \le i \le r_1 - 1 \\
\dot{x}_{r_1}^1 &= x_{r_1+1}^1 u_1 + \sum\limits_{j=1}^{m} f_{r_1 j}^1(x) \phi_{r_1 j}^{d_1}(t, x_0) u_0 \\
&\vdots \\
\dot{x}_i^m &= x_{i+1}^m u_0 + \sum\limits_{j=1}^{m} f_{ij}^m(x) \phi_{ij}^{d_m}(t, x_0) u_0, \\
&\qquad 1 \le i \le r_m - 1 \\
\dot{x}_{r_m}^m &= u_m + \sum\limits_{j=1}^{m} f_{r_m j}^m(x) \phi_{r_m j}^{d_m}(t, x_0) u_0
\end{cases} \tag{41}$$

where $x = (x_1, \cdots, x_{r_m})^{\mathrm{T}}$, the functions $d_0(t)$, $\phi_{ij}^{d_k}(t, x_0)$ ($i = 1, \cdots, r_k$) ($k, j = 1, \cdots, m$) denote the possible modeling error, neglected dynamics, or unknown functions. $f_{ij}^k(x)$ are linear functions with respect to x. It may be assumed that for every ($i = 1, \cdots, r_k$) ($j, k = 1, \cdots, m$), there are (known) positive A_{ij}^k and B_{ij}^k such that $A_{ij}^k \le \phi_{ij}^{d_k}(t, x_0) \le B_{ij}^k$ for all $t \ge 0$.

Summing up, there exist a lot of uncertain chained models which can be derived from practice. One of our tasks is to discover them and then discuss their stabilization problems.

4 Robust stabilization for the uncertain chained systems of NWMR

For the uncertain chained systems of NWMR mentioned in the last section, Wang et al.[45−48] first considered the stabilization problems for the uncertain system (7) of type $(2, 0)$ robot which is the equivalent system of (20). Then, the robust stabilization problems in particular case are discussed for the uncertain chained systems of type $(1, 1)$[49] and type $(1, 2)$[50] robots, respectively. The main results are addressed in this section.

4.1 Robust regulation and tracking control for uncertain chained system of type (2, 0) robot

In this subsection, the robust regulation (or stabilization)[45−47] and dynamic feedback tracking control[48] for (7) are introduced simply.

4.1.1 Robust regulation

In order to robustly regulate (7)[45−46], three cases were discussed as follows

1) θ_0 known, α_1 and α_2 unknown;
2) θ_0 unknown, α_1 and α_2 known;
3) θ_0 unknown, $\alpha_1 = \alpha_2 = \alpha$ unknown.

In each case, the controller was obtained by using different coordinate transformations and exploiting a new two-step technique. For example, in the case of 3), it was assumed that $0 < \alpha \le \bar{\alpha}$. Taking the following coordinate transformation from (x_m, y_m) to (z_1, z_2)[46]

$$\left[\begin{array}{c} z_1 \\ z_2 \end{array}\right] = \left[\begin{array}{cc} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{array}\right] \left[\begin{array}{c} x_m \\ y_m \end{array}\right] \tag{42}$$

one obtains

$$\left\{\begin{array}{lcl} \dot{z}_1 & = & -z_2\omega + \alpha v \cos\theta_0 \\ \dot{z}_2 & = & z_1\omega + \alpha v \sin\theta_0 \\ \dot{\theta} & = & \omega \end{array}\right.$$

Step 1. Taking the control input

$$\left\{\begin{array}{lcl} v & = & k_1 z_1 + k_2 z_2 \\ \omega & = & \omega \ \ (\text{constant}) \end{array}\right. \tag{43}$$

where k_1, k_2 are given in three cases respectively: $0 \le \theta_0 \le \frac{\pi}{2}$; $\frac{\pi}{2} \le \theta_0 \le \pi$; $\sigma \le \theta_0 \le \pi - \sigma$ (σ is a known small positive number), we can design the controller such that $z_1(t)$, $z_2(t)$ converge to zero as $t \to \infty$.

Step 2. When the absolute values of $z_1(t)$ and $z_2(t)$ are made as small as desired, the following controller is used

$$v = 0, \quad \omega = -a\theta \tag{44}$$

where a is a positive gain. Then, $\theta(t)$ converges to zero.

Summing up, system (7) can be robustly regulated by using the controllers (43) and (44)[46]. The other two cases were discussed in [45].

In [47], the time varying smooth dynamic feedback robust regulation was investigated for (7). It was assumed that

1) θ_0 known and $\alpha_1 = \alpha_2 = \alpha$ unknown;
2) $\underline{\alpha} \le \alpha \le \bar{\alpha}$, $\underline{\alpha}$ and $\bar{\alpha}$ are positive known parameters.

Then, the dynamic feedback regulation system can be described as

$$\left\{\begin{array}{lcl} \dot{z}_1 & = & -z_2\omega + \alpha v \\ \dot{z}_2 & = & z_1\omega \\ \dot{\theta} & = & \omega \\ \dot{\omega} & = & u_1 \\ \dot{v} & = & u_2 \end{array}\right. \tag{45}$$

where u_1 and u_2 are the generalized force and generalized torque, respectively.

In order to solve the dynamic feedback regulation problem for system (45), a new two-step technique[47] was used to design the controllers. The first step is to design u_1 such that ω remains a non-zero constant and then design u_2 to make z_1, z_2 and v as small as desired in a limited time. The second step is to design u_1 such that θ and ω converge to zero as t goes to infinity, while u_2 is designed to keep z_1, z_2 and v to have smaller variation.

Remark 1. Under the assumptions in [47], system (7) can be converted into the following system by substituting $\theta - \theta_0$ for θ and using (42)

$$\left\{\begin{array}{lcl} \dot{z}_1 & = & -z_2\omega + \alpha v \\ \dot{z}_2 & = & z_1\omega \\ \dot{\theta} & = & \omega \end{array}\right. \tag{46}$$

For (46), time-varying smooth regulation controllers were proposed in [45]. The controllers are velocities of kinematic systems. However, the design of generalized force and generalized torque is much more practical. So, the dynamic feedback regulation problem was investigated in [47].

4.1.2 Dynamic feedback tracking control

For system (7), the dynamic feedback tracking control problem was discussed in [48]. Taking the coordinate transformation from (x_m, y_m, θ) to (x_1, x_2, x_3), then letting $e_i = x_i - x_{id}$ ($i = 1, 2, 3$), we have

$$\left[\begin{array}{c} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{array}\right] = \left[\begin{array}{c} d(v - v_d) + e_2\omega + x_{2d}(\omega - \omega_d) \\ -e_1\omega - x_{1d}(\omega - \omega_d) \\ \omega - \omega_d \end{array}\right] \tag{47}$$

where v_d, ω_d are the respective linear and angular velocities of the desired WMR and x_{1d}, x_{2d}, x_{3d} are the desired states.

It was assumed that the desired trajectory was generated from a prerecorded set of images taken by the fixed camera and

1) θ_0 known, $\alpha_1 = \alpha_2 = d$ unknown;
2) v_d and ω_d are bounded, x_{1d}, x_{2d}, x_{3d} and their derivatives and the second derivative of x_{1d} are bounded. There exists a known positive number V_d such that $|v_d(t)| \le V_d$;
3) $\dot{x}_{1d} \nrightarrow 0$ as $t \to \infty$.

By choosing the feedback controller[48] as

$$\left[\begin{array}{c} \dot{p} \\ \omega \\ v \end{array}\right] = \left[\begin{array}{c} -k_2 p - e_3 + e_2 x_{1d} - e_1 x_{2d} \\ \omega_d + p \\ -k_1 e_1 - V_d \ \mathrm{sgn}\ e_1 \end{array}\right] \tag{48}$$

and using the Barbalat theorem and Lyapunov technique, we can make the states of the closed-loop system consisting of (47) and (48) converge to zero as t goes to infinity if d and v_d are unknown.

4.2 Robust exponential regulation for uncertain chained system of type (1, 1) robot

As shown in Section 3, the uncertain chained system of type (1, 1) robot was described as (25). In order to solve its stabilization problem[49], it was assumed that

1) $\theta_0 = 0$ and $\gamma_1 = \gamma_2 = \alpha$ unknown;

2) For positive unknown visual parameter α, there exist known positive numbers $\underline{\alpha}$ and $\bar{\alpha}$ such that $\underline{\alpha} \le \alpha \le \bar{\alpha}$.

For (25), if $x_0(0) \neq 0$, by using the strategy of state-scaling

$$y_1 = \frac{x_1}{x_0^2}, \quad y_2 = \frac{x_2}{x_0}, \quad y_3 = x_3 \tag{49}$$

and then choosing the controller as

$$\left\{\begin{array}{rcl} u_0 &=& -\lambda_0 x_0 \\ u_1 &=& k_1 y_1 + k_2 y_2 + k_3 y_3 \end{array}\right. \tag{50}$$

(25) can be rewritten as the following compact matrix form

$$\dot{Y} = [A + B(t)]Y \tag{51}$$

where

$$A = \begin{bmatrix} 2\lambda_0 & -\lambda_0 & 0 \\ 0 & \lambda_0 & -\lambda_0\alpha \\ k_1 & k_2 & k_3 \end{bmatrix}$$

$$B(t) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\lambda_0 x_0^2(\alpha - 1) \\ 0 & 0 & 0 \end{bmatrix}$$

By using Lemma 1 (see Section 4.3) and the Routh-Hurwits stability criterion[59], (51) can be exponentially stabilized if k_1, k_2, k_3 satisfy the following group of inequalities

$$\left\{\begin{array}{l} \lambda_0 > 0 \\ k_3 + 3\lambda_0 < 0 \\ 3k_3 + \alpha k_2 + 2\lambda_0 > 0 \\ 2k_3 + 2\alpha k_2 + \alpha k_1 > 0 \\ (k_3 + 3\lambda_0)(3k_3 + \alpha k_2 + 2\lambda_0) < \\ \qquad \lambda_0(2k_3 + 2\alpha k_2 + \alpha k_1) \end{array}\right. \tag{52}$$

If $x_0(0) = 0$, take $u_0 = k$ (non-zero constant). Then, $x_0(t) = kt$ holds. $x_0(t)$ will be not zero after a time limit T ($T > 0$). Hence, system (51) can be stabilized exponentially by switching to (50) and (52).

To sum up, choose λ_0, k_1, k_2, k_3 such that (52) holds and design the controller as (50). System (25) can be exponentially stabilized[49] by using state-scaling and switching technology under the assumptions in this subsection.

4.3 Robust exponential regulation for uncertain chained system of type (1, 2) robot

The uncertain chained system of type (1, 2) robot can be described as (32). Our objective is to design u_0, u_1, and u_2 to make (32) exponentially stabilizable under two assumptions below.

Assumption 1. $\theta_0 = 0$ and $\alpha_1 = \alpha_2 = r$ unknown;

Assumption 2. For positive unknown visual parameter r, there exist known positive r_1 and r_2 such that

$$r_1 \le r \le r_2 \tag{53}$$

Under the Assumptions 1 and 2, (32) can be rewritten as

$$\left\{\begin{array}{rcl} \dot{x}_0 &=& u_0 \\ \dot{x}_1 &=& [rx_2 + (r-1)\,x_3]u_0 \\ \dot{x}_2 &=& u_1 + [(r-1)\,x_1 + (1-r)x_4]u_0 \\ \dot{x}_3 &=& [(1-r)\,x_1 + rx_4]u_0 \\ \dot{x}_4 &=& u_2 + [(r-1)\,x_2 + (r-1)x_3]u_0 \end{array}\right. \tag{54}$$

In order to discuss the stabilization of system (54), we need to introduce the following lemmas.

Lemma 1. Consider time-varying linear system[60] defined by

$$\dot{x} = (A + B(t))x \tag{55}$$

where $x \in \mathbf{R}^n$ is the state vector. If $A \in \mathbf{R}^{n\times n}$ is a Hurwitz matrix, and for every element in $B(t) \in \mathbf{R}^{n\times n}$, it satisfies $b_{ij}(t) \to 0$ $(t \to \infty)$ exponentially $(i, j = 1, 2, \cdots)$. Then, system (55) is exponentially stable.

Lemma 2. If $A \in \mathbf{R}^{4\times 4}$, the characteristic polynomial of matrix A is specified as follows

$$|\lambda I - A| = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 \tag{56}$$

A is a Hurwitz matrix[59] if and only if $a_1 > 0$, $a_3 > 0$, $a_4 > 0$ and $\Delta_3 = a_1a_2a_3 - a_1^2a_4 - a_0a_3^2 > 0$.

Remark 2. It is worth noting that Lemma 1 is a corollary of the conclusion on [60]. Lemma 2 is a direct application of the well known Routh-Hurwitz stability criterion[59].

4.3.1 Controller design

Now, let us consider the controller design for (54).

Under the assumptions in this subsection, choose control input

$$u_0 = -\lambda_0 x_0$$

where λ_0 is a positive parameter. We have

$$\begin{array}{rcl} \dot{x}_0 &=& -\lambda_0 x_0 \\ x_0(t) &=& x_0(0)\mathrm{e}^{-\lambda t}, \quad t \ge 0 \end{array} \tag{57}$$

where $x_0(0)$ is the initial value of $x_0(t)$ when $t = 0$.

If $x_0(0) = 0$, take $u_0 = k$ (non zero constant). Then $x_0(t) = kt$. $x_0(t)$ will be not zero after a time limit T.

If $x_0(0) \neq 0$, then $x_0(t) \neq 0$ holds by (57). Let

$$\left\{\begin{array}{rcl} y_1 &=& \dfrac{x_1}{x_0} \\ y_2 &=& x_2 \\ y_3 &=& \dfrac{x_3}{x_0} \\ y_4 &=& x_4 \end{array}\right. \tag{58}$$

We have

$$\left\{\begin{array}{rcl} \dot{y}_1 &=& \lambda_0 y_1 - r\lambda_0 y_2 + (1-r)\lambda_0 x_0 y_3 \\ \dot{y}_2 &=& u_1 + (1-r)\lambda_0 x_0^2 y_1 + (r-1)\lambda_0 x_0 y_4 \\ \dot{y}_3 &=& (r-1)\lambda_0 x_0 y_1 + \lambda_0 y_3 - r\lambda_0 y_4 \\ \dot{y}_4 &=& u_2 + (1-r)\lambda_0 x_0 y_2 + (1-r)\lambda_0 x_0^2 y_3 \end{array}\right. \tag{59}$$

Take control inputs u_1 and u_2 as follows,

$$\left\{\begin{array}{rcl} u_1 &=& k_1y_1 + k_2y_2 + k_3y_3 + k_4y_4 \\ u_2 &=& p_1y_1 + p_2y_2 + p_3y_3 + p_4y_4 \end{array}\right.$$

and then substitute them into (59). We have

$$\dot{Y} = [A + B(t)]Y \tag{60}$$

where

$$A = \begin{bmatrix} \lambda_0 & -r\lambda_0 & 0 & 0 \\ k_1 & k_2 & k_3 & k_4 \\ 0 & 0 & \lambda_0 & -r\lambda_0 \\ p_1 & p_2 & p_3 & p_4 \end{bmatrix}$$

$$B(t) = \begin{bmatrix} 0 & 0 & b_{13} & 0 \\ b_{21} & 0 & 0 & -b_{13} \\ -b_{13} & 0 & 0 & 0 \\ 0 & b_{13} & b_{21} & 0 \end{bmatrix}$$

$Y = [y_1, y_2, y_3, y_4]^{\rm T}$, $b_{13} = (1-r)\lambda_0 x_0$, $b_{21} = (1-r)\lambda_0 x_0^2$, k_i and p_i $(i = 1, 2, 3, 4)$ are parameters to be designed.

Lemma 3. For system (60), choose k_i, p_i $(i = 1, 2, 3, 4)$ and a, b, c, d to satisfy the following conditions

$$\begin{array}{lll} k_1 = a\lambda_0, & k_2 = b\lambda_0, & k_3 = k_4 = 0 \\ p_3 = c\lambda_0, & p_4 = d\lambda_0, & p_1 = p_2 = 0 \end{array} \tag{61}$$

$$b < -1, \quad d < -1, \quad a > \frac{-b}{r_1}, \quad c > \frac{-d}{r_1} \tag{62}$$

Then, A is a Hurwitz matrix, and system (60) is exponentially stabilizable.

Proof. For system (60), denote the characteristic polynomial of matrix A as (56). With the choice of (61) and (62), it is obviously seen that

$$a > \frac{-b}{r_1} > \frac{-b}{r}, \quad c > \frac{-d}{r_1} > \frac{-d}{r}$$

Denote

$$\begin{array}{ll} Q = ar + b > 0, & B = b + 1 < 0 \\ C = cr + d > 0, & D = d + 1 < 0 \end{array}$$

Then, we have

$$\begin{aligned} a_1 &= -(b+d+2)\lambda_0 = -(B+D)\lambda_0 > 0 \\ a_2 &= [(ar+b) + (cr+d) + (b+1)(d+1)]\lambda_0^2 = \\ &\quad [Q + C + BD]\lambda_0^2 > 0 \\ a_3 &= -[(ar+b)(d+1) + (cr+d)(b+1)]\lambda_0^3 = \\ &\quad -[QD + BC]\lambda_0^3 > 0 \\ a_4 &= [(ar+b)(cr+d)\lambda_0^4 = QC\lambda_0^4 > 0 \\ \Delta_3 &= [(B+D)(Q+C+BD)(QD+BC) - \\ &\quad (B+D)^2QC - (QD+BC)^2]\lambda_0^6 = \\ &\quad BD[(Q-C)^2 + QBD + B^2C + \\ &\quad QD^2 + CBD]\lambda_0^6 > 0 \end{aligned}$$

Hence, A is a Hurwitz matrix. Because $b_{ij}(t)$ in $B(t)$ converges to zero exponentially as t goes to infinity, system (60) is exponentially stabilized by Lemma 1. □

To sum up, we have the following main result.

Theorem 1. In view of the assumptions and lemmas above, system (54) can be exponentially stabilized if the controller is chosen as

$$\left\{ \begin{aligned} u_0 &= -\lambda_0 x_0 \\ u_1 &= k_1 y_1 + k_2 y_2 + k_3 y_3 + k_4 y_4 \\ u_2 &= p_1 y_1 + p_2 y_2 + p_3 y_3 + p_4 y_4 \end{aligned} \right. \tag{63}$$

where $\lambda_0 > 0$, k_i and p_i $(i = 1, 2, 3, 4)$ satisfy (61) and (62) respectively.

Proof. If $x_0(0) \neq 0$, by the arguments above and Lemma 3, x_0 and y_1, y_2, y_3, y_4 converge to zero exponentially as t goes to infinity for system (60). (58) can be used to deduce that x_1, x_2, x_3, x_4 converge to zero exponentially too as t goes to infinity. Then, system (54) can be stabilized exponentially.

If $x_0(0) = 0$, take $u_0 = k$ (nonzero constant), then $x_0(t) = kt$. It is obvious that $x_0(t)$ will be not zero after a time limit T. Hence, switch to (63), (61) and (62). System (54) can be stabilized finally.

Therefore, system (54) can be exponentially stabilized by using state-scaling and switching strategies. □

4.3.2 Simulation

The simulations are conducted for systems (29) and (54).

For system (54), let the initial value is $[x_0(0),\ x_1(0),\ x_2(0),\ x_3(0),\ x_4(0)] = [0.7854\,\text{rad},\ -0.8247\,\text{cm},\ -0.5\,\text{cm},\ 0.5498\,\text{cm},\ 0.6\,\text{cm}]$. Because $x_0(0) = 0.7854 \neq 0$, so $x_0 = x_0(t) \neq 0$ due to formula (57). Take state transformation (58) and choose control input u_i $(i = 0, 1, 2)$ as (63). k_i and p_i $(i = 1, 2, 3, 4)$ satisfy (61) and (62) with parameters $\lambda_0 = 1$, $k_1 = 5$, $k_2 = -2$, $p_3 = 6$, $p_4 = -3$, $r_1 = 1$, $r_2 = 3$, $r = 1.5$. Then, the trajectories of states x_i $(i = 0, 1, 2, 3, 4)$ are obtained. They are shown in Figs. 3 and 4 below. The trajectories of control inputs u_i $(i = 0,\ 1,\ 2)$ can also be obtained which are shown in Fig. 5.

For system (29), the trajectories of x_m, y_m, θ, β_1, and β_2 with respect to time are obtained by using (30), (31) and the results or method above. They are shown in Figs. 6 ∼ 8 respectively. Simulation results illustrate that all trajectories converge to the origin smoothly and rapidly in a short time. This demonstrate the effectiveness of the proposed control strategy and robust exponential stabilization.

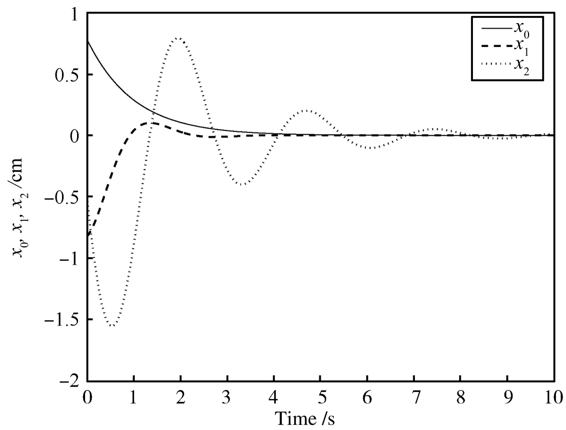

Fig. 3 The trajectories of state x_0, x_1, x_2 with respect to time

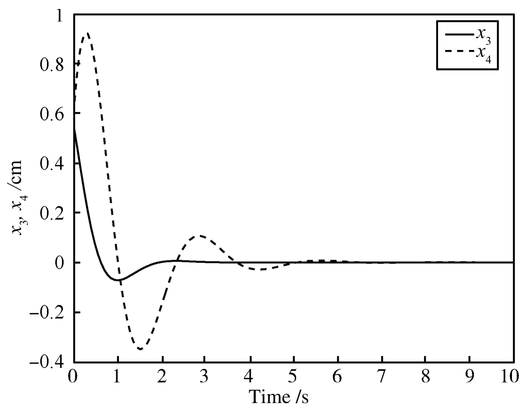

Fig. 4 The trajectories of state x_3 and x_4 with respect to time

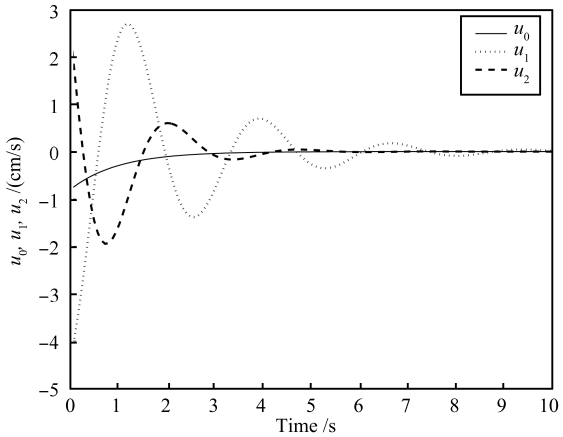

Fig. 5 The trajectories of input u_0, u_1, u_2 with respect to time

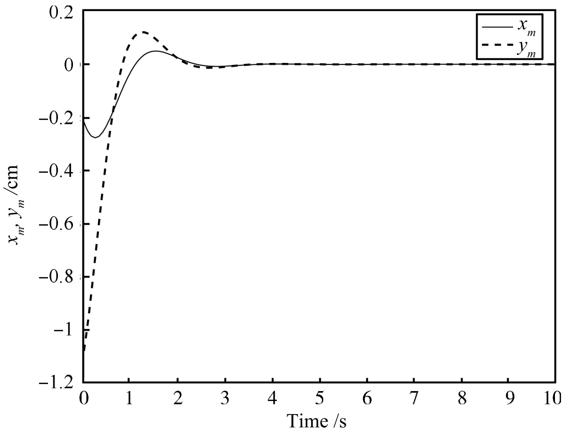

Fig. 6 The trajectories of x_m and y_m with respect to time

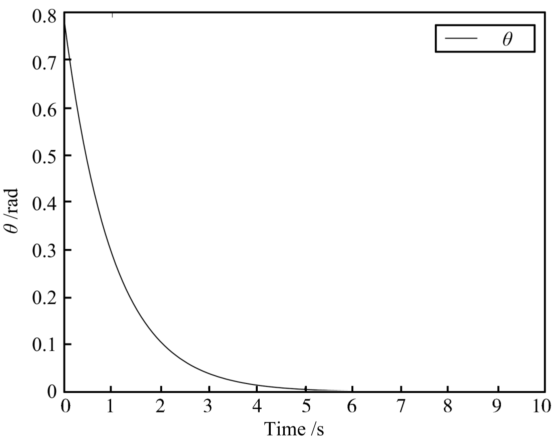

Fig. 7 The trajectory of θ with respect to time

5 Conclusions and future work

In this paper, we provide a summary of recent development of the robust stabilizing problems with uncertainties. It consists of the various uncertain chained models, control methodologies, and results proposed over past ten years. Based on the visual feedback, we obtain a few new exciting uncertain chained models by using the state and input transformations for nonholonomic kinematic systems of four types of NWMR, a car towing a single trailer and towing n trailers. Then, several new general uncertain chained

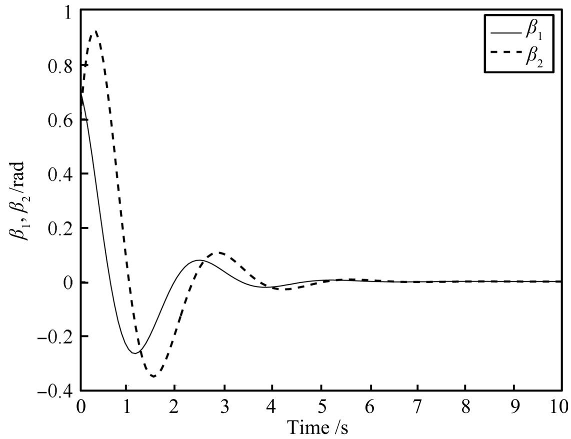

Fig. 8 The trajectories of β_1 and β_2 with respect to time

models are proposed in addition. Novel time-varying controllers are presented to stabilize the uncertain chained systems by exploiting a kind of new two-step technique or using switching technology for type $(2, 0)$, type $(1, 1)$, and type $(1, 2)$ robots. Simulation results demonstrate the effectiveness of the proposed control strategy and robust exponential stabilization for type $(1, 2)$ robot.

As presented in Section 3, it is obviously seen that there are a lot of uncertain chained systems in practice. They do not satisfy the triangularity conditions. For type $(1, 1)$ and type $(1, 2)$ robots, the stabilization problems in Section 4 are only discussed in particular case. As for other cases such as $\theta_0 \neq 0$ and $\alpha_1 \neq \alpha_2$ unknown, they will be discussed in the future. The stabilizing problems of systems (38) $\sim$ (41) are still open.

References

1 Bloch A M, Reyhanoglu M, McClamroch N H. Control and stabilization of nonholonomic dynamic systems. *IEEE Transactions on Automatic Control*, 1992, **37**(11): 1746−1757

2 Kolmanovsky I, McClamroch N H. Developments in nonholonomic control problems. *IEEE Control Systems Magazine*, 1995, **15**(6): 20−36

3 Murray R M, Sastry S S. Nonholonomic motion planning: steering using sinusoids. *IEEE Transactions on Automatic Control*, 1993, **38**(5): 700−716

4 Huo W, Ge S S. Exponential stabilization of nonholonomic systems: an ENI approach. *International Journal of Control*, 2001, **74**(15): 1492−1500

5 Tian Y P, Li S H. Exponential stabilization of nonholonomic dynamic systems by smooth time-varying control. *Automatica*, 2002, **38**(7): 1139−1146

6 Wang J, Qu Z H, Hull R A, Martin J. Cascaded feedback linearization and its application to stabilization of nonholonomic systems. *Systems and Control Letters*, 2007, **56**(4): 285−295

7 Ren Yan-Qing, Ma Bao-Li. Global stabilization of a class of nonholonomic systems. *Acta Automatica Sinica*, 2007, **33**(9): 979−984 (in Chinese)

8 Sordalen O J, Egeland O. Exponential stabilization of nonholonomic chained systems. *IEEE Transactions on Automatic Control*, 1995, **40**(1): 35−49

9 Walsh G C, Bushnell L G. Stabilization of multiple input chained form control systems. *Systems and Control Letters*, 1995, **25**(3): 227−234

10 Astolfi A, Schaufelberger W. State and output feedback stabilization of multiple chained systems with discontinuous control. *Systems and Control Letters*, 1997, **32**(1): 49−56

11 Jiang Z P, Nijmeijer H. A recursive technique for tracking control of nonholonomic systems in chained form. *IEEE Transactions on Automatic Control*, 1999, **44**(2): 265−279

12 Sun Z D, Ge S S, Huo W, Lee T H. Stabilization of nonholonomic chained systems via nonregular feedback linearization. *Systems and Control Letters*, 2001, **44**(4): 279−289

13 Qu Z H, Wang J, Plaisted C E, Hull R A. Global-stabilizing near-optimal control design for nonholonomic chained systems. *IEEE Transactions on Automatic Control*, 2006, **51**(9): 1440−1456

14 Wang C L. Semiglobal practical stabilization of nonholonomic wheeled mobile robots with saturated inputs. *Automatica*, 2008, **44**(3): 816−822

15 Ma B L, Tso S K. Unified controller for both trajectory tracking and point regulation of second-order nonholonomic chained systems. *Robotics and Autonomous Systems*, 2008, **56**(4): 317−323

16 Wu Y Q, Wang B, Zong G D. Finite-time tracking controller design for nonholonomic system with extended chained form. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 2005, **50**(11): 798−802

17 Cao Ke-Cai. Global κ-exponential tracking control of nonholonomic systems in chained-form by output feedback. *Acta Automatica Sinica*, 2009, **35**(5): 568−576

18 Campion G, Bastin G, Dandrea-Novel B. Structural properties and classification of kinematic and dynamic models of wheeled mobile robots. *IEEE Transactions on Robotics and Automation*, 1996, **12**(1): 47−62

19 Sordalen O J. Conversion of the kinematics of a car with n trailers into a chained form. In: Proceedings of the IEEE International Conference on Robot and Automation. Atlanta, USA: IEEE, 1993. 382−387

20 Leroquais W, d′Andréa-Novel B. Transformation of the kinematic models of restricted mobility wheeled mobile robots with a single platform into chain forms. In: Proceedings of the 34th IEEE Conference on Decision and Control. New Orleans, USA: IEEE, 1995. 3811−3816

21 Tilbury D, Sordalen O J, Bushnell L, Sastry S S. A multisteering trailer system: conversion into chained form using dynamic feedback. *IEEE Transactions on Robot and Automation*, 1995, **11**(6): 807−818

22 Wang Y N, Peng J Z, Sun W, Yu H S, Zhang H. Robust adaptive tracking control of robotic systems with uncertainties. *Journal of Control Theory and Applications*, 2008, **6**(3): 281−286

23 Hu Y M, Ge S S, Su C Y. Stabilization of uncertain nonholonomic systems via time-varying sliding mode control. *IEEE Transactions on Automatic Control*, 2004, **49**(5): 757−763

24 Ma B L, Tso S K. Robust discontinuous exponential regulation of dynamic nonholonomic wheeled mobile robots with parameter uncertainties. *International Journal of Robust Nonlinear Control*, 2008, **18**(9): 960−974

25 Dong W J, Farrell J A. Decentralized cooperative control of multiple nonholonomic dynamic systems with uncertainty. *Automatica*, 2009, **45**(3): 706−710

26 Li Z, Ge S S, Adams M, Wijesoma W S. Robust adaptive control of uncertain force/motion constrained nonholonomic mobile manipulators. *Automatica*, 2008, **44**(3): 776−784

27 Wu J B, Xu G H, Yin Z P. Robust adaptive control for a nonholonomic mobile robot with unknown parameters. *Journal of Control Theory and Applications*, 2009, **7**(2): 212−218

28 Ju G L, Wu Y Q, Sun W H. Adaptive output feedback asymptotic stabilization of nonholonomic systems with uncertainties. *Nonlinear Analysis: Theory, Methods and Applications*, 2009, **71**(11): 5106−5117

29 Maini M, Morin P, Pomet J B, Samson C. On the robust stabilization of chained systems by continuous feedback. In: Proceedings of the 38th IEEE Conference on Decision and Control. Phoenix, USA: IEEE, 1999. 3472−3477

30 Jiang Z P, Pomet J B. Combining backstepping and time-varying techniques for a new set of adaptive controllers. In: Proceedings of the 33rd IEEE Conference on Decision and Control. Florida: IEEE, 1994. 2207−2212

31 Hespanha J P, Liberzon S, Morse A S. Towards the supervisory control of uncertain nonholonomic systems. In: Proceedings of the American Control Conference. San Diego, USA: IEEE, 1999. 3520−3524

32 Do K D, Pan J. Adaptive global stabilization of nonholonomic systems with strong nonlinear drifts. *Systems and Control Letters*, 2002, **46**(3): 195−205

33 Ge S S, Wang Z P, Lee T H. Stabilization of uncertain nonholonomic systems using adaptive backstepping technique. In: Proceedings of the IEEE International Symposium on Intelligent Control. Washington D. C., USA: IEEE, 2002. 862−867

34 Ge S S, Wang Z P, Lee T H. Adaptive stabilization of uncertain nonholonomic systems by state and output feedback. *Automatica*, 2003, **39**(8): 1451−1460

35 Wang Qiang-De, Wei Chun-Ling. Robust adaptive control of nonholonomic systems with nonlinear parameterization. *Acta Automatica Sinica*, 2007, **33**(4): 399−403

36 Gao F Z, Yuan F S, Yao H J. Robust adaptive control for nonholonomic systems with nonlinear parameterization. *Nonlinear Analysis: Real World Applications*, 2010, **11**(4): 3242−3250

37 Yuan Z P, Wang Z P, Chen Q J. Adaptive output feedback control of uncertain nonholonomic systems with strong nonlinear drifts. In: Proceedings of the IEEE International Conference on Robotics and Automation. Kobe, Japan: IEEE, 2009. 3694−3699

38 Zheng X Y, Wu Y Q. Adaptive output feedback stabilization for nonholonomic systems with strong nonlinear drifts. *Nonlinear Analysis: Theory, Methods and Applications*, 2009, **70**(2): 904−920

39 Wang Z P, Ge S S, Lee T H. Robust adaptive neural network control of uncertain nonholonomic systems with strong nonlinear drifts. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 2004, **34**(5): 2048−2059

40 Ma Bao-Li. Robust smooth time-varying exponential stabilization of dynamic nonholonomic mobile cart with parameter uncertainties. *Acta Automatica Sinica*, 2005, **31**(2): 314−319

41 Jiang Z P. Robust exponential regulation of nonholonomic systems with uncertainties. *Automatica*, 2000, **36**(2): 189−209

42 Xi Z R, Feng G, Jiang Z P, Cheng D Z. A switching algorithm for global exponential stabilization of uncertain chained systems. *IEEE Transactions on Automatic Control*, 2003, **48**(10): 1793−1798

43 Xi Z R, Feng G, Jiang Z P, Cheng D Z. Output feedback exponential stabilization of uncertain chained systems. *Journal of the Franklin Institute*, 2007, **344**(1): 36−57

44 Hong Y G, Wang J K, Xi Z R. Stabilization of uncertain chained form systems within finite settling time. *IEEE Transactions on Automatic Control*, 2005, **50**(9): 1379−1384

45 Wang C L, Niu W B, Li Q S, Jia Q W. Visual servoing based regulation of nonholonomic mobile robots with uncalibrated monocular camera. In: Proceedings of the IEEE International Conference on Control and Automation. Guangzhou, China: IEEE, 2007. 214−219

46 Wang C L, Mei Y C, Liao Q W. Rubust regulation of nonholonomic mobile robots with visual servoing. In: Proceedings of the 7th World Congress on Intelligent Control and Automation. Chongqing, China: IEEE, 2008. 2153−2158

47 Wang C L, Liang Z Y, Jia Q W. Dynamic feedback robust stabilization of nonholonomic mobile robots based on visual servoing. *Journal of Control Theory and Applications*, 2010, **8**(2): 139−144

48 Wang C L, Mei Y C, Liang Z Y, Jia Q W. Dynamic feedback tracking control of non-holonomic mobile robots with unknown camera parameters. *Transactions of the Institute of Measurement and Control*, 2010, **32**(2): 155−169

49 Wang C L, Liang Z Y, Du J M, Liang S L. Robust stabilization of nonholonomic moving robots with uncalibrated visual parameters. In: Proceedings of the American Control Conference. St. Louis, USA: IEEE, 2009. 1347−1352

50 Liang Z Y, Wang C L, Sun Y, Yang Y M, Liu Y H. Uncalibrated visual servoing feedback based exponential stabilization of nonholonomic mobile robots. In: Proceedings of the IEEE International Conference on Robotics and Biomimetics. Guilin, China: IEEE, 2009. 671−676

51 Guo X L, Mu X W, Li L. κ-exponential stabilization of uncertain high-order nonholonomic chained systems. In: Proceedings of the 26th Chinese Control Conference. Zhangjiajie, Hunan, China: IEEE, 2007. 19−22

52 Allen P K, Timcenko A, Yoshimi B, Michelman P. Automated tracking and grasping of a moving object with a robotic hand-eye system. *IEEE Transactions on Robotics and Automation*, 1993, **9**(2): 152−165

53 Dixon W E, Dawson D M, Zergeroglu E, Behal A. Adaptive tracking control of a wheeled mobile robot via an uncalibrated camera system. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 2001, **31**(3): 341−352

54 Piepmeier J A, McMurray G V, Lipkin H. Uncalibrated dynamic visual servoing. *IEEE Transactions on Robotics and Automation*, 2004, **20**(1): 143−147

55 Fang Y C, Dixon W E, Dawson D M, Chawda P. Homography-based visual servo regulation of mobile robots. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 2005, **35**(5): 1041−1050

56 Chen J, Dixon W E, Dawson D, McIntyre M. Homography-based visual servo tracking control of a wheeled mobile robot. *IEEE Transactions on Robotics*, 2006, **22**(2): 406−415

57 Liu Y H, Wang H S, Wang C Y, Lam K K. Uncalibrated visual servoing of robots using a depth-independent interaction matrix. *IEEE Transactions on Robotics*, 2006, **22**(4): 804−817

58 Wang H S, Liu Y H, Zhou D X. Adaptive visual servoing using point and line features with an uncalibrated eye-in-hand camera. *IEEE Transactions on Robotics*, 2008, **24**(4): 843−856

59 Dorf R C, Bishop H R. *Modern Control System*. Pearson Education, New Jersey: Prentice-Hall, Inc, 1986. 2

60 Slotine J J, Li W P. *Applied Nonlinear Control*. New Jersey: Prentice Hall, 1991. 115−116

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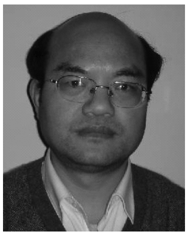

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