Stability and Stabilization of Networked Control Systems with Bounded Packet Dropout

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Abstract In this paper, the stability and stabilization problems of a class of networked control systems (NCSs) with bounded packet dropout are investigated. An iterative approach is proposed to model the NCSs with bounded packet dropout as Markovian jump linear systems (MJLSs). The transition probabilities of MJLSs are partly unknown due to the complexity of network. The system under consideration is more general, which covers the systems with completely known and completely unknown transition probabilities as two spacial cases. Moreover, both sensor-to-controller and controller-to-actuator packet dropouts are considered simultaneously. Sufficient conditions for stochastic stability and stabilization of the underlying systems are derived via linear matrix inequalities (LMIs) formulation. Lastly, two illustrative examples are given to demonstrate the effectiveness of the proposed results.

Key words Networked control system (NCS), packet dropout process, stochastically stable, linear matrix inequality (LMI), transition probabilities matrix

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Networked control systems (NCSs) are feedback control systems with control loops closed via digital communication channels. Compared with the traditional point-to-point wiring, the use of the communication channels can reduce the costs of cables and power, simplify the installation and maintenance of the whole system, and increase the reliability. The NCSs have many industrial applications in automobiles, manufacturing plants, aircrafts, and HVAC systems[1]. However, the insertion of communication networks in feedback control loops makes the NCSs analysis and synthesis complex; see [2−4] and the references therein, where much attention was paid to the delayed data packets of an NCS due to network transmissions. In fact, data packets through networks suffer not only transmission delays, but also, possibly, packet dropout[5−6]; the latter is a potential source of instability and poor performance in NCSs because of the critical real-time requirement in control systems. How such packet dropout affects stability and performance of NCSs is the focused issue in this paper.

Packet dropout is one of the most important and special issues of NCSs. Hence, the effect of packet dropout on the stability and performance of NCSs has received great attention[7−12]. Two effective approaches have been adopted to deal with the packet dropout. The first is asynchronous dynamical systems (ADSs) approach[7−9]. Based on ADSs theory, a criterion to check whether the NCS is stable at a certain rate of packet dropout and the maximum packet dropout rate under which the overall system remains stable were proposed in [7]. However, the stability condition and controller design given in [7] were derived based on the assumption that packet dropout exists only in the sensor-to-controller side. Recently, some results were obtained in [8−9], where ADSs were introduced to model NCSs with packet dropouts on both sensor-to-controller and controller-to-actuator sides. The other is switched system approach[10−11]. The packet dropout process was modeled as an arbitrary but finite switching signal in [10−11]. Based on the switching systems theory, the stability and stabilization conditions were derived via linear matrix inequalities (LMIs) formulation.

On the other hand, Markovian jump linear systems (MJLSs) have been extensively studied in the past decades[12−16]. An iterative approach was proposed to model NCSs with Markovian packet dropout processes as MJLSs in [17−19]. Based on MJLSs theory, the stability and stabilization conditions were derived via LMIs formulation. As a dominant factor, the transition probabilities in the jumping process determine the system behavior to a large extent. The analysis and synthesis results in [17−19] were based on the assumption of the complete knowledge of the transition probabilities. However, in almost all types of communication networks, either the variation of delays or the packet dropouts are vague and random in different running periods of networks. Thus, all or part of the elements in the transition probabilities matrix are hard or costly to obtain. To the best of our knowledge, the stability and stabilization problems for NCSs with partly unknown transition probabilities have not been fully investigated to date. Especially, for the case where both sensor-to-controller and controller-to-actuator packet dropouts are considered simultaneously, very few results related to NCSs are available in the existing literature, which motivates the study of this paper.

In this paper, the stability and stabilization problems of a class of NCSs with bounded packet dropout are studied. NCSs with bounded packet dropout are modeled as MJLSs with partly unknown transition probabilities. The sufficient conditions for stochastic stability and stabilization of the underlying systems are derived via LMIs formulation. Lastly, an illustrative example is given to demonstrate the effectiveness of the proposed results.

This paper is organized as follows. An iterative method to model NCSs with bounded packet dropout as MJLSs is proposed in Section 1; the stochastic stability and stabilization conditions for NCSs are derived via LMIs in Section 2; Section 3 provides two numerical examples to illustrate the effectiveness of our results; Finally, Section 4 gives some concluding remarks.

1 Problem formulation and preliminaries

A typical NCS as depicted in Fig. 1 consists of three components: a nominal plant to be controlled, a network such as the Internet, and a controller. In this paper, it is assumed that the nominal plant is described by

$$x(k+1) = Ax(k) + Bu(k) \tag{1}$$

where $x(k) \in \mathbf{R}^n$ is the state and $u(k) \in \mathbf{R}^m$ is the input. A and B are known real constant matrices with appropriate dimensions.

We make the following assumptions about the NCS:

1) Networks exist between sensor and controller, and between controller and actuator;

2) The sensor is clock driven, the controller and the actuator are event driven;

3) The data are transmitted in a single packet at each time step.

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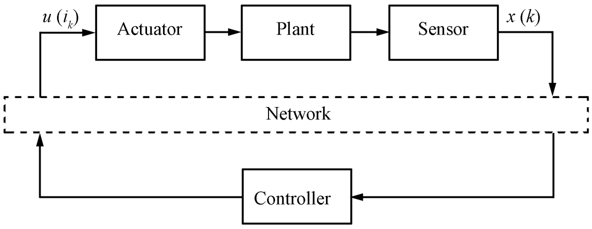

Fig. 1 Illustration of NCSs over communication network

Let $\mathcal{I} = \{i_1, i_2, \cdots\}$, which is a subsequence of $\mathbf{N} = \{1, 2, \cdots\}$, and denote the sequence of time points of successful data transmission from sensor to actuator. The state feedback controller law is

$$u(k) = Kx(k) \tag{2}$$

where $K \in \mathbf{R}^{m\times n}$ is to be designed. From the viewpoint of the zero-order hold, the control input is

$$u(l) = u(i_k) = Kx(i_k), \quad i_k \le l \le i_{k+1} - 1$$

Thus, the closed-loop system is

$$x(l+1) = Ax(l) + BKx(i_k), \quad i_k \le l \le i_{k+1} - 1 \tag{3}$$

From the closed-loop system (3), we can obtain

$$x(i_{k+1}) = \left(A^{i_{k+1}-i_k} + \sum_{r=0}^{i_{k+1}-i_k-1} A^r BK\right) x(i_k), \quad i_k \in \mathcal{I} \tag{4}$$

Define the packet dropout process as follows:

$$r(i_k) = i_{k+1} - i_k \tag{5}$$

Then, the closed-loop system (4) can be rewritten as an MJLS:

$$x(i_{k+1}) = \left(A^{r(i_k)} + \sum_{r=0}^{r(i_k)-1} A^r BK\right) x(i_k), \quad i_k \in \mathcal{I} \tag{6}$$

The packet dropout process $\{r(i_k), i_k \ge 0\}$ is described by a discrete-time homogeneous Markov chain, which takes values in the finite state space $\mathcal{S} = \{1, 2, \cdots, s\}$ with mode transition probabilities:

$$\pi_{ij} = \mathbb{P}(r(i_{k+1}) = j | r(i_k) = i) \ge 0, \quad \forall\, i, j \in \mathcal{S}$$

where

$$\sum_{j=1}^{s} \pi_{ij} = 1$$

and

$$s = \max_{i_k\in\mathcal{I}} (i_{k+1} - i_k)$$

The transition probabilities matrix is defined as:

$$\Pi = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1s} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{s1} & \pi_{s2} & \cdots & \pi_{ss} \end{bmatrix}$$

Remark 1. It is worth pointing out that the packet dropout process $\{r(i_k),\ i_k \ge 0\}$ includes both sensor-to-controller and controller-to-actuator packet dropouts. However, the results in [7, 10, 18] were derived based on the assumption that packet dropout exists only in the sensor-to-controller side.

It is noticed that the ideal knowledge about the transition probabilities of the packet dropout process are definitely expected to simplify the system analysis and design. However, the likelihood of obtaining such available knowledge is actually questionable, and the cost is probably expensive due to the complexity of networks. Hence, it is necessary to discuss packet dropout process with partly unknown transition probabilities, i.e., some elements of matrix Π are unknown. For instant, for system (6) with $s = 4$, the transition probabilities matrix Π may be as

$$\Pi = \begin{bmatrix} \pi_{11} & \circ & \pi_{13} & \circ \\ \circ & \circ & \circ & \pi_{24} \\ \pi_{31} & \circ & \pi_{33} & \circ \\ \circ & \circ & \pi_{43} & \pi_{44} \end{bmatrix}$$

where "$\circ$" represents the inaccessible element.

We have the following definitions for the packet dropout process (5).

Definition 1. Packet dropout process (5) is said to be partly-Markovian if part of elements of the transition probabilities matrix Π are unknown.

Definition 2. Packet dropout process (5) is said to be completely-Markovian if all the elements of the transition probabilities matrix Π are known.

Definition 3. Packet dropout process (5) is said to be arbitrary if all the elements of the transition probabilities matrix Π are unknown.

Denote

$$\mathcal{S} = \mathcal{S}^i_{\mathcal{K}} + \mathcal{S}^i_{\mathcal{UK}}, \quad \forall\, i \in \mathcal{S}$$

with

$$\begin{aligned} \mathcal{S}^i_{\mathcal{K}} &= \{j : \pi_{ij} \text{ is known}\} \\ \mathcal{S}^i_{\mathcal{UK}} &= \{j : \pi_{ij} \text{ is unknown}\} \end{aligned} \tag{7}$$

If $\mathcal{S}^i_{\mathcal{K}} \ne \emptyset$, it is further described as

$$\mathcal{S}^i_{\mathcal{K}} = (\mathcal{K}^i_1, \cdots, \mathcal{K}^i_m), \quad 1 \le m \le N \tag{8}$$

where $\mathcal{K}^i_m \in \mathbf{N}$ represents the m-th known element with the index $\mathcal{K}^i_m$ in the i-th row of matrix Π.

We have the following definition of stochastic stability for system (6).

Definition 4. System (6) is said to be stochastically stable if, for any initial condition $x_0 \in \mathbf{R}^n$ and $r_0 \in \mathcal{S}$, the following inequality holds

$$\mathrm{E}\left\{\sum_{k=0}^{\infty} \|x(k)\|^2 | x_0, r_0\right\} < \infty$$

To this end, the following lemmas will be essential for the proofs in the next section and their proofs can be found in the cited references.

Lemma 1[17]**.** System (6) is stochastically stable if and only if there exist a set of symmetric and positive definite matrices P_i, $i \in \mathcal{S}$ satisfying

$$\sum_{j=1}^{s} \pi_{ij} \left(A^j + B_j K\right)^{\mathrm{T}} P_j \left(A^j + B_j K\right) - P_i < 0$$

where

$$B_j = \sum_{r=0}^{j-1} A^r B \tag{9}$$

Lemma 2[20]**.** For any matrices $U \in \mathbf{R}^{n\times n}$ and $V \in \mathbf{R}^{n\times n}$, if the matrix V satisfies $V > 0$, then we have

$$UV^{-1}U^{\mathrm{T}} \geq U + U^{\mathrm{T}} - V$$

Lemma 3[11]**.** For a given symmetric matrix $W = \begin{bmatrix} W_{11} & W_{12} \\ W_{12}^{\mathrm{T}} & W_{22} \end{bmatrix}$, where $W_{11} \in \mathbf{R}^{p\times p}$, $W_{22} \in \mathbf{R}^{q\times q}$, and $W_{12} \in \mathbf{R}^{p\times q}$, the following three conditions are mutually equivalent:

1) $W < 0$;

2) $W_{11} < 0,\ W_{22} - W_{12}^{\mathrm{T}} W_{11}^{-1} W_{12} < 0$;

3) $W_{22} < 0,\ W_{11} - W_{12} W_{22}^{-1} W_{12}^{\mathrm{T}} < 0$.

2 Main results

In this section, we will develop the stability and stabilization results for the closed-loop NCS (6). The following theorem presents a sufficient condition for the stochastic stability of the considered system with partly unknown transition probabilities.

Theorem 1. Consider the closed-loop NCS (6) with partly-Markovian packet dropout process. If there exists matrix $P_i > 0,\ i \in \mathcal{S}$ such that

$$\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij}(A^j + B_jK)^{\mathrm{T}} P_j (A^j + B_jK) - \left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij}\right) P_i < 0 \tag{10}$$

$$\left(A^j + B_jK\right)^{\mathrm{T}} P_j \left(A^j + B_jK\right) - P_i < 0, \quad \forall\, j \in \mathcal{S}_{\mathcal{UK}}^i \tag{11}$$

where B_j is defined in (9), then system (6) is stochastically stable.

Proof. Based on Lemma 1, we know that system (6) is stochastically stable if

$$\sum_{j=1}^{s} \pi_{ij} \left(A^j + B_jK\right)^{\mathrm{T}} P_j \left(A^j + B_jK\right) - P_i < 0 \tag{12}$$

Note that

$$\sum_{j\in\mathcal{S}} \pi_{ij} = 1$$

Then, we can rewrite the left-hand side of (12) as

$$\Xi_i = \sum_{j\in\mathcal{S}} \pi_{ij} \left(A^j + B_jK\right)^{\mathrm{T}} P_j \left(A^j + B_jK\right) - \left(\sum_{j\in\mathcal{S}} \pi_{ij}\right) P_i$$

Thus, from (7) we have

$$\begin{aligned}
\Xi_i = &\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij} \left(A^j + B_jK\right)^{\mathrm{T}} P_j \left(A^j + B_jK\right) + \\
&\sum_{j\in\mathcal{S}_{\mathcal{UK}}^i} \pi_{ij} \left(A^j + B_jK\right)^{\mathrm{T}} P_j \left(A^j + B_jK\right) - \\
&\left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij}\right) P_i - \left(\sum_{j\in\mathcal{S}_{\mathcal{UK}}^i} \pi_{ij}\right) P_i = \\
&\sum_{j\in\mathcal{S}_{\mathcal{UK}}^i} \pi_{ij} \left(\left(A^j + B_jK\right)^{\mathrm{T}} P_j \left(A^j + B_jK\right) - P_i\right) + \\
&\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij} \left(A^j + B_jK\right)^{\mathrm{T}} P_j \left(A^j + B_jK\right) - \\
&\left(\textstyle\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij}\right) P_i
\end{aligned}$$

Since $\pi_{ij} \geq 0, \forall\, j \in \mathcal{S}_{\mathcal{UK}}^i$, it is straightforward that $\Xi_i < 0$ if (10) and (11) hold. Therefore, system (6) is stochastically stable against the partly unknown transition probabilities (7). □

Remark 2. It is noticed that if $\mathcal{S}_{\mathcal{UK}}^i = \emptyset,\ \forall\, i \in \mathcal{S}$, the underlying system is the one with completely known transition probabilities, which is the Markovian packet dropout process with known transition probabilities[17−19]. On the other hand, if $\mathcal{S}_{\mathcal{K}}^i = \emptyset,\ \forall\, i \in \mathcal{S}$, the underlying system is the one with completely unknown transition probabilities, which is an arbitrary packet dropout process[17, 19].

The following theorem gives a sufficient stochastic stabilization condition for discrete-time system (1) controlled by (3) over network with a partly-Markovian packet dropout process.

Theorem 2. Consider the closed-loop NCS (6) with partly-Markovian packet dropout process. If there exists matrix $X_i > 0,\ i \in \mathcal{S},\ G \in \mathbf{R}^{n\times n}$ and $Y \in \mathbf{R}^{m\times n}$ such that

$$\begin{bmatrix} -G - G^{\mathrm{T}} + \left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij}\right)^{-1} X_i & \mathcal{L}_{\mathcal{K}}^i \\ * & -\mathcal{X}_{\mathcal{K}}^i \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} -G - G^{\mathrm{T}} + X_i & (A^jG + B_jY)^{\mathrm{T}} \\ * & -X_j \end{bmatrix} < 0, \quad \forall\, j \in \mathcal{S}_{\mathcal{UK}}^i \tag{14}$$

where

$$\begin{aligned}
\mathcal{L}_{\mathcal{K}}^i \ = \ &\Big[\sqrt{\pi_{i\mathcal{K}_1^i}}(A^{\mathcal{K}_1^i}G + B_{\mathcal{K}_1^i}Y)^{\mathrm{T}} \ \cdots \\
&\sqrt{\pi_{i\mathcal{K}_m^i}}(A^{\mathcal{K}_m^i}G + B_{\mathcal{K}_m^i}Y)^{\mathrm{T}}\Big]
\end{aligned} \tag{15}$$

$$\mathcal{X}_{\mathcal{K}}^i = \mathrm{diag}\left\{X_{\mathcal{K}_1^i}, \quad \cdots, \quad X_{\mathcal{K}_m^i}\right\}, \quad \forall\, j \in \mathcal{S}_{\mathcal{K}}^i \tag{16}$$

with $\mathcal{K}_1^i, \cdots, \mathcal{K}_m^i$ described in (8), then NCS (6) is stochastically stable. Moreover, if the LMIs (13) and (14) have solutions, an admissible controller gain is given by

$$K = YG^{-1} \tag{17}$$

Proof. From (13), (14), and Lemma 3, we have

$$\begin{aligned}
&\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij}(A^jG + B_jY)^{\mathrm{T}} P_j (A^jG + B_jY) \ - \\
&\quad G - G^{\mathrm{T}} + \left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij}\right)^{-1} X_i < 0, \quad \forall\, j \in \mathcal{S}_{\mathcal{K}}^i
\end{aligned}$$

$$\begin{aligned}
&\left(A^jG + B_jY\right)^{\mathrm{T}} P_j \left(A^jG + B_jY\right) \ - \\
&\quad G - G^{\mathrm{T}} + X_i < 0, \quad \forall\, j \in \mathcal{S}_{\mathcal{UK}}^i
\end{aligned}$$

Denote $P_i^{-1} = X_i > 0$. By Lemma 2, we can obtain

$$-G^{\mathrm{T}} \left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij} P_i\right) G \leq -G - G^{\mathrm{T}} + \left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i} \pi_{ij}\right)^{-1} X_i$$

$$-G^{\mathrm{T}} P_i G \leq -G - G^{\mathrm{T}} + X_i$$

Then, we have

$$\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}(A^jG+B_jY)^{\mathrm{T}}P_j(A^jG+B_jY)-$$
$$G^{\mathrm{T}}\left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}P_i\right)G\ \le$$
$$\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}(A^jG+B_jY)^{\mathrm{T}}P_j(A^jG+B_jY)-$$
$$G-G^{\mathrm{T}}+\left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}\right)^{-1}X_i,\quad \forall\, j\in\mathcal{S}_{\mathcal{K}}^i$$
$$\left(A^jG+B_jY\right)^{\mathrm{T}}P_j\left(A^jG+B_jY\right)-G^{\mathrm{T}}P_iG\ \le$$
$$\left(A^jG+B_jY\right)^{\mathrm{T}}P_j\left(A^jG+B_jY\right)-$$
$$G-G^{\mathrm{T}}+X_i,\quad \forall\, j\in\mathcal{S}_{\mathcal{UK}}^i$$

Therefore,

$$\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}(A^jG+B_jY)^{\mathrm{T}}P_j(A^jG+B_jY)-$$
$$G^{\mathrm{T}}\left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}P_i\right)G<0,\quad \forall\, j\in\mathcal{S}_{\mathcal{K}}^i$$
$$\left(A^jG+B_jY\right)^{\mathrm{T}}P_j\left(A^jG+B_jY\right)-$$
$$G^{\mathrm{T}}P_iG<0,\quad \forall\, j\in\mathcal{S}_{\mathcal{UK}}^i$$

Let $Y=KG$. Then, we have

$$\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}(A^jG+B_jKG)^{\mathrm{T}}P_j(A^jG+B_jKG)-$$
$$G^{\mathrm{T}}\left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}P_i\right)G<0,\quad \forall\, j\in\mathcal{S}_{\mathcal{K}}^i$$
$$\left(A^jG+B_jKG\right)^{\mathrm{T}}P_j\left(A^jG+B_jKG\right)-$$
$$G^{\mathrm{T}}P_iG<0,\quad \forall\, j\in\mathcal{S}_{\mathcal{UK}}^i$$

which yields

$$G^{\mathrm{T}}\left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}(A^j+B_jK)^{\mathrm{T}}P_j(A^j+B_jK)\right)G-$$
$$G^{\mathrm{T}}\left(\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}P_i\right)G<0,\quad \forall\, j\in\mathcal{S}_{\mathcal{K}}^i$$
$$G^{\mathrm{T}}\left(\left(A^j+B_jK\right)^{\mathrm{T}}P_j\left(A^j+B_jK\right)-P_i\right)G<0,$$
$$\forall\, j\in\mathcal{S}_{\mathcal{UK}}^i$$

which is equivalent to

$$\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}(A^j+B_jK)^{\mathrm{T}}P_j(A^j+B_jK)-\sum_{j\in\mathcal{S}_{\mathcal{K}}^i}\pi_{ij}P_i<0$$
$$\left(A^j+B_jK\right)^{\mathrm{T}}P_j\left(A^j+B_jK\right)-P_i<0,\quad \forall\, j\in\mathcal{S}_{\mathcal{UK}}^i$$

Thus, if (13) and (14) hold, (10) and (11) will be satisfied by Theorem 1. In view of Theorem 1, system (6) is stochastically stable. Moreover, the desired controller gain is given by (17). □

Remark 3. From the development in the above theorems, one can clearly see that our obtained stability and stabilization conditions actually cover the results for completely-Markovian packet dropout process and arbitrary packet dropout process.

3 Numerical example

In this section, a numerical example and simulations are given to illustrate the effectiveness of the proposed methods. Let us consider the nominal continuous time system with no disturbance input[21]:

$$\dot{x}(t)=\bar{A}x(t)+\bar{B}u(t)$$

where

$$\bar{A}=\begin{bmatrix}-1 & 0 & -0.5\\ 1 & -0.5 & 0\\ 0 & 0 & 0.5\end{bmatrix},\quad \bar{B}=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

When the plant is sampled with a sampling period $T=0.5\,\mathrm{s}$, the discretized system is system (1) with

$$A=\begin{bmatrix}0.6065 & 0 & -0.2258\\ 0.3445 & 0.7788 & -0.0536\\ 0 & 0 & 1.2840\end{bmatrix},\quad B=\begin{bmatrix}-0.0582\\-0.0093\\0.5681\end{bmatrix}$$

Both the continuous time system and discretized system are unstable because the eigenvalues of $\bar{A}$ are $-0.5,-1,0.5$ and the eigenvalues of A are $0.7788, 0.6065, 1.2840$. Furthermore, we assume that the packet dropout upper bound is $s=4$ and the transition probabilities matrix is as follows:

$$\Pi=\begin{bmatrix}0.3 & \circ & 0.1 & \circ\\ \circ & \circ & 0.3 & 0.2\\ \circ & 0.1 & \circ & 0.3\\ 0.2 & \circ & \circ & \circ\end{bmatrix}$$

Applying Theorem 2, we can obtain

$$X_1=\begin{bmatrix}0.3242 & -0.0532 & 0.0343\\ -0.0532 & 0.4437 & 0.0472\\ 0.0343 & 0.0472 & 0.3594\end{bmatrix}$$
$$X_2=\begin{bmatrix}0.3670 & -0.0698 & 0.0498\\ -0.0698 & 0.5212 & 0.0571\\ 0.0498 & 0.0571 & 0.3999\end{bmatrix}$$
$$X_3=\begin{bmatrix}0.3095 & -0.0599 & 0.0497\\ -0.0599 & 0.4339 & 0.0530\\ 0.0497 & 0.0530 & 0.3258\end{bmatrix}$$
$$X_4=\begin{bmatrix}0.1671 & -0.0280 & 0.0227\\ -0.0280 & 0.2264 & 0.0280\\ 0.0227 & 0.0280 & 0.1814\end{bmatrix}$$

and

$$Y=\begin{bmatrix}-0.0267 & -0.0465 & -0.5157\end{bmatrix}$$
$$G=\begin{bmatrix}0.1671 & -0.0280 & 0.0227\\ -0.0280 & 0.2264 & 0.0280\\ 0.0227 & 0.0280 & 0.1814\end{bmatrix}$$

Therefore, the desired controller gain is given by

$$K=YG^{-1}=\begin{bmatrix}0.0584 & 0.0295 & -0.9048\end{bmatrix}$$

Secondly, let us consider the linearized state-space model of motion about the upward unstable equilibrium position of a pendulum:

$$\dot{x}(t) = \bar{A}x(t) + \bar{B}u(t)$$

where

$$\bar{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 63.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -33.31 & 0 & 0 & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} 0 \\ -520.72 \\ 0 \\ 804.13 \end{bmatrix}$$

When the plant is sampled with a sampling period $T = 0.005\,\mathrm{s}$, the discretized system is system (1) with

$$A = \begin{bmatrix} 1.0008 & 0.0050 & 0.0000 & 0.0000 \\ 0.3163 & 1.0008 & 0.0000 & 0.0000 \\ -0.0004 & -0.0000 & 1.0000 & 0.0050 \\ -0.1666 & -0.0004 & 0.0000 & 1.0000 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.0065 \\ -2.6043 \\ 0.0101 \\ 4.0210 \end{bmatrix}$$

The packet dropout upper bound is $s = 4$ and the transition probabilities matrix is as follows

$$\Pi = \begin{bmatrix} 0.3 & \circ & 0.1 & \circ \\ \circ & \circ & 0.3 & 0.2 \\ \circ & 0.1 & \circ & 0.3 \\ 0.2 & \circ & \circ & \circ \end{bmatrix}$$

Applying Theorem 2, we can obtain the controller gain, which is given by

$$K = \begin{bmatrix} 34.9978 & 3.5963 & 18.1167 & 2.2212 \end{bmatrix}$$

4 Conclusions

In this paper, the stability and stabilization problems of a class of NCSs with bounded packet dropout are investigated. The main contribution of this paper is that both sensor-to-controller and controller-to-actuator packet dropouts have been taken into account. Moreover, the elements of the transition probabilities matrix are partly unknown. The system under consideration is more general, which covers the systems with completely known and completely unknown transition probabilities as two spacial cases. The sufficient conditions for stochastic stability and stabilization of the underlying systems are derived via LMIs formulation. Lastly, two illustrative examples are given to demonstrate the effectiveness of the proposed results.

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