

# Passive Dynamic Object Manipulation: Preliminary Definition and Examples

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**Abstract** In this work, we introduce a category of dynamic manipulation processes, namely passive dynamic object manipulation, according to which an object is manipulated passively. Specifically, we study passive dynamic manipulation here. We define the main concept, discuss the challenges, and talk about the future directions. Like other passive robotic systems, there are no actuators in these systems. The object follows a path and travels along it under the effect of its own weight, as well as the interaction force applied by each manipulator on it. We select some simple examples to show the concept. For each example, dynamic equations of motion are derived and the stability of the process is taken into account. In this direction, some rules are derived under which we ensure that the manipulation process does not fail. Simulations support this idea.

**Key words** Passive systems, dynamic manipulation, passive manipulation, robotic system

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Dynamic manipulation is a main branch of robotics in which an object is manipulated dynamically using robotic systems. Important topics in dynamic manipulation include dynamic equations, control and stability of the process. The nature of control indicates that we should actuate the system using some actuators to force the system to follow our preferred trajectory. However, system “actuation” means energy consumption in order to force the system in the desired manner. The less energy consumption we have in the system, the more efficiency can be obtained. Passive systems are usually more efficient than active systems.

Here, we introduce a new branch of dynamic manipulation, namely passive dynamic object manipulation (PDM). In PDM, we manipulate objects by using no actuators and force the objects to follow a desired trajectory. An example of passive dynamic object manipulation can be seen in Fig. 1 where children slide down a slide in the playground. This task can be interpreted as passive manipulating of children by the slide. Many examples of such passive manipulation occur daily around us.



Fig. 1 An example of passive dynamic object manipulation

Our study is about passive dynamic systems, where a device operating passively, can dynamically be efficient since it needs no energy for stabilization or control, and only requires power to recover small energy losses. The most fundamental cause of this energy loss is impact, which can be found in all passive systems. In most passive-dynamic studies, power comes from the potential energy gained by the decreasing height of the center of mass of the system,

e.g., moving down a ramp. Gravitational power is an easy-to-implement factor for other simple low-power sources. In a way, the passive dynamic approach is the opposite of the trajectory control approach, which tends to use control actuation to force a system against its natural dynamic tendencies.

Passive systems have a long history in robotics. Most passive systems designed and manufactured up to now are passive walkers and runners. Simple two-legged passive walking toys were designed at least a century ago as in [1–2]. These toys operate on principles described by well-known concepts, but their analysis has only been possible recently by using modern computers, because of nonlinear nature of Newton’s laws, as applied to such walking machines.

Biped ramp walkers travel down a gentle slope and they walk in a somewhat stable, passive, three-dimensional gait. More recently, Coleman et al.<sup>[3]</sup> and Mombaur et al.<sup>[4]</sup> have demonstrated different kinds of passive walkers that walk very well. Their Tinkertoy walker is stable only when in motion. Also, Collins et al.<sup>[5]</sup> has designed an efficient biped walking robot based on passive ramp walker mechanisms.

McGeer effectively started the modern incarnation of the passive dynamic approach to locomotion. Having noted that the Wright brothers mastered gliding first, then added a small amount of power to make successful powered airplanes, he used the development of airplanes as inspiration<sup>[6]</sup>. In this view, passive dynamic ramp walkers are the gliders of walking robots. McGeer developed these free-motion designs using a nonlinear stability analysis based on numerical simulation of the Newton-Euler equations of motion. These studies led to his completely passive designs, implemented both in simulation and as walking machines built of bars and hinges. The McGeer machines, which have remarkably human-looking gaits, are more energy efficient than other walking robots, and are inherently stable with respect to small disturbances<sup>[6]</sup>.

This work is also a kind of nonprehensile manipulation (manipulation without a form- or force-closure grasp). Using dynamic nonprehensile manipulation (DNM), we can manipulate an object too large or heavy to be grasped and lifted by eliminating the gripper. Therefore, the structure of the manipulator is simplified. DNM allows a manipulator to control multiple parts simultaneously, using whatever surfaces of the manipulator that are available. If we define the workspace of a robot as the set of reachable states for

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an object manipulated by the robot, the size of the robot's workspace is effectively increased by throwing the object to points outside the robot's kinematic workspace<sup>[7]</sup>.

Lynch et al.<sup>[8]</sup> introduced some planning methods for manipulation of polygonal objects on the surface of 1-DOF manipulator. According to their methods, the object can be moved forward away from the arm's pivot. Tabata et al.<sup>[9]</sup> studied a tossing problem where a 1-DOF manipulator tosses a circular object out of its workspace into a desired position with a free orientation. To move the object out of the arm's workspace, the release velocity should be high, which results in a large catching impact.

Other examples of DNM include dynamic object manipulation of a disc using two planar manipulation<sup>[10]</sup>, dynamic manipulation of objects using multiple manipulators<sup>[11]</sup>, dynamic manipulation of multibody objects<sup>[12]</sup>, kinematic manipulation such as rolling an object between two plates<sup>[13–14]</sup>, quasi-static manipulation such as pushing with point contact<sup>[15]</sup> or pushing parts on a conveyor belt using a robot with just one joint<sup>[16]</sup>, and dynamic manipulation such as dynamic rolling<sup>[17]</sup> and snatching, rolling, and throwing<sup>[18]</sup>.

## 1 Motivation and outline

Simplicity and high efficiency for mechanical systems is the first and main motivation for studying passive systems. PDM is no exception. Second, in such passive systems, the control of speed and position is not a problem since the process is performed autonomously. In addition, the simple structure of the machine helps one to understand the concept.

Dynamic manipulation is a challenging subject in robotics. Dynamics, control, and stability are the most important issues in the systems categorized in this range. Impact usually is a major part of such systems. Therefore, most of these systems can be categorized as nonlinear systems with impulse effects. To study control and stability of such nonlinear systems is very complicated and need be paid much attention. We study the possibility of introducing a new branch of manipulation systems, which are stable and do not need any control actuation. Our main goal is to define some passive systems capable of manipulating objects. Here, "object" is anything that is to be manipulated passively. "Passive manipulation" is the process of such motion without consuming any net energy. Moreover, "manipulator" in PDM is the device that performs any task on the "object".

Some of the advantages of such passive manipulation systems in comparison with active ones are:

- 1) PDM systems are less energy-consuming and more efficient.
- 2) There is no need to consume cost, energy, and time to study control strategy in PDM systems.
- 3) In a successful design for a PDM system, stability is guaranteed. However, the stability region of passive systems is usually very limited.
- 4) PDM may construct a structure to design optimal and active manipulation systems. This is achieved by adding small actuation to passive systems in order that they follow some favorite trajectories close to their natural behavior.

Although passive systems are very simple and comprehensible, the first step in designing such systems, i.e., to find appropriate parameters for the system so that it works stably, is very important and time-consuming. Here, the main challenge is to show the possibility of designing some

simple kinds of PDM. If so, what are the stability conditions for the PDM systems to act properly?

Future directions of this study include finding a general framework and approach to design and study PDM systems. This can be divided into several sub-problems such as selection of optimal system's parameters and stability issues. The study of manipulation of multibody passive objects using multilink passive manipulators is also an issue on which the authors are currently working. In addition, it is very interesting to abridge passive manipulation with passive locomotion. Actually, the current work is part of a more extensive, ongoing study to find relation between dynamic passive manipulation and dynamic passive bipedal locomotion. In fact, we are interested in defining passive locomotion as a special case of passive manipulation.

Here, we try to introduce the idea of passive dynamic object manipulation with the aid of some examples. Since our main goal is to open discussion for future works, the examples are chosen for their simplicity. The examples do not belong to any special class of dynamic manipulation. In the following sections, we introduce the concept and discuss our way to approach it, followed by a few examples for demonstration purposes.

## 2 Concepts and planning

As discussed before, the idea of passive dynamic object manipulation has arisen from our interest in designing systems, which have no actuators but still able to manipulate objects. From the view of passive systems, this idea is not novel as other researchers have used it to design and analyzed other passive systems such as passive walkers. We focus on systems including passive manipulators, which are able to redirect the path of an object. In fact, these manipulators impose some constraints on the object's trajectory. Here, we define manipulators as anything that can passively manipulate an object. For example, when a ball strikes the wall and rebounds, we assume that the wall is a passive manipulator, which changes the path of the ball.

The motivation of moving an object (the ball) along its path comes from gravity. In addition, impact is an inevitable occurrence in this kind of passive system. These two factors are equivalent during the process, i.e., the energy dissipated by impact will be given back to the system by gravity. Usually, impact occurs in a semi-periodic manner. Between two impacts, the system tries to retrieve its energy. We are interested in designing a system in such a way that the cycles between consecutive impacts are similar. Then, we can claim that our system has a periodic and stable behavior. Detailed analysis of stability of dynamical systems usually involves complicated formulation and computation. In this paper, we will not deal with this intricate topic. In the following section, we offer some examples of PDM systems.

## 3 Examples of PDM

Here, we offer three simple examples of PDM: a frictionless bouncing ball, a bouncing ball with friction, and the passive manipulation of a polygonal object. We analyze the examples and derive some conditions for the systems to be periodic and stable.

### 3.1 Example 1: frictionless bouncing ball

Suppose that we want to design a system including a ball bouncing on different level surfaces. The goal for the ball is to negotiate down stairs without any increase in velocity, i.e., after each impact, the ball starts a new cycle the same

as the previous one (see Fig. 2). Actually, this example can be seen as a hopping problem, but we categorize it as a passive dynamic object manipulation, because there is no actuator acting on the ball. We suppose that the stairs manipulate the ball in a passive manner. This point of view will help us to analyze and design a passive manipulation system in future work, since impact is the main part of a passive system.

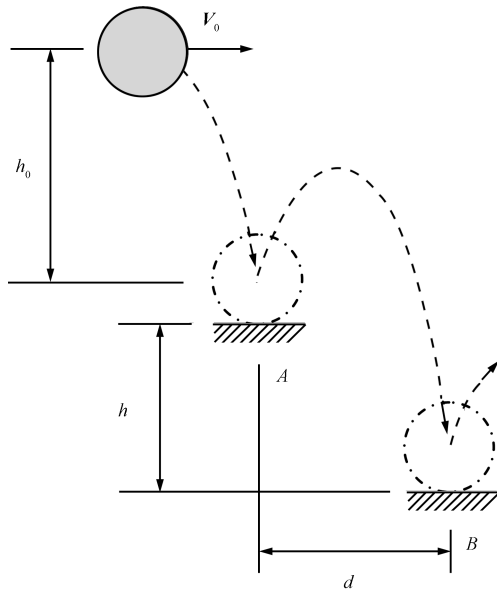


Fig. 2 Two consecutive impact points A and B and corresponding parameters when a ball bounces down stairs

Considering that the surfaces are frictionless and the impacts between ball and surfaces are not purely elastic (with restitution coefficient of  $0 < \varepsilon < 1$ ), we are interested in the conditions of the system and the proper arrangement of the stairs which will allow the ball to have a periodic trajectory between each step. By our definition, a step or period occurs between two consecutive impacts of the ball with sequential stairs. Fig. 2 depicts two consecutive impact points. Their vertical distance is  $h$  and horizontal distance is  $d$ . We may model each period with the two impact points of A and B. For the process to be periodic, it is necessary for the object to have equal impact velocity at both impact points A and B. In addition, rebound velocity at both impact points should be equal. Suppose that the ball impacts the stair at point A with the velocity of  ${}^A\mathbf{V}$ . Also, suppose that its vertical velocity before impact at point A is  ${}^A V_y^-$  and afterward it has a rebound vertical velocity of  ${}^A V_y^+$ . As the impact is not purely elastic, these two velocities are related as

$${}^A V_y^+ + \varepsilon \times {}^A V_y^- = 0 \tag{1}$$

where  $\varepsilon$  is the restitution coefficient of surfaces. Moreover, let  ${}^A V_x^-$  and  ${}^A V_x^+$  denote the horizontal velocity of the object just before and just after impact at point A, respectively. Since the surfaces are frictionless, we have no reaction in horizontal direction and therefore there is no change in the momentum of the ball. Therefore, we have

$${}^A V_x^+ = {}^A V_x^- \tag{2}$$

Let us denote vertical velocity of the ball before and after impact at point B by  ${}^B V_y^-$  and  ${}^B V_y^+$ , respectively. If we

want the ball to have the same impact velocity at impact points A and B,  ${}^B V_y^-$  should be equal to  ${}^A V_y^-$ . Therefore, from (1), we should have

$${}^A V_y^+ + \varepsilon \times {}^B V_y^- = 0 \tag{3}$$

On the other hand, supposing that we have no air resistance acting on the ball during free flight, we may derive a relation for  $h$  (see Fig. 2) as

$$h = \frac{({}^B V_y^-)^2 - ({}^A V_y^+)^2}{2g} \tag{4}$$

which after combining with (3) will be

$$h = \frac{({}^A V_y^-)^2}{2g} (1 - \varepsilon^2) \tag{5}$$

In the initial state, suppose that the ball drops down with the initial height of  $h_0$  and initial horizontal velocity of  $V_0$ . Equation (2) shows that there is no change in the horizontal velocity of the ball during impacts. This is true for other moments of the process as well, because there is no horizontal force acting on the ball. Therefore, we can write that

$${}^A V_x^+ = {}^A V_x^- = {}^B V_x^+ = {}^B V_x^- = V_0 \tag{6}$$

Applying the energy law between initial state and the first impact point, e.g., point A, gives us

$$h_0 = \frac{({}^A V_y^-)^2}{2g} \tag{7}$$

Combining (7) with (5) results in

$$h = (1 - \varepsilon^2) h_0 \tag{8}$$

which determines the vertical distance of two sequential stairs. Moreover, by computing time  $t$  between two impacts, i.e.,  $t = ({}^B V_y^- - {}^A V_y^+)/g$ , and using (3) and (6), we can easily derive the horizontal distance of the consecutive impact point,  $d$  (see Fig. 2). That is

$$d = \sqrt{\frac{2h_0}{g}} (1 + \varepsilon) V_0 \tag{9}$$

### 3.2 Example 2: bouncing ball with friction

In this example, we repeat the previous study with the exception that we have some friction in the contact surfaces of the system (see Fig. 3). We assume that in this example,  $r$  is the ball's radius. Furthermore,  $m$  and  $J$  denote ball's mass and its moment of inertia about the center of mass (COM), respectively. In addition, we further make an assumption that the ball is rolling without slip during impact. This can be achieved with enough friction to prevent the ball from slipping.

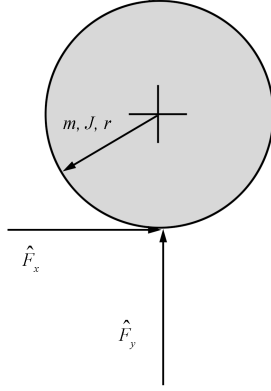


Fig. 3 The ball's parameters ( $m$  is the ball's mass,  $J$  is the ball's moment of inertia about center of mass, and  $r$  is the ball's radius.) and impulse forces acting on the ball during impact

Results for changing vertical velocities during impact are the same as those derived in Example 1. Therefore, we focus on horizontal direction. During impact, two impulse reactions of  $\hat{F}_x$  and  $\hat{F}_y$  act on ball (see Fig. 3). These impulse forces change the linear and angular momentum of the ball during impacts. Suppose that this impact occurs at point  $A$ . For the horizontal direction we have

$$\hat{F}_x = m \left( {}^A V_x^+ - {}^A V_x^- \right) \quad (10)$$

Assuming that the ball has an angular velocity of  ${}^A \omega^-$  and  ${}^A \omega^+$  just before and just after impact, respectively, we may write a relation for them as

$$\hat{F}_x \times r = J \left( {}^A \omega^+ - {}^A \omega^- \right) \quad (11)$$

The constraint of rolling the ball without slip gives us an additional relation of

$${}^A V_x^+ = -r \times {}^A \omega^+ \quad (12)$$

therefore, combining (10) ~ (12) gives us

$${}^A V_x^+ = \left( \frac{mr^2}{mr^2 + J} \right) \left( {}^A V_x^- - \frac{J}{mr} {}^A \omega^- \right) \quad (13)$$

and

$${}^A \omega^+ = \left( \frac{mr}{mr^2 + J} \right) \left( {}^A V_x^- - \frac{J}{mr} {}^A \omega^- \right) \quad (14)$$

These two equations obviously indicate that the velocities after impact, i.e., linear velocity in the  $x$ -direction and angular velocity, are different from velocities before impact only in the first impact and in the case of

$$V_{x,0} \neq r \times \omega_0 \quad (15)$$

where  $V_{x,0}$  and  $\omega_0$  are the initial linear and angular velocities of the ball, respectively. Since after each impact, (12) is valid for the velocities, and because the velocities after each impact are the velocities before the next impact, for the impact  $i$ ,  $i = B, C, \dots$  (impact points), we may write that

$${}^i V_x^- = r \times {}^i \omega^- \quad (16)$$

Substituting (16) in (13) and (14) gives us

$${}^i V_x^+ = {}^i V_x^-, \quad {}^i \omega^- = {}^i \omega^+, \quad i = B, C, \dots \quad (17)$$

In the case of initial conditions  $h_0$ ,  $V_0$ , and  $\omega_0$  to be initial height, initial horizontal linear velocity, and initial angular velocity, respectively, (8) is still valid for this example. For  $d$ , we may easily obtain

$$d = \sqrt{\frac{2h_0}{g}} (1 + \varepsilon) \left( \frac{mr^2}{mr^2 + J} \right) \left( V_0 - \frac{J}{mr} \omega_0 \right) \quad (18)$$

### 3.3 Example 3: passive dynamic manipulation of a polygonal object

In this example, we model a dynamic manipulation system including a polygonal object and a series of 1-DOF passive manipulators (see Fig. 4). The goal of designing such a system is to manipulate the object in a (semi) periodic manner. For simplicity, we assume the object to be a rectangular object. Assumptions in this example include:

- 1) All manipulators have same structures;
- 2) The position of manipulator  $i = 1, 2, \dots$  with respect to manipulator  $i+1$  is the same as that of manipulator  $i+1$  with respect to the manipulator  $i+2$ ;
- 3) Manipulators do not collide with each other;
- 4) The impact between object and each manipulator is a plastic (purely inelastic) impact, i.e.,  $\varepsilon = 0$ ;
- 5) At the first impact, the object lands on the middle point of the first manipulator's palm;
- 6) There is enough friction in the surfaces to prevent the object slipping on the manipulator;
- 7) The object impacts each manipulator at the angle  $\alpha$  and leaves it, i.e., impacts the next manipulator, at the angle  $\beta$  (see Fig. 4, angles are measured clockwise with respect to the vertical direction);
- 8) The object's mass is  $m$ , the palm's mass is  $M$ , and two other links are mass-less. In addition, the length of each link of the manipulator is  $l$ .

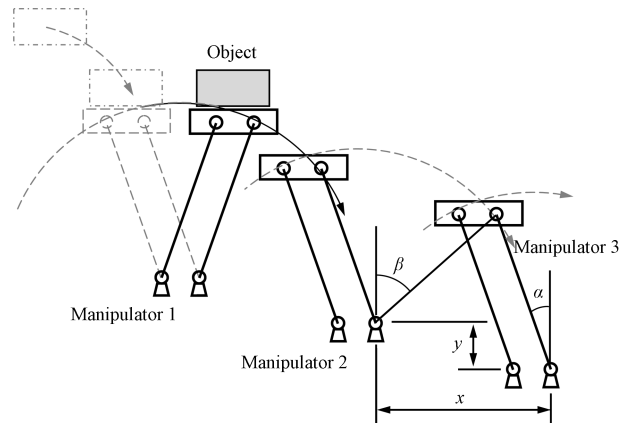


Fig. 4 Passive manipulation of a rectangular object using some serial passive manipulations

We begin with analyzing the impact between the object and the motion-less palm of the first manipulator in its initial configuration (see Fig. 5). Suppose that the absolute velocity vector of the object in the moment of impact is

$$\mathbf{V} = V_x \mathbf{i} + V_y \mathbf{j} \quad (19)$$

where  $\mathbf{i}$  and  $\mathbf{j}$  are unit vectors in the  $x$ - $y$  plane. In the case of impact, all impulse reactions in the  $n$ -direction are endured by the links ( $n$ - $t$  coordinate system is shown in Fig. 5). We are actually interested in the  $t$ -direction motion of the object. There is no force or impulse load acting on the integrated system of palm-object in the  $t$ -direction. The

absolute velocity of the object just before impact,  $V_t^-$ , in the  $t$ -direction can be written as

$$V_t^- = \mathbf{V} \times \mathbf{e}_t = V_x \cos \alpha + V_y \sin \alpha \quad (20)$$

where  $\mathbf{e}_t$  is the unit vector in the  $t$ -direction. So regarding Assumption 4), we can write an equation for the velocity of the object and the palm just after the impact,  $V_t^+$ , by using the linear momentum conservation for the integrated system of object-palm in the  $t$ -direction:

$$V_t^+ = \frac{m}{m+M} V_t^- = \frac{m}{m+M} (V_x \cos \alpha + V_y \sin \alpha) \quad (21)$$

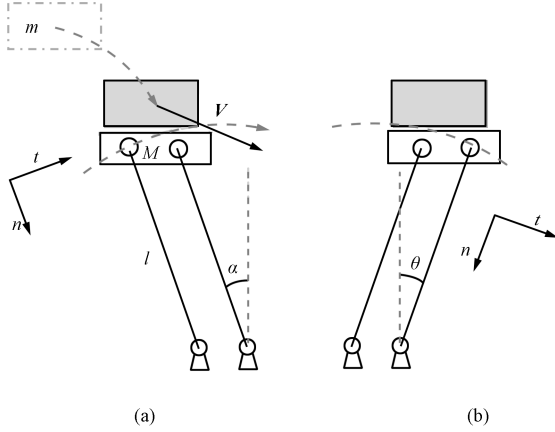


Fig. 5 Two snapshots corresponding to the manipulation of rectangular object using a 1-DOF manipulator  
(a) Impact between the object with absolute velocity  $\mathbf{V}$  and the manipulator in its initial configuration;  
(b) Manipulator tends to rotate clockwise after satisfying conditions (22) and (23)

For the manipulation process to be continued, it is necessary for the manipulator to rotate clockwise until the object releases. This means that the object should reach the top point in its traveling trajectory. Therefore, we should have some condition like

$$V_t^+ > \sqrt{2gl(1 - \cos \alpha)} \quad (22)$$

and therefore

$$V_x \cos \alpha + V_y \sin \alpha > \frac{m+M}{m} \sqrt{2gl(1 - \cos \alpha)} \quad (23)$$

If the process satisfies condition (23), the manipulator reaches its vertical configuration and afterward, because of gravity, it tends to rotate clockwise and falls down (see Fig. 5 (b)). For every configuration corresponding to angle  $\theta$  (Fig. 5 (b)), the magnitude of absolute velocity of both object and palm,  $V_\theta$ , can be easily derived from conservation of energy law as

$$|V_\theta| = V_t(\theta) = l\dot{\theta} = \frac{\sqrt{\left(\frac{m(V_x \cos \alpha + V_y \sin \alpha)}{m+M}\right)^2 + 2gl(\cos \alpha - \cos \theta)}}{1} \quad (24)$$

and for  $\theta = \beta$ , we clearly have

$$|V_\beta| = \frac{\sqrt{\left(\frac{m(V_x \cos \alpha + V_y \sin \alpha)}{m+M}\right)^2 + 2gl(\cos \alpha - \cos \beta)}}{1} \quad (25)$$

and

$$\mathbf{V}_\beta = |V_\beta| \cos \beta \mathbf{i} - |V_\beta| \sin \beta \mathbf{j} \quad (26)$$

Now, whenever the second impact occurs (where  $\theta = \beta$ ), the velocity  $\mathbf{V}_\beta$  in (26) plays the role of  $\mathbf{V}$  in (19). From now on, we may consider

$$V_x = |V_\beta| \cos \beta, \quad V_y = -|V_\beta| \sin \beta \quad (27)$$

For the passive manipulation process to be periodic,  $\mathbf{V}_\beta$  described by (26) and (27) should satisfy (23). Each value of  $\beta$ , which satisfies this condition, can be chosen for the separation of the object from current manipulator and occurrence of the next impact. It is not required for all  $\beta$  in all manipulators to be the same for continuation of the manipulation process, but if we are interested in periodic cycles, we may impose such a constraint. Then for the impact  $i$ ,  $i = 2, 3, \dots$ , we may rewrite (25) as

$$|V_\beta| = \sqrt{\left(\frac{m|V_\beta|(\cos \beta \cos \alpha - \sin \beta \sin \alpha)}{m+M}\right)^2 + 2gl(\cos \alpha - \cos \beta)} = \sqrt{\left(\frac{m}{m+M}\right)^2 \cos^2(\beta - \alpha) |V_\beta|^2 + 2gl(\cos \alpha - \cos \beta)} \quad (28)$$

That is,

$$|V_\beta|^2 = \frac{2gl(\cos \alpha - \cos \beta)}{\left(1 - \left(\frac{m}{m+M}\right)^2 \cos^2(\beta - \alpha)\right)} \quad (29)$$

and with a little calculation, we will obtain

$$|V_\beta| = \sqrt{\frac{2(m+M)^2 gl(\cos \alpha - \cos \beta)}{M^2 + 2mM + m^2 \sin^2(\beta - \alpha)}} \quad (30)$$

Now, every  $\alpha$  and  $\beta$  which evaluate  $|V_\beta|$  in such a way that it satisfies (30), (27) and afterward condition (23) are acceptable. Having determined  $\alpha$  and  $\beta$ , we may now determine the vertical and horizontal distance of two consecutive manipulators (see Fig. 4) as

$$\begin{aligned} x &= l(\sin \alpha + \sin \beta) \\ y &= -l(\cos \alpha - \cos \beta) \end{aligned} \quad (31)$$

According to Assumption 2), we suppose that  $x$  and  $y$  are the system's parameters and do not change for individual manipulators. However, it is notable that, if there is any inaccuracy in  $x$  and  $y$  in (31), it does not mean that the process cannot be successful. In fact, (22) and (23) are the key equations to check if the manipulation is successful in any step. Fig. 6 (a) shows an exact-assembled manipulation system while Fig. 6 (b) illustrates an inaccurate-assembled one. In this figure, the red-colored manipulator has some offsets in  $x$  and  $y$  with respect to the values computed from (31). It is seen that the manipulation continues in spite of these offsets. Furthermore, the acceptable offsets in  $x$  and  $y$  should let the manipulation process be performed physically. For example, Fig. 6 (c) depicts a very inaccurately assembled manipulation system. In this system, the red-colored manipulator physically cannot pass the object to the next manipulator. Therefore, the whole process refuses to continue.

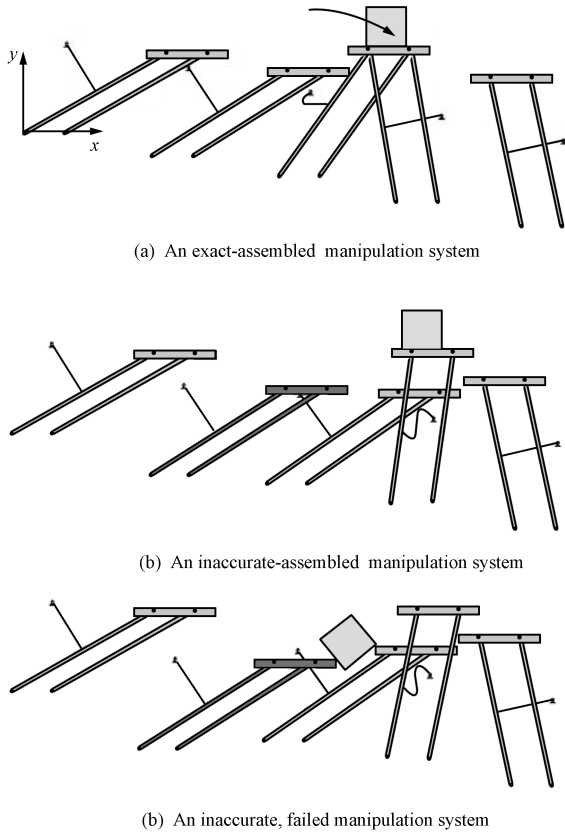


Fig. 6 Passive manipulation of a rectangular object using a series of 1-DOF passive manipulators

#### 4 Simulation and results

In this section, we simulate the examples discussed in the previous section. All simulations have been done with 2-D working model software. For the first example of a friction-less bouncing ball, we assembled and ran a system in the software with the initial conditions

$$h_0 = 1 \text{ m}, \quad V_0 = 2 \text{ m/s} \quad (32)$$

and system parameters of

$$\begin{aligned} \mu &= 0 \text{ (no friction)}, \quad \varepsilon = 0.8 \\ h &= 0.36 \text{ m}, \quad d = 1.626 \text{ m} \end{aligned} \quad (33)$$

where  $h$  and  $d$  are calculated based on (7) and (8). Fig. 7 depicts the  $y$ -direction displacement of the ball in this simulation. It is seen that the behavior of the ball is quite periodic in the  $y$ -direction as we expected. We did not plot  $x$ -direction diagram of the ball, as it did not add any valuable information.

For the second example, i.e., a bouncing ball with friction, the initial conditions are as follows

$$h_0 = 1 \text{ m}, \quad V_0 = 3 \text{ m/s}, \quad \omega_0 = 0.5 \text{ rad/s} \quad (34)$$

By choosing

$$\begin{aligned} \mu &= 0.5 \text{ (system with friction)}, \quad \varepsilon = 0.8 \\ r &= 0.5 \text{ m}, \quad m = 0.283 \text{ kg}, \quad J = 0.035 \text{ kg} \cdot \text{m}^2 \end{aligned} \quad (35)$$

where  $\mu$  is the coefficient of coulomb friction, and  $r$ ,  $m$ , and  $J$  are defined in Fig. 3, the parameters  $d$  and  $h$  of the

system could be obtained from (18) and (7) as

$$h = 0.36 \text{ m}, \quad d = 1.565 \text{ m} \quad (36)$$

The  $y$ -direction linear velocity of this example is the same as that of the previous example (see Fig. 7), but the angular velocity and  $x$ -direction linear velocity of the ball in the current example is depicted in Figs. 8 and 9, respectively. It is obvious that the angular velocity and  $x$ -direction linear velocity of the ball have jumps during the first impact, and afterward these do not change, as discussed in the previous section; please refer to (13) ~ (17) and corresponding discussion. After the first impact, they experience no change during the process. In this case, their constant values can be obtained from (17).

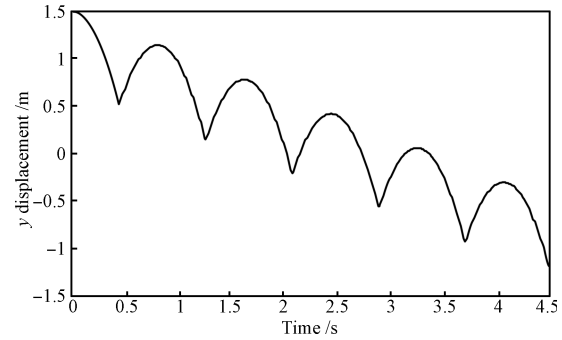


Fig. 7  $y$ -position of the ball in both with-friction and friction-less mode

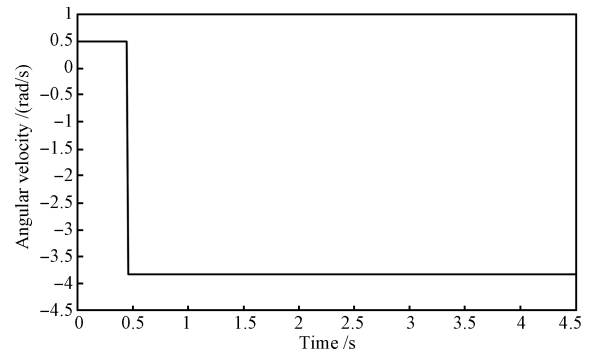


Fig. 8 Angular velocity of the ball in a friction mode

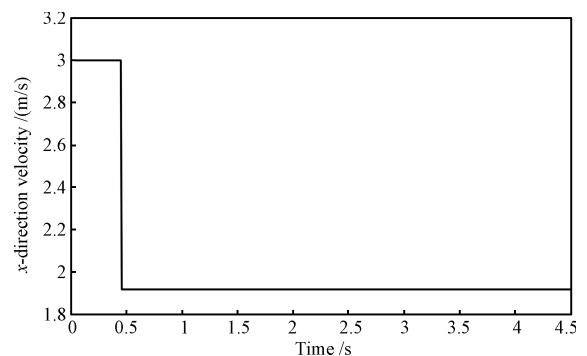


Fig. 9  $x$ -direction linear velocity of the ball in friction mode

For the system of rectangular object manipulation using a series of 1-DOF manipulators, we again use 2-D working model to simulate the results. In this direction, we choose the base of inertial frame  $x$ - $y$  placed on the first pivot of

the first manipulator (see Fig. 6). For the object and each manipulator, we choose

$$m = 1 \text{ kg}, \quad M = 0.1 \text{ kg}, \quad l = 2 \text{ m} \quad (37)$$

Then, we consider that the rectangular object has initial conditions as

$$\begin{aligned} (x_0, y_0) &= (-1.25, 3.05) \text{ m} \\ (V_{x0}, V_{y0}) &= (3, 0) \text{ m/s} \end{aligned} \quad (38)$$

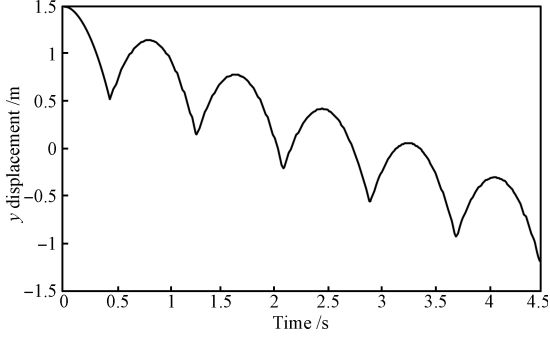


Fig. 7  $y$ -position of the ball in both with-friction and friction-less mode

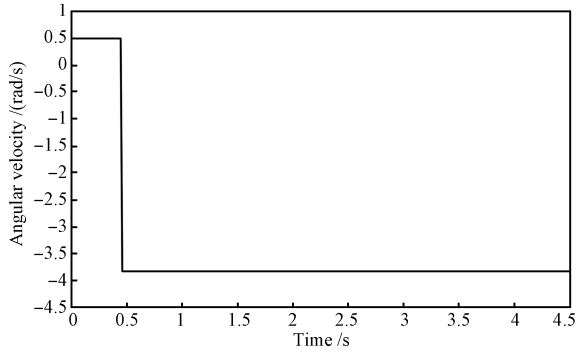


Fig. 8 Angular velocity of the ball in a friction mode

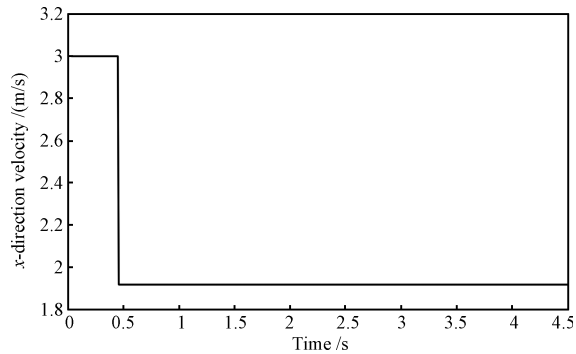


Fig. 9  $x$ -direction linear velocity of the ball in friction mode

In addition, we consider that the impact is purely inelastic and the surfaces have enough friction to prevent the object from slipping on the palms. In other words,

$$\varepsilon = 0, \quad \mu = 0.8 \quad (39)$$

By these values, we choose  $\alpha$  and  $\beta$  in such a way that (23) is satisfied. Then, we have

$$\alpha = 0.2 \text{ rad}, \quad \beta = 0.6 \text{ rad} \quad (40)$$

In this way, the distance of two consecutive manipulators can be obtained from (31) as

$$\begin{aligned} x &= l(\sin \alpha + \sin \beta) = 1.5266 \text{ m} \\ y &= l(\cos \alpha - \cos \beta) = 0.3095 \text{ m} \end{aligned} \quad (41)$$

As depicted in Fig. 6 (a), we assembled the system accurately and ran the simulation for five steps. Fig. 10 shows variation of  $y$ -direction linear velocity of the object with respect to the time  $t$ . It is obvious that the behavior of the object in this direction is periodic after the first impact. In the first impact, the vertical velocity of the object is depending on the initial conditions, i.e., (38). In addition, Figs. 11 and 12 depict the  $x$ -direction linear velocity and  $y$ -direction displacement of the rectangular object during the passive manipulation process done by a series of 1-DOF passive manipulators, respectively. They show the periodic behavior of the system after the first impact. In addition, Fig. 13 illustrates the actual path of the object in the  $x$ - $y$  plane. It is notable that we ran the simulation with a variety of initial velocities and all simulations led to similar results.

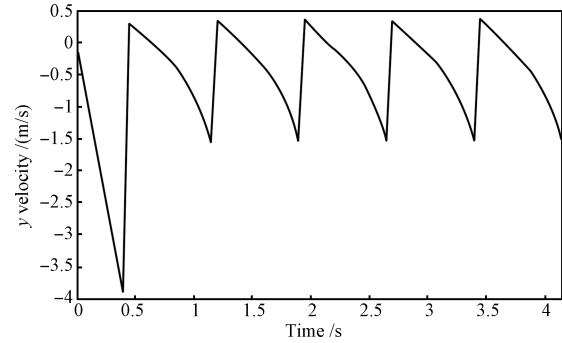


Fig. 10  $y$ -direction linear velocity of the rectangular object in passive manipulation using a series of 1-DOF manipulators

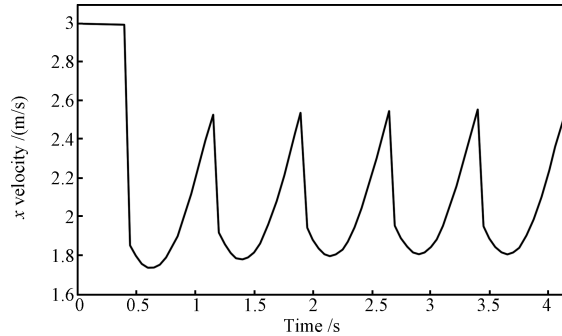


Fig. 11  $x$ -direction linear velocity of the rectangular object in passive manipulation using a series of 1-DOF manipulators

As we discussed before, we also let the system be assembled inaccurately. This way, we chose  $x$  and  $y$  in (41) in a way that they have corresponding offsets for the second manipulator as (see Fig. 6 (b)):

$$\Delta x = 0.1 \text{ m}, \quad \Delta y = -0.2 \text{ m} \quad (42)$$

Then, we can see in Fig. 14 that the manipulation can continue while  $x$  and  $y$  in (41) experience some mismatches. In this figure, the path of the object is also included for comparison. It is obvious that the object, after being passed from inaccurately assembled manipulator, has a periodic behavior.

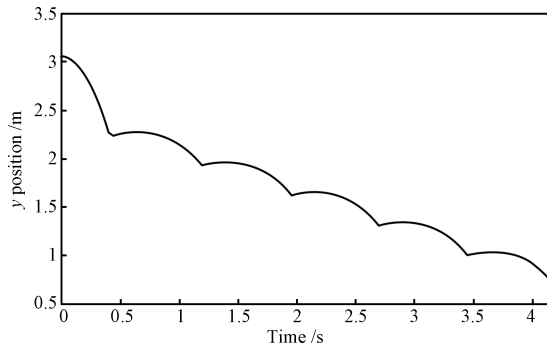


Fig. 12  $y$ -direction displacement of the rectangular object in passive manipulation using a series of 1-DOF manipulators

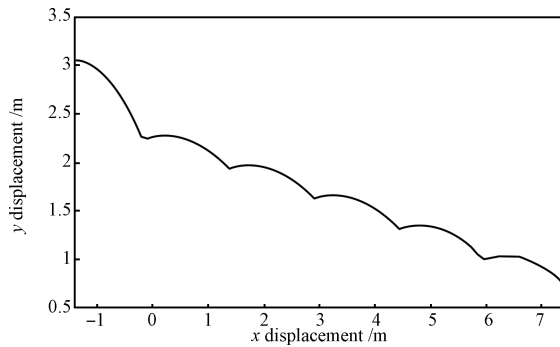


Fig. 13 Actual displacement of the rectangular object in passive manipulation corresponding to Fig. 6 (a)

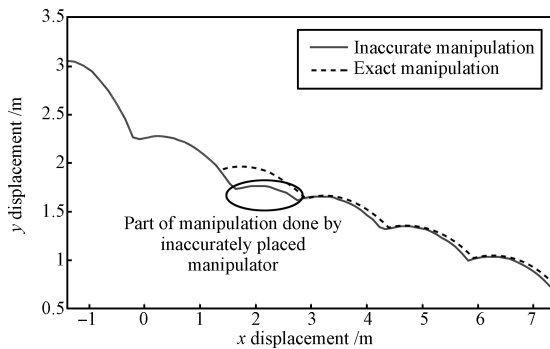


Fig. 14 Actual displacement of the rectangular object corresponding to Fig. 6 (b) (One of the manipulators (the second one) is inaccurately placed.)

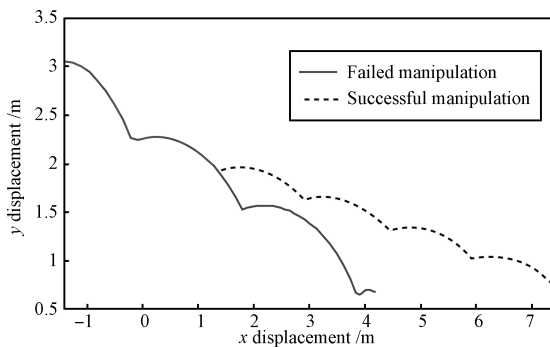


Fig. 15 Actual displacement of the rectangular object corresponding to Fig. 6 (c) (One of the manipulators (the second one) is very inaccurately placed; manipulation is failed.)

In another experiment, we chose (see Fig. 6 (c))

$$\Delta x = 0 \text{ m}, \quad \Delta y = -0.4 \text{ m} \quad (43)$$

for the second manipulator and ran the simulation. These offsets are too much to let the inaccurately assembled manipulator pass the object to the next manipulator successfully. Therefore, the whole process failed and the object stopped moving among manipulators. Fig. 15 is the corresponding result for this simulation.

## 5 Conclusion

In this paper, we discuss a new topic in robotics, namely passive dynamic object manipulation. A passive dynamic manipulation system performs manipulation of an object without the aid of actuators. As this was the first time that such a discussion was opened, we tried to express the concept by using some very simple examples and giving appropriate analysis. Examples included a bouncing ball moving down friction-less stairs and stairs with friction, and rectangular objects manipulated by serial passive 1-DOF manipulators. Finally, we support our formulations with the aid of simulations. During simulation, we saw that the manipulation of the rectangular object could continue even in the presence of some inaccuracy in system parameters.

## References

- 1 Fallis G T. Walking Toy, U.S. Patent No. 376588, 1888
- 2 Bechstein B U. Improvements in and relating to Toys, U.K. Patent No. 7453, 1912
- 3 Coleman M J, Ruina A. An uncontrolled walking toy that cannot stand still. *Physical Review Letters*, 1998, **80**(16): 3658–3661
- 4 Coleman M J, Garcia M, Mombaur K, Ruina A. Prediction of stable walking for a toy that cannot stand still. *Physical Review E*, 2001, **64**(2): 022901
- 5 Collins S, Ruina A, Tedrake R, Wisse M. Efficient bipedal robots based on passive-dynamic walkers. *Science*, 2005, **307**(5712): 1082–1085
- 6 McGeer T. Passive dynamic walking. *The International Journal of Robotic Researches*, 1990, **9**(2): 62–82
- 7 Lynch K M, Black C K. Recurrence, controllability, and stabilization of juggling. *IEEE Transactions on Robotics and Automation*, 2001, **17**(2): 113–124
- 8 Lynch K M, Mason M T. Dynamic manipulation with a one joint robot. In: Proceedings of the IEEE International Conference on Robotics and Automation. Albuquerque, USA: IEEE, 1997. 359–366
- 9 Lynch K M, Mason M T. Stable pushing: mechanics, controllability, and planning. *The International Journal of Robotic Researches*, 1996, **15**(6): 533–556
- 10 Tabata T, Aiyama Y. Tossing manipulation by 1 degree-of-freedom manipulator. In: Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems. Maui, USA: IEEE, 2001. 132–137
- 11 Beigzadeh B, AhmadAbadi M N, Meghdari A. Two dimensional dynamic manipulation of a disc using two manipulators. In: Proceedings of the IEEE International Conference on Mechatronics and Automation. Luoyang, China: IEEE, 2006. 1191–1196
- 12 Beigzadeh B, Meghdari A, Beigzadeh Y. Dynamic manipulation of objects using a series of manipulators. In: Proceedings of the International Mechanical Engineering Congress and Exposition. San Diego, USA: ASME, 2007
- 13 Bicchi A, Sorrentino R. Dexterous manipulation through rolling. In: Proceedings of the IEEE International Conference on Robotics and Automation. Nagoya, Japan: IEEE, 1995. 452–457
- 14 Marigo A, Bicchi A. Rolling bodies with regular surface: controllability theory and application. *IEEE Transactions on Automatic Control*, 2000, **45**(9): 1586–1599



- 15 Akella S, Huang W H, Lynch K M, Mason M T. Parts feeding on a conveyor with a one joint robot. *Algorithmica*, 2000, **26**(3–4): 313–344
- 16 Arai H, Khatib O. Experiments with dynamic skills. In: Proceedings of the 1994 Japan-USA Symposium on Flexible Automation. Kobe, Japan: ASME, 1994. 81–84
- 17 Beigzadeh B, Meghdari A, Sohrabpour S. Control and manipulation of multibody objects. In: Proceedings of the 10th Biennial Conference on Engineering Systems Design and Analysis. Istanbul, Turkey: ASME, 2010
- 18 Lynch K M, Mason M T. Dynamic nonprehensile manipulation: controllability, planning, and experiments. *The International Journal of Robotic Research*, 1999, **18**(1): 64–92



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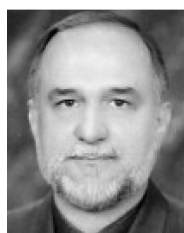
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