On Fault-tolerant Control of Dynamic Systems with Actuator Failures and External Disturbances

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Abstract This paper extends and improves the existing result on fault-tolerant control (FTC) of dynamic systems with actuator failures and external disturbances in several aspects. More specifically, the proposed method does not involve solving Lyapunov equation that contains time-varying and unknown variables associated with actuator failures; one does not need to analytically estimate the bound on the actuator failure factors in designing and implementing the proposed control scheme; the developed FTC is able to attenuate both bounded and unbounded external disturbances under actuator failures. To some extent, the results presented here include the existing results as a special case and the resultant control algorithms are fault-independent in that there is no need for explicit fault information in terms of its magnitude (size), or time instance of the fault occurrence, thus, is more user-friendly for control design and more feasible for implementation as compared with the existing work.

Key words Fault-tolerant control (FTC), actuator failure, robust adaptive control, unbounded disturbance

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Fault-tolerant control (FTC) has been viewed as one of the most promising methods to increase system safety and reliability, and has thus received considerable attention from the control and system engineering research communities in the last couple of decades. The main objective of fault-tolerant control is to maintain the specified performance of a system in the presence of faults. Most existing approaches for FTC broadly fall into two categories: the passive and the active approaches[1−6], which can also be classified as fault detection and diagnosis (FDD)-independent methods[7−17] or FDD-dependent methods[18−30].

In general, there is no universal approach to accommodating system failures, yet different FTC schemes lead to different control performances, depending on the nature of the faults and the system on which the faults impact[5−16, 18−24]. It is noted that since FDD-independent FTC methods do not require fault detection and diagnosis, the resultant control schemes have simpler structure and demand less online computations. Furthermore, in contrast to FDD-based methods, FDD-independent methods have the advantage of avoiding misdiagnosis of faults or false alarm phenomena.

In this work, we focus on FDD-independent approach for FTC. For a linear time-invariant system with actuator failures and bounded disturbances, Jin and Yang[31] have recently proposed a robust adaptive FTC method to stabilize the system asymptotically. While the proposed control scheme circumvents some of the typical disadvantages of those FTC schemes as mentioned in [31], it suffers from several drawbacks. For instance, 1) One needs to analytically determine the parameters α and β (see [31] for the variable definitions) which rely on the information of the actuator effectiveness variables $\rho_i(t)$; 2) Design of the FTC involves solving Lyapunov equation that contains time-varying and uncertain variables associated with actuator failures; 3) The stability analysis ignores the term $\dot{\rho}_i(t)\tilde{K}_{1,i}^{\rm T}\Gamma^{-1}\tilde{K}_{1,i}$ in the derivative of the Lyapunov function, which disappears only for the special class of actuator failures in which the actuator efficiency factors are constant or slowly time-varying; 4) The condition $B_2\rho(t)K_1 = B_2K$ assumed in deriving the control strategy is rather restrictive, as such a condition does not hold in general (i.e., a time-varying matrix does not generally equal a constant one); and 5) Only bounded disturbance can be attenuated with their method.

The purpose of this paper is to present a control scheme that extends and improves the work of [31] in that all the aforementioned shortcomings are removed. More specifically, the FTC control scheme developed herein does not need to solve a Lyapunov equation that contains uncertain and time-varying actuator failure variables, nor does it demand any explicit fault information to design or implement the control strategy. Furthermore, both time-varying actuator failures and unbounded external disturbances can be accommodated with the proposed control algorithms. Overall, the FTC presented herein is more user-friendly for design and more feasible for implementation.

1 Fault-tolerant control problem

Consider the stabilization problem of the following dynamic system under actuator faults and external disturbances:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B(\boldsymbol{u}_a(t) + \boldsymbol{\omega}(\boldsymbol{x}, t)) \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ is the state, $\boldsymbol{u}_a(t) \in \mathbf{R}^m$ denotes the actual control input to the system, $\boldsymbol{\omega}(\boldsymbol{x}, t) \in \mathbf{R}^m$ models external disturbances acting on the system, A and B are known real constant matrices with appropriate dimensions.

In the presence of actuator failures (such as outage, loss of effectiveness, stuck condition or combination of all), the actual control input $\boldsymbol{u}_a(t)$ which is able to impact the system is not the same as the designed control input $\boldsymbol{u}(t)$ designed in general. They are, instead, related through

$$\boldsymbol{u}_a(t) = \rho(t)\boldsymbol{u}(t) + \boldsymbol{E}(t) \tag{2}$$

where $\rho(t) = {\rm diag}\{\rho_i^j(t)\}$ is a diagonal matrix with $\rho_i^j(t) \in (0, 1]$ $(i = 1, 2, \cdots, m, j = 1, 2, \cdots, L)$ being the unknown and time-varying scalar function called actuator efficiency factor[31], or "health indicator"[17], the index j denotes the j-th faulty mode, L is the number of total faulty modes, $\boldsymbol{E}(t)$ denotes a vector function reflecting the portion of the control action produced by the actuator that is completely out of control. The system is the same as that considered in [13] without external disturbances, and the same as in

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[31] except that the disturbance does not need to be bound by a constant, and that $\boldsymbol{E}(t)$ can be time-varying.

The type of actuator failures considered in this work are listed in the table below.

Table 1 Fault model

Type of actuator failures	$\rho_i(t)$	$\boldsymbol{E}(t)$
Healthy actuator	1	0
Loss of effectiveness only	<1	0
Loss of effectiveness and partially out of control	<1	Time-varying
Partially stuck, partially adjustable	<1	Constant
Completely stuck	0	Constant

In order for the system to admit a feasible FTC, the following assumptions are imposed:

Assumption 1. All the states of the system are available at every instant.

Assumption 2. (A, B) is controllable in that there exists a constant matrix K_0 such that the matrix $A-BK_0$ is Hurwitz.

Assumption 3. The unparameterizable stuck-actuator fault and external disturbances are piecewise continuous bounded functions, that is, there exist unknown positive constants $a_{_E}$ and a_ω such that

$$\|\boldsymbol{E}(t)\| \le a_{_E} < \infty, \quad \|\boldsymbol{\omega}(\cdot)\| \le a_\omega < \infty$$

respectively.

Assumption 4. For the system under consideration, there exist some constants $\alpha > 0$ and $\beta > 0$ such that for all possible actuator faults, the following relation holds:

$$\alpha\|B^{\mathrm{T}}P\boldsymbol{x}\|^2 \le \beta\|B^{\mathrm{T}}P\boldsymbol{x}\sqrt{\rho}\|^2$$

where

$$\sqrt{\rho} = \mathrm{diag}\Big\{\sqrt{(\rho_i^j(t))}\Big\}, \quad \rho_i^j(t) \in [0,1],$$

$$i = 1,2,\cdots,m;\ j = 1,2,\cdots,L$$

Remark 1. The first two assumptions imposed here are rather standard in addressing the system stabilization problem. Assumption 3 confines the external disturbances to be bounded by a constant, which will be relaxed later. Assumption 4, slightly less restrictive than the one used in [31], sets constraint on the actuation faults that a feasible FTC is able to deal with. Clearly, such a condition is well justified if all the actuators with faults are still functional (i.e., $\rho_i^j(t) \ne 0$), whereas the extreme fault in which all the actuators completely fail to work (i.e., $\rho_i^j(t) = 0$), makes the assumption invalid.

Let

$$\bar{A} = A - BK_0 \tag{3}$$

Since (A, B) is controllable, one can choose K_0 properly such that $\bar{A}$ is Hurwitz. Namely, for any given $Q = Q^{\mathrm{T}} > 0$, there exists a symmetric and positive definite matrix P such that

$$-Q = \bar{A}^{\mathrm{T}}P + P\bar{A} \tag{4}$$

Since A and B are available and $\bar{A}$ can be specified as Hurwitz by the designer, K_0 can be determined directly from (3) and P can be readily solved from the Lyapunov equation (4) for a given $Q = Q^{\mathrm{T}} > 0$.

2 Fault-tolerant control

2.1 Robust fault-tolerant control

In this section, a robust fault-tolerant control of the following form

$$\boldsymbol{u}(t) = -K_0\boldsymbol{x} + \boldsymbol{K}(t) \tag{5a}$$

is proposed, where K_0 is chosen such that $A - BK_0$ is Hurwitz, and $\boldsymbol{K}(t)$ is generated by

$$\boldsymbol{K}(t) = -\frac{a}{\lambda_m}(1+2\|K_0\boldsymbol{x}\|)\frac{B^{\mathrm{T}}P\boldsymbol{x}}{\|B^{\mathrm{T}}P\boldsymbol{x}\|} \tag{5b}$$

with $0 < \lambda_m \le \alpha/\beta$ being a constant, where $\alpha > 0$ and $\beta > 0$ are suitable constants such that

$$\alpha\|B^{\mathrm{T}}P\boldsymbol{x}\|^2 \le \beta\|B^{\mathrm{T}}P\boldsymbol{x}\sqrt{\rho}\|^2 \tag{5c}$$

and

$$a = \max\{1, a_{_E} + a_\omega\}$$

Theorem 1. Under Assumptions 1 ∼ 3, the system described by (1) subject to (2) is asymptotically stable if the FTC as given in (5) is applied.

Proof. When the system is subject to the actuator failure as described in (2), its dynamic behavior becomes

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + B(\rho(t)\boldsymbol{u}(t) + \boldsymbol{E}(t) + \boldsymbol{\omega}(\cdot)) \tag{6}$$

With the proposed control (5), one has

$$\begin{aligned}\dot{\boldsymbol{x}} = A\boldsymbol{x} + B[\rho(t)(-K_0\boldsymbol{x} + \boldsymbol{K}(t)) + \boldsymbol{E}(t) + \boldsymbol{\omega}(\cdot)] &= \\ (A - BK_0)\boldsymbol{x} + B[\rho(t)\boldsymbol{K}(t) + L(t)] &= \\ \bar{A}\boldsymbol{x} + B[\rho(t)\boldsymbol{K}(t) + L(t)]\end{aligned} \tag{7}$$

where

$$L(\cdot) = (I - \rho)K_0\boldsymbol{x} + \boldsymbol{E}(t) + \boldsymbol{\omega}(\cdot)$$

By Assumption 3,

$$\|L(\cdot)\| \le 2\|K_0\boldsymbol{x}\| + \|\boldsymbol{E}(t)\| + \|\boldsymbol{\omega}\| \le a(1+2\|K_0\boldsymbol{x}\|)$$

where $a = \max\{1, a_{_E} + a_\omega\}$.

It can be shown that

$$(B^{\mathrm{T}}P\boldsymbol{x})^{\mathrm{T}}L \le a(1+2\|K_0\boldsymbol{x}\|)\|B^{\mathrm{T}}P\boldsymbol{x}\| \tag{8}$$

Choose a Lyapunov function candidate

$$V = \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}P\boldsymbol{x}$$

It follows that

$$\begin{aligned}\dot{V} = &-\frac{1}{2}\boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} + (B^{\mathrm{T}}P\boldsymbol{x})^{\mathrm{T}} \times \\ &\left[-\rho\frac{a}{\lambda_m}(1+2\|K_0\boldsymbol{x}\|)\frac{B^{\mathrm{T}}P\boldsymbol{x}}{\|B^{\mathrm{T}}P\boldsymbol{x}\|}\right] + (B^{\mathrm{T}}P\boldsymbol{x})^{\mathrm{T}}L \le \\ &-\frac{1}{2}\boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} - \frac{a}{\lambda_m}\frac{(1+2\|K_0\boldsymbol{x}\|)}{\|B^{\mathrm{T}}P\boldsymbol{x}\|}(B^{\mathrm{T}}P\boldsymbol{x})^{\mathrm{T}}\rho(B^{\mathrm{T}}P\boldsymbol{x}) + \\ &a(1+2\|K_0\boldsymbol{x}\|)\|B^{\mathrm{T}}P\boldsymbol{x}\|\end{aligned} \tag{9}$$

where the Lyapunov equation (4) is used.

From (5c), it holds that

$$(B^{\mathrm{T}}P\boldsymbol{x})^{\mathrm{T}}\rho(B^{\mathrm{T}}P\boldsymbol{x}) \ge \frac{\alpha}{\beta}\|B^{\mathrm{T}}P\boldsymbol{x}\|^2 \ge \lambda_m\|B^{\mathrm{T}}P\boldsymbol{x}\|^2$$

then with certain manipulation, the last two terms of (9) are canceled out to lead to

$$\dot{V} \le -\frac{1}{2}\boldsymbol{x}^{\mathrm{T}}Q\boldsymbol{x} \le 0$$

Then, by Lyapunov stability theory, global stabilization with disturbance rejection is ensured with the proposed FTC. □

Remark 2. Note that (5c), similar to the one used in [31], holds for the faulty modes considered, unless all the actuators completely fail to work. The use of (5c) avoids the direct identification of the magnitude of the actuator failures.

Remark 3. Two parameters need to be determined in implementing the control scheme (5), i.e., a and λ_m (i.e., α and β). This implies that one needs to know certain information on the fault model and external disturbances. The next result removes such a requirement.

2.2 Robust adaptive fault-tolerant control

The control scheme that is independent of explicit information on faults and disturbances is proposed as follows:

$$\boldsymbol{u}(t) = -K_0\boldsymbol{x}(t) + \hat{\boldsymbol{K}}(t) \tag{10a}$$

where K_0 is chosen such that $A - BK_0$ is Hurwitz, and $\hat{\boldsymbol{K}}(t)$ is generated by

$$\hat{\boldsymbol{K}}(t) = -\frac{\hat{a}(t)\varphi(\boldsymbol{x})B^{\rm T}P\boldsymbol{x}}{\|B^{\rm T}P\boldsymbol{x}\|} \tag{10b}$$

with

$$\varphi(\boldsymbol{x}) = 1 + 2\|K_0\boldsymbol{x}\| \tag{10c}$$

and

$$\dot{\hat{a}}(t) = \gamma\varphi(\boldsymbol{x})\|B^{\rm T}P\boldsymbol{x}\|, \quad \gamma > 0 \tag{10d}$$

Theorem 2. Under Assumptions $1 \sim 3$, the system with actuator failures and bounded disturbances as described by (1) and (2) is asymptotically stable if the FTC (10) is applied.

Proof. Consider the following Lyapunov function candidate:

$$V = \frac{1}{2}\boldsymbol{x}^{\rm T}P\boldsymbol{x} + \frac{1}{\lambda_m\gamma}(a - \hat{a}\lambda_m)^2$$

where $\gamma > 0$ is a constant related to the adaptive rate chosen by the designer and $\lambda_m > 0$ is the constant defined as before. Then, it is straightforward to show that

$$\begin{aligned}\dot{V} = &-\dot{\boldsymbol{x}}^{\rm T}P\boldsymbol{x} + \boldsymbol{x}^{\rm T}P\dot{\boldsymbol{x}} + 2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1}) = \\ &[\bar{A}\boldsymbol{x} + B(\rho\hat{\boldsymbol{K}} + L)^{\rm T}]P\boldsymbol{x} + \boldsymbol{x}^{\rm T}P[\bar{A}\boldsymbol{x} + B(\rho\hat{\boldsymbol{K}} + L)] + \\ &2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1}) = \\ &\boldsymbol{x}^{\rm T}(\bar{A}^{\rm T}P + P\bar{A})\boldsymbol{x} + 2\boldsymbol{x}^{\rm T}PB(\rho\hat{\boldsymbol{K}} + L) + \\ &2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1})\end{aligned} \tag{11}$$

By Lyapunov equation (4), one obtains

$$\begin{aligned}\dot{V} = &-\boldsymbol{x}^{\rm T}Q\boldsymbol{x} + 2\boldsymbol{x}^{\rm T}PB\rho\left[-\frac{\hat{a}(t)(1 + 2\|K_0\boldsymbol{x}\|)(B^{\rm T}P\boldsymbol{x})}{\|B^{\rm T}P\boldsymbol{x}\|}\right] + \\ &2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1})\end{aligned} \tag{12}$$

Using $(B^{\rm T}P\boldsymbol{x})^{\rm T}\rho(B^{\rm T}P\boldsymbol{x}) \geq \lambda_m\|B^{\rm T}P\boldsymbol{x}\|^2$, it can be readily shown that

$$\begin{aligned}\dot{V} \leq &-\boldsymbol{x}^{\rm T}Q\boldsymbol{x} - 2\|B^{\rm T}P\boldsymbol{x}\|\lambda_m\hat{a}(1 + 2\|K_0\boldsymbol{x}\|) + \\ &2(B^{\rm T}P\boldsymbol{x})^{\rm T}L + 2(a - \hat{a}\lambda_m)(-\dot{\hat{a}}\gamma^{-1})\end{aligned} \tag{13}$$

In light of (8) and the updating law (10d), the last three terms can be canceled from (13) to get

$$\dot{V} \leq -\boldsymbol{x}^{\rm T}Q\boldsymbol{x} \leq 0$$

Therefore, $\boldsymbol{x} \in L_2 \cap L_\infty$ and $\hat{a} \in L_\infty$, leading to $\boldsymbol{u}(t) \in L_\infty$ and $\dot{\boldsymbol{x}}(t) \in L_\infty$. Thus, $\boldsymbol{x}$ is uniformly continuous, which allows Barbalat lemma[32] to be used to conclude that $\boldsymbol{x} \to 0$ as $t \to \infty$. □

Remark 4. While the control scheme (10) bears some similarities as compared with the one developed in [31], it avoids all the aforementioned shortcomings of [31]:

1) In designing the FTC, one only needs to solve the Lyapunov equation (4) to determine the matrix P and such a process does not involve the uncertain and time-varying effectiveness factor $\rho(t)$.

2) There is no need to assume that $B_2\rho(t)K_1 = B_2K$, which, in general, does not hold for constant K_1 and K because the left side of the equation is time-varying and the right side is constant.

3) Because of the introduction of λ_m, the Lyapunov candidate function avoids using $\rho_i(t)\tilde{K}_{1,i}^{\rm T}\Gamma^{-1}\tilde{K}_{1,i}$, thus the term of $\dot{\rho}_i(t)\tilde{K}_{1,i}^{\rm T}\Gamma^{-1}\tilde{K}_{1,i}$ ignored in the stability analysis in [31] does not occur here. Although λ_m is used in stability analysis, it is not included in control design, thus one does not need to analytically estimate such a parameter. In other words, the proposed control does not involve the parameters α and β as in [31], thus no additional information on the fault model is required in control design and implementation.

The design and implementation steps can be summarized as follows:

a) Choose $\gamma > 0$;
b) Choose K_0 such that $\bar{A} = A - BK_0$ is Hurwitz;
c) Given $Q > 0$, solve (4) to get P;
d) Compute $\dot{\hat{a}}(t) = \gamma(1 + 2\|K_0\boldsymbol{x}\|)\|B^{\rm T}P\boldsymbol{x}\|$;
e) Compute $\hat{\boldsymbol{K}}(t) = -\frac{\hat{a}(t)(1+2\|K_0\boldsymbol{x}\|)B^{\rm T}P\boldsymbol{x}}{\|B^{\rm T}P\boldsymbol{x}\|}$;
f) $\boldsymbol{u}(t) = -K_0\boldsymbol{x}(t) + \hat{\boldsymbol{K}}(t)$.

2.3 Extension to unbounded disturbances

The result can be extended to address the case when the external disturbances acting on the system are not bounded by a constant, rather they are bounded by

$$\|\boldsymbol{\omega}\| \leq a'_\omega\psi_\omega(\boldsymbol{x}) \tag{14}$$

where $a'_\omega \geq 0$ is a constant and $\psi_\omega(\boldsymbol{x})$ is a known function satisfying $\psi_\omega(\boldsymbol{x}) \in L_\infty$ for $\forall\ \boldsymbol{x} \in L_\infty$. Furthermore, it can be extended to a class of nonlinear systems of the form

$$\dot{\boldsymbol{x}} = \boldsymbol{N}(\boldsymbol{x}) + B(\boldsymbol{u}_a + \boldsymbol{\omega}(\boldsymbol{x}, t)) \tag{15a}$$

with

$$\boldsymbol{N}(\boldsymbol{x}) = A\boldsymbol{x} + B\xi(\boldsymbol{x}) \tag{15b}$$

subject to actuator failures as modeled by (2).

Let

$$L(\cdot) = \xi(\boldsymbol{x}) + \boldsymbol{E}(t) + \boldsymbol{\omega}(\boldsymbol{x}, t) + (I - \rho)K_0\boldsymbol{x}$$

Then if $\|\xi(\boldsymbol{x})\| \leq a_\xi\psi_\xi(\boldsymbol{x})$ for some unknown constant $a_\xi > 0$ and known function $\psi_\xi(\boldsymbol{x})$, one has

$$\begin{aligned}\|L(\cdot)\| \leq &2\|K_0\boldsymbol{x}\| + \|\boldsymbol{E}\| + \|\boldsymbol{\omega}\| + \|\xi\| \leq \\ &a(\psi_\xi(\boldsymbol{x}) + \psi_\omega(\boldsymbol{x}) + 1 + 2\|K_0\boldsymbol{x}\|) = a\psi(\boldsymbol{x})\end{aligned} \tag{16}$$

where

$$\psi(\boldsymbol{x}) = 1 + \psi_\xi(\boldsymbol{x}) + \psi_\omega(\boldsymbol{x}) + 2\|K_0\boldsymbol{x}\|$$

and

$$a = \max(1, a_E, a'_\omega, a_\xi)$$

The following result states that the nonlinear system (15) can be asymptotically stabilized by a control strategy similar to (10).

Theorem 3. Consider the dynamic system (15) subject to the actuator failures as modeled in (2). Let Assumptions 1 and 2 hold. The system is asymptotically stable if the following FTC is applied:

$$\boldsymbol{u}(t) = -K_0\boldsymbol{x}(t) + \hat{\boldsymbol{K}}(t) \tag{17a}$$

$$\hat{\boldsymbol{K}}(t) = -\hat{a}(t)\psi(\boldsymbol{x})\frac{B^{\mathrm{T}}P\boldsymbol{x}}{\|B^{\mathrm{T}}P\boldsymbol{x}\|} \tag{17b}$$

$$\dot{\hat{a}}(t) = \gamma\psi(\boldsymbol{x})\|B^{\mathrm{T}}P\boldsymbol{x}\|, \quad \gamma > 0 \tag{17c}$$

Proof. It can be proved following the same lines as in the proof of Theorem 2. □

The overall control block diagram is depicted in Fig. 1, and the following comments are in order.

Remark 5. Obviously, the system considered in [31] is a special case of the system studied in this paper, i.e., it corresponds to the case that $\xi(\cdot) + \omega(\cdot)$ in (15) is bounded by a constant. Namely, the proposed FTC (17) is able to deal with both bounded and unbounded disturbances.

Remark 6. Note that the proposed fault-tolerant control schemes (5), (10), and (17) contain a structure of the form $\frac{B^{\mathrm{T}}P\boldsymbol{x}}{\|B^{\mathrm{T}}P\boldsymbol{x}\|}$, which might cause discontinuity (chattering) as $\boldsymbol{x}$ gets closer to zero. To ensure smooth and bounded control action, a simple and feasible solution is to replace $\frac{B^{\mathrm{T}}P\boldsymbol{x}}{\|B^{\mathrm{T}}P\boldsymbol{x}\|}$ with $\frac{B^{\mathrm{T}}P\boldsymbol{x}}{\|B^{\mathrm{T}}P\boldsymbol{x}\|+\epsilon_0}$ (ε_0 is a small number). Also, to prevent parameter estimation drifting, one can use the modified adaptive update algorithm of the form $\dot{\hat{a}}(t) = -\sigma\hat{a} + \gamma\psi(\boldsymbol{x})\|B^{\mathrm{T}}P\boldsymbol{x}\|$, with $\sigma > 0$, $\gamma > 0$. In this case, ultimately uniformly bounded stability instead of asymptotical stability is achieved.

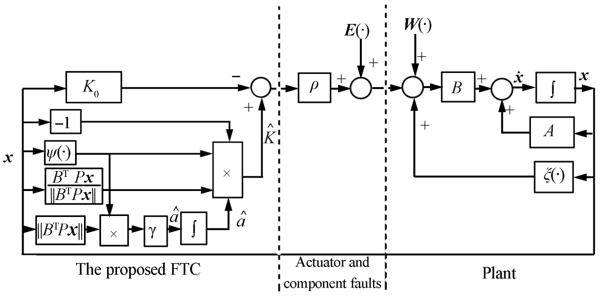

Fig. 1 Block diagram of the proposed control scheme

3 Simulation verification

To test the effectiveness of the proposed method, two examples are considered. The first one is a rocket fairing structural-acoustic model (single-mode) taken from [13] with bounded external disturbances as considered in [31]. The second one is a nonlinear system with unbounded disturbance.

3.1 Example 1

We consider a rocket fairing structural-acoustic model with external disturbance input added[13]:

$$A = \begin{bmatrix} 0 & 1 & 0.0802 & 1.0415 \\ -0.1980 & -0.1150 & -0.0318 & 0.3 \\ -3.0500 & 1.1800 & -0.4650 & 0.9 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1.55 & 075 \\ 0.975 & 0.8 & 0.85 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The disturbances that enter into the system at the beginning ($t \geq 0$) are $\boldsymbol{\omega}(t) = [-7.5\sin(0.1t) + 5, 10\sin(0.1t) - 5, 5\sin(0.1t)+2.5]^{\mathrm{T}}$ and the initial system states are $\boldsymbol{x}(0) = [0, 1, -0.5, -1]^{\mathrm{T}}$. In applying the control scheme (10), one needs to determine the control parameter matrix K_0, which can be easily obtained as follows:

$$K_0 = \begin{bmatrix} 2.5956 & -0.2730 & -1.0617 & -0.8154 \\ 2.4081 & -0.4127 & -1.2002 & -0.9280 \\ 2.0027 & -0.0839 & -0.7777 & -0.5500 \end{bmatrix}$$

To prevent estimation from drifting, the parameter $\hat{a}(t)$ is updated by $\dot{\hat{a}}(t) = -\sigma\hat{a}+\gamma\psi(\boldsymbol{x})\|B^{\mathrm{T}}P\boldsymbol{x}\|$, where $\sigma = 0.13$, $\hat{a}(0) = 0$, and $\gamma = 50$.

The actuator efficiency variables for each of the three control channels simulated are the same as that considered in [31]. Namely, the system operates normally until $t = 8$ s, at which time some faults in the actuators occur: the first actuator is stuck at $\boldsymbol{E}(t) = [10+3\sin(t)+2\cos(0.5t), 0, 0]^{\mathrm{T}}$ (uncontrollable) and the third actuator faced loss of effectiveness by $\rho_3(t) = 1.24 - 0.03t$, while the second actuator has no fault during the whole operation process.

The simulation results in terms of stabilization of the four states are presented in Fig. 2. The estimated parameter $\hat{a}$ is shown in Fig. 3. The results confirm the theoretical prediction.

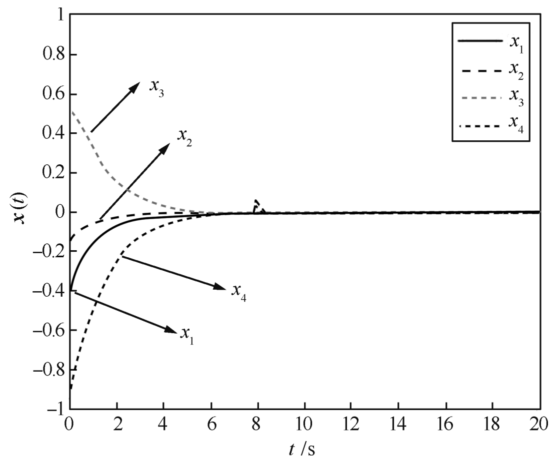

Fig. 2 System responses under the action of the proposed FTC (10)

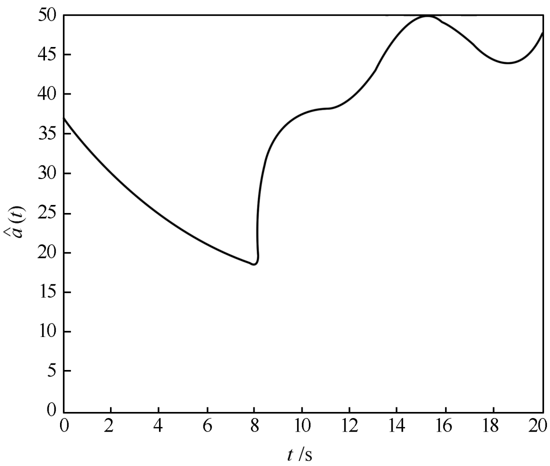

Fig. 3 Updating of $\hat{a}$

3.2 Example 2

The second example is a nonlinear system with state-dependent disturbance,

$$\ddot{x} = \sin(x)\dot{x} + \cos(\dot{x})x + b(u + x^2 \sin(t))$$

which can be converted into (15) with

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ b \end{bmatrix}$$

$$\xi(x, \dot{x}) = \sin(x)\dot{x} + \cos(\dot{x})x, \quad \omega(x, t) = x^2 \sin(t)$$

The control scheme developed in [31] is inapplicable in this case as the external disturbance is not bounded by a constant. However, it can be easily dealt with by the proposed control scheme (17), where the control parameters can be chosen quite arbitrarily as

$$\boldsymbol{K}_0 = [2, 3], \ \ \hat{a}(0) = 0, \quad \gamma = 2, \quad b = 1$$

The following fault mode is simulated,

$$\rho(t) = \begin{cases} 1, & 0 \le t \le 6 \\ \dfrac{14 - t}{8}, & 6 < t \le 10 \\ 0.5, & 10 < t \le 12 \\ 0.75, & 12 < t \le 18 \\ 0.4, & t > 18 \end{cases}$$

which is illustrated in Fig. 4.

It is worth-mentioning that the fault mode considered is fast time-varying, which might not be diagnosed by any FDD method in a timely manner, yet most FDD-dependent methods demand heavy analytic computations in determining the corresponding control parameters, whereas the proposed control scheme is able to handle such fault gracefully. One can observe from Fig. 5 that the proposed FTC (17) is able to ensure high control precision for the system in the presence of actuator fault and state-dependent external disturbance.

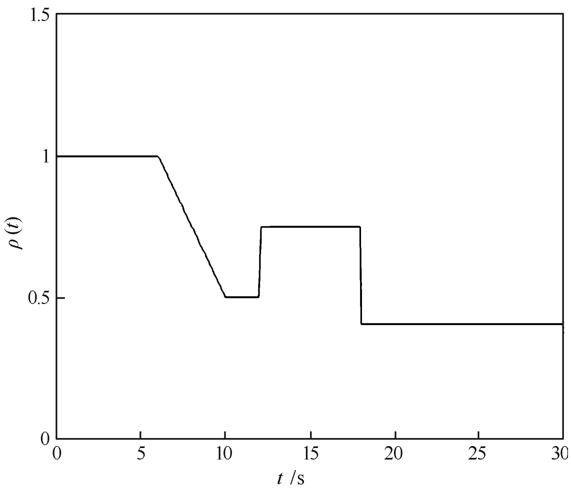

Fig. 4 Profile of the time-varying actuator efficiency variables

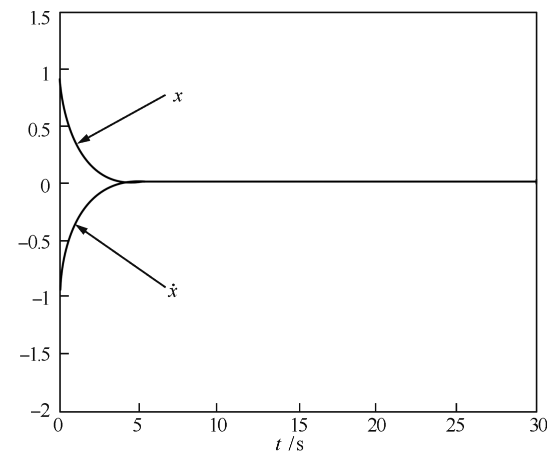

Fig. 5 System responses under the action of the proposed FTC (17)

4 Conclusion

This article presents a method for fault-tolerant control of dynamic systems with actuator failures and external disturbances, which extends and improves the result in [31]. The proposed method is FDD-independent in the sense that it does not require any explicit information about the faults in terms of the fault magnitude (size) and time instant of the fault occurrence, as long as all the actuators are functional (although with faults). As a result, it is more feasible and more user-friendly to design and implement the proposed control scheme as compared with the existing work. Both theoretical analysis and numerical simulations validate the benefits and effectiveness of the proposed approach.

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