Delayed-state-feedback Exponential Stabilization of Stochastic Markovian Jump Systems with Mode-dependent Time-varying State Delays

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Abstract In this paper, an improved mean-square exponential stability condition and delayed-state-feedback controller for stochastic Markovian jump systems with mode-dependent time-varying state delays are obtained. First, by constructing a modified Lyapunov-Krasovskii functional, a mean-square exponential stability condition for the above systems is presented in terms of linear matrix inequalities (LMIs). Here, the decay rate can be a finite positive constant in a range and the derivative of time-varying delays is only required to have an upper bound which is not required to be less than 1. Then, based on the proposed stability condition, a delayed-state-feedback controller is designed. Finally, numerical examples are presented to illustrate the effectiveness of the theoretical results.

Key words Stochastic systems, time-varying delay, exponential stability, Markov chain

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In practice, many dynamical systems have different structures due to random and abrupt variations, such as random failures of the components, changes in the interconnection of subsystems, sudden environment changes, and so on[1−2]. As we know, Markovian jump systems, which are firstly introduced in [3] and modeled by a set of subsystems with transitions among all the modes governed by a Markov chain taking values in a finite set, are often employed to describe the above dynamical systems. This class of systems can be regarded as a special case of hybrid systems, since the states take continuous values and the jumping parameters take discrete values in a system simultaneously. A great deal of attention has been devoted to the study of Markovian jump systems. Among various research subjects, the stability and control of Markovian jump systems are significant research areas[4−13].

It has been recognized that time delays, which are frequently encountered in practical systems, are the main cause of instability and unsatisfactory performance[14]. Therefore, the study on Markovian jump systems with time-delays has attracted much attention in the past years. The stabilization problem of Markovian jump systems with time-delays is studied in [1, 6, 12, 15]. It can be clearly seen that the time delays are independent of the system modes in the afore-mentioned references. As a matter of fact, in many engineering applications, random delays are unavoidably encountered, for instance, the utilization of a multi-user network with random demands affecting the network traffic could result in random delays in the feedback loop[16]. Thus, the mode-dependent time delays should be taken into account for Markovian jump systems. The problem of stabilization for discrete-time and continuous-time Markovian jump systems with mode-dependent time-delays has been considered in [2, 17] and [16, 18−20], respectively.

On the other hand, in many branches of science and industry, the signal transmission is usually a noisy process which happens as a result of random fluctuations due to probabilistic reasons[21−23]. Hence, Markovian jump systems with time-delays are often corrupted by the noise which can be approximated by Brownian motion. Recently, this class of systems, that is stochastic Markovian jump systems with time-delays, has received much attention. As far as the stability problem is concerned, many results for Markovian jump systems with time-delays have been extended to the stochastic case[4, 24−28]. The sufficient condition of asymptotic stability is presented in [4] by using convergence theorem of nonnegative semimartingales. Moreover, [24−28] investigate the problem of mean-square exponential stability. In addition, due to the time spent in transmissions, state-feedback controller is usually subject to delays. Therefore, the stabilization problem under this case has received considerable attention. Recently, [29] considers this problem for stochastic systems with constant time-delays.

Although the afore-mentioned results are shown to be effective when solving the mean-square exponential stability problem, the decay rate usually needs to satisfy some constraints. Then, under these restricted conditions, the decay rate can be obtained by the following two steps. First, by solving linear matrix inequalities (LMIs), one can achieve the Lyapunov-Krasovskii functional matrices. Second, by using the knowledge of Lyapunov-Krasovskii functional matrices to solve a transcendental equation or several inequalities, the value of decay rate can thus be obtained[20, 24−29]. Consequently, due to the inherent property of the corresponding transcendental equation, the equation on decay rate has only one unique solution. Therefore, the decay rate will be a fixed value and cannot be adjusted to cater design specifications[20, 24−28]. For the case when the decay rate satisfies several inequalities, the computation process aiming to obtain the decay rate is very complex[29]. References [30−31] also observe this problem and propose a new method to achieve the exponential stability criterion for time delay systems. However, this proposed approach cannot be applied to systems with time-varying delays. Besides, the derivative of time-varying delays is constrained to be less than 1 in [12, 24−25, 27, 29]. As a result, the obtained results in [12, 24−25, 27, 29] are invalid when the derivative of time-varying delays equals to or is greater than 1. To the best of the authors′ knowledge, there is relatively little attention paid to the problem of delayed-state-

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feedback exponential stabilization for stochastic Markovian jump systems with mode-dependent time-varying state delays.

Motivated by the above observations, in this paper, we will not only deal with the mean-square exponential stability problem, but will also design a delayed-state-feedback controller for stochastic Markovian jump systems with mode-dependent time-varying delays. In order to relax constraints on the decay rate and the derivative of time-varying delay, by choosing a novel Lyapunov-Krasovskii functional and making full use of the information of both the lower and upper bound of time-varying delay, we develop an approach to simultaneously weaken the above-mentioned constraints on the decay rate and the derivative of time-varying delay. Then, the sufficient conditions for mean-square exponential stability and delayed-state-feedback controller design are proposed in terms of LMIs. Here, the derivative of mode-dependent time-varying delays only needs to have an upper bound which is not required to be less than 1. The decay rate can be finite positive value in a range. Moreover, the suboptimal upper bound of the decay rate can also be computed by convex optimization algorithm conveniently from the obtained LMIs. Finally, two numerical examples are provided to verify the validity of the results obtained in this paper.

Notations. Throughout this paper, $\mathbf{R}^n$ denotes the n dimensional Euclidean space. The superscript "T" denotes matrix transposition and "-1" denotes inverse matrix. "$*$" denotes transpose of the corresponding sub-matrix. The notation $X > Y$, where X and Y are symmetric matrices, means that $X-Y$ is positive definite. $|\cdot|$ denotes Euclidean norm of vectors and $\|\cdot\|$ denotes the spectral norm of matrices. $a \vee b$ denotes $\max\{a, b\}$. $\mathcal{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all P-null sets). $\mathrm{E}\{\cdot\}$ is the mathematical expectation with respect to probability measure P. $C([a,b];\mathbf{R}^n)$ denotes the family of continuous functions from $[a,b]$ to $\mathbf{R}^n$. $C^b_{\mathcal{F}_0}([a,b];\mathbf{R}^n)$ denotes the family of all bounded, $\mathcal{F}_0$-measurable and $C([a,b];\mathbf{R}^n)$-valued random variables, where $a \leq b$. If $x(t)$ is a $\mathbf{R}^n$-valued stochastic process on $t \in [-\bar{\tau},\infty)$, we let $x_t = \{x(t+\theta) : -\bar{\tau} \leq \theta \leq 0\}$ for $t \geq 0$, which is regarded as a $C([-\bar{\tau},0];\mathbf{R}^n)$-valued stochastic process.

1 Problem formulation

Consider the following class of stochastic Markovian jump systems with mode-dependent time-varying state delays:

$$\begin{aligned}\mathrm{d}x(t) = &[A(r_t)x(t) + A_d(r_t)x(t-\tau(r_t,t)) + C(r_t)u(t)]\mathrm{d}t + \\ &[B(r_t)x(t) + B_d(r_t)x(t-\tau(r_t,t))]\mathrm{d}\omega(t)\\ x(t) = &\phi(t), \quad t \in [-\bar{\tau}, 0] \end{aligned}\tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state, $\omega(t)$ is a one-dimensional Brownian motion defined on probability space $(\Omega, \mathcal{F}, P)$ with $\mathrm{E}(\omega(t)) = 0, \mathrm{E}(\omega^2(t)) = t$[32], $u(t) \in \mathbf{R}^p$ is the control input, $\{r_t\}_{t\geq 0}$ is a continuous-time Markov chain taking values in a finite set $\mathcal{S} = \{1, 2, \cdots, N\}$. Let $\Pi = \{\pi_{ij} : i, j \in \mathcal{S}\}$ be the density matrix of Markov chain $\{r_t\}_{t\geq 0}$. Thus, $\pi_{ij} \geq 0$ for $i \neq j$ and $\pi_{ii} = -\sum_{j=1,j\neq i}^{N} \pi_{ij}$. Furthermore, the transition probability of Markov chain $\{r_t\}_{t\geq 0}$ can be described as

$$P\{r_{t+\triangle} = j | r_t = i\} = \begin{cases} \pi_{ij}\triangle + o(\triangle), & i \neq j \\ 1 + \pi_{ii}\triangle + o(\triangle), & i = j \end{cases}$$

where $\triangle > 0$ and $\lim_{\triangle\to 0}(o(\triangle)/\triangle) = 0$. $\tau(r_t, t)$ denotes the mode-dependent time-varying delay when the mode is in r_t. When $r_t = i, i \in \mathcal{S}$, $\tau(r_t, t)$ is denoted by $\tau_i(t)$, which satisfies $0 \leq \underline{\tau}_i \leq \tau_i(t) \leq \bar{\tau}_i < \infty$ and $\dot{\tau}_i(t) \leq d_i$. In (1), $\bar{\tau} = \max\{\bar{\tau}_i, i \in \mathcal{S}\}$, $\phi(t) \in C^b_{\mathcal{F}_0}([-\bar{\tau}, 0];\mathbf{R}^n)$ is the initial data. In addition, we assume that Markov chain $\{r_t\}_{t\geq 0}$ is independent of Brownian motion $\omega(t)$. $A(r_t), A_d(r_t), C(r_t), B(r_t)$, and $B_d(r_t)$ are known real matrix functions of r_t with appropriate dimensions.

In order to avoid complicated notations, for each possible $r_t = i$, $i \in \mathcal{S}$, a matrix $N(r_t)$ will be denoted by N_i. For example, $A(r_t)$ and $A_d(r_t)$ are denoted by A_i and A_{di}, respectively.

Throughout this paper, we adopt the following definition.

Definition 1. The stochastic Markovian jump system (1) is said to achieve mean-square exponential stability with decay rate β if, when $u(t) = 0$, any $\phi(t) \in C^b_{\mathcal{F}_0}([-\bar{\tau}, 0];\mathbf{R}^n)$ and initial mode $r_0 \in \mathcal{S}$, there exist constant scalars $b > 0$ and $\beta > 0$ such that

$$\mathrm{E}\{|x(t,\phi,r_0)|^2\} \leq b \sup_{-\bar{\tau}\leq\theta\leq 0} |\phi(\theta)|^2 \mathrm{e}^{-\beta t}$$

where $x(t, \phi, r_0)$ denotes the solution of system (1) at time t under the initial conditions $\phi(\cdot)$ and r_0, and β is called the decay rate.

It is well known that $\{x_t, r_t\}_{t\geq 0}$ is a $C([-\bar{\tau}, 0];\mathbf{R}^n) \times \mathcal{S}$-valued Markov process[25]. Its weak infinitesimal generator $\mathcal{L}$, acting on functional $V(\cdot,\cdot,\cdot) : C([-\bar{\tau}, 0];\mathbf{R}^n) \times \mathcal{S} \times \mathbf{R}_+ \to \mathbf{R}_+$, is defined by the following formula:

$$\begin{aligned}\mathcal{L}V(x_t, i, t) = \lim_{\Delta\to 0^+} \frac{1}{\Delta}\{&\mathrm{E}[V(x_{t+\Delta}, r_{t+\Delta}, t+\Delta)|x_t, r_t = i] - \\ &V(x_t, i, t)\}\end{aligned}$$

The main purpose of the rest of this paper is to obtain sufficient conditions such that the following two requirements are satisfied:

1) The stochastic Markovian jump system (1) is mean-square exponentially stable.

2) Design a delayed-state-feedback controller

$$u(t) = K(r_t)x(t-\tau(r_t,t)) \tag{2}$$

which can mean-square exponentially stabilize system (1), where $K(r_t)$ is a controller gain matrix function to be determined later.

For simplicity, let us introduce some notations as follows:

$$\begin{aligned}\underline{\tau} = \min\{\underline{\tau}_i, i \in \mathcal{S}\}, \quad \eta = \max\{|\pi_{ii}|, i \in \mathcal{S}\}\\ \varphi(t) = A(r_t)x(t) + A_d(r_t)x(t-\tau(r_t,t))\\ \psi(t) = B(r_t)x(t) + B_d(r_t)x(t-\tau(r_t,t))\end{aligned}$$

Before giving the main results, we first present the following lemmas, which are important for the proof of main theorems.

Lemma 1[33]**.** If for any constant matrix $M \in \mathbf{R}^{n\times n}$, $M = M^{\mathrm{T}} > 0$, scalars $b > a > 0$, and vector function $f(\cdot) : [a, b] \to \mathbf{R}^n$, the integrations in the following are well defined, then one has the following inequality:

$$\{\int_a^b f(s)\mathrm{d}s\}^{\mathrm{T}} M \{\int_a^b f(s)\mathrm{d}s\} \leq (b-a)\int_a^b f^{\mathrm{T}}(s)Mf(s)\mathrm{d}s$$

Lemma 2[32]**.** For any $a, b \in \mathbf{R}, a \leq b$ and $f(s) \in \mathcal{L}_2[a, b]$, we have

$$\mathrm{E}\int_a^b f(s)\mathrm{d}\omega(s) = 0$$

$$\mathrm{E}\{(\int_a^b f(s)\mathrm{d}\omega(s))^{\mathrm{T}}(\int_a^b f(s)\mathrm{d}\omega(s))\} = \mathrm{E}\int_a^b |f(s)|^2\mathrm{d}s$$

Lemma 3 (Gronwall-Bellman lemma)[34]**.** Let $v(t)$ and $w(t)$ be real functions of t, Let $w(t) \geq 0$ and c be a real constant. If $v(t) \leq c + \int_0^t w(s)v(s)\mathrm{d}s$, then $v(t) \leq c\mathrm{e}^{\int_0^t w(s)\mathrm{d}s}$.

2 Main results

In this section, we first present a delay-range-dependent and decay-rate-dependent sufficient condition, which guarantees the mean-square exponential stability for the following system:

$$\begin{aligned} \mathrm{d}x(t) =& [A(r_t)x(t) + A_d(r_t)x(t-\tau(r_t,t))]\mathrm{d}t + \\ & [B(r_t)x(t) + B_d(r_t)x(t-\tau(r_t,t))]\mathrm{d}\omega(t) \\ x(t) =& \phi(t), \quad t \in [-\bar{\tau}, 0] \end{aligned} \tag{3}$$

Theorem 1. For given finite constants $\beta > 0, \bar{\tau} > \underline{\tau} \geq 0, \eta > 0$, and $d_i, i \in \mathcal{S}$, if there exist matrices $Q > 0, Q_1 > 0, Q_2 > 0, R > 0, P_i > 0$ and any appropriately dimensional matrices $M_{1i}, M_{2i}, N_{ki}, k = 1, 2, 3$, for each $i \in \mathcal{S}$, such that the following LMI holds:

$$\left[\begin{array}{ccccccccccc} \Theta_{1i} & \Theta_{2i} & N_{3i}^{\mathrm{T}} & -M_{1i} & -M_{1i} & -N_{1i} & B_i^{\mathrm{T}}P_i & A_i^{\mathrm{T}}R & B_i^{\mathrm{T}} & M_{1i} & N_{1i} \\ * & \Theta_{3i} & -N_{3i}^{\mathrm{T}} & -M_{2i} & -M_{2i} & -N_{2i} & B_{di}^{\mathrm{T}}P_i & A_{di}^{\mathrm{T}}R & B_{di}^{\mathrm{T}} & M_{2i} & N_{2i} \\ * & * & -\mathrm{e}^{-\beta\underline{\tau}}Q_1 & 0 & 0 & -N_{3i} & 0 & 0 & 0 & 0 & N_{3i} \\ * & * & * & -\mathrm{e}^{-\beta\bar{\tau}}Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\dfrac{1}{\bar{\tau}-\underline{\tau}}R & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\dfrac{1}{\bar{\tau}}R & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -P_i & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\dfrac{\beta}{\mathrm{e}^{\beta\bar{\tau}}-1}R & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\dfrac{\beta}{\mathrm{e}^{\beta\bar{\tau}}-1}I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & * & * & -I \end{array}\right] < 0 \tag{4}$$

where

$$\begin{aligned} \Theta_{1i} =& \sum_{j=1}^{N}\pi_{ij}P_j + \beta P_i + P_iA_i + A_i^{\mathrm{T}}P_i + \mathrm{e}^{\beta\bar{\tau}}Q + \\ & \frac{\eta}{\beta}(\mathrm{e}^{\beta(2\bar{\tau}-\underline{\tau})} - \mathrm{e}^{\beta\bar{\tau}})Q + N_{1i}^{\mathrm{T}} + N_{1i} + Q_1 + Q_2 \\ \Theta_{2i} =& P_iA_{di} + M_{1i} - N_{1i} + N_{2i}^{\mathrm{T}} \\ \Theta_{3i} =& [(d_i - 1) \vee ((d_i-1)\mathrm{e}^{\beta(\bar{\tau}-\underline{\tau})})]Q - N_{2i}^{\mathrm{T}} - N_{2i} + \\ & M_{2i}^{\mathrm{T}} + M_{2i} \end{aligned}$$

then system (3) is mean-square exponentially stable with decay rate β.

Proof. Firstly, we choose a Lyapunov-Krasovskii functional candidate for any $t \geq \bar{\tau}$ as follows:

$$\begin{aligned} V(x_t, r_t, t) =& \\ & \mathrm{e}^{\beta t}V_1(x_t, r_t) + V_2(x_t, r_t, t) + V_3(x_t, r_t, t) + \\ & V_4(x_t, r_t, t) + V_5(x_t, r_t, t) + V_6(x_t, r_t, t) \end{aligned} \tag{5}$$

where

$$\begin{aligned} V_1(x_t, r_t) =& x^{\mathrm{T}}(t)P(r_t)x(t) \\ V_2(x_t, r_t, t) =& \int_{t-\tau(r_t,t)}^{t}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s \\ V_3(x_t, r_t, t) =& \int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t}\mathrm{e}^{\beta(s-\theta)}(\varphi^{\mathrm{T}}(s)R\varphi(s) + \\ & \psi^{\mathrm{T}}(s)\psi(s))\mathrm{d}s\mathrm{d}\theta \\ V_4(x_t, r_t, t) =& \eta\int_{-\bar{\tau}}^{-\underline{\tau}}\int_{t+\theta}^{t}\mathrm{e}^{\beta(s-\theta-\underline{\tau}+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s\mathrm{d}\theta \\ V_5(x_t, r_t, t) =& \int_{t-\underline{\tau}}^{t}\mathrm{e}^{\beta s}x^{\mathrm{T}}(s)Q_1x(s)\mathrm{d}s \\ V_6(x_t, r_t, t) =& \int_{t-\bar{\tau}}^{t}\mathrm{e}^{\beta s}x^{\mathrm{T}}(s)Q_2x(s)\mathrm{d}s \end{aligned}$$

For each $r_t = i$, $i \in \mathcal{S}$, by Itô formula[32] and the definition of weak infinitesimal generator, it can be verified that

$$\begin{aligned} \mathrm{d}V(x_t, i, t) =& \mathcal{L}V(x_t, i, t)\mathrm{d}t + 2\mathrm{e}^{\beta t}x^{\mathrm{T}}(t)P_i\psi(t)\mathrm{d}\omega(t) = \\ & [\beta\mathrm{e}^{\beta t}V_1(x_t, i) + \mathrm{e}^{\beta t}\mathcal{L}V_1(x_t, i) + \mathcal{L}V_2(x_t, i, t) + \\ & \mathcal{L}V_3(x_t, i, t) + \mathcal{L}V_4(x_t, i, t) + \mathcal{L}V_5(x_t, i, t) + \\ & \mathcal{L}V_6(x_t, i, t)]\mathrm{d}t + 2\mathrm{e}^{\beta t}x^{\mathrm{T}}(t)P_i\psi(t)\mathrm{d}\omega(t) \end{aligned} \tag{6}$$

with

$$\begin{aligned} \mathcal{L}V_1(x_t, i) =& x^{\mathrm{T}}(t)(\sum_{j=1}^{N}\pi_{ij}P_j + 2P_iA_i)x(t) + 2x^{\mathrm{T}}(t)P_iA_{di} \times \\ & x(t-\tau_i(t)) + [B_ix(t) + B_{di}x(t-\tau_i(t))]^{\mathrm{T}}P_i \times \\ & [B_ix(t) + B_{di}x(t-\tau_i(t))] \\ \mathcal{L}V_2(x_t, i, t) =& \sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s + \\ & \mathrm{e}^{\beta(t+\bar{\tau})}x^{\mathrm{T}}(t)Qx(t) - \mathrm{e}^{\beta(t-\tau_i(t)+\bar{\tau})}(1-\dot{\tau}_i(t)) \times \\ & x^{\mathrm{T}}(t-\tau_i(t))Qx(t-\tau_i(t)) \\ \mathcal{L}V_3(x_t, i, t) =& \frac{\mathrm{e}^{\beta(t+\bar{\tau})} - \mathrm{e}^{\beta t}}{\beta}[\varphi^{\mathrm{T}}(t)R\varphi(t) + \psi^{\mathrm{T}}(t)\psi(t)] - \\ & \int_{t-\bar{\tau}}^{t}\mathrm{e}^{\beta t}[\varphi^{\mathrm{T}}(s)R\varphi(s) + \psi^{\mathrm{T}}(s)\psi(s)]\mathrm{d}s \\ \mathcal{L}V_4(x_t, i, t) =& \frac{\eta(\mathrm{e}^{\beta(t+2\bar{\tau}-\underline{\tau})} - \mathrm{e}^{\beta(t+\bar{\tau})})}{\beta}x^{\mathrm{T}}(t)Qx(t) - \\ & \eta\int_{t-\bar{\tau}}^{t-\underline{\tau}}\mathrm{e}^{\beta(t-\underline{\tau}+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s \\ \mathcal{L}V_5(x_t, i, t) =& \mathrm{e}^{\beta t}x^{\mathrm{T}}(t)Q_1x(t) - \\ & \mathrm{e}^{\beta(t-\underline{\tau})}x^{\mathrm{T}}(t-\underline{\tau})Q_1x(t-\underline{\tau}) \end{aligned}$$

and

$$\begin{aligned} \mathcal{L}V_6(x_t, i, t) =& \mathrm{e}^{\beta t}x^{\mathrm{T}}(t)Q_2x(t) - \\ & \mathrm{e}^{\beta(t-\bar{\tau})}x^{\mathrm{T}}(t-\bar{\tau})Q_2x(t-\bar{\tau}) \end{aligned}$$

By using Lemma 1, we can get

$$-\int_{t-\bar{\tau}}^{t}\varphi^{\mathrm{T}}(s)R\varphi(s)\mathrm{d}s=-\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi^{\mathrm{T}}(s)R\varphi(s)\mathrm{d}s-\int_{t-\tau_i(t)}^{t}\varphi^{\mathrm{T}}(s)R\varphi(s)\mathrm{d}s\leq -\frac{1}{\bar{\tau}-\underline{\tau}}(\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s)^{\mathrm{T}}R(\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s)-\frac{1}{\bar{\tau}}(\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s)^{\mathrm{T}}R(\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s) \quad (7)$$

Noting $\pi_{ij}\geq 0$, for $j\neq i$ and $\pi_{ii}\leq 0$, we have

$$\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s=\sum_{j\neq i}\pi_{ij}\int_{t-\tau_j(t)}^{t}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s-|\pi_{ii}|\int_{t-\tau_i(t)}^{t}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s\leq \sum_{j\neq i}\pi_{ij}\int_{t-\bar{\tau}}^{t}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s-|\pi_{ii}|\int_{t-\underline{\tau}}^{t}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s=|\pi_{ii}|\int_{t-\bar{\tau}}^{t-\underline{\tau}}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s\leq \eta\mathrm{e}^{\beta(t-\underline{\tau}+\bar{\tau})}\int_{t-\bar{\tau}}^{t-\underline{\tau}}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s \quad (8)$$

Moreover, by using

$$-(1-\dot{\tau}_i(t))\mathrm{e}^{\beta(t+\bar{\tau}-\tau_i(t))}\leq \{(d_i-1)\vee[(d_i-1)\mathrm{e}^{\beta(\bar{\tau}-\underline{\tau})}]\}\mathrm{e}^{\beta t}$$

where $\{(d_i-1)\vee[(d_i-1)\mathrm{e}^{\beta(\bar{\tau}-\underline{\tau})}]\}=\max\{(d_i-1),(d_i-1)\mathrm{e}^{\beta(\bar{\tau}-\underline{\tau})}\}$, combining with (6) ∼ (8), we obtain

$$\mathcal{L}V(x_t,i,t)\leq \mathrm{e}^{\beta t}\{x^{\mathrm{T}}(t)[\beta P_i+\sum_{j=1}^{N}\pi_{ij}P_j+2P_iA_i+Q_1+Q_2+B_i^{\mathrm{T}}P_iB_i+\mathrm{e}^{\beta\bar{\tau}}Q+\frac{\eta(\mathrm{e}^{\beta(2\bar{\tau}-\underline{\tau})}-\mathrm{e}^{\beta\bar{\tau}})}{\beta}Q+\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}\times(A_i^{\mathrm{T}}RA_i+B_i^{\mathrm{T}}B_i)]x(t)+2x^{\mathrm{T}}(t)[P_iA_{di}+B_i^{\mathrm{T}}P_iB_{di}+\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}(A_i^{\mathrm{T}}RA_{di}+B_i^{\mathrm{T}}B_{di})]x(t-\tau_i(t))+x^{\mathrm{T}}(t-\tau_i(t))\times[\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}(A_{di}^{\mathrm{T}}RA_{di}+B_{di}^{\mathrm{T}}B_{di})+[(d_i-1)\vee((d_i-1)\times\mathrm{e}^{\beta(\bar{\tau}-\underline{\tau})})]Q+B_{di}^{\mathrm{T}}P_iB_{di}]x(t-\tau_i(t))-\mathrm{e}^{-\beta\underline{\tau}}x^{\mathrm{T}}(t-\underline{\tau})Q_1\times x(t-\underline{\tau})-\mathrm{e}^{-\beta\bar{\tau}}x^{\mathrm{T}}(t-\bar{\tau})Q_2x(t-\bar{\tau})-\int_{t-\bar{\tau}}^{t}\psi^{\mathrm{T}}(s)\psi(s)\mathrm{d}s-\frac{1}{\bar{\tau}-\underline{\tau}}(\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s)^{\mathrm{T}}R(\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s)-\frac{1}{\bar{\tau}}(\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s)^{\mathrm{T}}R(\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s)\} \quad (9)$$

On the other hand, from (3), it is clear that

$$[x^{\mathrm{T}}(t)M_{1i}+x^{\mathrm{T}}(t-\tau_i(t))M_{2i}][x(t-\tau_i(t))-x(t-\bar{\tau})-\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s-\int_{t-\bar{\tau}}^{t-\tau_i(t)}\psi(s)\mathrm{d}\omega(s)]=0 \quad (10)$$

and

$$[x^{\mathrm{T}}(t)N_{1i}+x^{\mathrm{T}}(t-\tau_i(t))N_{2i}+x^{\mathrm{T}}(t-\underline{\tau})N_{3i}][x(t)-x(t-\tau_i(t))-\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s-\int_{t-\tau_i(t)}^{t}\psi(s)\mathrm{d}\omega(s)]=0 \quad (11)$$

In addition, we can also have the following two estimates

$$-2[x^{\mathrm{T}}(t)M_{1i}+x^{\mathrm{T}}(t-\tau_i(t))M_{2i}]\int_{t-\bar{\tau}}^{t-\tau_i(t)}\psi(s)\mathrm{d}\omega(s)\leq [x^{\mathrm{T}}(t)M_{1i}+x^{\mathrm{T}}(t-\tau_i(t))M_{2i}][M_{1i}^{\mathrm{T}}x(t)+M_{2i}^{\mathrm{T}}x(t-\tau_i(t))]+(\int_{t-\bar{\tau}}^{t-\tau_i(t)}\psi(s)\mathrm{d}\omega(s))^{\mathrm{T}}\int_{t-\bar{\tau}}^{t-\tau_i(t)}\psi(s)\mathrm{d}\omega(s) \quad (12)$$

and

$$-2[x^{\mathrm{T}}(t)N_{1i}+x^{\mathrm{T}}(t-\tau_i(t))N_{2i}+x^{\mathrm{T}}(t-\underline{\tau})N_{3i}]\times\int_{t-\tau_i(t)}^{t}\psi(s)\mathrm{d}\omega(s)\leq [x^{\mathrm{T}}(t)N_{1i}+x^{\mathrm{T}}(t-\tau_i(t))N_{2i}+x^{\mathrm{T}}(t-\underline{\tau})N_{3i}][N_{1i}^{\mathrm{T}}x(t)+N_{2i}^{\mathrm{T}}x(t-\tau_i(t))+N_{3i}^{\mathrm{T}}x(t-\underline{\tau})]+(\int_{t-\tau_i(t)}^{t}\psi(s)\mathrm{d}\omega(s))^{\mathrm{T}}\int_{t-\tau_i(t)}^{t}\psi(s)\mathrm{d}\omega(s) \quad (13)$$

For the second terms in (12) and (13), using Lemma 2, we also have

$$\mathrm{E}(\int_{t-\bar{\tau}}^{t-\tau_i(t)}\psi(s)\mathrm{d}\omega(s))^{\mathrm{T}}\int_{t-\bar{\tau}}^{t-\tau_i(t)}\psi(s)\mathrm{d}\omega(s)+\mathrm{E}(\int_{t-\tau_i(t)}^{t}\psi(s)\mathrm{d}\omega(s))^{\mathrm{T}}\int_{t-\tau_i(t)}^{t}\psi(s)\mathrm{d}\omega(s)=\mathrm{E}\int_{t-\bar{\tau}}^{t-\tau_i(t)}\psi^{\mathrm{T}}(s)\psi(s)\mathrm{d}s+\mathrm{E}\int_{t-\tau_i(t)}^{t}\psi^{\mathrm{T}}(s)\psi(s)\mathrm{d}s=\mathrm{E}\int_{t-\bar{\tau}}^{t}\psi^{\mathrm{T}}(s)\psi(s)\mathrm{d}s \quad (14)$$

Substituting (10) ∼ (13) and (14) into (9) yields

$$\mathrm{E}\mathcal{L}V(x_t,i,t)\leq \mathrm{E}\mathrm{e}^{\beta t}\{x^{\mathrm{T}}(t)[\beta P_i+\sum_{j=1}^{N}\pi_{ij}P_j+2P_iA_i+Q_1+Q_2+B_i^{\mathrm{T}}P_iB_i+\mathrm{e}^{\beta\bar{\tau}}Q+\frac{\eta(\mathrm{e}^{\beta(2\bar{\tau}-\underline{\tau})}-\mathrm{e}^{\beta\bar{\tau}})}{\beta}Q+\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}\times(A_i^{\mathrm{T}}RA_i+B_i^{\mathrm{T}}B_i)]x(t)+2x^{\mathrm{T}}(t)[P_iA_{di}+B_i^{\mathrm{T}}P_iB_{di}+\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}(A_i^{\mathrm{T}}RA_{di}+B_i^{\mathrm{T}}B_{di})]x(t-\tau_i(t))+x^{\mathrm{T}}(t-\tau_i(t))[B_{di}^{\mathrm{T}}P_iB_{di}+\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}(A_{di}^{\mathrm{T}}RA_{di}+B_{di}^{\mathrm{T}}B_{di})+$$

$$[(d_i-1)\vee((d_i-1)\mathrm{e}^{\beta(\bar{\tau}-\underline{\tau})})]Q]x(t-\tau_i(t))-\mathrm{e}^{-\beta\underline{\tau}}x^{\mathrm{T}}(t-\underline{\tau})Q_1x(t-\underline{\tau})-\mathrm{e}^{-\beta\bar{\tau}}x^{\mathrm{T}}(t-\bar{\tau})Q_2x(t-\bar{\tau})-\int_{t-\bar{\tau}}^{t}\psi^{\mathrm{T}}(s)\psi(s)\mathrm{d}s-\frac{1}{\bar{\tau}-\underline{\tau}}(\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s)^{\mathrm{T}}R\times\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s-\frac{1}{\bar{\tau}}(\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s)^{\mathrm{T}}R\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s+2[x^{\mathrm{T}}(t)M_{1i}+x^{\mathrm{T}}(t-\tau_i(t))M_{2i}][x(t-\tau_i(t))-x(t-\bar{\tau})-\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s-\int_{t-\bar{\tau}}^{t-\tau_i(t)}\psi(s)\mathrm{d}\omega(s)]+2[x^{\mathrm{T}}(t)N_{1i}+x^{\mathrm{T}}(t-\tau_i(t))N_{2i}+x^{\mathrm{T}}(t-\underline{\tau})N_{3i}][x(t)-x(t-\tau_i(t))-\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s-\int_{t-\tau_i(t)}^{t}\psi(s)\mathrm{d}\omega(s)]\}\leq\mathrm{E}\mathrm{e}^{\beta t}\xi^{\mathrm{T}}(t)\Gamma_i\xi(t)$$

where

$$\Gamma_i=\begin{bmatrix}\hat{\Theta}_{1i} & \hat{\Theta}_{2i} & N_{3i}^{\mathrm{T}}+N_{1i}N_{3i}^{\mathrm{T}} & -M_{1i} & -M_{1i} & -N_{1i}\\ * & \hat{\Theta}_{3i} & -N_{3i}^{\mathrm{T}}+N_{2i}N_{3i}^{\mathrm{T}} & -M_{2i} & -M_{2i} & -N_{2i}\\ * & * & -\mathrm{e}^{-\beta\underline{\tau}}Q_1+N_{3i}N_{3i}^{\mathrm{T}} & 0 & 0 & -N_{3i}\\ * & * & * & -\mathrm{e}^{-\beta\bar{\tau}}Q_2 & 0 & 0\\ * & * & * & * & -\frac{1}{\bar{\tau}-\underline{\tau}}R & 0\\ * & * & * & * & * & -\frac{1}{\bar{\tau}}R\end{bmatrix}$$

$$\xi(t)=[x^{\mathrm{T}}(t),x^{\mathrm{T}}(t-\tau_i(t)),x^{\mathrm{T}}(t-\underline{\tau}),x^{\mathrm{T}}(t-\bar{\tau}),(\int_{t-\bar{\tau}}^{t-\tau_i(t)}\varphi(s)\mathrm{d}s)^{\mathrm{T}},(\int_{t-\tau_i(t)}^{t}\varphi(s)\mathrm{d}s)^{\mathrm{T}}]^{\mathrm{T}}$$

$$\hat{\Theta}_{1i}=\Theta_{1i}+M_{1i}M_{1i}^{\mathrm{T}}+N_{1i}N_{1i}^{\mathrm{T}}+B_i^{\mathrm{T}}P_iB_i+\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}(A_i^{\mathrm{T}}RA_i+B_i^{\mathrm{T}}B_i)$$

$$\hat{\Theta}_{2i}=\Theta_{2i}+M_{1i}M_{2i}^{\mathrm{T}}+N_{1i}N_{2i}^{\mathrm{T}}+B_i^{\mathrm{T}}P_iB_{di}+\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}(A_i^{\mathrm{T}}RA_{di}+B_i^{\mathrm{T}}B_{di})$$

$$\hat{\Theta}_{3i}=\Theta_{3i}+M_{2i}M_{2i}^{\mathrm{T}}+N_{2i}N_{2i}^{\mathrm{T}}+B_{di}^{\mathrm{T}}P_iB_{di}+\frac{\mathrm{e}^{\beta\bar{\tau}}-1}{\beta}(A_{di}^{\mathrm{T}}RA_{di}+B_{di}^{\mathrm{T}}B_{di})$$

By employing Schur complement and (4), it follows that $\mathrm{E}(\mathcal{L}V(x_t,i,t))<0$ for each $i\in\mathcal{S}$.

For all $t\geq\bar{\tau}$, the Dynkin's formula[32] provides

$$\mathrm{E}\int_{\bar{\tau}}^{t}\mathcal{L}V(x_s,r_s,s)\mathrm{d}s=\mathrm{E}V(x_t,r_t,t)-\mathrm{E}V(x_{\bar{\tau}},r_{\bar{\tau}},\bar{\tau})$$

So, from (5), it is clear that

$$\mathrm{E}V(x_t,r_t,t)\leq\mathrm{E}V(x_{\bar{\tau}},r_{\bar{\tau}},\bar{\tau})=\mathrm{E}\{\mathrm{e}^{\beta\bar{\tau}}x^{\mathrm{T}}(\bar{\tau})P(r_{\bar{\tau}})x(\bar{\tau})+\int_{\bar{\tau}-\tau(r_{\bar{\tau}},\bar{\tau})}^{\bar{\tau}}\mathrm{e}^{\beta(s+\bar{\tau})}x^{\mathrm{T}}(s)Q\times x(s)\mathrm{d}s+\int_{-\bar{\tau}}^{0}\int_{\bar{\tau}+\theta}^{\bar{\tau}}\mathrm{e}^{\beta(s-\theta)}(\varphi^{\mathrm{T}}(s)R\varphi(s)+\psi^{\mathrm{T}}(s)\times\psi(s))\mathrm{d}s\mathrm{d}\theta+\eta\int_{-\bar{\tau}}^{-\underline{\tau}}\int_{\bar{\tau}+\theta}^{\bar{\tau}}\mathrm{e}^{\beta(s-\theta-\underline{\tau}+\bar{\tau})}x^{\mathrm{T}}(s)Qx(s)\mathrm{d}s\mathrm{d}\theta+\int_{\bar{\tau}-\underline{\tau}}^{\bar{\tau}}\mathrm{e}^{\beta s}x^{\mathrm{T}}(s)Q_1x(s)\mathrm{d}s+\int_{0}^{\bar{\tau}}\mathrm{e}^{\beta s}x^{\mathrm{T}}(s)Q_2x(s)\mathrm{d}s\}\quad(15)$$

For $t>0$, following from system (3), we obtain

$$x(t)=x(0)+\int_0^t\varphi(s)\mathrm{d}s+\int_0^t\psi(s)\mathrm{d}\omega(s)$$

Let

$$\mu_1=\max_{i\in\mathcal{S}}\{\|A_i\|\},\ \mu_2=\max_{i\in\mathcal{S}}\{\|B_i\|\}$$
$$\mu_3=\max_{i\in\mathcal{S}}\{\|A_{di}\|\},\ \mu_4=\max_{i\in\mathcal{S}}\{\|B_{di}\|\}$$

From Lemma 2, we have the following formulae:

$$\mathrm{E}x^{\mathrm{T}}(0)\int_0^t\psi(s)\mathrm{d}\omega(s)=0\quad(16a)$$

$$\mathrm{E}[\int_0^t\psi(s)\mathrm{d}\omega(s)]^{\mathrm{T}}[\int_0^t\psi(s)\mathrm{d}\omega(s)]=\mathrm{E}\int_0^t|\psi(s)|^2\mathrm{d}s\quad(16b)$$

By combining (16) and Lemma 1, for $0<t\leq\bar{\tau}$, it is not difficult to check that

$$\mathrm{E}|x(t)|^2=\mathrm{E}|x(0)+\int_0^t\varphi(s)\mathrm{d}s+\int_0^t\psi(s)\mathrm{d}\omega(s)|^2\leq\mathrm{E}\{|x(0)|^2+|\int_0^t\varphi(s)\mathrm{d}s|^2+\int_0^t|\psi(s)|^2\mathrm{d}s+|x(0)|^2+|\int_0^t\varphi(s)\mathrm{d}s|^2+|\int_0^t\varphi(s)\mathrm{d}s|^2+\int_0^t|\psi(s)|^2\mathrm{d}s\}\leq 2\mathrm{E}|x(0)|^2+3\bar{\tau}\mathrm{E}\int_0^t|\varphi(s)|^2\mathrm{d}s+2\mathrm{E}\int_0^t|\psi(s)|^2\mathrm{d}s\leq 2\sup_{-\bar{\tau}\leq\theta\leq 0}|\phi(\theta)|^2+2(3\mu_3^2\bar{\tau}+2\mu_4^2)\bar{\tau}\sup_{-\bar{\tau}\leq\theta\leq 0}|\phi(\theta)|^2+2\int_0^t(3\mu_1^2\bar{\tau}+2\mu_2^2)\mathrm{E}|x(s)|^2\mathrm{d}s=2(1+3\mu_3^2\bar{\tau}^2+2\mu_4^2\bar{\tau})\sup_{-\bar{\tau}\leq\theta\leq 0}|\phi(\theta)|^2+2\int_0^t(3\mu_1^2\bar{\tau}+2\mu_2^2)\mathrm{E}|x(s)|^2\mathrm{d}s\quad(17)$$

By using Gronwall-Bellman lemma, for $0<t\leq\bar{\tau}$, it follows from (17) that

$$\mathrm{E}|x(t)|^2\leq b_1\sup_{-\bar{\tau}\leq\theta\leq 0}|\phi(\theta)|^2\quad(18)$$

where $b_1=2(1+3\mu_3^2\bar{\tau}^2+2\mu_4^2\bar{\tau})\mathrm{e}^{2\bar{\tau}(3\mu_1^2\bar{\tau}+2\mu_2^2)}$.

Moreover, from (15) and (18), we also have

$$\mathrm{E}V(x_t,r_t,t)\leq\mathrm{e}^{\beta\bar{\tau}}\max_{i\in\mathcal{S}}\{\|P_i\|\}b_1\sup_{-\bar{\tau}\leq\theta\leq 0}|\phi(\theta)|^2+b_1\|Q\|\sup_{-\bar{\tau}\leq\theta\leq 0}|\phi(\theta)|^2\int_{\bar{\tau}-\tau(r_{\bar{\tau}},\bar{\tau})}^{\bar{\tau}}\mathrm{e}^{\beta(s+\bar{\tau})}\mathrm{d}s+2(\mu_1^2b_1\|R\|+\mu_3^2\|R\|+\mu_2^2b_1+\mu_4^2)\sup_{-\bar{\tau}\leq\theta\leq 0}|\phi(\theta)|^2\times$$

$$\int_{-\bar\tau}^{0}\int_{\bar\tau+\theta}^{\bar\tau}\mathrm{e}^{\beta(s-\theta)}\mathrm{d}s\mathrm{d}\theta+b_1\|Q_1\|\sup_{-\bar\tau\le\theta\le 0}|\phi(\theta)|^2\times$$
$$\int_{\bar\tau-\underline{\tau}}^{\bar\tau}\mathrm{e}^{\beta s}\mathrm{d}s+b_1\|Q_2\|\sup_{-\bar\tau\le\theta\le 0}|\phi(\theta)|^2\int_{0}^{\bar\tau}\mathrm{e}^{\beta s}\mathrm{d}s+$$
$$\eta b_1\|Q\|\sup_{-\bar\tau\le\theta\le 0}|\phi(\theta)|^2\int_{-\bar\tau}^{-\underline{\tau}}\int_{\bar\tau+\theta}^{\bar\tau}\mathrm{e}^{\beta(s-\theta-\underline{\tau}+\bar\tau)}\mathrm{d}s\mathrm{d}\theta=$$
$$\mathrm{e}^{\beta\bar\tau}(\max_{i\in\mathcal{S}}\{\|P_i\|\}b_1+b_1\|Q\|\frac{\mathrm{e}^{\beta\bar\tau}-\mathrm{e}^{\beta(\bar\tau-\tau(r_{\bar\tau},\bar\tau))}}{\beta}+$$
$$2(\mu_1^2b_1\|R\|+\mu_3^2\|R\|+\mu_2^2b_1+\mu_4^2)\frac{\mathrm{e}^{\beta\bar\tau}-1-\beta\bar\tau}{\beta^2}+$$
$$b_1\frac{1-\mathrm{e}^{-\beta\underline{\tau}}}{\beta}\|Q_1\|+b_1\frac{1-\mathrm{e}^{-\beta\bar\tau}}{\beta}\|Q_2\|+\eta b_1\|Q\|\times$$
$$\mathrm{e}^{\beta(\bar\tau-\underline{\tau})}\frac{\mathrm{e}^{\beta\bar\tau}-\mathrm{e}^{\beta\underline{\tau}}-\beta(\bar\tau-\underline{\tau})}{\beta^2})\sup_{-\bar\tau\le\theta\le 0}|\phi(\theta)|^2$$

In addition, we observe

$$\mathrm{E}V(x_t,r_t,t)\ge\frac{1}{\max_{i\in\mathcal{S}}\{\|P_i^{-1}\|\}}\mathrm{e}^{\beta t}\mathrm{E}|x(t)|^2$$

Hence, for $t>\bar\tau$, it can be concluded that

$$\mathrm{E}|x(t)|^2\le\mathrm{e}^{-\beta t}\max_{i\in\mathcal{S}}\{\|P_i^{-1}\|\}\mathrm{e}^{\beta\bar\tau}b_2\sup_{-\bar\tau\le\theta\le 0}|\phi(\theta)|^2\qquad(19)$$

where $b_2=\max_{i\in\mathcal{S}}\{\|P_i\|\}b_1+b_1\|Q\|\frac{\mathrm{e}^{\beta\bar\tau}-\mathrm{e}^{\beta(\bar\tau-\tau(r_{\bar\tau},\bar\tau))}}{\beta}+2(\mu_2^2b_1+\mu_1^2b_1\|R\|+\mu_3^2\|R\|+\mu_4^2)\frac{\mathrm{e}^{\beta\bar\tau}-1-\beta\bar\tau}{\beta^2}+\eta b_1\|Q\|\mathrm{e}^{\beta(\bar\tau-\underline{\tau})}\frac{\mathrm{e}^{\beta\bar\tau}-\mathrm{e}^{\beta\underline{\tau}}-\beta(\bar\tau-\underline{\tau})}{\beta^2}+b_1\frac{1-\mathrm{e}^{-\beta\underline{\tau}}}{\beta}\|Q_1\|+b_1\frac{1-\mathrm{e}^{-\beta\bar\tau}}{\beta}\|Q_2\|$.

For $0<t\le\bar\tau$, we also have

$$\mathrm{E}|x(t)|^2\le\mathrm{e}^{-\beta t}\mathrm{e}^{\beta\bar\tau}b_1\sup_{-\bar\tau\le\theta\le 0}|\phi(\theta)|^2\qquad(20)$$

Evidently, for $t>0$, taking (19) and (20) into account, we conclude that

$$\mathrm{E}|x(t)|^2\le b\sup_{-\bar\tau\le\theta\le 0}|\phi(\theta)|^2\mathrm{e}^{-\beta t}$$

where $b=\max\{\mathrm{e}^{\beta\bar\tau}\max_{i\in\mathcal{S}}\{\|P_i^{-1}\|\}b_2,\mathrm{e}^{\beta\bar\tau}b_1\}$

Therefore, system (3) is mean-square exponentially stable with decay rate β. □

Remark 1. In Theorem 1, the derivative of time-varying delay $\tau_i(t)$ only has an upper bound d_i, where d_i can be any finite constant. It is more general than the assumptions on time-varying delays in [12, 24−25, 27, 29]. Furthermore, the decay rate in many literatures needs to satisfy two constraints: LMI and transcendental equation or LMI and a set of inequalities[20,24−29]. As pointed out in the introduction section, the decay rate satisfying transcendental equation will be a fixed constant and cannot be chosen conveniently according to practical design specifications. And the constraint satisfying a set of inequalities adds more difficulty and complexity in the process of computing. Moreover, the obtained results in both of the cases cannot or are very difficult to tell us whether the system can possess a larger and suboptimal decay rate than the computed value. Here, decay rate β in Theorem 1 only needs to satisfy LMI (4) and there is no additional equation or multi-inequality constraint. Since the obtained sufficient condition is not only delay-range-dependent but also decay-rate-dependent, the suboptimal upper bound of decay rate β can be computed by convex optimization algorithm conveniently. This will introduce more flexibility when choosing decay rate β. In Example 1, we will compute the suboptimal upper bound of decay rate β.

Remark 2. In this paper, the time-varying delay $\tau_i(t)$ is assumed to be interval time-varying delay and the lower bound $\underline{\tau}_i$ is not less than 0. This hypothesis on $\tau_i(t)$ is more general. Furthermore, when $\underline{\tau}_i=0$, the obtained sufficient condition in Theorem 1 has less conservatism than the existing results in [15, 24, 35], which can be verified by Example 1. In addition, when the mode-dependent time-varying delay $\tau_i(t)\equiv\tau(t)$, for any $i\in\mathcal{S}$, Theorem 1 is also effective for stochastic Markovian jump systems with time-varying delays.

Now, based on the result obtained in Theorem 1, the following theorem is devoted to designing a delayed-state-feedback controller of form (2) that can exponentially stabilize system (1).

Theorem 2. For given finite constants $\beta>0,\bar\tau>\underline{\tau}\ge 0,\eta>0,d_i,i\in\mathcal{S}$ and tuning matrices $\tilde M_{1i}$, $\tilde M_{2i},\tilde N_{ki},k=1,2,3$, if there exist matrices $\tilde Q>0,\tilde Q_1>0,\tilde Q_2>0,\tilde R>0,X_i>0,Y_i$, for each $i\in\mathcal{S}$, satisfying LMI (21) as shown at the top of next page, where

$$\begin{aligned}\tilde\Theta_{1i}=&\ \pi_{ii}X_i+\beta X_i+A_iX_i+X_iA_i^{\mathrm{T}}+\tilde N_{1i}X_i+X_i\tilde N_{1i}^{\mathrm{T}}\\ \tilde\Theta_{2i}=&\ A_{di}\tilde Q+C_iY_i+\tilde M_{1i}\tilde Q-\tilde N_{1i}\tilde Q+X_i\tilde N_{2i}^{\mathrm{T}}\\ \tilde\Theta_{3i}=&\ [(d_i-1)\vee((d_i-1)\mathrm{e}^{\beta(\bar\tau-\underline{\tau})})]\tilde Q-\tilde Q\tilde N_{2i}^{\mathrm{T}}-\tilde N_{2i}\tilde Q+\\ &\ \tilde Q\tilde M_{2i}^{\mathrm{T}}+\tilde M_{2i}\tilde Q\\ \Phi_i=&\ [\sqrt{\pi_{i1}}X_i,\cdots,\sqrt{\pi_{ii-1}}X_i,\sqrt{\pi_{ii+1}}X_i,\cdots,\sqrt{\pi_{iN}}X_i,\\ &\ X_i,X_i,X_i]\\ \Psi_i=&\ \mathrm{diag}\{X_1,\cdots,X_{i-1},X_{i+1},\cdots,X_N\\ &\ \frac{\beta}{\eta\mathrm{e}^{\beta(2\bar\tau-\underline{\tau})}-\eta\mathrm{e}^{\beta\bar\tau}}\tilde Q,\tilde Q_1,\tilde Q_2\}\end{aligned}$$

then the closed-loop system consisting of (1) and (2) is mean-square exponentially stable. Moreover, the gain matrix K_i can be chosen as $K_i=Y_i\tilde Q^{-1}$ for each $i\in\mathcal{S}$.

Proof. From system (1) and controller (2), the closed-loop system can be presented as follows:

$$\begin{aligned}\mathrm{d}x(t)=&[A(r_t)x(t)+(A_d(r_t)+C(r_t)K(r_t))\times\\ &x(t-\tau(r_t,t))]\mathrm{d}t+[B(r_t)x(t)+\\ &B_d(r_t)x(t-\tau(r_t,t))]\mathrm{d}\omega(t)\end{aligned}\qquad(21)$$

By Schur complement, LMI (22) implies that the following LMI holds:

$$\left[\begin{array}{cccccc}\Lambda_i & \tilde\Theta_{2i} & X_i\tilde N_{3i}^{\mathrm{T}} & -\tilde M_{1i}\tilde Q_2 & -\tilde M_{1i}\tilde R & -\tilde N_{1i}\tilde R\\ * & \tilde\Theta_{3i} & -\tilde Q\tilde N_{3i}^{\mathrm{T}} & -\tilde M_{2i}\tilde Q_2 & -\tilde M_{2i}\tilde R & -\tilde N_{2i}\tilde R\\ * & * & -\mathrm{e}^{-\beta\underline{\tau}}\tilde Q_1 & 0 & 0 & -\tilde N_{3i}\tilde R\\ * & * & * & -\mathrm{e}^{-\beta\bar\tau}\tilde Q_2 & 0 & 0\\ * & * & * & * & -\frac{1}{\bar\tau-\underline{\tau}}\tilde R & 0\\ * & * & * & * & * & -\frac{1}{\bar\tau}\tilde R\\ * & * & * & * & * & *\\ * & * & * & * & * & *\\ * & * & * & * & * & *\\ * & * & * & * & * & *\\ * & * & * & * & * & *\end{array}\right.$$

$$
\begin{bmatrix}
\tilde\Theta_{1i} & \tilde\Theta_{2i} & X_i\tilde N_{3i}^{\mathrm T} & -\tilde M_{1i}\tilde Q_2 & -\tilde M_{1i}\tilde R & -\tilde N_{1i}\tilde R & X_iB_i^{\mathrm T} & X_iA_i^{\mathrm T} & X_iB_i^{\mathrm T} & \tilde M_{1i} & \tilde N_{1i} & \Phi_i \\
* & \tilde\Theta_{3i} & -\tilde Q\tilde N_{3i}^{\mathrm T} & -\tilde M_{2i}\tilde Q_2 & -\tilde M_{2i}\tilde R & -\tilde N_{2i}\tilde R & \tilde QB_{di}^{\mathrm T} & \tilde QA_{di}^{\mathrm T}+Y_i^{\mathrm T}C_i^{\mathrm T} & \tilde QB_{di}^{\mathrm T} & \tilde M_{2i} & \tilde N_{2i} & 0 \\
* & * & -\mathrm e^{-\beta\underline{\tau}}\tilde Q_1 & 0 & 0 & -\tilde N_{3i}\tilde R & 0 & 0 & 0 & 0 & \tilde N_{3i} & 0 \\
* & * & * & -\mathrm e^{-\beta\bar\tau}\tilde Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\frac{1}{\bar\tau-\underline{\tau}}\tilde R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\frac{1}{\bar\tau}\tilde R & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -X_i & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\frac{\beta}{\mathrm e^{\beta\bar\tau}-1}\tilde R & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\frac{\beta}{\mathrm e^{\beta\bar\tau}-1}I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\Psi_i
\end{bmatrix} < 0 \tag{22}
$$

$$
\left.\begin{matrix}
X_iB_i^{\mathrm T} & X_iA_i^{\mathrm T} & X_iB_i^{\mathrm T} & \tilde M_{1i} & \tilde N_{1i} \\
\tilde QB_{di}^{\mathrm T} & \tilde QA_{di}^{\mathrm T}+Y_i^{\mathrm T}C_i^{\mathrm T} & \tilde QB_{di}^{\mathrm T} & \tilde M_{2i} & \tilde N_{2i} \\
0 & 0 & 0 & 0 & \tilde N_{3i} \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-X_i & 0 & 0 & 0 & 0 \\
* & -\frac{\beta}{\mathrm e^{\beta\bar\tau}-1}\tilde R & 0 & 0 & 0 \\
* & * & -\frac{\beta}{\mathrm e^{\beta\bar\tau}-1}I & 0 & 0 \\
* & * & * & -I & 0 \\
* & * & * & * & -I
\end{matrix}\right] < 0 \tag{23}
$$

where $\Lambda_i = \tilde\Theta_{1i} + \Phi_i\Psi_i^{-1}\Phi_i^{\mathrm T}$.

For each $i \in \mathcal S$, denote

$$
\begin{aligned}
&P_i = X_i^{-1},\ Q = \tilde Q^{-1},\ Q_1 = \tilde Q_1^{-1},\ Q_2 = \tilde Q_2^{-1},\ R = \tilde R^{-1} \\
&M_{1i} = X_i^{-1}\tilde M_{1i},\ M_{2i} = \tilde Q^{-1}\tilde M_{2i},\ N_{1i} = X_i^{-1}\tilde N_{1i} \\
&N_{2i} = \tilde Q^{-1}\tilde N_{2i},\ N_{3i} = \tilde Q_1^{-1}\tilde N_{3i},\ K_i = Y_i\tilde Q^{-1}
\end{aligned}
$$

Pre- and post-multiplying LMI (23) by $\mathrm{diag}\{X_i^{-1}, \tilde Q^{-1}, \tilde Q_1^{-1}, \tilde Q_2^{-1}, \tilde R^{-1}, \tilde R^{-1}, X_i^{-1}, \tilde R^{-1}, I, I, I\}$ and its transpose, respectively, we can rewrite LMI (23) as follows:

$$
\begin{bmatrix}
\Theta_{1i} & \bar\Theta_{2i} & N_{3i}^{\mathrm T} & -M_{1i} & -M_{1i} & -N_{1i} & B_i^{\mathrm T}P_i & A_i^{\mathrm T}R & B_i^{\mathrm T} & M_{1i} & N_{1i} \\
* & \Theta_{3i} & -N_{3i}^{\mathrm T} & -M_{2i} & -M_{2i} & -N_{2i} & B_{di}^{\mathrm T}P_i & (A_{di}+C_iK_i)^{\mathrm T}R & B_{di}^{\mathrm T} & M_{2i} & N_{2i} \\
* & * & -\mathrm e^{-\beta\underline{\tau}}Q_1 & 0 & 0 & -N_{3i} & 0 & 0 & 0 & 0 & N_{3i} \\
* & * & * & -\mathrm e^{-\beta\bar\tau}Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\frac{1}{\bar\tau-\underline{\tau}}R & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\frac{1}{\bar\tau}R & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -P_i & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\frac{\beta}{\mathrm e^{\beta\bar\tau}-1}R & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\frac{\beta}{\mathrm e^{\beta\bar\tau}-1}I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0
$$

where $\bar\Theta_{2i} = P_i(A_{di} + C_iK_i) + M_{1i} - N_{1i} + N_{2i}^{\mathrm T}$.

Hence, from Theorem 1, by replacing A_{di} in (4) with $A_{di} + C_iK_i$, the closed-loop system (22) is mean-square exponentially stable. This completes the proof of Theorem 2. □

3 Numerical examples

Example 1. In this example, we consider the following system

$$\dot x(t) = A(r_t)x(t) + A_d(r_t)x(t-\tau(t))$$

with system matrices as follows:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},\quad A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},\quad A_{d2} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

where $0 < \tau(t) < \bar\tau, \dot\tau(t) < d$. The parameter $\Pi = \{\pi_{ij}\}$ is given by

$$\Pi = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$$

With above parameters and $\beta = 0.1, \eta = 2$, Table 1 presents the comparison results with respect to [15, 24, 35]. From this table, it can be easily concluded that the achieved maximum value of $\bar\tau$ in this paper is larger than the ones in [15, 24, 35], which also shows that the result obtained in Theorem 1 is less conservative than the ones in [15, 24, 35]. Moreover, when let $\bar\tau = 0.8$ and $d = 1.5$, by convex optimization algorithm, it is easy to obtain that the suboptimal upper bound of decay rate is 0.8065.

Table 1 The achieved maximum values of $\bar{\tau}$ corresponding to different values of d

d	0.9	1	1.5
$\bar{\tau}$ by [24]	0.9279	-	-
$\bar{\tau}$ by [35]	0.7529	0.7529	0.7255
$\bar{\tau}$ by [15]	0.9359	0.8886	0.8886
$\bar{\tau}$ by Theorem 1	1.0428	1.0428	1.0428

Example 2. Consider the time-varying delay stochastic system with Markovian jump parameters in the form of (1) with two modes, that is, $\mathcal{S} = \{1, 2\}$. The system matrices are shown as follows:

$$A_1 = \begin{bmatrix} -0.5 & 0 \\ -1 & 0.5 \end{bmatrix}, A_{d1} = \begin{bmatrix} 1.5 & 0 \\ -0.1 & -0.5 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0.2 & 0 \\ 1 & -0.5 \end{bmatrix}, B_{d1} = \begin{bmatrix} -0.5 & 0 \\ -1 & 0.3 \end{bmatrix}, C_1 = \begin{bmatrix} 1.2 \\ -5 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -1 & 0 \\ 0.2 & -1 \end{bmatrix}, A_{d2} = \begin{bmatrix} 1 & 0.5 \\ 0 & 0.3 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -0.2 & 0 \\ 0.1 & 0 \end{bmatrix}, B_{d2} = \begin{bmatrix} -0.1 & 0 \\ 0.5 & -0.3 \end{bmatrix}, C_2 = \begin{bmatrix} 2.2 \\ 1.2 \end{bmatrix}$$

The parameter $\Pi = \{\pi_{ij}\}$ is given by

$$\Pi = \begin{bmatrix} -1.8 & 1.8 \\ 2 & -2 \end{bmatrix}$$

If we let $\tau_1(t) = (\mathrm{e}^{-2.4t} + 0.2)/2, \tau_2(t) = (\mathrm{e}^{-1.5t} + 0.5)/3$, then one has $d_1 = 1.2, d_2 = 0.5, \underline{\tau} = 0.1, \bar{\tau} = 0.6$. With the above parameters and $\beta = 0.1, \eta = 2$, Figs. 1 and 2 show the operation modes and the state responses of open-loop system in two modes with an arbitrary initial state, respectively.

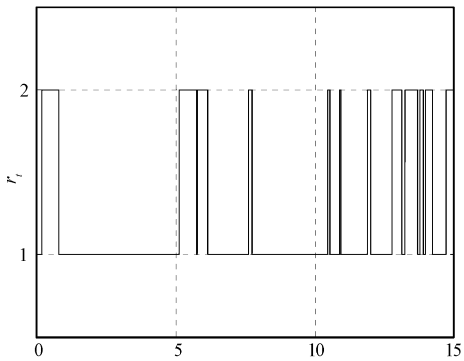

Fig. 1 The operation modes

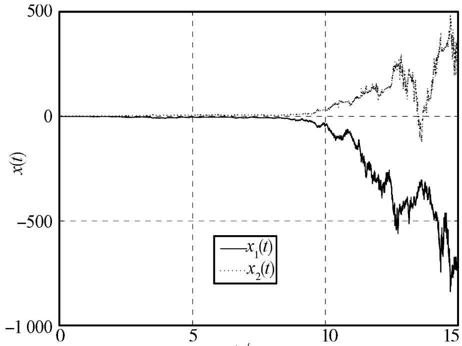

Fig. 2 The state responses of open-loop system

From Fig. 2, it is easy to see that this system is unstable. We choose

$$N_{11} = \begin{bmatrix} 0 & -0.05 \\ 0 & -0.1 \end{bmatrix}, N_{12} = \begin{bmatrix} 0 & -0.01 \\ 0 & -0.1 \end{bmatrix}, N_{21} = \begin{bmatrix} 0 & 0.01 \\ 0 & 0.17 \end{bmatrix}$$

$$N_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}, N_{31} = \begin{bmatrix} 0.02 & 0 \\ -0.01 & -1 \end{bmatrix}$$

$$N_{32} = \begin{bmatrix} 0 & 0.1 \\ -0.1 & 1.5 \end{bmatrix}, M_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, M_{12} = \begin{bmatrix} 0 & -0.05 \\ 0 & -0.1 \end{bmatrix}$$

$$M_{21} = \begin{bmatrix} 0 & -0.1 \\ 0.01 & -2 \end{bmatrix}, M_{22} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By Matlab LMI tool box and Theorem 2, we obtain the following delayed-state-feedback controller gain matrices:

$$K_1 = \begin{bmatrix} -0.7604 & -0.0683 \end{bmatrix}, K_2 = \begin{bmatrix} -0.5766 & -0.2004 \end{bmatrix}$$

Figs. 3 and 4 present the operation modes and the state response of closed-loop system in two modes with initial condition $[-1, 1]^{\mathrm{T}}$, respectively. Consequently, we can verify that the designed delayed-state-feedback controller is effective for exponentially stabilizing the system with decay rate 0.1.

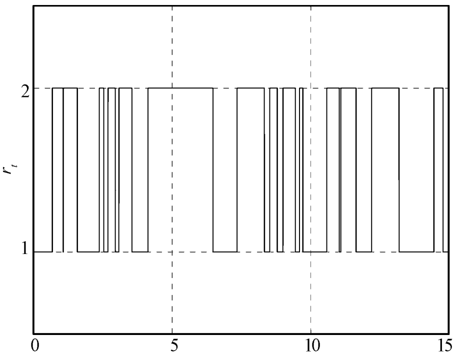

Fig. 3 The operation modes

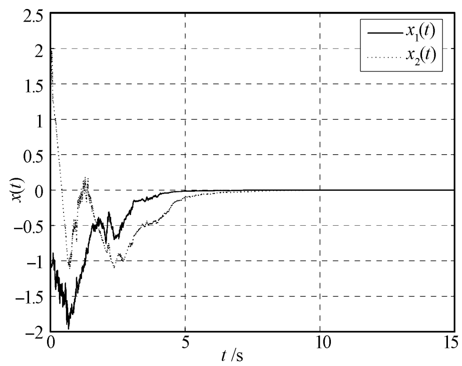

Fig. 4 The state responses of closed-loop system

4 Conclusion

In this paper, we have investigated the stochastic Markovian jump systems with time-varying delays, in which the time-varying delays are dependent on the system modes. By constructing a suitable Lyapunov-Krasovskii functional, sufficient condition for mean-square exponential stability has been proposed. Based on this newly established stability criterion, a delayed-state-feedback controller which can exponentially stabilize the stochastic Markovian jump systems has been presented. There is no additional equation or multi-inequality constraint on decay rate β and the derivative of time-varying delay only needs to satisfy $\dot{\tau}_i(t) < d_i$, where d_i is a constant. Finally, numerical examples have been provided to demonstrate the effectiveness of the obtained results in this paper.

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