An Improved H_∞ Filter Design for Nonlinear System with Time-delay via T-S Fuzzy Models

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Abstract This paper is concerned with the filter design for nonlinear systems with time-varying delay via Takagi-Sugeno fuzzy model approach. Some sufficient conditions of the existence of fuzzy H_∞ filter are established through constructing an improved Lyapunov functional candidate, which could overcome the conservatism of the existing ones. The main technique used is the free weighting matrix method combined with a matrix decoupling approach. An illustrative example is given to show the effectiveness of the method.

Key words Fuzzy system model, H_∞ filter, linear matrix inequality (LMI), time-varying delays control, T-S fuzzy system
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Nonlinear filtering is of both theoretical and practical importance in signal processing, and this area has kept attracting researchers for decades. So far, various methodologies have been developed for the filter designs[1−2], such as Kalman filter[3−4], H_∞ filter[4−10], and so on. The Kalman filter is based on the assumption that the systems are exactly known and their disturbances are stationary Gaussian noises with known statistics, while the H_∞ filter can determine an asymptotically stable filter without a certain signal model. Moreover, the H_∞ filter is designed by minimizing state estimation error for the worst-case bounded disturbances and noises. Recently, considerable attention has been paid to H_∞ filtering for linear systems[5−6]. As known, T-S fuzzy model[7, 11−14] has become a popular and effective approach to control complex and ill-defined systems for which the application of conventional techniques is infeasible. In recent years, T-S fuzzy model approach has been extended to H_∞ filter design[8, 14−18]. In [17−18], an H_∞ filtering methodology for nonlinear discrete-time systems with multiple time delays was proposed. In [10], a delay-independent linear matrix inequality (LMI) approach was proposed for exponential H_∞ filter design for T-S fuzzy delayed systems. In [16], a delay-dependent design scheme was proposed for T-S fuzzy delayed systems. The method therein is suitable for the case that the filter form is of the extended Kalman filtering type. The work in [9, 19] was concerned with H_∞ filtering of nonlinear continuous-time state-space models with time-varying delays via T-S fuzzy model approach. However, during the filter design to estimate the upper bound of the derivative of Lyapunov functional, some useful terms were ignored. For example, the derivative of $\int_{-\tau_0}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\hat{Z}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s\mathrm{d}\theta$ and $\int_{-\tau(t)}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\hat{Z}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s\mathrm{d}\theta$ were often estimated as $\tau_0\dot{\boldsymbol{\eta}}^{\mathrm{T}}(t)\hat{Z}\dot{\boldsymbol{\eta}}(t)-\int_{t-\tau(t)}^{t}\dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\hat{Z}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s$ and the term $-\int_{t-\tau_0}^{t-\tau(t)}\dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\hat{Z}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s$ was ignored. Therefore, there is room for further investigation to reduce the conservativeness of the filter design. This motivates the current research.

In this paper, some new delay-dependent conditions for H_∞ filter are proposed in terms of LMIs through constructing a new Lyapunov functional that is different from the existing ones in [9, 19] and adopting new free-weighing matrix[10, 20−22]. An example is used to compare with the existing results to demonstrate the effectiveness of the proposed method.

This paper is organized as follows. The problem formulation is presented in Section 1. In Section 2, the main results on the H_∞ filter analysis are presented based on the T-S fuzzy model. In Section 3, a numerical example is used to illustrate the effectiveness of the proposed scheme. Finally, conclusion is given in Section 4.

Notations. Throughout this paper, I denotes the identity matrix, notation $X>0$ $(X\geq 0)$, for $X\in\mathbf{R}^{n\times n}$, means that matrix X is real symmetric positive definite (positive semi-definite). If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations. The symbol "*" in a matrix $X\in\mathbf{R}^{n\times n}$ stands for the transposed elements in the symmetric positions. The family of continuous functions defined on $[-h,0]$ is denoted by $C[-h,0]$.

1 Problem formulation

Consider the following continuous-time T-S model with time-varying delay:

Plant rule i:

IF $\xi_1(t)$ is M_{i1}, $\cdots$, and $\xi_p(t)$ is M_{ip}, THEN

$$\begin{aligned}
\dot{\boldsymbol{x}}(t)&=A_i\boldsymbol{x}(t)+A_{\tau i}\boldsymbol{x}(t-\tau(t))+B_i\boldsymbol{w}(t)\\
\boldsymbol{y}(t)&=C_i\boldsymbol{x}(t)+C_{\tau i}\boldsymbol{x}(t-\tau(t))+D_i\boldsymbol{w}(t)\\
\boldsymbol{z}(t)&=E_i\boldsymbol{x}(t)+E_{\tau i}\boldsymbol{x}(t-\tau(t))\\
\boldsymbol{x}(t)&=\boldsymbol{\varphi}(t),\quad \forall t\in[-\tau_0,0]
\end{aligned}\tag{1}$$

where M_{ij} is the fuzzy set, $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the state vector, $\boldsymbol{y}(t)\in\mathbf{R}^m$ is the measurement vector, $\boldsymbol{z}(t)\in\mathbf{R}^p$ stands for the signal vector to be estimated, $\boldsymbol{w}(t)\in\mathbf{R}^q$ is the disturbance variable which belongs to $L_2[0,\infty)$; A_i, $A_{\tau i}$, B_i, C_i, $C_{\tau i}$, D_i, E_i, and $E_{\tau i}$ are some constant real matrices of appropriate dimensions, where $i=1,2,\cdots,r$ and r is the number of IF-THEN rules; $\xi_1(t),\cdots,\xi_p(t)$ are the premise variables; $\boldsymbol{\varphi}(t)$ is a vector-valued initial continuous function defined on the interval $[-\tau_0,0]$, and $\tau(t)$ is a time-varying delay satisfying the inequalities below:

$$0\leq\tau(t)\leq\tau_0,\quad \dot{\tau}(t)\leq d \tag{2}$$

where τ_0 and d are two scalars. The fuzzy system (1) is supposed to have singleton fuzzifier, product inference, and centroid defuzzifier. The final output of the fuzzy system is inferred as follows:

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$$\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \sum_{i=1}^{r} \mu_i[A_i\boldsymbol{x}(t) + A_{\tau i}\boldsymbol{x}(t-\tau(t)) + B_i\boldsymbol{w}(t)] \\
\boldsymbol{y}(t) &= \sum_{i=1}^{r} \mu_i[C_i\boldsymbol{x}(t) + C_{\tau i}\boldsymbol{x}(t-\tau(t)) + D_i\boldsymbol{w}(t)] \\
\boldsymbol{z}(t) &= \sum_{i=1}^{r} \mu_i[E_i\boldsymbol{x}(t) + E_{\tau i}\boldsymbol{x}(t-\tau(t))] \\
\boldsymbol{x}(t) &= \boldsymbol{\varphi}(t), \quad \forall t \in [-\tau_0, 0]
\end{aligned} \tag{3}$$

where $\boldsymbol{\xi}(t) = (\xi_1(t), \xi_2(t), \cdots, \xi_p(t))$, $\mu_i(\boldsymbol{\xi}(t)) = \beta_i(\boldsymbol{\xi}(t))/\sum_{j=1}^{r}\beta_j(\boldsymbol{\xi}(t))$, $\beta_i(\boldsymbol{\xi}(t)) = \prod_{i=1}^{p} M_{ij}(\boldsymbol{\xi}(t))$, and $\boldsymbol{\xi}(t)$ are the premise variables. $M_{ij}(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_{ij}. It is easy to find that:

$$\beta_i(\boldsymbol{\xi}(t)) \geq 0,\ i = 1, 2, \cdots, r, \quad \sum_{j=1}^{r} \beta_j(\boldsymbol{\xi}(t)) > 0, \quad \forall t$$

Therefore, $\mu_i(\boldsymbol{\xi}(t)) \geq 0$, for $i = 1, 2, \cdots, r$ and $\sum_{j=1}^{r}\mu_j(\boldsymbol{\xi}(t)) = 1$, $\forall t$. In this paper, we denote that $\mu_i = \mu_i(\boldsymbol{\xi}(t))$.

The i-th rule of a fuzzy H_∞ filter is given as follows:

Rule i: IF $\xi_1(t)$ is M_{i1}, $\cdots$, and $\xi_p(t)$ is M_{ip}, THEN

$$\begin{aligned}
\dot{\hat{\boldsymbol{x}}}(t) &= \sum_{i=1}^{r} \mu_i[A_{fi}\hat{\boldsymbol{x}}(t) + B_{fi}\boldsymbol{y}(t)],\ \hat{\boldsymbol{x}}(0) = \hat{\boldsymbol{x}}_0 \\
\hat{\boldsymbol{z}}(t) &= \sum_{i=1}^{r} \mu_i C_{fi}\hat{\boldsymbol{x}}(t)
\end{aligned} \tag{4}$$

where $A_{fi}, B_{fi}, C_{fi}, i = 1, 2, \cdots, r$ are the filter parameters to be designed. Thus, from (3) and (4), the filter error system can be written in the following form:

$$\begin{aligned}
\dot{\boldsymbol{\eta}}(t) &= \hat{A}(t)\boldsymbol{\eta}(t) + \hat{A}_\tau(t)\boldsymbol{\eta}(t-\tau(t)) + \hat{B}(t)\boldsymbol{w}(t) \\
\boldsymbol{e}(t) &= \boldsymbol{z}(t) - \hat{\boldsymbol{z}}(t) = \hat{E}(t)\boldsymbol{\eta}(t) + \hat{E}_\tau(t)\boldsymbol{\eta}(t-\tau(t))
\end{aligned} \tag{5}$$

where $\boldsymbol{\eta}(t) = [\boldsymbol{x}^{\rm T}(t), \hat{\boldsymbol{x}}^{\rm T}(t)]$ and $\boldsymbol{\eta}(0) = [\boldsymbol{\varphi}^{\rm T}(t), \hat{\boldsymbol{x}}_0^{\rm T}]$ for $\forall t \in [-\tau_0, 0]$,

$$\hat{A}(t) = \sum_{i,j=1}^{r} \mu_i\mu_j \begin{bmatrix} A_j & 0 \\ B_{fi}C_j & A_{fi} \end{bmatrix} = \begin{bmatrix} A(t) & 0 \\ B_f(t)C(t) & A_f(t) \end{bmatrix}$$

$$\hat{A}_\tau(t) = \sum_{i,j=1}^{r} \mu_i\mu_j \begin{bmatrix} A_{\tau j} & 0 \\ B_{fi}C_{\tau j} & 0 \end{bmatrix} = \begin{bmatrix} A_\tau(t) & 0 \\ B_f(t)C_\tau(t) & 0 \end{bmatrix}$$

$$\hat{B}(t) = \sum_{i,j=1}^{r} \mu_i\mu_j \begin{bmatrix} B_j \\ B_{fi}D_j \end{bmatrix} = \begin{bmatrix} B(t) \\ B_f(t)D(t) \end{bmatrix}$$

$$\hat{E}(t) = \sum_{i,j=1}^{r} \mu_i\mu_j \begin{bmatrix} E_j & C_{fi} \end{bmatrix} = \begin{bmatrix} E(t) & C_f(t) \end{bmatrix}$$

$$\hat{E}_\tau(t) = \sum_{i=1}^{r} \mu_i \begin{bmatrix} E_{\tau j} & 0 \end{bmatrix} = \begin{bmatrix} E_\tau(t) & 0 \end{bmatrix}$$

So far, the fuzzy H_∞ filtering problem for system (3) can be expressed as follows.

Given a prescribed level of noise attention $\gamma > 0$, find a suitable filter in the form of (4) satisfying the following requirements:

1) The filter error system (5) with $w = 0$ is asymptotically stable;

2) The following H_∞ performance is satisfied:

$$\int_0^L \|\boldsymbol{e}(t)\|^2 {\rm d}t \leq \gamma^2 \int_0^L \|\boldsymbol{w}(t)\|^2 {\rm d}t \tag{6}$$

for all $L > 0$ and $\boldsymbol{w}(t) \in L_2[0, \infty)$ under zero initial conditions. If this is the case, we say that the H_∞ filter design problem is solved.

2 Main results

Similar to [19], we construct an approved Lyapunov functional candidate to establish the sufficient conditions of the existence of fuzzy H_∞ filter. Our main results are as follows.

Lemma 1. For nonlinear system (3) with (2) and a prescribed real number $\gamma > 0$, if there exist matrices $\hat{P} > 0$, $\hat{Q} > 0$, $\hat{Z} > 0$ and matrix functions $A_f(t)$, $B_f(t)$, $C_f(t)$, $\hat{Y}(t)$, $\hat{T}(t)$, $\hat{U}(t)$ such that the matrix inequality (7) holds for a given scalar $\delta > 0$, then the filtering error system (5) is asymptotically stable and satisfies the H_∞ performance index, where

$$\begin{aligned}
\varphi_{11} &= \hat{P}\hat{A}(t) + \hat{A}^{\rm T}(t)\hat{P} + \hat{Q} + \hat{Y}(t) + \hat{Y}^{\rm T}(t) \\
\varphi_{12} &= \hat{P}\hat{A}_\tau(t) - \hat{Y}(t) + \hat{T}^{\rm T}(t) \\
\varphi_{22} &= -(1-d)\hat{Q} - \hat{T}(t) - \hat{T}^{\rm T}(t)
\end{aligned}$$

Proof. The proof is given in Appendix. □

Lemma 1 provides a sufficient condition for the existence of an asymptotically stable filter with H_∞ performance. However, there exist some coupled matrix variables in the matrix inequality. In order to decouple the variables in (7), we will use decoupling technique similar to that in [6, 9, 19]. In such a way, inequality (7) can be equivalently transformed into another form as follows:

Lemma 2. There exist matrices $\hat{P} > 0$, $\hat{Q} > 0$, $\hat{Z} > 0$ and matrix functions $A_f(t)$, $B_f(t)$, $C_f(t)$, $\hat{Y}(t)$, $\hat{T}(t)$, $\hat{U}(t)$ such that (7) holds if and only if there exist matrices $P > 0$, $F > 0$, $Q > 0$, $Z > 0$ and matrix functions $\bar{A}_f(t)$, $\bar{B}_f(t)$, $\bar{C}_f(t)$, $Y(t)$, $T(t)$, $U(t)$ such that inequality (8) holds, where

$$\begin{aligned}
(1,1) &= \phi_{11} + \phi_{11}^{\rm T} + Q + Y(t) + Y^{\rm T}(t) \\
(1,2) &= \phi_{12} - Y(t) + T^{\rm T}(t) \\
(2,2) &= -(1-d)Q - T(t) - T^{\rm T}(t)
\end{aligned}$$

$$\phi_{11} = \begin{bmatrix} PA(t) + \bar{B}_f(t)C(t) & \bar{A}_f(t) \\ FA(t) + \bar{B}_f(t)C(t) & \bar{A}_f(t) \end{bmatrix}$$

$$\phi_{12} = \begin{bmatrix} PA_\tau(t) + \bar{B}_f(t)C_\tau(t) & 0 \\ FA_\tau(t) + \bar{B}_f(t)C_\tau(t) & 0 \end{bmatrix}$$

$$\phi_{13} = \begin{bmatrix} PB(t) + \bar{B}_f(t)D(t) \\ FB(t) + \bar{B}_f(t)D(t) \end{bmatrix},\ \phi_{16} = \begin{bmatrix} E^{\rm T}(t) \\ -\bar{C}_f^{\rm T}(t) \end{bmatrix}$$

$$\phi_{26} = \begin{bmatrix} E_\tau^{\rm T}(t) \\ 0 \end{bmatrix},\ \phi_{55} = \begin{bmatrix} P & F \\ F & F \end{bmatrix}$$

$$\Theta(t)=\begin{bmatrix}\varphi_{11} & \varphi_{12} & \hat{P}\hat{B}(t)+\hat{U}^{\mathrm{T}}(t) & \sqrt{\tau_0}\hat{Y}(t) & \sqrt{\tau_0}\hat{A}^{\mathrm{T}}(t)\hat{P} & \hat{E}^{\mathrm{T}}(t)\\ * & \varphi_{22} & -\hat{U}^{\mathrm{T}}(t) & \sqrt{\tau_0}\hat{T}(t) & \sqrt{\tau_0}\hat{A}_{\tau}^{\mathrm{T}}(t)\hat{P} & \hat{E}_{\tau}^{\mathrm{T}}(t)\\ * & * & -\gamma^2 I & \sqrt{\tau_0}\hat{U}(t) & \sqrt{\tau_0}\hat{B}^{\mathrm{T}}(t)\hat{P} & 0\\ * & * & * & -k\hat{Z} & 0 & 0\\ * & * & * & * & \dfrac{-2\delta\hat{P}+\delta^2\hat{Z}}{k} & 0\\ * & * & * & * & * & -I\end{bmatrix}<0 \tag{7}$$

$$\Xi(t)=\begin{bmatrix}(1,1) & (1,2) & \phi_{13}+\hat{U}^{\mathrm{T}}(t) & \sqrt{\tau_0}\hat{Y}(t) & \sqrt{\tau_0}\phi_{11}^{\mathrm{T}} & \phi_{16}\\ * & (2,2) & -\hat{U}^{\mathrm{T}}(t) & \sqrt{\tau_0}\hat{T}(t) & \sqrt{\tau_0}\phi_{12}^{\mathrm{T}} & \phi_{26}\\ * & * & -\gamma^2 I & \sqrt{\tau_0}\hat{U}(t) & \sqrt{\tau_0}\phi_{13}^{\mathrm{T}} & 0\\ * & * & * & -kZ & 0 & 0\\ * & * & * & * & \dfrac{-2\delta\phi_{55}+\delta^2 Z}{k} & 0\\ * & * & * & * & * & -I\end{bmatrix}<0 \tag{8}$$

$$\Xi_{ij}=\begin{bmatrix}(1,1) & (1,2) & \phi_{13}+U_i^{\mathrm{T}} & \sqrt{\tau_0}Y_i & \sqrt{\tau_0}\phi_{11}^{\mathrm{T}} & \phi_{16}\\ * & (2,2) & -U_i^{\mathrm{T}} & \sqrt{\tau_0}T_i & \sqrt{\tau_0}\phi_{12}^{\mathrm{T}} & \phi_{26}\\ * & * & -\gamma^2 I & \sqrt{\tau_0}U_i & \sqrt{\tau_0}\phi_{13}^{\mathrm{T}} & 0\\ * & * & * & -kZ & 0 & 0\\ * & * & * & * & \dfrac{-2\delta\phi_{55}+\delta^2 Z}{k} & 0\\ * & * & * & * & * & -I\end{bmatrix}<0 \tag{9}$$

Proof. (Necessity) Suppose that (8) holds for $\hat{P}>0$, $\hat{Q}>0$, $\hat{Z}>0$, and matrix functions $A_f(t)$, $B_f(t)$, $C_f(t)$, $\hat{Y}(t)$, $\hat{T}(t)$, and $\hat{U}(t)$. Partition $\hat{P}$ as

$$\hat{P}=\begin{bmatrix}P & S\\ S^{\mathrm{T}} & W\end{bmatrix} \tag{10}$$

where $P>0$, $W>0$ and S is invertible (otherwise, S can be made invertible through slight perturbation). Let

$$H=\begin{bmatrix}I & 0\\ 0 & SW^{-1}\end{bmatrix} \tag{11}$$

Pre- and post-multiplying (7) by $\mathrm{diag}\{H,H,I,H,H,I\}$ and its transpose, respectively, produces (8) with the variable changing: $F=SW^{-1}S^{\mathrm{T}}$, $Q=H\hat{Q}H^{\mathrm{T}}$, $Z=H\hat{Z}H^{\mathrm{T}}$, $U=\hat{U}H^{\mathrm{T}}$, $Y(t)=H\hat{Y}(t)H^{\mathrm{T}}$, $T(t)=H\hat{T}(t)H^{\mathrm{T}}$, $\bar{A}_f(t)=SA_f(t)W^{-1}S^{\mathrm{T}}$, $\bar{B}_f(t)=SB_f(t)$, $\bar{C}_f(t)=C_f(t)W^{-1}S^{\mathrm{T}}$. This proves necessity.

(Sufficiency) Suppose that (8) holds for $P>0$, $F>0$, $Q>0$, $Z>0$ and matrix functions $\bar{A}_f(t)$, $\bar{B}_f(t)$, $\bar{C}_f(t)$, $Y(t)$, $T(t)$, and $U(t)$. Choose two matrices $W>0$ and S invertible such that $F=SW^{-1}S^{\mathrm{T}}$. Let $\hat{P}$ and H be defined as in (10) and (11), respectively. Then, $\hat{P}>0$ due to $\phi_{55}>0$ as inferred by (8). Pre and post-multiplying (8) by $\mathrm{diag}\{H^{-1},H^{-1},I,H^{-1},H^{-1},I\}$ and its transpose yields (7) with the variable changing: $\hat{Q}=H^{-1}QH^{-\mathrm{T}}$, $\hat{Z}=H^{-1}ZH^{-\mathrm{T}}$, $\hat{U}=UH^{-\mathrm{T}}$, $\hat{Y}=H^{-1}Y(t)H^{-\mathrm{T}}$, $\hat{T}=H^{-1}T(t)H^{-\mathrm{T}}$, $A_f(t)=S^{-1}\bar{A}_f(t)S^{-\mathrm{T}}W$, $B_f(t)=S^{-1}\bar{B}_f(t)$, $C_f(t)=\bar{C}_f(t)S^{-\mathrm{T}}W$. □

Theorem 1. The H_∞ filter system (5) is asymptotically stable and satisfies the H_∞ performance index if there exist matrices $P>0$, $F>0$, $Q>0$, $Z>0$, $\bar{A}_{fi}$, $\bar{B}_{fi}$, $\bar{C}_{fi}$, Y_i, T_i, U_i, $i=1,2,\cdots,r$ such that the following LMIs hold for given scalars $k>0$ and $\delta>0$:

$$\Xi_{ij}+\Xi_{ji}<0,\quad i\le j \tag{12}$$

where Ξ_{ij} are given as in (9). The filter parameters in (4) are given by

$$A_{fi}=F^{-1}\bar{A}_{fi}, B_{fi}=F^{-1}\bar{B}_{fi}, C_{fi}=\bar{C}_{fi},\quad i=1,2,\cdots,r \tag{13}$$

where

$$(1,1)=\phi_{11}+\phi_{11}^{\mathrm{T}}+Q+Y_i+Y_i^{\mathrm{T}}$$
$$(1,2)=\phi_{12}-Y_i+T_i^{\mathrm{T}}$$
$$(2,2)=-(1-d)Q-T_i-T_i^{\mathrm{T}}$$
$$\phi_{11}=\begin{bmatrix}PA_j+\bar{B}_{fi}C_j & \bar{A}_{fi}\\ FA_j+\bar{B}_{fi}C_j & \bar{A}_{fi}\end{bmatrix}$$
$$\phi_{12}=\begin{bmatrix}PA_{\tau j}+\bar{B}_{fi}C_{\tau j} & 0\\ FA_{\tau j}+\bar{B}_{fi}C_{\tau j} & 0\end{bmatrix}$$
$$\phi_{13}=\begin{bmatrix}PB_j+\bar{B}_{fi}D_j\\ FB_j+\bar{B}_{fi}D_j\end{bmatrix},\ \phi_{16}=\begin{bmatrix}E_j^{\mathrm{T}}\\ -\bar{C}_{fi}^{\mathrm{T}}\end{bmatrix}$$
$$\phi_{26}=\begin{bmatrix}E_{\tau j}^{\mathrm{T}}\\ 0\end{bmatrix},\ \phi_{55}=\begin{bmatrix}P & F\\ F & F\end{bmatrix}$$

Proof. Set

$$\bar{A}_f(t)=\sum_{i=1}^{r}\mu_i\bar{A}_{fi},\ \bar{B}_f(t)=\sum_{i=1}^{r}\mu_i\bar{B}_{fi}$$
$$\bar{C}_f(t)=\sum_{i=1}^{r}\mu_i\bar{C}_{fi},\ Y(t)=\sum_{i=1}^{r}\mu_i Y_i$$
$$T(t)=\sum_{i=1}^{r}\mu_i T_i,\ U_i(t)=\sum_{i=1}^{r}\mu_i U_i$$

From (14), we have

$$\Xi(t)=\sum_{i=1}^{r}\mu_i^2\Xi_{ii}+\sum_{i<j=1}^{r}\mu_i\mu_j(\Xi_{ij}+\Xi_{ji})<0$$

Table 1 Minimum index γ for $d = 0.3$

Method	$\delta = 0.7$		$\delta = 1$		$\delta = 2$		$\delta = 5$		$\delta = 20$	
	[19]	Theorem 1	[19]	Theorem 1	[19]	Theorem 1	[19]	Theorem 1	[19]	Theorem 1
$\tau_0 = 0.5$	0.59	0.21	0.38	0.21	0.35	0.22	0.34	0.21	0.37	0.22
$\tau_0 = 0.6$	1.03	0.23	0.43	0.22	0.36	0.23	0.34	0.22	0.45	0.24
$\tau_0 = 0.8$	11.98	0.35	0.83	0.24	0.38	0.24	0.35	0.24	1.01	0.25
$\tau_0 = 1$	-	0.68	2.22	0.29	0.41	0.26	0.37	0.26	-	0.27
k		0.58		0.58		0.58		2.01		5.01

By virtue of Lemmas 1 and 2, the H_∞ filter design problem is solvable, and the filter matrix functions are given by $A_f(t) = S^{-1}\bar{A}_f(t)S^{-\mathrm{T}}W$, $B_f(t) = S^{-1}\bar{B}_f(t)$, $C_f(t) = \bar{C}_f(t)S^{-\mathrm{T}}W$, where matrices $W > 0$ and S are such that $F = SW^{-1}S^{\mathrm{T}}$. Or equivalently, under transformation $S^{-\mathrm{T}}W\hat{\boldsymbol{x}}(t)$, the filter matrix functions can be of the following forms:

$$
\begin{aligned}
A_f(t) &= S^{-\mathrm{T}}W(S^{-1}\bar{A}_f(t)S^{-\mathrm{T}}W)W^{-1}S^{\mathrm{T}} = F^{-1}\bar{A}_f(t)\\
B_f(t) &= S^{-\mathrm{T}}W(S^{-1}\bar{B}_f(t)) = F^{-1}\bar{B}_f(t)\\
C_f(t) &= (\bar{C}_f(t)S^{-\mathrm{T}}W)W^{-1}S^{\mathrm{T}} = \bar{C}_f(t)
\end{aligned}
$$

Hence, the filter parameters in (4) are given by (13). □

Remark 1. If $k = 1$, then our results are reduced to those that can be seen in [19]. So, by choosing suitable k, our results could overcome the conservatism of those in [19].

3 Simulation

In this section, a numerical example is used to test the effectiveness of the proposed method.

Example. Consider the following system cited from [19]:

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \sum_{i=1}^{2}\mu_i(\boldsymbol{\xi}(t))[A_i\boldsymbol{x}(t) + A_{\tau i}\boldsymbol{x}(t-\tau(t)) + B_i\boldsymbol{w}(t)]\\
\boldsymbol{y}(t) &= \sum_{i=1}^{2}\mu_i(\boldsymbol{\xi}(t))[C_i\boldsymbol{x}(t) + C_{\tau i}\boldsymbol{x}(t-\tau(t)) + D_i\boldsymbol{w}(t)]\\
\boldsymbol{z}(t) &= \sum_{i=1}^{2}\mu_i(\boldsymbol{\xi}(t))[E_i\boldsymbol{x}(t) + E_{\tau i}\boldsymbol{x}(t-\tau(t))]
\end{aligned}
$$

where

$$A_1 = \begin{bmatrix} -2.1 & 0.1\\ 1 & -2 \end{bmatrix},\ A_2 = \begin{bmatrix} -1.9 & 0\\ -0.2 & -1.1 \end{bmatrix}$$

$$A_{\tau 1} = \begin{bmatrix} -1.1 & 0.1\\ -0.8 & -0.9 \end{bmatrix},\ A_{\tau 2} = \begin{bmatrix} -0.9 & 0\\ -1.1 & -1.2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1\\ -0.2 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.3\\ 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},\ C_2 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}$$

$$C_{\tau 1} = \begin{bmatrix} -0.8 & 0.6 \end{bmatrix},\ C_{\tau 2} = \begin{bmatrix} -0.2 & 1 \end{bmatrix}$$

$$D_1 = 0.3,\ D_2 = -0.6$$

$$E_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix},\ E_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}$$

$$E_{\tau 1} = \begin{bmatrix} 0.1 & 0 \end{bmatrix},\ E_{\tau 2} = \begin{bmatrix} 0 & 0.2 \end{bmatrix}$$

According to Theorem 1, for given $(d, \tau_0, \delta, k) = (0.3, 0.5, 1, 0.58)$ solve LMI (12) and get the minimum attenuation level $\gamma = 0.21$ and a set of feasible solutions as follows:

$$F = \begin{bmatrix} 0.1991 & -0.1227\\ -0.1227 & 0.2078 \end{bmatrix}$$

$$\bar{A}_{f1} = \begin{bmatrix} -0.9880 & 0.2366\\ 0.8052 & -0.6834 \end{bmatrix}$$

$$\bar{A}_{f2} = \begin{bmatrix} -0.3791 & 0.2101\\ 0.1677 & -0.4949 \end{bmatrix}$$

$$\bar{B}_{f1} = \begin{bmatrix} -0.5804\\ 0.4180 \end{bmatrix},\ \bar{B}_{f2} = \begin{bmatrix} -0.0843\\ 0.2127 \end{bmatrix}$$

$$\bar{C}_{f1} = \begin{bmatrix} -0.9962 & 0.5098 \end{bmatrix}$$

$$\bar{C}_{f2} = \begin{bmatrix} 0.1168 & -0.2795 \end{bmatrix}$$

Furthermore, the H_∞ filter parameter matrices are computed from (13) as

$$A_{f1} = \begin{bmatrix} -4.0462 & -1.3164\\ 1.4864 & -4.0657 \end{bmatrix}$$

$$A_{f2} = \begin{bmatrix} -2.2104 & -0.6467\\ -0.4976 & -2.7630 \end{bmatrix}$$

$$B_{f1} = \begin{bmatrix} -2.6327\\ 0.4576 \end{bmatrix},\ B_{f2} = \begin{bmatrix} 0.3257\\ 1.2159 \end{bmatrix}$$

$$C_{f1} = \begin{bmatrix} -0.9962 & 0.5098 \end{bmatrix}$$

$$C_{f2} = \begin{bmatrix} 0.1168 & -0.2795 \end{bmatrix}$$

To compare with the recently developed fuzzy H_∞ filter, we consider different (τ_0, δ) by choosing k to find the minimum index δ for $d = 0.3$. The corresponding results are summarized in Table 1.

The fuzzy H_∞ filtering approaches proposed by us and those proposed in [19] are used to design the fuzzy H_∞ filter for this system. According to Theorem 1, we choose $d = 0.3$ and suitable k, then run the simulation for the case. In the case, we find the minimum index γ for the given different τ_0 and δ by choosing a suitable scalar k from Table 1, respectively. When $\tau_0 = 1,\ \delta = 0.7$ or $\tau_0 = 1,\ \delta = 20$, we can get the feasible solution by Theorem 1, but [19] can not. Furthermore, we can also find a smaller γ than that in Table 1 through choosing a more suitable k. It can be clearly seen that our approach produces less conservative results than the existing ones in [19] .

4 Conclusion

This paper deals with the problem of H_∞ filter design for nonlinear systems through T-S fuzzy models. By constructing a new Lyapunov functional and using free-weighting matrix approach, the H_∞ filter design scheme is proposed. A numerical example is used to illustrate the design procedure and the effectiveness of the proposed method. And the approach of H_∞ filter design in this paper can be also used in other H_∞ filtering fields.

Appendix The Proof of Lemma 1

In this paper, we choose the Lyapunov function candidate different from that in [9, 19]:

$$V(\boldsymbol{\eta}(t)) = \boldsymbol{\eta}^{\mathrm{T}}(t)\hat{P}\boldsymbol{\eta}(t) + \int_{t-\tau(t)}^{t} \boldsymbol{\eta}^{\mathrm{T}}(s)\hat{Q}\boldsymbol{\eta}(s)\mathrm{d}s + k\int_{-\tau_0}^{0}\int_{t+\theta}^{t} \dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\hat{Z}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s\mathrm{d}\theta \tag{14}$$

where $\hat{P} > 0$, $\hat{Q} > 0$, $\hat{Z} > 0$, $k > 0$. As in the proof of [16−17], by the Leibniz-Newton formula, one has

$$\boldsymbol{\eta}^{\mathrm{T}}(t-\tau(t)) = \boldsymbol{\eta}(t) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{\eta}}(s)\mathrm{d}s$$

Thus, for appropriately dimensioned matrices $\hat{Y}(t)$, $\hat{T}(t)$, and $\hat{U}(t)$, the following equalities are true:

$$2[\boldsymbol{\eta}^{\mathrm{T}}(t)\hat{Y}(t) + \boldsymbol{\eta}^{\mathrm{T}}(t-\tau(t))\hat{T}(t) + \boldsymbol{w}^{\mathrm{T}}(t)\hat{U}(t)]\times [\boldsymbol{\eta}(t) - \boldsymbol{\eta}(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{\eta}}(s)\mathrm{d}s] \equiv 0$$

With the above equation, differentiating (14) along the trajectories of system (5) yields

$$\begin{aligned}
&\dot{V}(\boldsymbol{\eta}(t)) = 2\boldsymbol{\eta}^{\mathrm{T}}(t)\hat{P}\dot{\boldsymbol{\eta}}(t) + \boldsymbol{\eta}^{\mathrm{T}}(t)\hat{Q}\boldsymbol{\eta}(t) - \\
&\quad (1-\dot{\tau}(t))\boldsymbol{\eta}^{\mathrm{T}}(t-\tau(t))\hat{Q}\boldsymbol{\eta}(t-\tau(t)) + \\
&\quad k\tau_0\dot{\boldsymbol{\eta}}^{\mathrm{T}}(t)\hat{Z}\dot{\boldsymbol{\eta}}(t) - k\int_{t-\tau_0}^{t} \dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\hat{Z}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s \le \\
&\quad 2\boldsymbol{\eta}^{\mathrm{T}}(t)\hat{P}[\hat{A}(t)\boldsymbol{\eta}(t) + \hat{A}_\tau(t)\boldsymbol{\eta}(t-\tau(t)) + \hat{B}(t)\boldsymbol{w}(t)] + \\
&\quad \boldsymbol{\eta}^{\mathrm{T}}(t)\hat{Q}\boldsymbol{\eta}(t) - (1-d)\boldsymbol{\eta}^{\mathrm{T}}(t-\tau(t))\hat{Q}\boldsymbol{\eta}(t-\tau(t)) + \\
&\quad k\tau_0\dot{\boldsymbol{\eta}}^{\mathrm{T}}(t)\hat{Z}\dot{\boldsymbol{\eta}}(t) - k\int_{t-\tau(t)}^{t} \dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)\hat{Z}\dot{\boldsymbol{\eta}}(s)\mathrm{d}s + \\
&\quad 2[\boldsymbol{\eta}^{\mathrm{T}}(t)\hat{Y}(t) + \boldsymbol{\eta}^{\mathrm{T}}(t-\tau(t))\hat{T}(t) + \boldsymbol{w}^{\mathrm{T}}(t)\hat{U}(t)]\times \\
&\quad [\boldsymbol{\eta}(t) - \boldsymbol{\eta}(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{\eta}}(s)\mathrm{d}s]
\end{aligned}$$

Furthermore, a straightforward computation gives

$$\begin{aligned}
&\dot{V}(\boldsymbol{\eta}(t)) - \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) - \gamma^2\boldsymbol{w}^{\mathrm{T}}(t)\boldsymbol{w}(t) \le \\
&\quad \boldsymbol{\zeta}^{\mathrm{T}}(t)[\Omega(t) + \tau_0 M(t)(k\hat{Z})^{-1}M^{\mathrm{T}}(t)]\boldsymbol{\zeta}(t) - \\
&\quad \int_{t-\tau(t)}^{t} [\boldsymbol{\zeta}^{\mathrm{T}}(t)M(t) + \dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)(k\hat{Z})](k\hat{Z})^{-1}\times \\
&\quad [M(t)\boldsymbol{\zeta}^{\mathrm{T}}(t) + (k\hat{Z})\dot{\boldsymbol{\eta}}^{\mathrm{T}}(s)]\mathrm{d}s
\end{aligned} \tag{15}$$

where $\tau > 0$ and

$$\begin{aligned}
&\boldsymbol{\zeta}^{\mathrm{T}}(t) = [\boldsymbol{\eta}^{\mathrm{T}}(t), \boldsymbol{\eta}^{\mathrm{T}}(t-\tau(t)), \boldsymbol{w}^{\mathrm{T}}(t)] \\
&M^{\mathrm{T}}(t) = [\hat{Y}(t), \hat{T}(t), \hat{U}(t)] \\
&\theta_{11} = \hat{P}\hat{A}(t) + \hat{A}^{\mathrm{T}}(t)\hat{P} + \hat{Q} + \hat{Y}(t) + \hat{Y}^{\mathrm{T}}(t) + \hat{E}^{\mathrm{T}}(t)\hat{E}(t) \\
&\theta_{12} = \hat{P}\hat{A}_\tau(t) - \hat{Y}(t) + \hat{T}^{\mathrm{T}}(t) + \hat{E}^{\mathrm{T}}(t)\hat{E}_\tau(t) \\
&\theta_{22} = -(1-d)\hat{Q} - \hat{T}(t) - \hat{T}^{\mathrm{T}}(t) + \hat{E}_\tau^{\mathrm{T}}(t)\hat{E}_\tau(t)
\end{aligned}$$

$$\Omega(t) = \begin{bmatrix} \theta_{11} & \theta_{12} & \hat{P}\hat{B}(t) + \hat{U}^{\mathrm{T}}(t) \\ * & \theta_{22} & -\hat{U}^{\mathrm{T}}t) \\ * & * & -\gamma^2 I \end{bmatrix} + k\tau_0[\hat{A}(t), \hat{A}_\tau(t), \hat{B}(t)]^{\mathrm{T}}\hat{Z}[\hat{A}(t), \hat{A}_\tau(t), \hat{B}(t)] \tag{16}$$

From $(\delta\hat{Z} - \hat{P})\hat{Z}^{-1}(\delta\hat{Z} - \hat{P}) \ge 0$, one has $-\hat{P}(k\hat{Z})^{-1}\hat{P} \ge (-2\delta\hat{P} + \delta^2\hat{Z})/k$ holds for any scalar $k > 0$ and $\delta > 0$. By this inequality, it can be verified that (8) implies $\tilde{\Omega}(t) < 0$, where $\tilde{\Omega}(t) < 0$ is a matrix derived from (8) by change the (5, 5) block $(-2\delta\hat{P} + \delta^2\hat{Z})/k$ to $-\hat{P}(k\hat{Z})^{-1}\hat{P}$. Obviously, by Schur complement $\tilde{\Omega}(t) < 0$ is equivalent to $\Omega(t) + \tau_0 M(t)(k\hat{Z})^{-1}M^{\mathrm{T}}(t) < 0$. Consequently, it follows from (15), $V(\boldsymbol{\eta}(t))|_{t=0} = 0$ and $V(\boldsymbol{\eta}(t))|_{t=L} \ge 0$ that

$$\int_0^L (\|\boldsymbol{e}(t)\|^2 - \gamma^2\|\boldsymbol{w}(t)\|^2)\mathrm{d}t + V(\boldsymbol{\eta}(t))|_{t=L} - V(\boldsymbol{\eta}(t))|_{t=0} \le 0 \tag{17}$$

which implies that (6) holds. Hence, H_∞ performance is verified.

In addition, it can be clearly seen from (8) that the time derivative of $V(\boldsymbol{\eta}(t))$ along the solution of (5) when $\boldsymbol{w} = \boldsymbol{0}$ satisfies $\dot{V}(\boldsymbol{\eta}(t)) < 0$. As a result, the asymptotic stability of system (5) follows immediately when $\boldsymbol{w} = \boldsymbol{0}$. □

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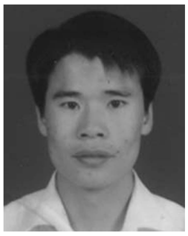

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