Convergent Stabilization Conditions of Discrete-time 2-D T-S Fuzzy Systems via Improved Homogeneous Polynomial Techniques

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Abstract This paper is concerned with the problem of stabilization of the Roesser type discrete-time 2-D T-S fuzzy system via some improved homogeneous polynomial techniques. First, a novel kind of non-quadratic control scheme is proposed to stabilize the underlying 2-D T-S fuzzy system, thus less conservative stabilization conditions are attained by applying two kinds of improved homogeneous polynomial techniques. As the degree of the homogeneous polynomially parameter-dependent matrix increases, these attained sufficient conditions may be asymptotically necessary in a convergent sense. Second, for the sake of further reducing conservatism, a new right-hand-side slack variables introducing approach which suits the homogenous polynomial setting is also proposed. Finally, a numerical example is provided to illustrate the effectiveness of the proposed methods.

Key words Roesser model, 2-D discrete-time systems, linear matrix inequality (LMI), non-quadratic stabilization, homogeneous polynomial matrix

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The search for more general and efficient control design techniques for nonlinear systems has been an important issue of great interest in control theory and its applications. In this direction, Takagi-Sugeno (T-S) fuzzy system[1] plays an important role. Based on the T-S fuzzy model, many stability conditions for the nonlinear systems have also been investigated in [2−5] and most of the existing results are derived by applying a single quadratic Lyapunov function and the usual parallel distributed compensation (PDC) control scheme, which tend to give more conservative conditions. To overcome this drawback, different Lyapunov functions have been proposed: piecewise Lyapunov functions using a partition of the state space have been investigated in [6]; another family of functions considers the use of non-quadratic Lyapunov functions[7−9]. Furthermore, relaxed stabilization conditions are also given in [10−11] by considering the information of some idiographic membership functions (MFs). More recently, all kinds of existing relaxed conditions for stability and stabilization of discrete-time T-S model are recalled in [12].

Over the past three decades, the two-dimensional (2-D) systems[13−14] have received considerable attention due to their extensive applications, such as those in image data processing and transmission, thermal process, signal filtering, etc. Recently, the 2-D system theory is also frequently used as an analysis tool to some problems, e.g., iterative learning control[15], repetitive process control[16], and PI control of discrete linear repetitive processes[17]. In [18], the problem of H_∞ control for 2-D discrete state delay systems described by the second Fornasini-Marchesini (FM) state-space model has also been investigated. Due to the application in modeling hybrid systems, H_∞ filtering for 2-D Markovian jump systems has also been investigated in [19]. Moreover, stability analysis of 2-D discrete systems described by the FM second model with state saturation was studied in [20]. However, the aforementioned results are only for linear 2-D systems. As well known, most of the actual 2-D systems are nonlinear and the above results do not work in the nonlinear case[21].

Recently, based on the T-S fuzzy modeling approach, non-quadratic stabilization conditions for a class of nonlinear Roesser type 2-D systems have been proposed by applying some new fuzzy relaxed techniques in [21]. The authors in [21] extended both the non-quadratic control scheme[7] and the right-hand-side slack variables introducing approach[2−3] to the 2-D T-S fuzzy setting. With several kinds of linear matrix inequality (LMI) relaxations for stabilizing the underlying 2-D T-S systems available in [21], it seems that the challenge now is to improve the quality of the relaxations. For instance, in terms of efficiency, i.e., the ability that the same results are obtained with less computational burden; or the easiness that a given stability condition is extended to cope with similar problems. As stated in [12, 21], there is still some conservatism to be lifted if we change "something" of either the control law or the form of introducing additional variables.

In this paper, the problem of stabilization for Roesser type discrete-time 2-D T-S fuzzy systems will be investigated via some improved homogeneous polynomial techniques. First, with the sake of further releasing the conservatism, convergent stabilizations are obtained by using both the novel kind of non-PDC scheme and some improved homogeneous polynomial techniques. Because more extra degrees of freedom are introduced, it is worth noting that the conservatism will be gradually reduced as the degrees of some control gain matrix parameters increase. Second, a new right-hand-side slack variables introducing approach which suits the homogenous polynomial setting is also proposed to accelerate the convergence rate of these attained results, i.e., attaining equivalent or less conservative stabilization conditions by using the same degrees of some control gain matrix parameters.

The rest of this paper is organized as follows. Following the introduction, system description and some preliminaries are given in Section 1. In Section 2, new non-quadratic stabilization conditions are proposed by using both the novel non-quadratic control scheme and some improved homogeneous polynomial techniques. With the purpose of further reducing the conservatism, a new kind of right-hand-side slack variables introducing techniques is also investigated which suits to the homogeneous matrix polyno-

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mial setting in Section 3. In Section 4, an example is given to demonstrate the effectiveness of the results proposed in Sections 2 and 3. Finally, some conclusions are drawn in Section 5.

For simplicity, the notations used are fairly standard. For example, $X > 0$ (or $X \geq 0$) means the matrix X is symmetric and positive (semi-positive) definite. For a square matrix E, He(E) is defined as $E + E^{\mathrm{T}}$. X^{T} denotes the transpose of X. The symbol I represents the identity matrix with appropriate dimension. "$*$" in a symmetric matrix denotes the transposed element in the symmetric position. For a matrix P, $\min(P)$ and $\max(P)$ means the smallest and largest eigenvalue of P, respectively. $\mathbf{Z}^+$ denotes the set of non-negative integers $\{0, 1, 2, \cdots\}$. $M!$ represents factorial, i.e., $M! = M(M-1)(M-2)\cdots(2)(1)$ for $M \in \mathbf{Z}^+$ with $0! = 1$. C^1 denotes the set of continuous functions with derivatives of one order.

1 Problem statement

1.1 Discrete-time 2-D T-S fuzzy model[21]

Consider a class of Roesser type discrete-time nonlinear 2-D systems described as follows:

$$\boldsymbol{x}^+(s,l) = \mathcal{Z}(\boldsymbol{x}(s,l)) + \mathcal{S}(\boldsymbol{x}(s,l))\boldsymbol{u}(s,l) \tag{1}$$

$$\boldsymbol{x}^h(0,l) = \boldsymbol{f}(l), \quad \boldsymbol{x}^v(s,0) = \boldsymbol{g}(s) \tag{2}$$

with

$$\boldsymbol{x}(s,l) = \begin{bmatrix} \boldsymbol{x}^h(s,l) \\ \boldsymbol{x}^v(s,l) \end{bmatrix}, \ \boldsymbol{x}^+(s,l) = \begin{bmatrix} \boldsymbol{x}^h(s+1,l) \\ \boldsymbol{x}^v(s,l+1) \end{bmatrix}$$

where $\boldsymbol{x}^h(\cdot)$ is the horizonal state in $\mathbf{R}^{n_1}$; $\boldsymbol{x}^v(\cdot)$ is the vertical state in $\mathbf{R}^{n_2}$; $\boldsymbol{u}(\cdot)$ is the control input in $\mathbf{R}^m$; $\mathcal{Z}(\cdot)$ and $\mathcal{S}(\cdot)$ are general nonlinear functions satisfying $\mathcal{Z}, \mathcal{S} \in C^1$. s, l are two integers in $\mathbf{Z}^+$; $\boldsymbol{f}(l)$ and $\boldsymbol{g}(s)$ are corresponding boundary conditions along two independent directions.

Extending the usual 1-D T-S fuzzy modeling method to the 2-D case, a discrete-time 2-D T-S fuzzy model described by the following rules is proposed to represent discrete-time nonlinear 2-D systems (1):

$$\begin{aligned} &\text{IF } z_1(s,l) \text{ is } M_{i1}, \ \cdots, \text{ and } z_L(s,l) \text{ is } M_{iL}, \text{ THEN,} \\ &\boldsymbol{x}^+(s,l) = A_i\boldsymbol{x}(s,l) + B_i\boldsymbol{u}(s,l), \ i = 1, \cdots, r \qquad (3) \\ &\boldsymbol{x}^h(0,l) = \boldsymbol{f}(l), \boldsymbol{x}^v(s,0) = \boldsymbol{g}(s) \end{aligned}$$

with

$$A_i = \begin{bmatrix} A_i^{11} & A_i^{12} \\ A_i^{21} & A_i^{22} \end{bmatrix}, \ B_i = \begin{bmatrix} B_i^1 \\ B_i^2 \end{bmatrix}$$

where $z_p(s,l)$, $p = 1, \cdots, L$ are the premise variables, M_{ip} is the fuzzy set, and r is the number of IF-THEN rules. $A_i^{11} \in \mathbf{R}^{n_1 \times n_1}, A_i^{12} \in \mathbf{R}^{n_1 \times n_2}, A_i^{21} \in \mathbf{R}^{n_2 \times n_1}, A_i^{22} \in \mathbf{R}^{n_2 \times n_2}, B_i^1 \in \mathbf{R}^{n_1 \times m}, B_i^2 \in \mathbf{R}^{n_2 \times m}$, respectively.

By using product of inference, singleton fuzzifier, and center-average defuzzifier, the overall discrete-time 2-D T-S fuzzy systems can be expressed as follows:

$$\boldsymbol{x}^+(s,l) = \sum_{i=1}^r h_i(z(s,l))\{A_i\boldsymbol{x}(s,l) + B_i\boldsymbol{u}(s,l)\} \tag{4}$$

$$\boldsymbol{x}^h(0,l) = \boldsymbol{f}(l), \quad \boldsymbol{x}^v(s,0) = \boldsymbol{g}(s)$$

where $h_i(z(s,l)) = \mu_i(z(s,l))/\sum_{i=1}^r \mu_i(z(s,l))$, $\mu_j(z(s,l)) = \Pi_{m=1}^L M_{mj}(z(s,l))$.

Denote $X_r = \sup\{\|\boldsymbol{x}(s,l)\| : r = s + l\}$, and we first give out the definition of asymptotic stability for system (4).

Definition 1. The discrete-time 2-D T-S fuzzy systems (4) is asymptotically stable if $\lim_{r\to\infty} X_r = 0$ with the initial and boundary conditions (2).

In this paper, for a matrix X, the following notations will be adopted for simplicity:

$$h_i = h_i(z(s,l)), \ X_z = \sum_{i=1}^r h_iX_i, \ X_z^{-1} = \left(\sum_{i=1}^r h_iX_i\right)^{-1} \tag{5}$$

1.2 Existing stabilization conditions via non-PDC scheme and PDLF

In [21], a kind of non-quadratic controller is proposed to attain less conservative stabilization conditions than the usual PDC scheme as follows:

$$\boldsymbol{u}(s,l) = F_zG_z^{-1}\boldsymbol{x}(s,l) \tag{6}$$

where $F_i = \begin{bmatrix} F_i^1 & F_i^2 \end{bmatrix}$ and $G_i = \begin{bmatrix} G_i^1 & 0 \\ 0 & G_i^2 \end{bmatrix}$.

Lemma 1[21]**.** The discrete-time 2-D T-S fuzzy system (4) with the non-quadratic controller (6) is asymptotically stable if there exist appropriately dimensional matrices $P_i > 0, F_i, G_i, R_{ii}^{mn}, R_{ij}^{mn} = (R_{ji}^{mn})^{\mathrm{T}}$, with

$$P_i = \begin{bmatrix} P_i^1 & 0 \\ 0 & P_i^2 \end{bmatrix}, P_i^1 \in \mathbf{R}^{n_1 \times n_1}, P_i^2 \in \mathbf{R}^{n_2 \times n_2}$$

$$R_{ii}^{mn} = \begin{bmatrix} R_{ii}^{mn}(1,1) & \cdots & R_{ii}^{mn}(1,4) \\ * & \ddots & \vdots \\ * & * & R_{ii}^{mn}(4,4) \end{bmatrix}$$

$$R_{ij}^{mn} = \begin{bmatrix} R_{ij}^{mn}(1,1) & R_{ij}^{mn}(1,2) & R_{ij}^{mn}(1,3) & R_{ij}^{mn}(1,4) \\ R_{ij}^{mn}(2,1) & R_{ij}^{mn}(2,2) & R_{ij}^{mn}(2,3) & R_{ij}^{mn}(2,4) \\ R_{ij}^{mn}(3,1) & R_{ij}^{mn}(3,2) & R_{ij}^{mn}(3,3) & R_{ij}^{mn}(3,4) \\ R_{ij}^{mn}(4,1) & R_{ij}^{mn}(4,2) & R_{ij}^{mn}(4,3) & R_{ij}^{mn}(4,4) \end{bmatrix}$$

such that the following LMIs hold:

$$\Upsilon_{ii}^{mn} > R_{ii}^{mn}, \quad i, m, n = 1, \cdots, r \tag{7}$$

$$\Upsilon_{ij}^{mn} + \Upsilon_{ji}^{mn} > R_{ij}^{mn} + R_{ji}^{mn}, \tag{8}$$

$$i \neq j, \ i, j, m, n = 1, \cdots, r$$

$$R^{mn} = \begin{bmatrix} R_{11}^{mn} & R_{12}^{mn} & \cdots & R_{1r}^{mn} \\ R_{21}^{mn} & R_{22}^{mn} & \cdots & R_{2r}^{mn} \\ \vdots & \vdots & \ddots & \vdots \\ R_{r1}^{mn} & R_{r2}^{mn} & \cdots & R_{rr}^{mn} \end{bmatrix} > 0, \tag{9}$$

$$m, n = 1, \cdots, r$$

where for $i, j, m, n = 1, \cdots, r$, we have

$$\Upsilon_{ij}^{mn} = \begin{bmatrix} P_i^1 & 0 & \Upsilon_{ij}^{mn}(1,3) & \Upsilon_{ij}^{mn}(1,4) \\ * & P_i^2 & \Upsilon_{ij}^{mn}(2,3) & \Upsilon_{ij}^{mn}(2,4) \\ * & * & \Upsilon_{ij}^{mn}(3,3) & 0 \\ * & * & * & \Upsilon_{ij}^{mn}(4,4) \end{bmatrix}, \text{ and}$$

$$\Upsilon_{ij}^{mn}(1,3) = (A_i^{11}G_j^1 + B_i^1F_j^1)^{\mathrm{T}}$$
$$\Upsilon_{ij}^{mn}(1,4) = (A_i^{21}G_j^1 + B_i^2F_j^1)^{\mathrm{T}}$$
$$\Upsilon_{ij}^{mn}(2,3) = (A_i^{12}G_j^2 + B_i^1F_j^2)^{\mathrm{T}}$$
$$\Upsilon_{ij}^{mn}(2,4) = (A_i^{22}G_j^2 + B_i^2F_j^2)^{\mathrm{T}}$$
$$\Upsilon_{ij}^{mn}(3,3) = G_m^1 + (G_m^1)^{\mathrm{T}} - P_m^1$$
$$\Upsilon_{ij}^{mn}(4,4) = G_n^2 + (G_n^2)^{\mathrm{T}} - P_n^2$$

Remark 1. In [21], the authors have investigated the problem of stabilization of the discrete-time 2-D T-S fuzzy model (4) by applying quadratic Lyapunov function and non-quadratic Lyapunov function, respectively. Based on an improved non-quadratic control scheme (6) for the underlying 2-D fuzzy system, less conservative stabilization condition than those in the existing literature was proposed by Lemma 1. However, it is worth noting that MFs play an important part in system (4); hence, how to make good use of the information of them in the process of controller synthesis seems meaningful and interesting[21]. Furthermore, the underlying 2-D T-S system′s information is propagated along two independent directions and this fact makes the problem of stabilization more complicated, especially for the case of non-quadratic stabilization.

1.3 Homogeneous matrix polynomial and useful lemmas

Firstly, two useful lemmas which will play an important part in the derivation of our results are given as follows.

Lemma 2[7]**.** For two symmetric matrices $P > 0, P_+ > 0$, the inequality $A^{\mathrm{T}}P_+A - P < 0$ holds, if there exists a matrix G such that $\begin{bmatrix} P & * \\ GA & G+G^{\mathrm{T}}-P_+ \end{bmatrix} > 0$.

Lemma 3 (Polya′s theorem)[22]**.** Let $F(\xi) = F(\alpha_1, \alpha_1, \cdots, \alpha_N)$ be a real homogenous polynomial which is positive $\forall \alpha \in \Delta_N$. Then, for a sufficiently large $d \in \mathbf{Z}^+$, the product $(\alpha_1+\alpha_2+\cdots+\alpha_N)^dF(\alpha)$ has all its coefficients strictly positive, where Δ_N is defined as follows:

$$\Delta_N = \left\{\alpha \in \mathbf{R}^N; \sum_{i=1}^{N} \alpha_i = 1; \alpha \geq 0\right\} \tag{10}$$

The following definitions are needed, which are consistent with those in [23].

Define $\mathcal{K}(g)$ as the set of N-tuples obtained as all possible combinations of $k_1k_2\cdots k_N$, $k_i \in \mathbf{Z}^+, i = 1, \cdots, N$ such that $k_1 + k_2 + \cdots + k_N = g$. $\alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N}, \alpha \in \triangle_N, k_i \in \mathbf{Z}^+, i = 1, 2, \cdots, N$ are the monomials, $k = k_1k_2\cdots k_N$, and $P_k \in \mathbf{R}^{n\times n}, \forall k \in \mathcal{K}(g)$ are matrix-valued coefficients. Here, by definition, $\mathcal{K}(g)$ is the set of N-tuples obtained as all possible combinations of nonnegative integers $k_i, i = 1, 2, \cdots, N$, such that $k_1 + k_2 + \cdots + k_N = g$. Since the number of vertices is equal to N, the number of elements in $\mathcal{K}(g)$ is given by

$$J(g) = \frac{(N+g-1)!}{g!(N-1)!} \tag{11}$$

To give an example, for homogeneous polynomials of degree $g = 4$ with $N = 2$ variables, the possible values of the partial degrees are $\mathcal{K}(4) = 04, 13, 22, 31, 40$, $J(4) = 5$, corresponding to the generic form $P_4(\alpha) = \alpha_2^4P_{04} + \alpha_1\alpha_2^3P_{13} + \alpha_1^2\alpha_2^2P_{22} + \alpha_1^3\alpha_2P_{31} + \alpha_1^4P_{40}$.

By definition, for N-tuples k and k', one writes $k \geq k'$ if $k_i \geq k_i'$ $(i = 1, \cdots, N)$. The usual operations of summation, $k + k'$, and subtraction, $k - k'$(whenever $k \geq k'$), are defined component-wise. Moreover, let us define some new mathematical notations as follows:

$$e_i = 0\cdots 0 \underbrace{1}_{i\text{-th}} 0\cdots 0,\ rec(e_i) = i$$
$$\pi(k) = (k_1!)(k_2!)\cdots(k_N!) \tag{12}$$

2 Convergent stabilization conditions via a novel non-quadratic control scheme

As well known, MFs play important parts in the T-S fuzzy systems. Thus, there is a chance to further reduce the conservatism if we consider information of MFs in the process of controller design.

With the purpose of further releasing the conservatism, new stabilization conditions for system (4) will be proposed by using a novel non-PDC scheme, while some improved homogeneous matrix polynomial techniques are also applied in this section. Here, the novel non-quadratic control law, named as homogeneous polynomially non-quadratic control law (HPNQCL), is designed as follows:

$$\boldsymbol{u}(s,l) = F_g(h)\left(\sum_{i=1}^{r} h_iG_i\right)^{-1}\boldsymbol{x}(s,l) = F_g(h)G_z^{-1}\boldsymbol{x}(s,l) \tag{13}$$

where $F_g(h)$ is a homogenous polynomially parameter-dependent matrix of degree g, $g \in \mathbf{Z}^+$ denoted by

$$F_g(h) = \sum_{k\in\mathcal{K}(g)} h_1^{k_1}\cdots h_r^{k_r}F_k, \quad k = k_1k_2\cdots k_r \tag{14}$$

F_k, $k \in \mathcal{K}(g)$ and G_i, $i = 1, \cdots, r$ are appropriately dimensional matrices to be determined and have the following matrix structures:

$$F_k = \begin{bmatrix} F_k^1 & F_k^2 \end{bmatrix}, G_i = \begin{bmatrix} G_i^1 & 0 \\ 0 & G_i^2 \end{bmatrix} \tag{15}$$

Based on the HPNQCL (13), the closed-loop system could be developed as follows:

$$\boldsymbol{x}^+(s,l) = (A_z + B_zF_g(h)G_z^{-1})\boldsymbol{x}(s,l) \tag{16}$$

Theorem 1. The discrete-time 2-D T-S fuzzy system (4) with the HPNQCL (13) is asymptotically stable if there exist appropriately dimensional matrices $P_i > 0, G^{ij}$, P^{ij}, $i = 1, 2, \cdots, r$, $j = 1, 2, \cdots, r$ and $F_{k'}$ $(k' \in \mathcal{K}(g))$, with

$$P_i = \begin{bmatrix} P_i^1 & 0 \\ 0 & P_i^2 \end{bmatrix},\ P_i^1 \in \mathbf{R}^{n_1\times n_1},\ P_i^2 \in \mathbf{R}^{n_2\times n_2}$$
$$G^{ij} = \begin{bmatrix} G_i^1 & 0 \\ 0 & G_j^2 \end{bmatrix},\ P^{ij} = \begin{bmatrix} P_i^1 & 0 \\ 0 & P_j^2 \end{bmatrix}$$

such that the following LMIs hold,

$$L_k^{mn}(h) = \sum_{k'\in\mathcal{K}(g), k\geq k'} \begin{bmatrix} 0 & * \\ B_{rec(k-k')}F_{k'} & 0 \end{bmatrix} + \sum_{i,j\in\{1,\cdots,r\}, k-e_i-e_j\geq 0} \frac{(g-1)!}{\pi(k-e_i-e_j)} \times \begin{bmatrix} P_i & * \\ A_iG_j & G^{mn} + G^{mn\mathrm{T}} - P^{mn} \end{bmatrix} > 0,$$
$$\forall\ k \in \mathcal{K}(g+1), m, n \in \{1, 2, \cdots, r\} \tag{17}$$

Proof. Consider a non-quadratic Lyapunov function for discrete-time 2-D T-S systems as follows:

$$V(\boldsymbol{x}(s,l)) = \boldsymbol{x}^{\mathrm{T}}(s,l)G_z^{-\mathrm{T}}P_zG_z^{-1}\boldsymbol{x}(s,l) \tag{18}$$

First, let us check the existence of G_z^{-1}. Noting that if these conditions of Theorem 1 hold true, we have inequalities with (17): $G^{mn} + (G^{mn})^{\mathrm{T}} - P^{mn} > 0$, $m, n = 1, \cdots, r$. Therefore, $\sum_{i=1}^{r} h_i(G_i + G_i^{\mathrm{T}} - P_i) > 0$, which ensures that G_z^{-1} exists.

Second, we check the non-quadratic Lyapunov function (18)′s validity. We can write

$$\boldsymbol{x}^{\mathrm{T}}(s,l)\underline{\lambda}G_z^{-\mathrm{T}}G_z^{-1}\boldsymbol{x}(s,l) \le V \le \boldsymbol{x}^{\mathrm{T}}(s,l)\overline{\lambda}G_z^{-\mathrm{T}}G_z^{-1}\boldsymbol{x}(s,l) \tag{19}$$

where $\underline{\lambda} = \min_z(P_z)$ and $\overline{\lambda} = \max_z(P_z)$.

As $(G_z^{-\mathrm{T}}G_z^{-1})^{-1} = G_zG_z^{\mathrm{T}}$ and with $\underline{\mu} = \min_z(G_zG_z^{\mathrm{T}})$ and $\overline{\mu} = \max_z(G_zG_z^{\mathrm{T}})$, (18) becomes $\underline{\lambda}\overline{\mu}^{-1}\|\boldsymbol{x}(s,l)\|^2 \le V \le \overline{\lambda}\underline{\mu}^{-1}\|\boldsymbol{x}(s,l)\|^2$ that ensures $V(\boldsymbol{x}(s,l))$ to be a candidate Lyapunov function.

Then, its variation is written as

$$\triangle V(\boldsymbol{x}(s,l)) = \boldsymbol{x}^{\mathrm{T}}(s,l)[(A_z+B_zF_g(h)G_z^{-1})^{\mathrm{T}}G_{z+}^{-\mathrm{T}}P_{z+}G_{z+}^{-1}\times (A_z+B_zF_g(h)G_z^{-1})-G_z^{-\mathrm{T}}P_zG_z^{-1}]\boldsymbol{x}(s,l) \tag{20}$$

where

$$G_{z+} = \begin{bmatrix} \sum\limits_{i=1}^{r} h_i(z(s+1,l))G_i^1 & 0 \\ 0 & \sum\limits_{j=1}^{r} h_j(z(s,l+1))G_j^2 \end{bmatrix}$$

$$P_{z+} = \begin{bmatrix} \sum\limits_{i=1}^{r} h_i(z(s+1,l))P_i^1 & 0 \\ 0 & \sum\limits_{j=1}^{r} h_j(z(s,l+1))P_j^2 \end{bmatrix}$$

Here, $h_i(z(s+1,l))$ and $h_j(z(s,l+1))$ are two different one-step ahead MFs produced by the fact that the 2-D systems′s information is propagated along two independent directions. Therefore, in the following derivation of the new non-quadratic stabilization conditions via the HPNQCL, we have to consider this difference via solving more LMIs as a tradeoff.

Then, $\triangle V(\boldsymbol{x}(s,l)) < 0$ holds if we have the following inequality:

$$(A_z + B_zF_g(h)G_z^{-1})^{\mathrm{T}}G_{z+}^{-\mathrm{T}}P_{z+}G_{z+}^{-1}(A_z + B_zF_g(h)G_z^{-1}) - G_z^{-\mathrm{T}}P_zG_z^{-1} < 0 \tag{21}$$

Pre- and post-multiplying (21) by G_z^{T} and G_z, it is easy to verify that the system (4) with the non-quadratic controller (13) is asymptotically stable if we have the following inequality:

$$(G_z^{\mathrm{T}}A_z^{\mathrm{T}}+F_g(h)^{\mathrm{T}}B_z^{\mathrm{T}})G_{z+}^{-\mathrm{T}}P_{z+}G_{z+}^{-1}(A_zG_z+B_zF_g(h))-P_z< 0 \tag{22}$$

and using Lemma 2 with $A = G_{z+}^{-1}(A_zG_z - B_zF_g(h))$, leads to

$$\begin{bmatrix} P_z & * \\ A_zG_z + B_zF_g(h) & G_{z+} + G_{z+}^{\mathrm{T}} - P_{z+} \end{bmatrix} = \sum_{m=1}^{r}\sum_{n=1}^{r} h_m(z(s+1,l))h_n(z(s,l+1))\times \left(\begin{bmatrix} P_z & * \\ A_zG_z + B_zF_g(h) & G^{mn} + G^{mn\mathrm{T}} - P^{mn} \end{bmatrix}\right) > 0 \tag{23}$$

where $G^{mn} = \begin{bmatrix} G_m^1 & 0 \\ 0 & G_n^2 \end{bmatrix}$ and $P^{mn} = \begin{bmatrix} P_m^1 & 0 \\ 0 & P_n^2 \end{bmatrix}$.

Let

$$L^{mn}(h) = \begin{bmatrix} P_z & * \\ A_zG_z + B_zF_g(h) & G^{mn} + G^{mn\mathrm{T}} - P^{mn} \end{bmatrix}$$

and (23) becomes:

$$\sum_{m=1}^{r}\sum_{n=1}^{r} h_m(z(s+1,l))h_n(z(s,l+1))L^{mn}(h) > 0 \tag{24}$$

On the other hand, we have

$$L^{mn}(h) = \sum_{k\in\mathcal{K}(g+1)} h_1^{k_1}\cdots h_r^{k_r}\times \Bigg(\sum_{k'\in\mathcal{K}(g),k\ge k'} \begin{bmatrix} 0 & * \\ B_{rec(k-k')}F_{k'} & 0 \end{bmatrix} + \sum_{i,j\in\{1,\cdots,r\},k-e_i-e_j\ge 0} \frac{(g-1)!}{\pi(k-e_i-e_j)}\times \begin{bmatrix} P_i & * \\ A_iG_j & G^{mn} + G^{mn\mathrm{T}} - P^{mn} \end{bmatrix}\Bigg) = \sum_{k\in\mathcal{K}(g+1)} h_1^{k_1}\cdots h_r^{k_r}L_k^{mn}(h) \tag{25}$$

Thus, if $L_k^{mn}(h) > 0$ for all $k \in \mathcal{K}(g+1)$ hold, $L^{mn}(h) > 0$ evidently holds. In other words, (24) holds if those LMIs (17) hold, which guarantee the asymptotic stability for the closed-loop system (16). □

Remark 2. In Theorem 1, new non-quadratic stabilization condition for fuzzy 2-D system (4) is proposed by applying the HPNQCL. It is worth noting that the HPNQCL reduces to the usual non-quadratic control scheme (6) when $g = 1$, i.e., the usual non-quadratic control scheme is a special case of the HPNQCL. Moreover, although more additional variables introduce extra degrees of freedom, the conservatism will be gradually reduced as the value of g increases, which proved that for a given polynomial structure, the sufficient condition may be asymptotically necessary in a convergent sense. This fact will also be illustrated by a numerical example in Section 4.

To further reduce the conservatism, we will exploit the generalization of the Polya′s theorem for the case of positive polynomials with matrix-valued coefficients as follows.

Theorem 2. The discrete-time 2-D T-S fuzzy system (4) with the HPNQCL (13) is asymptotically stable if there exist appropriately dimensional matrices $P_i > 0, G_{ij}, P_{ij}$, $i = 1,2,\cdots,r$, $j = 1,2,\cdots,r$, and F_k, $k \in \mathcal{K}(g)$, with

$$P_i = \begin{bmatrix} P_i^1 & 0 \\ 0 & P_i^2 \end{bmatrix},\ P_i^1 \in \mathbf{R}^{n_1\times n_1},\ P_i^2 \in \mathbf{R}^{n_2\times n_2}$$

$$G^{ij} = \begin{bmatrix} G_i^1 & 0 \\ 0 & G_j^2 \end{bmatrix},\ P^{ij} = \begin{bmatrix} P_i^1 & 0 \\ 0 & P_j^2 \end{bmatrix}$$

such that the following LMIs hold,

$$\sum_{\substack{k'\in\mathcal{K}(d),\\ k\ge k'}} \Bigg\{ \sum_{\substack{i\in\{1,\cdots,r\},\\ k_i>k_i'}} \frac{d!}{\pi(k')}\begin{bmatrix} 0 & * \\ B_iF_{k-k'-e_i} & 0 \end{bmatrix} + \sum_{\substack{i,j\in\{1,\cdots,r\}\\ k-k'-e_i-e_j\ge 0}} \frac{d!}{\pi(k')}\frac{(g-1)!}{\pi(k-k'-e_i-e_j)}\times \begin{bmatrix} P_i & * \\ A_iG_j & G^{mn} + G^{mn\mathrm{T}} - P^{mn} \end{bmatrix}\Bigg\} > 0, \tag{26}$$

$$\forall\ k \in \mathcal{K}(g+d+1), m,n \in \{1,2,\cdots,r\}, d \in \mathbf{Z}^+$$

Proof. From the proof of Theorem 1, we know that the closed-loop 2-D system is asymptotically stable if the following matrix inequality holds:

$$L^{mn}(h) > 0,\ m,n \in \{1,2,\cdots,r\} \tag{27}$$

where $L^{mn}(h)$ has been defined in (24).

On the other hand, using the fact that $h_1+\cdots+h_r=1$, we have $L^{mn}(h)=(h_1+h_2+\cdots+h_r)^d L^{mn}(h), d\in\mathbf{Z}^+$. Thus, we have that (27) is equivalent to

$$(h_1+\cdots+h_r)^d L^{mn}(h)=\sum_{k\in\mathcal{K}(g+d+1)} h_1^{k_1}h_2^{k_2}\cdots h_r^{k_r}T_k^{mn}>0,\quad \forall k\in\mathcal{K}(g+d+1) \tag{28}$$

where T_k^{mn} is equivalent to the left side of (26).

From (28), if we have $T_k^{mn}<0$ for all $k\in\mathcal{K}(g+d+1)$ and $m,n\in\{1,2,\cdots,r\}$, while d has some fixed value, (27) evidently holds. In other words, the discrete-time 2-D system (4) is asymptotically stable via the fuzzy controller (13) under the LMI-based condition (26).

Furthermore, using the Polya′s theorem, we have

$$L^{mn}(h)>0\Leftrightarrow T_k^{mn}>0,\ \exists\, d\in\mathbf{Z}^+ \tag{29}$$

It means that (26) becomes asymptotically sufficient and necessary conditions for (27) as the value of d increases. □

Remark 3. Using the Polya′s theorem, a systematic procedure for constructing a family of linear matrix inequalities conditions of precision is given. As the value of d increases, more LMIs provide less conservative sufficient conditions for stabilizing the underlying 2-D T-S fuzzy systems. Moreover, necessity in some sense will be attained through a relaxation while d tends to ∞. In other words, Theorem 2 provides a kind of convergent stabilization condition.

3 Reducing conservatism via an improved right-hand-side slack variables introducing approach

In [2], the authors firstly proposed a kind of right-hand-side slack variables introducing approach by collecting the interactions among those sub-systems in a single matrix. Several improved right-hand-side slack variables introducing approaches[3−4] have also been given out for conceiving less conservative stabilization conditions. As far as the slack variables introducing technique is concerned, Lemma 1 could be seen as an extension to the 2-D T-S fuzzy system based on the relaxed technique proposed in [3−4]. In the above section, two kinds of convergent stabilization conditions are proposed under the framework of homogenous matrix polynomial. In this case, conventional right-hand-side slack variables introducing approach[2−4] fails to work and hence an improved right-hand-side slack variables introducing approach which suits to the homogenous matrix polynomial setting is proposed as follows.

Theorem 3. The discrete-time 2-D T-S fuzzy system (4) with the HPNQCL (13) is asymptotically stable if there exist appropriately dimensional matrices $P_i>0, G_{ij}$, $i,j=1,2,\cdots,r$, F_k, $k\in\mathcal{K}(g)$, symmetric matrices $E_{iimn}^{k_1k_2\cdots k_r}$ and matrices $E_{ijmn}^{k_1k_2\cdots k_r}=\left(E_{jimn}^{k_1k_2\cdots k_r}\right)^{\rm T}, k\in\mathcal{K}(g+d-1)$, $i,j,m,n=1,2,\cdots,r$, $g,d\in\mathbf{Z}^+$ with

$$P_i=\begin{bmatrix} P_i^1 & 0\\ 0 & P_i^2\end{bmatrix},\ P_i^1\in\mathbf{R}^{n_1\times n_1},\ P_i^2\in\mathbf{R}^{n_2\times n_2}$$

$$G^{ij}=\begin{bmatrix} G_i^1 & 0\\ 0 & G_j^2\end{bmatrix},\ P^{ij}=\begin{bmatrix} P_i^1 & 0\\ 0 & P_j^2\end{bmatrix}$$

such that the following LMIs hold:

$$T_k^{mn}+\sum_{1\le i\le r}E_{iimn}^{k_1\cdots(k_i-2)\cdots k_r}+{\rm He}\left(\sum_{1\le i<j\le r}E_{ijmn}^{k_1\cdots(k_i-1)\cdots(k_j-1)\cdots k_r}\right)>0,\quad k\in\mathcal{K}(g+d+1);\ m,n\in\{1,2,\cdots,r\} \tag{30}$$

$$E_{mn}^{k_1k_2\cdots k_r}=\left[E_{ijmn}^{k_1k_2\cdots k_r}\right]_{r\times r}<0,\quad k\in\mathcal{K}(g+d-1);\ m,n\in\{1,2,\cdots,r\} \tag{31}$$

where $E_{iimn}^{k_1\cdots(k_i-2)\cdots k_r}=0$ for $k_i-2<0$, $E_{ijmn}^{k_1\cdots(k_i-1)\cdots(k_j-1)\cdots k_r}=0$ for $k_i-1<0$ or $k_j-1<0$, and T_k^{mn} are the same as in (28).

Proof. Pre- and post-multiplying (31) by $[h_1I\ h_2I\ \cdots\ h_rI]$ and its transpose, then we have

$$\sum_{1\le i\le N}h_i^2E_{iimn}^{k_1\cdots k_r}+{\rm He}\left(\sum_{1\le i<j\le r}h_ih_jE_{ijmn}^{k_1\cdots k_r}\right)<0,\quad k_1\cdots k_r\in\mathcal{K}(g+d-1) \tag{32}$$

Multiplying (30) by $h_1^{k_1}\cdots h_r^{k_r}$ $(k_1\cdots k_r=\mathcal{K}_l(g+d+1))$, thus it follows that

$$\begin{aligned}&h_1^{k_1}\cdots h_r^{k_r}\left\{T_k^{mn}+\sum_{1\le i\le r}E_{iimn}^{k_1\cdots(k_i-2)\cdots k_r}+{\rm He}\left(\sum_{1\le i<j\le r}E_{ijmn}^{k_1\cdots(k_i-1)\cdots(k_j-1)\cdots k_r}\right)\right\}=\\ &h_1^{k_1}\cdots h_r^{k_r}T_k^{mn}+h_1^{k_1}\cdots h_r^{k_r}\sum_{1\le i\le r}E_{iimn}^{k_1\cdots(k_i-2)\cdots k_r}+\\ &h_1^{k_1}\cdots h_r^{k_r}{\rm He}\left(\sum_{1\le i<j\le r}E_{ijmn}^{k_1\cdots(k_i-1)\cdots(k_j-1)\cdots k_r}\right)>0,\quad k\in\mathcal{K}(g+d+1)\end{aligned} \tag{33}$$

By summing (33) from $l=1$ to $J(g+d+1)$, we can obtain

$$\begin{aligned}&\sum_{l=1}^{J(g+d+1)}h_1^{k_1}\cdots h_r^{k_r}T_k^{mn}+\\ &\sum_{l=1}^{J(g+d+1)}\sum_{1\le i\le r}(h_1^{k_1}\cdots h_i^{k_i-2}\cdots h_r^{k_r})(h_i^2)E_{iimn}^{k_1\cdots(k_i-2)\cdots k_r}+\\ &\sum_{l=1}^{J(g+d+1)}{\rm He}\left(\sum_{1\le i<j\le r}h_1^{k_1}\cdots h_r^{k_r}E_{ijmn}^{k_1\cdots(k_i-1)\cdots(k_j-1)\cdots k_r}\right)>0,\quad k\in\mathcal{K}(g+d+1)\end{aligned} \tag{34}$$

Combining with (34) and the fact $E_{iimn}^{k_1\cdots(k_i-2)\cdots k_r}=0$ for $k_i-2<0$, $E_{ijmn}^{k_1\cdots(k_i-1)\cdots(k_j-1)\cdots k_r}=0$ for $k_i-1<0$ or

$k_j - 1 < 0$, it follows that

$$\sum_{l=1}^{J(g+d+1)} \sum_{1\le i\le r} (h_1^{k_1}\cdots h_i^{k_i-2}\cdots h_r^{k_r})(h_i^2)E_{iimn}^{k_1\cdots(k_i-2)\cdots k_r}+$$
$$\sum_{l=1}^{J(g+d+1)} \mathrm{He}\left(\sum_{1\le i<j\le r} h_1^{k_1}\cdots h_r^{k_r}E_{ijmn}^{k_1\cdots(k_i-1)\cdots(k_j-1)\cdots k_r}\right)=$$
$$\sum_{l=1}^{J(g+d-1)} (h_1^{k_1}\cdots h_i^{k_i}\cdots h_r^{k_r})\sum_{1\le i\le r}(h_i^2)E_{iimn}^{k_1\cdots(k_i)\cdots k_r}+$$
$$\sum_{l=1}^{J(g+d-1)} (h_1^{k_1}\cdots h_i^{k_i}\cdots h_j^{k_j}\cdots h_r^{k_r})\times$$
$$\mathrm{He}\left(\sum_{1\le i<j\le r} h_ih_jE_{ijmn}^{k_1\cdots(k_i)\cdots(k_j)\cdots k_r}\right)=$$
$$\sum_{l=1}^{J(g+d-1)} (h_1^{k_1}\cdots h_r^{k_r})\left\{\sum_{1\le i\le r}(h_i^2)E_{iimn}^{k_1\cdots k_r}+\right.$$
$$\left.\mathrm{He}\left(\sum_{1\le i<j\le r} h_ih_jE_{ijmn}^{k_1\cdots k_r}\right)\right\} \qquad (35)$$

Them, combining (30), (32), and (35), we have

$$\sum_{l=1}^{J(g+d+1)} h_1^{k_1}\cdots h_r^{k_r}T_k^{mn} > 0 \qquad (36)$$

which guarantees the asymptotic stability for the closed-loop system (16). □

Remark 4. Theorem 3 presents less conservative stabilization conditions by using a new right-hand-side slack variables introducing technique which suits to the homogeneous matrix polynomials setting. From (31), these interactions of some fuzzy sub-model′s combinations are collected in some single matrix. In some sense, it could be seemed as an extension of the relaxed technique firstly proposed in [2] and hence the convergence rate will be accelerated. Moreover, this fact will also be illustrated in Section 4.

4 Numerical example

Example. Consider the same discrete-time 2-D fuzzy systems as in [21]:

IF $\sin^2(x^v(k,l))$ is about 0, THEN

$$\begin{bmatrix} x^h(k+1,l)\\ x^v(k,l+1)\end{bmatrix} = A_1\begin{bmatrix} x^h(k,l)\\ x^v(k,l)\end{bmatrix} + B_1u(k,l)$$

IF $\sin^2(x^v(k,l))$ is about ∓ 1, THEN

$$\begin{bmatrix} x^h(k+1,l)\\ x^v(k,l+1)\end{bmatrix} = A_2\begin{bmatrix} x^h(k,l)\\ x^v(k,l)\end{bmatrix} + B_2u(k,l)$$

here, $A_1 = \begin{bmatrix} 1+a_1T_1 & a_1a_2T_1\\ T_2 & 1+a_2T_2\end{bmatrix}$, $B_1 = \begin{bmatrix} bT_1\\ 0\end{bmatrix}$, $A_2 = \begin{bmatrix} 1+a_1T_1 & (a_1a_2+a_0)T_1\\ T_2 & 1+a_2T_2\end{bmatrix}$, $B_2 = B_1$.

Then, the MFs of the attained discrete-time 2-D T-S fuzzy system become: $h_1(k,l) = 1 - \sin^2(x^v(k,l)), h_2(k,l) = \sin^2(x^v(k,l))$. Consider the following parameter values: $a_0 = -2$, $a_1 = -3$, $b = -1$, $T_1 = 0.5$, and $T_2 = 0.8$. We can calculate the feasible parameter intervals by evaluating the feasibility of the associated LMI problems with Lemma 1 and Theorems 1 ∼ 3 for varying values of a_2.

Table 1 shows the parameter feasible intervals of a_2 in which the fuzzy state feedback stabilizing controllers of the above system can be found by using those results provided in Lemma 1 and Theorems 1 ∼ 3 with different g or d, respectively. From Table 1, it can be seen that Theorem 3 provides the most relaxed results. Moreover, the feasible intervals attained by Theorems 1 ∼ 3 tend to be bigger as the value of g and d increase.

Table 1 Feasible parameter intervals of a_2

Methods	Feasible intervals
Usual PDC[21]	[−1.990, −0.512]
Theorem 1 of [21]	[−2.012, −0.494]
Lemma 1	[−2.492, −0.014]
Corollary 1 of [21]	[−2.292, −0.212]
Theorem 1 with $g = 2$	[−2.502, −0.013]
Theorem 1 with $g = 3$	[−2.546, −0.011]
Theorem 1 with $g = 4$	[−2.575, −0.010]
Theorem 1 with $g = 5$	[−2.592, −0.009]
Theorem 2 with $g = d = 2$	[−2.561, −0.012]
Theorem 2 with $g = d = 3$	[−2.579, −0.011]
Theorem 2 with $g = d = 4$	[−2.591, −0.009]
Theorem 2 with $g = d = 5$	[−2.596, −0.008]
Theorem 3 with $g = d = 2$	[−2.566, −0.011]
Theorem 3 with $g = d = 3$	[−2.583, −0.010]
Theorem 3 with $g = d = 4$	[−2.594, −0.008]
Theorem 3 with $g = d = 5$	[−2.610, −0.007]

Next, choosing $a_2 = -2.56$ which is feasible for Theorems 1 ∼ 3 but unfeasible for Lemma 1, and solving (30) and (31) with $d = g = 2$ by the Matlab LMI solver, the corresponding controller gain matrices are attained as follows:

$$F_{20} = \begin{bmatrix} -16.27 & 223.93\end{bmatrix},\ F_{11} = \begin{bmatrix} -10.36 & 210.26\end{bmatrix}$$
$$F_{02} = \begin{bmatrix} -7.69 & 156.83\end{bmatrix}$$
$$G_1 = \begin{bmatrix} 34.96 & 0\\ 0 & 81.47\end{bmatrix},\ G_2 = \begin{bmatrix} 19.74 & 0\\ 0 & 76.95\end{bmatrix}$$

Then, under the controller of (13), Figs. 1 and 2 show the evolution of two state $x^h(k,l)$ and $x^v(k,l)$, respectively, with the initial and boundary conditions to be

$$x^h(0,l) = 0.5,\ 0\le l\le 30,\ x^v(k,0) = 0.5,\ 0\le k\le 30$$
$$x^h(0,l) = 0.05,\ x^v(k,0) = 0.05,\ i,j > 30$$

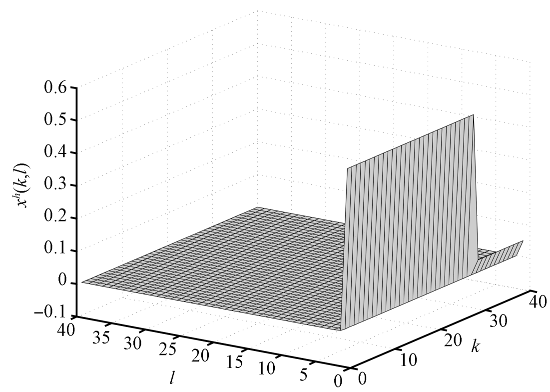

Fig. 1 Trajectory of the state $x^h(k,l)$

From Figs. 1 and 2, it is easy to see that the closed-loop 2-D T-S fuzzy system is asymptotically stable via the attained HPNQCL.

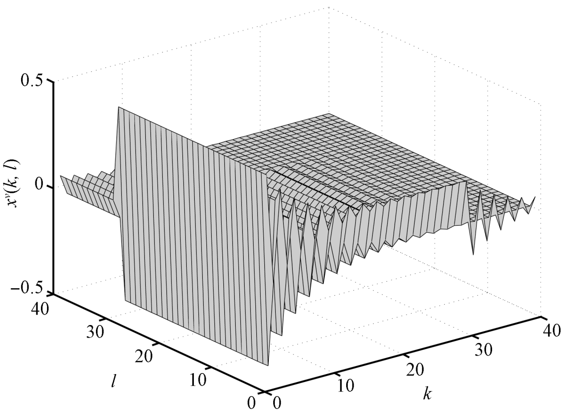

Fig. 2 Trajectory of the state $x^v(k,l)$

5 Conclusion

This paper has presented a kind of convergent stabilization method for Roesser type discrete-time 2-D T-S fuzzy system. Three kinds of relaxed stabilization conditions are proposed by applying a novel non-quadratic control scheme, homogenous polynomial techniques, and a new improved right-hand-side slack variables introducing approach. Numerical example shows the effectiveness of the proposed results.

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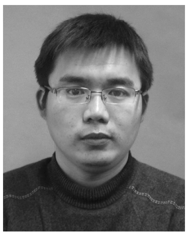

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