A New Exponential Stability Condition for Delayed Systems with Markovian Switching

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Abstract This paper discusses the problem of exponential mean-square stability analysis for delayed systems with Markovian switching. A mode-dependent Lyapunov-Krasovskii functional taking a rather general form is used. The statistical property of Markov process is used to derive the differential of the functional. Considering this and applying the free-weighting matrix technique, a new stability criterion is obtained by means of linear matrix inequalities. The effectiveness of the proposed result is numerically demonstrated by examples provided finally.

Key words Linear matrix inequality (LMI), Markov process, mean-square stability, time-delay system

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It is well known that the hysteresis is a fundamental aspect to the stability issue. Hence, the stability analysis problem for time-delay systems has been extensively investigated over the past several decades. Recent research efforts have been focused on the development of delay-dependent stability theory so as to reduce the conservatism caused by the length and varying rate of delay. On one hand, the Lyapunov-Krasovskii functional of the complete form that leads to a complicated system of partial differential equations can be computed approximately by dividing the delay segment into finer as required[1]. On the other hand, numerous works have been contributed to improve the analytical techniques for a kind of special functional via introducing some slack matrices[2−3]. It should be pointed out that some of the existing results are intrinsically equivalent to each other[4]. It is also worth noting that an equivalent approach to eliminate the time-varying delay has been developed in [5] on the basis of a convex combination condition. By using this approach, we can drop out the coarse estimation that time-varying delay is replaced by its range.

Systems with Markovian switching constitute a classical and absorbing branch of the research field of stochastic systems[6−8]. The distinction between Itô stochastic systems and systems with Markovian switching lies in the different interpretations for calculus, under which the state evolutions are understood, namely, the Itô and Riemann-Lebesgue interpretations, respectively. For the former, the Itô calculus rules, such as Itô′s formula and Itô isometry, and the statistical properties of Itô diffusions play a fundamental role[9−11]. As a consequence, the discretized computation approach to the complete functional would fail to be generalized for the Itô stochastic systems since it is based on the ordinary calculus rules, such as derivation, changing integral orders, and integrating by parts[1]. Conversely, for the latter, regardless of the internal random variations and the performance index in the statistical sense, its right-hand side satisfies the so-called Carathéodory condition and the state derivation could be well defined in the usual sense. This feature allows us to show that some method and technique developed in the deterministic framework can be available for the systems with Markovian switching[12−15].

The objective of this paper is to enhance the potential applicability of Lyapunov-Krasovskii method for the stability analysis of time-delay systems with Markovian switching. Given this, we derive a less conservative condition guaranteeing the exponential stability in the mean-square sense for the underlying system. To this end, the statistical property of Markov process is applied to compute the differential of the functional. In addition, the corresponding techniques are used to keep the generality of the functional as much as possible. Two numerical examples are provided finally to demonstrate the reduced conservatism of the proposed stability criterion.

Notation. $\{\Omega, \mathcal{F}, \mathcal{P}\}$ is a complete probability space, η_t denotes a right-continuous Markov chain defined on it and taking value in the finite state space $\Im = \{1, \cdots, s\}$. $\mathrm{E}\{\cdot\}$ represents the mathematical expectation. $\mathcal{C}([-h, 0], \mathbf{R}^n)$ stands for the Banach space constituted of all continuous functions from $[-h, 0]$ to $\mathbf{R}^n$ with the uniform norm. For a real symmetric matrix, we write $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$, respectively, for its maximum and minimum eigenvalues. "$*$" indicates the matrix block obtained by the symmetry.

1 System description and preliminaries

Consider the delayed system with Markovian switching as follows:

$$\begin{aligned} &\dot{x}(t) = A(\eta_t)x(t) + B(\eta_t)x(t - h(t)), \quad t \geq 0 \\ &x(t) = \phi(t), \quad -h \leq t \leq 0 \end{aligned} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector with the initial condition $\phi \in \mathcal{C}([-h, 0], \mathbf{R}^n)$. $h(t) \in [0, h]$ is the time-varying delay with $\dot{h}(t) \leq \mu$. η_t is generated by $\Pi = [\rho_{ij}]; i, j \in \Im$ with $\rho_{ij} \geq 0, i \neq j$ and $\rho_{ii} = -\sum_{j \neq i} \rho_{ij}$, and the transition probability is described as $\mathcal{P}(\eta_{t+\Delta} = j \mid \eta_t = i) = \begin{cases} \rho_{ij}\Delta + o(\Delta), & j \neq i \\ 1 + \rho_{ii}\Delta + o(\Delta), & j = i \end{cases}$, where $\lim_{\Delta \to 0^+} \frac{o(\Delta)}{\Delta} = 0$. Matrices $A_i = A(\eta_t = i)$ and $B_i = B(\eta_t = i)$ are of appropriate dimensions.

Definition 1. The infinitesimal generator of the solution to system (1) is defined as

$$\mathcal{L}V(x_t, \eta_t) = \lim_{\Delta \to 0^+} \frac{\mathrm{E}[V(x_{t+\Delta}, \eta_{t+\Delta}) \mid x_t, \eta_t] - V(x_t, \eta_t)}{\Delta} \tag{2}$$

Definition 2. System (1) is said to be exponentially stable in the mean square sense, if there exist constant scalars $\lambda > 0$ and $\kappa > 0$, such that

$$\mathrm{E}\left[|x(t; \eta_0, \phi)|^2\right] \leq \kappa \mathrm{E}\left[\sup_{-h \leq \theta \leq 0} |\phi(\theta)|^2\right] \mathrm{e}^{-\lambda t}, \quad t \geq 0$$

Lemma 1[1]**.** Given $h > 0$, for a vector $a(\theta)$ and a positive-definite matrix Z of appropriate dimensions, we have that

$$h \int_{-h}^{0} a^{\mathrm{T}}(\theta) Z a(\theta) \mathrm{d}\theta \geq \left[\int_{-h}^{0} a(\theta) \mathrm{d}\theta\right]^{\mathrm{T}} Z \left[\int_{-h}^{0} a(\theta) \mathrm{d}\theta\right]$$

Lemma 2[13]**.** $\lambda_{\max}\left(\sum\limits_{j \in \Im} \rho_{ij} Z_j\right) \geq 0$, provided that $Z_i > 0, i \in \Im$.

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2 Main results

Choose a Lyapunov-Krasovskii functional candidate as

$$V(x_t,\eta_t)=\sum_{k=1}^{6}V_k(x_t,\eta_t) \tag{3}$$

where

$$V_1(x_t,\eta_t)=x^{\mathrm{T}}(t)P(\eta_t)x(t) \tag{4}$$

$$V_2(x_t,\eta_t)=2x^{\mathrm{T}}(t)Q(\eta_t)\int_{t-h}^{t}x(\theta)\mathrm{d}\theta \tag{5}$$

$$V_3(x_t,\eta_t)=\int_{t-h}^{t}\mathrm{d}\theta\int_{t-h}^{t}x^{\mathrm{T}}(\theta)R(\eta_t)x(\sigma)\mathrm{d}\sigma \tag{6}$$

$$V_4(x_t,\eta_t)=\int_{t-h}^{t}x^{\mathrm{T}}(\theta)T(\eta_t)x(\theta)\mathrm{d}\theta \tag{7}$$

$$V_5(x_t,\eta_t)=\int_{t-h(t)}^{t}x^{\mathrm{T}}(\theta)S(\eta_t)x(\theta)\mathrm{d}\theta \tag{8}$$

$$V_6(x_t,\eta_t)=\int_{t-h}^{t}\mathrm{d}\theta\int_{t+\theta}^{t}\begin{bmatrix}x(\sigma)\\ \dot{x}(\sigma)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}X & Y\\ * & Z\end{bmatrix}\begin{bmatrix}x(\sigma)\\ \dot{x}(\sigma)\end{bmatrix}\mathrm{d}\sigma \tag{9}$$

For $S_i>0$ and $T_i>0$, the constructed functional (3) is positive-definite, provided that

$$\begin{bmatrix}X & Y\\ * & Z\end{bmatrix}>0 \tag{10}$$

and

$$\begin{bmatrix}P_i & Q_i\\ * & h^{-1}T_i+R_i\end{bmatrix}>0 \tag{11}$$

By Lemma 1, (11) can be observed as

$$\sum_{k=1}^{4}V_k(x_t,\eta_t=i)\geq \begin{bmatrix}x(t)\\ \int_t^{t-h}x(\theta)\mathrm{d}\theta\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}P_i & Q_i\\ * & h^{-1}T_i+R_i\end{bmatrix}\begin{bmatrix}x(t)\\ \int_t^{t-h}x(\theta)\mathrm{d}\theta\end{bmatrix} \tag{12}$$

Moreover, the action of infinitesimal generator (2) on each term of functional (3) could be expressed as

$$\mathcal{L}V_1(x_t,\eta_t=i)=2x^{\mathrm{T}}(t)P_i\dot{x}(t)+x^{\mathrm{T}}(t)\sum_{j\in\Im}\rho_{ij}P_jx(t) \tag{13}$$

$$\mathcal{L}V_2(x_t,\eta_t=i)=2\dot{x}^{\mathrm{T}}(t)Q_i\int_{t-h}^{t}x(\theta)\mathrm{d}\theta+2x^{\mathrm{T}}(t)Q_i\times [x(t)-x(t-h)]+x^{\mathrm{T}}(t)\sum_{j\in\Im}\rho_{ij}Q_j\int_{t-h}^{t}x(\theta)\mathrm{d}\theta \tag{14}$$

$$\mathcal{L}V_3(x_t,\eta_t=i)=2\left[x(t)-x(t-h)\right]^{\mathrm{T}}R_i\int_{t-h}^{t}x(\theta)\mathrm{d}\theta+\int_{t-h}^{t}\mathrm{d}\theta\int_{t-h}^{t}x^{\mathrm{T}}(\theta)\sum_{j\in\Im}\rho_{ij}R_jx(\sigma)\mathrm{d}\sigma \tag{15}$$

$$\mathcal{L}V_4(x_t,\eta_t=i)=x^{\mathrm{T}}(t)T_ix(t)-x^{\mathrm{T}}(t-h)T_ix(t-h)+\int_{t-h}^{t}x^{\mathrm{T}}(\theta)\sum_{j\in\Im}\rho_{ij}T_jx(\theta)\mathrm{d}\theta \tag{16}$$

$$\mathcal{L}V_5(x_t,\eta_t=i)=x^{\mathrm{T}}(t)S_ix(t)-(1-\dot{h}(t))x^{\mathrm{T}}(t-h(t))\times S_ix(t-h(t))+\int_{t-h(t)}^{t}x^{\mathrm{T}}(\theta)\sum_{j\in\Im}\rho_{ij}S_jx(\theta)\mathrm{d}\theta \tag{17}$$

$$\mathcal{L}V_6(x_t,\eta_t=i)=h\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}X & Y\\ * & Z\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}-\int_{t-h}^{t}\begin{bmatrix}x(\theta)\\ \dot{x}(\theta)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}X & Y\\ * & Z\end{bmatrix}\begin{bmatrix}x(\theta)\\ \dot{x}(\theta)\end{bmatrix}\mathrm{d}\theta \tag{18}$$

Clearly, (18) can be understood as well as in the deterministic sense; and then we no more need to concern with the proof of it. The proof of (13) $\sim$ (17) is given in Appendix. In the sequel, we can derive the following stability criterion.

Theorem 1. System (1) is exponentially stable in the mean square sense, if there exist $S_i>0, T_i>0$, and X,Y,Z subjected to condition (10), and P_i,Q_i,R_i subjected to condition (11), and $L_{i,q},H_{i,q}, i\in\Im, q=1,2,3$ and $M_{i,pq},N_{i,pq}, i\in\Im, p,q=1,2,3$ such that the following LMIs hold:

$$\Xi_i+h\Gamma_{i1}<0 \tag{19}$$

$$\Xi_i+h\Gamma_{i2}<0 \tag{20}$$

$$\Upsilon_{i1}=\begin{bmatrix}\Gamma_{i1} & \Omega_i & \Theta_{i1}\\ * & X-\sum\limits_{j\in\Im}\rho_{ij}(S_j+T_j) & Y\\ * & * & Z\end{bmatrix}\geq 0 \tag{21}$$

$$\Upsilon_{i2}=\begin{bmatrix}\Gamma_{i2} & \Omega_i & \Theta_{i2}\\ * & X-\sum\limits_{j\in\Im}\rho_{ij}T_j & Y\\ * & * & Z\end{bmatrix}\geq 0 \tag{22}$$

$$\sum_{j\in\Im}\rho_{ij}R_j\leq 0 \tag{23}$$

where

$$\Xi_i=\begin{bmatrix}\Xi_{i11} & \Xi_{i12} & \Xi_{i13}\\ * & \Xi_{i22} & \Xi_{i23}\\ * & * & \Xi_{i33}\end{bmatrix},\quad \Omega_i=\begin{bmatrix}\Omega_{i1}\\ \Omega_{i2}\\ \Omega_{i3}\end{bmatrix}$$

$$\Gamma_{i1}=\begin{bmatrix}M_{i11} & M_{i12} & M_{i13}\\ * & M_{i22} & M_{i23}\\ * & * & M_{i33}\end{bmatrix},\ \Gamma_{i2}=\begin{bmatrix}N_{i11} & N_{i12} & N_{i13}\\ * & N_{i22} & N_{i23}\\ * & * & N_{i33}\end{bmatrix}$$

$$\Theta_{i1}=\begin{bmatrix}L_{i1} & L_{i2} & L_{i3}\end{bmatrix}^{\mathrm{T}},\ \Theta_{i2}=\begin{bmatrix}H_{i1} & H_{i2} & H_{i3}\end{bmatrix}^{\mathrm{T}}$$

and

$$\Xi_{i11}=A_i^{\mathrm{T}}P_i+P_iA_i+L_{i1}^{\mathrm{T}}+L_{i1}+Q_i^{\mathrm{T}}+Q_i+S_i+T_i+\sum_{j\in\Im}\rho_{ij}P_j+h\left(A_i^{\mathrm{T}}ZA_i+X+YA_i+A_i^{\mathrm{T}}Y^{\mathrm{T}}\right)$$

$$\Xi_{i12}=L_{i2}-L_{i1}^{\mathrm{T}}+H_{i1}^{\mathrm{T}}+P_iB_i+h\left(A_i^{\mathrm{T}}ZB_i+YB_i\right)$$

$$\Xi_{i22}=H_{i2}^{\mathrm{T}}+H_{i2}-L_{i2}^{\mathrm{T}}-L_{i2}-(1-\mu)S_i+hB_i^{\mathrm{T}}ZB_i$$

$$\Xi_{i13}=-Q_i+L_{i3}-H_{i1}^{\mathrm{T}},\ \Xi_{i23}=H_{i3}-H_{i2}^{\mathrm{T}}-L_{i3}$$

$$\Xi_{i33}=-T_i-H_{i3}^{\mathrm{T}}-H_{i3}$$

$$\Omega_{i1}=A_i^{\mathrm{T}}Q_i+R_i+\sum_{j\in\Im}\rho_{ij}Q_j,\ \Omega_{i2}=B_i^{\mathrm{T}}Q_i,\ \Omega_{i3}=-R_i$$

Proof. Denote that

$$\psi(t)=\begin{bmatrix}x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-h(t)) & x^{\mathrm{T}}(t-h)\end{bmatrix}^{\mathrm{T}}$$

and

$$\chi_t(\theta) = \begin{bmatrix}\psi^{\mathrm{T}}(t) & x^{\mathrm{T}}(t+\theta) & \dot{x}^{\mathrm{T}}(t+\theta)\end{bmatrix}^{\mathrm{T}}$$

From (13) ~ (18), it follows that

$$\begin{aligned}
&\mathcal{L}V(x_t,\ \eta_t = i) \le \\
&\quad 2\dot{x}^{\mathrm{T}}(t)\left[P_i x(t) + Q_i\int_{t-h}^{t} x(\theta)\mathrm{d}\theta\right] + \\
&\quad x^{\mathrm{T}}(t)(S_i+T_i)x(t) - x^{\mathrm{T}}(t-h)T_i x(t-h) - \\
&\quad (1-\mu)x^{\mathrm{T}}(t-h(t))S_i x(t-h(t)) + \\
&\quad 2\left[x(t)-x(t-h)\right]^{\mathrm{T}}\left[Q_i x(t) + R_i\int_{t-h}^{t} x(\theta)\mathrm{d}\theta\right] + \\
&\quad h\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}X & Y\\ * & Z\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix} - \\
&\quad \int_{t-h}^{t}\begin{bmatrix}x(\theta)\\ \dot{x}(\theta)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}X & Y\\ * & Z\end{bmatrix}\begin{bmatrix}x(\theta)\\ \dot{x}(\theta)\end{bmatrix}\mathrm{d}\theta + \\
&\quad x^{\mathrm{T}}(t)\sum_{j\in\Im}\rho_{ij}P_j x(t) + \\
&\quad x^{\mathrm{T}}(t)\sum_{j\in\Im}\rho_{ij}Q_j\int_{t-h}^{t} x(\theta)\mathrm{d}\theta + \\
&\quad \int_{t-h}^{t}\mathrm{d}\theta\int_{t-h}^{t} x^{\mathrm{T}}(\theta)\sum_{j\in\Im}\rho_{ij}R_j x(\sigma)\mathrm{d}\sigma + \\
&\quad \int_{t-h(t)}^{t} x^{\mathrm{T}}(\theta)\sum_{j\in\Im}\rho_{ij}S_j x(\theta)\mathrm{d}\theta + \\
&\quad \int_{t-h}^{t} x^{\mathrm{T}}(\theta)\sum_{j\in\Im}\rho_{ij}T_j x(\theta)\mathrm{d}\theta + \\
&\quad 2\left[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t}\dot{x}(\theta)\mathrm{d}\theta\right]^{\mathrm{T}} \times \\
&\quad [L_{i1}x(t) + L_{i2}x(t-h(t)) + L_{i3}x(t-h)] + \\
&\quad 2\left[x(t-h(t)) - x(t-h) - \int_{t-h}^{t-h(t)}\dot{x}(\theta)\mathrm{d}\theta\right]^{\mathrm{T}} \times \\
&\quad [H_{i1}x(t) + H_{i2}x(t-h(t)) + H_{i3}x(t-h)] + \\
&\quad [h-h(t)]\psi^{\mathrm{T}}(t)\Gamma_{i2}\psi(t) - \int_{t-h}^{t-h(t)}\psi^{\mathrm{T}}(t)\Gamma_{i2}\psi(t)\mathrm{d}\theta + \\
&\quad h(t)\psi^{\mathrm{T}}(t)\Gamma_{i1}\psi(t) - \int_{t-h(t)}^{t}\psi^{\mathrm{T}}(t)\Gamma_{i1}\psi(t)\mathrm{d}\theta
\end{aligned}\tag{24}$$

Recalling (23) and rearranging the terms in (24) according to the system variables give

$$\begin{aligned}
&\mathcal{L}V(x_t,\eta_t = i) \le \\
&\quad \psi^{\mathrm{T}}(t)\left[\Xi_i + h(t)\Gamma_{i1} + (h-h(t))\Gamma_{i2}\right]\psi(t) - \\
&\quad \int_{-h(t)}^{0}\chi_t^{\mathrm{T}}(\theta)\Upsilon_{i1}\chi_t(\theta)\mathrm{d}\theta - \int_{-h}^{-h(t)}\chi_t^{\mathrm{T}}(\theta)\Upsilon_{i2}\chi_t(\theta)\mathrm{d}\theta
\end{aligned}\tag{25}$$

Furthermore, by the strictness of (19) and (20), they will remain true as Ξ_{i11} are replaced by $\Xi_{i11} + \gamma I_n$ uniformly for all $i \in \Im$ and for a properly chosen $\gamma > 0$. Thus, in view of the convexity on $h(t)$, it could be equivalently eliminated from the right-hand side of (25) by using the boundary conditions (19) and (20), respectively, according to the cases of $h(t) = h$ and $h(t) = 0$. Correspondingly, we can deduce that

$$\mathrm{E}[\mathcal{L}V(x_t,\eta_t)] \le -\gamma\mathrm{E}\left[|x(t)|^2\right],\ t \ge 0 \tag{26}$$

This, together with the fact that $\mathrm{E}\left[|x(t)|^2\right] \le \{\min_{i\in\Im}[\lambda_{\min}(P_i)]\}^{-1}\mathrm{E}[V(x_t,\eta_t)]$ and the Dynkin formula $\mathrm{E}[V(x_t,\eta_t)] = \mathrm{E}[V(x_0,\eta_0)] + \mathrm{E}[\int_0^t \mathcal{L}V(x_\sigma,\eta_\sigma)\mathrm{d}\sigma]$, yields

$$\mathrm{E}\left[|x(t)|^2\right] \le \left\{\min_{i\in\Im}[\lambda_{\min}(P_i)]\right\}^{-1} \times \left\{\mathrm{E}[V(x_0,\eta_0)] - \gamma\int_0^t \mathrm{E}\left[|x(\sigma)|^2\right]\mathrm{d}\sigma\right\},\ t \ge 0$$

Consequently, the proof can be completed by applying a Gronwall-type inequality[9]. □

Remark 1. Compared with the widely-used common functional method[11−12], the functional (3) is constructed in such a way that it keeps an individual part for each subsystem as much as possible. As a result, as shown in Appendix, we have an insight into the inherent randomness and its basic role in our method, and then the resulted stability conditions turn out to be less conservative. For example, the convex combination of the functional matrices with the coefficients ρ_{ij}, such as the imposed condition (23), could play a relaxing role, as well as the slack matrices.

Remark 2. The corresponding techniques are designed such that every derivative, with the aid of slack matrices and the boundary conditions on time-varying delay, would be conducted almost in an equivalent manner. In particular, a generalized Gronwall-type inequality gives the uniform decay rate directly. This would reduce the conservatism and complexity caused by the routine technique proving the stochastic stability[13,15], which, to the authors′ opinion, can be viewed as a Halanay-type inequality approach[10].

Example 1. Consider the delayed system with Markovian switching for the following parameters[12]:

$$A_1 = \begin{bmatrix}0.5 & -1\\ 0 & -3\end{bmatrix},\quad B_1 = \begin{bmatrix}0.5 & -0.2\\ 0.2 & 0.3\end{bmatrix}$$

$$A_2 = \begin{bmatrix}-5 & 1\\ 1 & 0.2\end{bmatrix},\quad B_2 = \begin{bmatrix}-0.3 & 0.5\\ 0.4 & -0.5\end{bmatrix},\quad \Pi = \begin{bmatrix}-7 & 7\\ 3 & -3\end{bmatrix}$$

Table 1 gives the calculated stability margin of delay for different varying rates.

Table 1 Stability margin of delay for various cases of Example 1

μ	0.0	0.1	0.5	0.9	3.0
Theorem 1	2.176	2.134	2.013	1.831	1.469
Reference [11]	1.349	1.337	1.326	1.249	0.823

Example 2. Consider the delayed system with Markovian switching for the following parameters[11]:

$$A_1 = \begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\ B_1 = \begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}$$

$$A_2 = \begin{bmatrix}-1 & 0.5\\ 0 & -1\end{bmatrix},\ B_2 = \begin{bmatrix}-1 & 0\\ 0.1 & -1\end{bmatrix},\ \Pi = \begin{bmatrix}-1 & 1\\ 2 & -2\end{bmatrix}$$

Table 2 gives the calculated stability margin of delay for various cases.

Table 2 Stability margin of delay for various cases of Example 2

μ	0.0	0.1	0.5	0.9	3.0
Theorem 1	4.845	3.895	2.153	1.673	1.658
Reference [11]	2.633	2.306	1.433	0.914	0.884

It is important to note that, by means of Lemma 2, we discuss about what kind of conditions could be imposed on the convex combination of functional matrices. For example, we cannot let $\sum_{j\in\Im}\rho_{ij}T_j\leq 0$. In this regard, Lemma 2 would serve as an implicit restriction that lets the obtained stability conditions reasonable.

3 Conclusion

A new Lyapunov-Krasovskii technique has been proposed to investigate the exponential stability of the delayed system with Markovian switching. The statistical property of Markov process has played a basic role in computing the differential of the constructed functional. Then, a delay-dependent criterion has been established by presenting some slack matrices and by equivalently eliminating time-varying delay via the convexity. Two examples have shown that the method may lead to an improvement over existing results for the stability margin of delay.

Appendix The Proof of (13) ~ (17)

Proof. One can refer [13] for the proof of (13), (16) ~ (17), while (14) ~ (15) are verified as follows.

By the statistical property of Markov process, we can have

$$\begin{aligned}
&\mathrm{E}\left[V_2(x_{t+\Delta},\eta_{t+\Delta})\mid x_t,\eta_t=i\right]=\\
&\quad 2\mathrm{E}\left[x^{\mathrm{T}}(t)Q(\eta_{t+\Delta})\int_{-h}^{0}x(t+\theta)\mathrm{d}\theta\mid x_t,\eta_t=i\right]+\\
&\quad 2\mathrm{E}\left[(x(t+\Delta)-x(t))^{\mathrm{T}}Q(\eta_{t+\Delta})\int_{-h}^{0}x(t+\Delta+\theta)\mathrm{d}\theta\mid x_t,\eta_t=i\right]+\\
&\quad 2\mathrm{E}\left[x^{\mathrm{T}}(t)Q(\eta_{t+\Delta})\left(\int_{-h}^{0}x(t+\Delta+\theta)\mathrm{d}\theta-\right.\right.\\
&\quad \left.\left.\int_{-h}^{0}x(t+\theta)\mathrm{d}\theta\right)\mid x_t,\eta_t=i\right]=\\
&\quad \sum_{j\in\Im}(\rho_{ij}\Delta+o(\Delta))\,x^{\mathrm{T}}(t)Q_j\int_{-h}^{0}x(t+\theta)\mathrm{d}\theta+V_2\left(x_t,\eta_t=i\right)+\\
&\quad 2\mathrm{E}\left[\Delta\dot{x}^{\mathrm{T}}(t)Q(\eta_{t+\Delta})\int_{-h}^{0}x(t+\Delta+\theta)\mathrm{d}\theta\mid x_t,\eta_t=i\right]+\\
&\quad 2\mathrm{E}\left[x^{\mathrm{T}}(t)Q(\eta_{t+\Delta})\left(\int_{0}^{\Delta}x(t+\theta)\mathrm{d}\theta-\right.\right.\\
&\quad \left.\left.\int_{-h}^{\Delta-h}x(t+\theta)\mathrm{d}\theta\right)\mid x_t,\eta_t=i\right]+o(\Delta)
\end{aligned}$$

and

$$\begin{aligned}
&\mathrm{E}\left[V_3(x_{t+\Delta},\eta_{t+\Delta})\mid x_t,\eta_t=i\right]=\\
&\quad \mathrm{E}\left[\int_{-h}^{0}\mathrm{d}\theta\int_{-h}^{0}x^{\mathrm{T}}(t+\theta)R(\eta_{t+\Delta})x(t+\sigma)\mathrm{d}\sigma\mid x_t,\eta_t=i\right]+\\
&\quad \mathrm{E}\left[\int_{-h}^{0}\mathrm{d}\theta\int_{-h}^{0}x^{\mathrm{T}}(t+\Delta+\theta)R(\eta_{t+\Delta})x(t+\Delta+\sigma)\mathrm{d}\sigma-\right.\\
&\quad \left.\int_{-h}^{0}\mathrm{d}\theta\int_{-h}^{0}x^{\mathrm{T}}(t+\theta)R(\eta_{t+\Delta})x(t+\Delta+\sigma)\mathrm{d}\sigma\mid x_t,\eta_t=i\right]+\\
&\quad \mathrm{E}\left[\int_{-h}^{0}\mathrm{d}\theta\int_{-h}^{0}x^{\mathrm{T}}(t+\theta)R(\eta_{t+\Delta})x(t+\Delta+\sigma)\mathrm{d}\sigma-\right.\\
&\quad \left.\int_{-h}^{0}\mathrm{d}\theta\int_{-h}^{0}x^{\mathrm{T}}(t+\theta)R(\eta_{t+\Delta})x(t+\sigma)\mathrm{d}\sigma\mid x_t,\eta_t=i\right]=\\
&\quad \sum_{j\in\Im}(\rho_{ij}\Delta+o(\Delta))\int_{-h}^{0}\mathrm{d}\theta\int_{-h}^{0}x^{\mathrm{T}}(t+\theta)R_jx(t+\sigma)\mathrm{d}\sigma+\\
&\quad \int_{-h}^{0}\mathrm{d}\theta\int_{-h}^{0}x^{\mathrm{T}}(t+\theta)R_ix(t+\sigma)\mathrm{d}\sigma+\\
&\quad \mathrm{E}\left[\left(\int_{0}^{\Delta}x^{\mathrm{T}}(t+\theta)\mathrm{d}\theta-\int_{-h}^{\Delta-h}x^{\mathrm{T}}(t+\theta)\mathrm{d}\theta\right)R(\eta_{t+\Delta})\times\right.\\
&\quad \left.\int_{-h}^{0}x(t+\Delta+\sigma)\mathrm{d}\sigma\mid x_t,\eta_t=i\right]+\\
&\quad \mathrm{E}\left[\int_{-h}^{0}x^{\mathrm{T}}(t+\theta)\mathrm{d}\theta R(\eta_{t+\Delta})\times\right.\\
&\quad \left.\left(\int_{0}^{\Delta}x(t+\sigma)\mathrm{d}\sigma-\int_{-h}^{\Delta-h}x(t+\sigma)\mathrm{d}\sigma\right)\mid x_t,\eta_t=i\right]=\\
&\quad \Delta\sum_{j\in\Im}\rho_{ij}\int_{-h}^{0}\mathrm{d}\theta\int_{-h}^{0}x^{\mathrm{T}}(t+\theta)R_jx(t+\sigma)\mathrm{d}\sigma+V_3\left(x_t,\eta_t=i\right)+\\
&\quad 2\Delta\left[(x(t)-x(t-h))^{\mathrm{T}}R_i\int_{-h}^{0}x(t+\sigma)\mathrm{d}\sigma\right]+o(\Delta)
\end{aligned}$$

Therefore, substituting the formulas above into (2) and computing the obtained expressions, respectively, gives (14) and (15) immediately. □

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