Adaptive Fuzzy Control for Unknown Nonlinear Systems with Perturbed Dead-zone Inputs

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Abstract Adaptive fuzzy control is used to control a class of unknown nonlinear systems with perturbed dead-zone inputs in this paper. A new dead-zone actuator model which contains time-varying and perturbed actuation gain is proposed. The dead-zone nonlinearity is treated as a linear-like term, a nonlinear term, and a disturbance-like term, by which the robustness of the system can be obtained by less control effort. Backstepping technique combined with nonlinearly parameterized fuzzy approximators is employed to derive the controller, which removes the restriction that fuzzy basis functions must be well-known for control design. It is proved in theory that the proposed controller guarantees the stability and desired tracking performance of the closed-loop system. A simulation example is also included to demonstrate the effectiveness of the controller.

Key words Adaptive control, fuzzy system, dead-zone, time-varying gain, perturbation, backstepping technique, nonlinear system

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Dead-zone nonlinearity is ubiquitous in many practical systems, for example, some mechanical and electrical components like valves and DC servo motors are all with dead-zone inputs. The existence of such a non-differential nonlinearity has caused many difficulties in control design since the dead-zone parameters are unknown in most cases. Many efforts have been made to deal with dead-zone nonlinearity, as it may cause severe deterioration of system performance in high precision control.

There are three main approaches to design control systems with dead-zone inputs. The first one is to construct an inverse dead-zone nonlinearity to minimize the effects of dead-zone; the second one is based on a group of fuzzy rules, which describe some raw knowledge of dead-zone characteristics; and the third one models dead-zone as a combination of a linear and a disturbance-like term, then robust control technique can be used to obtain the required control performance. The first approach is intuitive for control design and will be effective if the dead-zone parameters are all known. Following this approach, successful control was obtained for linear systems in [1] and nonlinear systems in [2], however, it is assumed that the dead-zone parameters are constants. The second approach was used to control some mechanical systems in [3−4]. It depends much on the experiences of operators or experts. When comprehensive rules about the dead-zone cannot be acquired, the approach will be rendered infeasible. The recent results[5−9] were obtained based on the third approach, which employed the upper bound of the disturbance-like term to achieve robustness of the controlled system. Though satisfactory performance was obtained, the design using the third approach is conservative to some extent.

The above mentioned results assume that the systems under control are well-known, but actually in many practical systems, the dynamics of the system are not completely known. Since Wang[10] proved that adaptive fuzzy systems are universal approximators, many control strategies have been proposed for unknown nonlinear systems based on adaptive fuzzy approximation[11−15]. These results were obtained with the restriction that the system is feedback linearizable. References [16−18] relax this restriction by employing backstepping technique to develop adaptive fuzzy tracking control. Though both single-input single-output (SISO) and multi-input multi-output (MIMO) nonlinear systems have been studied in [16−17] and [18], respectively, so far there is no result on control of unknown nonlinear systems with perturbed dead-zone inputs.

This paper proposes a control scheme for unknown nonlinear systems with dead-zone inputs. The considered systems are general: they are not required to be feedback linearizable; and the nonlinearities in the controlled plant are all unknown. Actually, actuators are not strictly linear even without dead-zone, but may be perturbed or time variant. A dead-zone model is proposed here with time-varying and perturbed actuation gain. The model is treated as a perturbed linear-like input, a nonlinear function, and a bounded disturbance-like term for control design. The width of the dead-zone is unknown and estimated explicitly by an adaptive law, so the control scheme has the capability to adapt to uncertainty of the width caused by changing conditions. Unknown functions in the design are approximated by nonlinearly parameterized adaptive fuzzy system and backstepping technique is employed to derive the controller. The proposed control scheme can guarantee the stability of the closed-loop system and satisfactory output tracking to the given reference signal.

The rest of this paper is organized as follows. The proposed adaptive fuzzy control scheme is introduced in Section 2. In Section 3, a simulation example illustrates the effectiveness of the proposed scheme. Finally, Section 4 concludes the paper.

1 Problem formulation

Consider the following nonlinear plant

$$\begin{aligned} \dot{x}_i &= f_i(\bar{\boldsymbol{x}}_i) + g_i(\bar{\boldsymbol{x}}_i)x_{i+1}, \quad 1 \le i \le n-1 \\ \dot{x}_n &= f_n(\bar{\boldsymbol{x}}_n) + g_n(\bar{\boldsymbol{x}}_n)\mathscr{D}(u) \\ y &= x_1 \end{aligned} \tag{1}$$

where $x_1,\ x_2,\ \cdots,\ x_n$ are available states of the system, $\bar{\boldsymbol{x}}_i = (x_1,\ \cdots,\ x_i)^{\mathrm{T}}$, and $\boldsymbol{x} = \bar{\boldsymbol{x}}_n = (x_1,\ \cdots,\ x_n) \in U \subseteq \mathbf{R}^n$ is the state vector, U is a compact set in $\mathbf{R}^n$. y is the system output, u_i is the designed control law, and $\mathscr{D}(u)$ is the output of the dead-zone actuator. The nonlinear functions $f_i(\bar{\boldsymbol{x}}_i) \in \mathbf{R}$ and $g_i(\bar{\boldsymbol{x}}_i) \in \mathbf{R}$ with $i = 1,\ \cdots,\ n$ are unknown but smooth.

The dead-zone characteristic considered in this paper is

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different from the existing literature because time variation and perturbation are taken into account here. The model of the dead-zone is described as follows:

$$\mathscr{D}(u) = \begin{cases} (m(t)+\phi(\boldsymbol{x}))(u-b), & u \geq b \\ 0, & -b < u < b \\ (m(t)+\phi(\boldsymbol{x}))(u+b), & u \leq -b \end{cases} \tag{2}$$

where $m(t)+\phi(\boldsymbol{x}) > 0$ with $m(t)$ being the time-varying slope and $\phi(\boldsymbol{x})$ being the perturbed term, $b > 0$ is the unknown width of the above dead-zone model. From a practical point of view, it is reasonable to make the following assumptions.

Assumption 1. There exist constants $\underline{m}$ and $\overline{m}$ which satisfy $0 < \underline{m} \leq m(t)+\phi(\boldsymbol{x}) \leq \overline{m}$.

Assumption 2. There exists a constant $\bar{b}$ such that $b \leq \bar{b}$.

Remark 1. Though $m(t)+\phi(\boldsymbol{x})$ and b are bounded by some constant values, they are not required to be known to the designer, but only used for analysis.

For the control design, we rewrite the dead-zone characteristic as

$$\mathscr{D}(u) = (m(t)+\phi(\boldsymbol{x}))u + \eta(\boldsymbol{x}, u, b) \tag{3}$$

with η (short for $\eta(\boldsymbol{x}, u, b)$) defined as

$$\eta = \begin{cases} -(m(t)+\phi(\boldsymbol{x}))b, & u \geq b \\ -(m(t)+\phi(\boldsymbol{x}))u, & -b < u < b \\ (m(t)+\phi(\boldsymbol{x}))b, & u \leq b \end{cases} \tag{4}$$

We further treat η as the sum of a hyperbolic tangent function and a bounded disturbance-like term, i.e.,

$$\eta = -(m(t)+\phi(\boldsymbol{x}))b\tanh(\frac{u}{b}) + \psi(\boldsymbol{x}) \tag{5}$$

where $\psi(\boldsymbol{x})$ satisfies

$$\begin{aligned} |\psi(\boldsymbol{x})| = |\eta + [m(t)+\phi(\boldsymbol{x})]b\tanh(\frac{u}{b})| \leq \\ [m(t)+\phi(\boldsymbol{x})]b[1-\tanh(1)] \end{aligned} \tag{6}$$

Then, from Assumptions 1 and 2, it is obvious that $\psi(\boldsymbol{x})$ is bounded.

The control objective is to design a feedback control law for u to ensure that all closed-loop signals are bounded and the plant output $y(t)$ tracks a given reference signal $y_r(t)$ as closely as possible though the nonlinearities of the system are unknown and the actuator is with time-varying perturbed dead-zone described as (2).

2 Adaptive fuzzy control design

2.1 Preliminaries

In this section, a new adaptive fuzzy control for the nonlinear system described by (1) is presented in detail. Because fuzzy logic systems with adjustable parameters are used to approximate the unknown system functions, we first show the approximation property of adaptive fuzzy system in the following lemma.

Lemma 1[10]**.** For any given real continuous function $F(\boldsymbol{x})$ on a compact set $\Omega \subseteq \mathbf{R}^n$, there exists a fuzzy logic system $Y(\boldsymbol{x}) = \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{x})$ such that $\forall \varepsilon > 0$,

$$\sup_{\boldsymbol{x}\in\Omega} \left| F(\boldsymbol{x}) - \boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{x}) \right| \leq \varepsilon \tag{7}$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \cdots, \theta_M)^{\mathrm{T}}$ is the vector of connection weights, $\boldsymbol{\xi}(\boldsymbol{x}) = (\xi_1(x), \xi_2(x), \cdots, \xi_M(x))^{\mathrm{T}}$ is the vector of fuzzy basis functions, and M is the number of fuzzy rules. Readers can refer [11] for more details.

In most existing designs, fuzzy basis functions are assumed to be known, this implies that all the fuzzy membership functions are certain for the described fuzzy sets. However, in many cases, the fuzzy membership functions are uncertain because there is no apriori knowledge available for them. In such situation, the membership function of the fuzzy set A_{ji} for x_i in the j-th rule can be defined by

$$\mu_{A_{ji}}(x_i) = \mathrm{e}^{-[\sigma_{ji}(x_i - c_{ji})]^2}$$

with σ_{ji} and c_{ji} unknown to the designer. This case is considered in our design. We choose the fuzzy basis function for j-th rule as

$$\xi_j(\bar{\boldsymbol{x}}_k, \boldsymbol{c}_j, \boldsymbol{\sigma}_j) = \prod_{i=1}^{k} \mu_{A_{ji}}(x_i) \tag{8}$$

where $\boldsymbol{c}_j = (c_{j1}, c_{j2}, \cdots, c_{jk})^{\mathrm{T}}$, $\boldsymbol{\sigma}_j = (\sigma_{j1}, \sigma_{j2}, \cdots, \sigma_{jk})^{\mathrm{T}}$ with $1 \leq k \leq n$. Denote $\boldsymbol{c}_j^i$ and $\boldsymbol{\sigma}_j^i$ as the corresponding vectors of $\boldsymbol{c}_j$ and $\boldsymbol{\sigma}_j$ in the i-th step design, and θ_j^i is the connection weight of the j-th rule in the i-th step. Suppose there are M_i rules in the i-th step design, define parameter vectors $\boldsymbol{\theta}^i = (\theta_1^i, \theta_2^i, \cdots \theta_{M_i}^i)^{\mathrm{T}}$, $\boldsymbol{c}^i = ({\boldsymbol{c}_1^i}^{\mathrm{T}}, {\boldsymbol{c}_2^i}^{\mathrm{T}}, \cdots, {\boldsymbol{c}_{M_i}^i}^{\mathrm{T}})^{\mathrm{T}}$ and $\boldsymbol{\sigma}^i = ({\boldsymbol{\sigma}_1^i}^{\mathrm{T}}, {\boldsymbol{\sigma}_2^i}^{\mathrm{T}}, \cdots, {\boldsymbol{\sigma}_{M_i}^i}^{\mathrm{T}})^{\mathrm{T}}$, where $i = 1, 2, \cdots, n$ corresponding to n step backstepping design respectively. $\boldsymbol{\theta}^{i*}$, $\boldsymbol{c}^{i*}$, and $\boldsymbol{\sigma}^{i*}$ denote the optimal parameters, which minimize the following expression:

$$\sup_{\boldsymbol{x}\in U} \left| F^i(\boldsymbol{x}) - {\boldsymbol{\theta}^i}^{\mathrm{T}}\boldsymbol{\xi}(\boldsymbol{x}, \boldsymbol{c}^i, \boldsymbol{\sigma}^i) \right|$$

It is obvious that fuzzy logic systems constructed by the fuzzy basis functions in the form of (8) are not linearly parameterized, which brings challenges to the control design.

Besides, the following lemmas and assumptions are needed for the design of the proposed controller.

Lemma 2[18]**.** Let $P(x_1, x_2, \cdots, x_n)$ be a real-value continuous function and satisfy $0 < a_m \leq P(x_1, x_2, \cdots, x_n) \leq a_M$ with a_m and a_M being two constants. Define functions $V(t)$ as follows:

$$V(t) = \int_0^{z(t)} \rho P(x_1, x_2, \cdots, x_{k-1}, \rho + \beta(t), x_{k+1}, \cdots, x_n)\mathrm{d}\rho$$

where $z(t)$ and $\beta(t)$ are real-value functions with $t \in [0, \infty)$. Then, the integral function $V(t)$ has the following properties:

1)
$$\frac{1}{2}a_m z^2(t) \leq V(t) \leq \frac{1}{2}a_M z^2(t)$$

2)
$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}V(t) = \\ &z(t)P(x_1, x_2, \cdots, x_{k-1}, z(t)+\beta(t), x_{k+1}, \cdots, x_n)\dot{z}(t) + \\ &\dot{\beta}(t)z(t)P(x_1, x_2, \cdots, x_{k-1}, z(t)+\beta(t), x_{k+1}, \cdots, x_n) + \\ &z^2(t)\int_0^1 \Big[\theta \sum_{i=1, i\neq k}^{n} \dot{x}_i(t)\frac{\partial}{\partial x_i}P(x_1, x_2, \cdots, x_{k-1}, z(t)+\beta(t), \\ &x_{k+1}, \cdots, x_n)\Big]\mathrm{d}\theta - z(t)\dot{\beta}(t)\int_0^1 P(x_1, x_2, \cdots, x_{k-1}, \\ &\theta z(t)+\beta(t), x_{k+1}, \cdots, x_n)\mathrm{d}\theta \end{aligned}$$

The proof of Lemma 2 can be found in [18].

Lemma 3. For any $\epsilon > 0$ and any $q \in \mathbf{R}$, the hyperbolic tangent function fulfills

$$0 \leq |q| - q\,\tanh\left(\frac{q}{\epsilon}\right) \leq \kappa\epsilon$$

where κ is a constant that satisfies $\kappa = \mathrm{e}^{-(\kappa+1)}$ (i.e., $\kappa \approx 0.2785$).

The proof of Lemma 3 is omitted due to space limitation.

Assumption 3. For system functions $g_i(\bar{\boldsymbol{x}}_i)$ $(1 \leq i \leq n)$, there exist positive constants g_l and g_u such that $g_{il} \leq |g_i(\bar{\boldsymbol{x}}_i)| \leq g_{iu}$.

From Assumption 3, it can be concluded that the unknown functions $g_i(\bar{\boldsymbol{x}}_i)$ are not zero. Without loss of generality, it is assumed that $g_i(\bar{\boldsymbol{x}}_i) > 0$.

Assumption 4. There exist constants $\bar{\theta}^i$, $\bar{c}^i$, and $\bar{\sigma}^i$ such that $\|\boldsymbol{\theta}^i\|_\infty \leq \bar{\theta}^i$, $\|\boldsymbol{c}^i\|_\infty \leq \bar{c}^i$, and $\|\boldsymbol{\sigma}^i\|_\infty \leq \bar{\sigma}^i$ for $i = 1, 2, \cdots, n$, where $\|\cdot\|_\infty$ denotes the infinity-norm of a vector.

2.2 Control design

Step 1. define $z_1 = x_1 - y_r$, then

$$\dot{z}_1 = f_1(\bar{x}_1) + g_1(\bar{x}_1)x_2 - \dot{y}_r \tag{9}$$

Consider a Lyapunov function candidate as

$$V_1 = \int_0^{z_1} \rho P_1(\rho + y_r)\mathrm{d}\rho + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\Gamma_{\theta^1}^{-1}\tilde{\boldsymbol{\theta}}^1 + \frac{1}{2}\tilde{\boldsymbol{c}}^{1\mathrm{T}}\Gamma_{c^1}^{-1}\tilde{\boldsymbol{c}}^1 + \frac{1}{2}\tilde{\boldsymbol{\sigma}}^{1\mathrm{T}}\Gamma_{\sigma^1}^{-1}\tilde{\boldsymbol{\sigma}}^1 + \frac{1}{2\gamma_1}\tilde{\delta}_1^2 \tag{10}$$

where $P_1(\rho + y_r) = g_1^{-1}(\rho + y_r)$, Γ_{θ^1}, Γ_{c^1}, and Γ_{σ^1} are positive definite matrices with proper dimensions, γ_1 is a positive constant, $\tilde{\boldsymbol{\theta}}^1 = \hat{\boldsymbol{\theta}}^1 - \boldsymbol{\theta}^{1*}$, $\tilde{\boldsymbol{c}}^1 = \hat{\boldsymbol{c}}^1 - \boldsymbol{c}^{1*}$, and $\tilde{\boldsymbol{\sigma}}^1 = \hat{\boldsymbol{\sigma}}^1 - \boldsymbol{\sigma}^{1*}$ with $\hat{\boldsymbol{\theta}}^1$, $\hat{\boldsymbol{c}}^1$, and $\hat{\boldsymbol{\sigma}}^1$ are the estimates of $\boldsymbol{\theta}^{1*}$, $\boldsymbol{c}^{1*}$, and $\boldsymbol{\sigma}^{1*}$, respectively; $\tilde{\delta}_1 = \hat{\delta}_1 - \delta_1^*$ with δ_1^* defined later, $\hat{\delta}_1$ is the estimate of δ_1^*.

From Lemma 2, the derivative of V_1 is

$$\begin{aligned}\dot{V}_1 = {} & z_1 g_1^{-1}\dot{z}_1 + \dot{y}_r z_1 g_1^{-1} - z_1\dot{y}_r\int_0^1 P_1(\vartheta z_1 + y_r)\mathrm{d}\vartheta + \\ & \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\Gamma_{\theta^1}^{-1}\dot{\hat{\boldsymbol{\theta}}}^1 + \tilde{\boldsymbol{c}}^{1\mathrm{T}}\Gamma_{c^1}^{-1}\dot{\hat{\boldsymbol{c}}}^1 + \tilde{\boldsymbol{\sigma}}^{1\mathrm{T}}\Gamma_{\sigma^1}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^1 + \frac{1}{\gamma_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 = \\ & z_1(x_2 + \Delta f_1) + \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\Gamma_{\theta^1}^{-1}\dot{\hat{\boldsymbol{\theta}}}^1 + \tilde{\boldsymbol{c}}^{1\mathrm{T}}\Gamma_{c^1}^{-1}\dot{\hat{\boldsymbol{c}}}^1 + \\ & \tilde{\boldsymbol{\sigma}}^{1\mathrm{T}}\Gamma_{\sigma^1}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^1 + \frac{1}{\gamma_1}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{11}$$

where $\Delta f_1 = g_1^{-1}(x_1)f_1(x_1) - \dot{y}_r\int_0^1 P_1(\vartheta z_1 + y_r)\mathrm{d}\vartheta$. According to Lemma 1, for a given ε_1 there exists a fuzzy logic system $\boldsymbol{\theta}^{1*\mathrm{T}}\xi(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})$ such that

$$\begin{aligned}\Delta f_1 = {} & \boldsymbol{\theta}^{1*\mathrm{T}}\boldsymbol{\xi}(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*}) + \varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*}) = \\ & \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\boldsymbol{\xi}}^1 - (\hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\theta}^{1*\mathrm{T}}\boldsymbol{\xi}^{1*}) + \varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})\end{aligned} \tag{12}$$

with $\varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})$ being the approximation error and $|\varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})| \leq \varepsilon_1$, $\hat{\boldsymbol{\xi}}^1 = \boldsymbol{\xi}(x_1, \hat{\boldsymbol{c}}^1, \hat{\boldsymbol{\sigma}}^1)$, and $\boldsymbol{\xi}^{1*} = \boldsymbol{\xi}(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*})$.

Define $\delta_1^* = \varepsilon_1 + \|\theta^{1*}\|_1$, $\hat{\xi}'_{c^1} = \frac{\partial\boldsymbol{\xi}(x_1, \boldsymbol{c}1, \boldsymbol{\sigma}^1)}{\partial\boldsymbol{c}^1} \mid (\boldsymbol{c}^1 = \hat{\boldsymbol{c}}^1, \boldsymbol{\sigma}^1 = \hat{\boldsymbol{\sigma}}^1)$ and $\hat{\xi}'_{\sigma^1} = \frac{\partial\boldsymbol{\xi}(x_1, \boldsymbol{c}^1, \boldsymbol{\sigma}^1)}{\partial\boldsymbol{\sigma}^1} \mid (\boldsymbol{c}^1 = \hat{\boldsymbol{c}}^1, \boldsymbol{\sigma}^1 = \hat{\boldsymbol{\sigma}}^1)$.

Then, by Taylor series expansion of $\boldsymbol{\xi}^{1*}$ at $(\hat{\boldsymbol{c}}^1, \hat{\boldsymbol{\sigma}}^1)$, one has

$$\begin{aligned}& \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\theta}^{1*\mathrm{T}}\boldsymbol{\xi}^{1*} = \\ & \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\boldsymbol{\xi}}^1 + \boldsymbol{\theta}^{1*\mathrm{T}}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \boldsymbol{\theta}^{1*\mathrm{T}}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 - \boldsymbol{\theta}^{1*\mathrm{T}}o(x_1, \tilde{\boldsymbol{c}}^1, \tilde{\boldsymbol{\sigma}}^1) = \\ & \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\boldsymbol{\xi}}^1 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\boldsymbol{c}^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 - \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 - \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 - \\ & \boldsymbol{\theta}^{1*\mathrm{T}}o(\cdot) = \\ & \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \\ & \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\xi}'_{c^1}\boldsymbol{c}^{1*} + \hat{\xi}'_{\sigma^1}\boldsymbol{\sigma}^{1*}) - \boldsymbol{\theta}^{1*\mathrm{T}}o(\cdot) = \\ & \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \\ & \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{c^1}\boldsymbol{c}^{1*} - \boldsymbol{\theta}^{1*\mathrm{T}}\hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 + \boldsymbol{\theta}^{1*\mathrm{T}}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\boldsymbol{\sigma}^{1*} - \\ & \boldsymbol{\theta}^{1*\mathrm{T}}\hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1 + \boldsymbol{\theta}^{1*\mathrm{T}}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \boldsymbol{\theta}^{1*\mathrm{T}}(\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\xi}^{1*} - \hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1) = \\ & \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \\ & \hat{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\xi}'_{c^1}\boldsymbol{c}^{1*} + \hat{\xi}'_{\sigma^1}\boldsymbol{\sigma}^{1*}) - \boldsymbol{\theta}^{1*\mathrm{T}}(\hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \\ & \boldsymbol{\theta}^{1*\mathrm{T}}(\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\xi}^{1*}) \leq \\ & \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \\ & \|\hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{c^1}\|_1\bar{\boldsymbol{c}}^1 + \|\hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\|_1\bar{\sigma}^1 + \|\hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1\|_1\bar{\theta}^1 + \|\boldsymbol{\theta}^{1*}\|_1\end{aligned} \tag{13}$$

where $o(\cdot) = o(x_1, \tilde{\boldsymbol{c}}^1, \tilde{\boldsymbol{\sigma}}^1)$, and $\|\hat{\boldsymbol{\xi}}^1 - \boldsymbol{\xi}^{1*}\|_\infty < 1$ is used.

According to (13), (11) can be rewritten as

$$\begin{aligned}\dot{V}_1 = {} & z_1[x_2 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\boldsymbol{\xi}}^1 + \varepsilon_1(x_1, \boldsymbol{c}^{1*}, \boldsymbol{\sigma}^{1*}) - \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \\ & \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) - \hat{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1) - \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\xi}'_{c^1}\boldsymbol{c}^{1*} + \hat{\xi}'_{\sigma^1} \times \\ & \boldsymbol{\sigma}^{1*}) + \boldsymbol{\theta}^{1*\mathrm{T}}o(x_1, \tilde{\boldsymbol{c}}^1, \tilde{\boldsymbol{\sigma}}^1)] + \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\Gamma_{\theta^1}^{-1}\dot{\hat{\boldsymbol{\theta}}}^1 + \tilde{\boldsymbol{c}}^{1\mathrm{T}}\Gamma_{c^1}^{-1}\dot{\hat{\boldsymbol{c}}}^1 + \\ & \tilde{\boldsymbol{\sigma}}^{1\mathrm{T}}\Gamma_{\sigma^1}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^1 + \tfrac{1}{\gamma_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 \leq \\ & z_1[x_2 + \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\boldsymbol{\xi}}^1 - \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) - \\ & \hat{\boldsymbol{\theta}}^{1\mathrm{T}}(\hat{\xi}'_{c^1}\tilde{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1)] + |z_1\omega_1| + |z_1\delta_1^*| + \tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\Gamma_{\theta^1}^{-1}\dot{\hat{\boldsymbol{\theta}}}^1 + \\ & \tilde{\boldsymbol{c}}^{1\mathrm{T}}\Gamma_{c^1}^{-1}\dot{\hat{\boldsymbol{c}}}^1 + \tilde{\boldsymbol{\sigma}}^{1\mathrm{T}}\Gamma_{\sigma^1}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^1 + \tfrac{1}{\gamma_1}\tilde{\delta}_1\dot{\hat{\delta}}_1\end{aligned} \tag{14}$$

where $\omega_1 = \|\hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{c^1}\|_1\bar{c}^1 + \|\hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\xi}'_{\sigma^1}\|_1\bar{\sigma}^1 + \|\hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 + \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1\|_1\bar{\theta}^1$.

Choose the virtual control in this step as

$$\alpha_1 = -q_1 z_1 - \hat{\boldsymbol{\theta}}^{1\mathrm{T}}\hat{\boldsymbol{\xi}}^1 - \omega_1\tanh\left(\frac{z_1\omega_1}{\pi_1}\right) - \hat{\delta}_1\tanh\frac{z_1\hat{\delta}_1}{\tau_1} \tag{15}$$

where q_1, π_1 and τ_1 are positive constants.

$$\begin{aligned}& \dot{\hat{\boldsymbol{\theta}}}^1 = \mathrm{Proj}[\Gamma_{\theta^1}z_1(\hat{\boldsymbol{\xi}}^1 - \hat{\xi}'_{c^1}\hat{\boldsymbol{c}}^1 - \hat{\xi}'_{\sigma^1}\hat{\boldsymbol{\sigma}}^1) - R_{\theta^1}\hat{\boldsymbol{\theta}}^1] \\ & \dot{\hat{\boldsymbol{c}}}^1 = \mathrm{Proj}[\Gamma_{c^1}z_1\hat{\xi}'^{\mathrm{T}}_{c^1}\hat{\boldsymbol{\theta}}^1 - R_{c^1}\hat{\boldsymbol{c}}^1] \\ & \dot{\hat{\boldsymbol{\sigma}}}^1 = \mathrm{Proj}[\Gamma_{\sigma^1}z_1\hat{\xi}'^{\mathrm{T}}_{\sigma^1}\hat{\boldsymbol{\theta}}^1 - R_{\sigma^1}\hat{\boldsymbol{\sigma}}^1] \\ & \dot{\hat{\delta}}_1 = \gamma_1 z_1 - r_1\hat{\delta}_1\end{aligned} \tag{16}$$

where R_{θ^1}, R_{c^1}, and R_{σ^1} are positive definite matrices with proper dimensions, r_1 is a positive real constant. $\mathrm{Proj}[\cdot]$ is the projection operator to ensure that $\|\boldsymbol{\theta}^i\|_\infty \leq \bar{\theta}^i$, $\|\boldsymbol{c}^i\|_\infty \leq \bar{c}^i$, and $\|\boldsymbol{\sigma}^i\|_\infty \leq \bar{\sigma}^i$ for $1 \leq i \leq n$. Let $z_2 = x_2 - z_1$, the following inequalities can be obtained with the help of Lemma 3.

$$\begin{aligned}\dot{V}_1 \leq {} & -q_1 z_1^2 + z_1 z_2 + |z_1\hat{\delta}_1| - z_1\hat{\delta}_1\tanh\left(\frac{z_1\delta_1}{\tau_1}\right) + |z_1\omega_1| - \\ & z_1\omega_1\tanh\left(\frac{z_1\omega_1}{\pi_1}\right) - \frac{1}{2}\tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\Gamma_{\theta^1}^{-1}R_{\theta^1}\tilde{\boldsymbol{\theta}}^1 + \frac{1}{2}\boldsymbol{\theta}^{1*\mathrm{T}}\Gamma_{\theta^1}^{-1} \times \\ & R_{\theta^1}\tilde{\boldsymbol{\theta}}^{1*} - \frac{1}{2}\tilde{\boldsymbol{c}}^{1\mathrm{T}}\Gamma_{c^1}^{-1}R_{c^1}\tilde{\boldsymbol{c}}^1 + \frac{1}{2}\boldsymbol{c}^{1*\mathrm{T}}\Gamma_{c^1}^{-1}R_{c^1}\boldsymbol{c}^{1*} - \frac{1}{2}\tilde{\boldsymbol{\sigma}}^{1\mathrm{T}} \times \\ & \Gamma_{\sigma^1}^{-1}R_{\sigma^1}\tilde{\boldsymbol{\sigma}}^1 + \frac{1}{2}\boldsymbol{\sigma}^{1*\mathrm{T}}\Gamma_{\sigma^1}^{-1}R_{\sigma^1}\boldsymbol{\sigma}^{1*} - \frac{1}{2\gamma_1}\tilde{\delta}_1^2 + \frac{1}{2\gamma_1}\delta_1^{*2} \leq\end{aligned}$$

$$
-q_1 z_1^2 - \frac{\lambda_{\theta^1}^{\min}}{2}\tilde{\boldsymbol{\theta}}^{1\mathrm{T}}\Gamma_{\theta^1}^{-1}\tilde{\boldsymbol{\theta}}^1 - \frac{\lambda_{c^1}^{\min}}{2}\tilde{\boldsymbol{c}}^{1\mathrm{T}}\Gamma_{c^1}^{-1}\tilde{\boldsymbol{c}}^1 - \frac{\lambda_{\sigma^1}^{\min}}{2}\tilde{\boldsymbol{\sigma}}^{1\mathrm{T}}\Gamma_{\sigma^1}^{-1}\tilde{\boldsymbol{\sigma}}^1 - \frac{1}{2\gamma_1}\tilde{\delta}_1^2 + z_1z_2 + \kappa(\pi_1+\tau_1) + \frac{1}{2}\boldsymbol{\theta}^{1*\mathrm{T}}\Gamma_{\theta^1}^{-1}R_{\theta^1}\tilde{\boldsymbol{\theta}}^{1*} + \frac{1}{2}\boldsymbol{c}^{1*\mathrm{T}}\Gamma_{c^1}^{-1}R_{c^1}\boldsymbol{c}^{1*} + \frac{1}{2}\boldsymbol{\sigma}^{1*\mathrm{T}}\Gamma_{\sigma^1}^{-1}R_{\sigma^1}\boldsymbol{\sigma}^{1*} + \frac{1}{2\gamma_1}\delta_1^{*2} \tag{17}
$$

where $\lambda_{\theta^1}^{\min}$, $\lambda_{c^1}^{\min}$, and $\lambda_{\sigma^1}^{\min}$ are the minimal eigenvalues of R_{θ^1}, R_{c^1}, and R_{σ^1}, respectively.

Step 2.

$$
\dot{z}_2 = f_2(\bar{\boldsymbol{x}}_2) + g_2(\bar{\boldsymbol{x}}_2)x_3 - \dot{\alpha}_1 \tag{18}
$$

Then, take a Lyapunov function candidate as

$$
V_2 = V_1 + \int_0^{z_2}\rho P_2(x_1,\rho+\alpha_1)\mathrm{d}\rho + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{2\mathrm{T}}\Gamma_{\theta^2}^{-1}\tilde{\boldsymbol{\theta}}^2 + \frac{1}{2}\tilde{\boldsymbol{c}}^{2\mathrm{T}}\Gamma_{c^2}^{-1}\tilde{\boldsymbol{c}}^2 + \frac{1}{2}\tilde{\boldsymbol{\sigma}}^{2\mathrm{T}}\Gamma_{\sigma^2}^{-1}\tilde{\boldsymbol{\sigma}}^2 + \frac{1}{2\gamma_2}\tilde{\delta}_2^2 \tag{19}
$$

with $P_2(x_1,\rho+\alpha_1) = g_2^{-1}(x_1,\rho+\alpha_1)$, Γ_{θ^2}, Γ_{c^2} and Γ_{σ^2} are positive definite matrices with proper dimensions, γ_2 is a positive constant, $\tilde{\boldsymbol{\theta}}^2 = \hat{\boldsymbol{\theta}}^2 - \boldsymbol{\theta}^{2*}$, $\tilde{\boldsymbol{c}}^2 = \hat{\boldsymbol{c}}^2 - \boldsymbol{c}^{2*}$, $\tilde{\boldsymbol{\sigma}}^2 = \hat{\boldsymbol{\sigma}}^2 - \boldsymbol{\sigma}^{2*}$, and $\tilde{\delta}_2 = \hat{\delta}_2 - \delta_2^*$.

$$
\begin{aligned}
\dot{V}_2 = {} & \dot{V}_1 + z_2 g_2^{-1}\dot{z}_2 + \dot{\alpha}_1 z_2 g_2^{-1} - z_2\dot{\alpha}_1\int_0^1 P_2(\vartheta z_2+\alpha_1)\mathrm{d}\vartheta + z_2^2\dot{x}_1\int_0^1\vartheta\frac{\partial P_2(\vartheta z_2+\alpha_1)}{\partial x_1}\mathrm{d}\vartheta + \boldsymbol{\theta}^{2\mathrm{T}}\Gamma_{\theta^2}^{-1}\dot{\hat{\boldsymbol{\theta}}}^2 + \tilde{\boldsymbol{c}}^{2\mathrm{T}}\Gamma_{c^2}^{-1}\dot{\hat{\boldsymbol{c}}}^2 + \\
& \tilde{\boldsymbol{\sigma}}^{2\mathrm{T}}\Gamma_{\sigma^2}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^2 + \frac{1}{\gamma_2}\tilde{\delta}_2\dot{\hat{\delta}}_2 = \\
& \dot{V}_1 + z_2(x_3+\Delta f_2) + \tilde{\boldsymbol{\theta}}^{2\mathrm{T}}\Gamma_{\theta^2}^{-1}\dot{\hat{\boldsymbol{\theta}}}^2 + \tilde{\boldsymbol{c}}^{2\mathrm{T}}\Gamma_{c^2}^{-1}\dot{\hat{\boldsymbol{c}}}^2 + \tilde{\boldsymbol{\sigma}}^{2\mathrm{T}}\Gamma_{\sigma^2}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^2 + \frac{1}{\gamma_2}\tilde{\delta}_2\dot{\hat{\delta}}_2
\end{aligned} \tag{20}
$$

where $\Delta f_2 = g_2^{-1}(\bar{\boldsymbol{x}}_2)f_2(\bar{\boldsymbol{x}}_2) - \dot{\alpha}_1\int_0^1 P_2(\vartheta z_2+\alpha_1)\mathrm{d}\vartheta + z_2\dot{x}_1\int_0^1\vartheta\frac{\partial P_2(\vartheta z_2+\alpha_1)}{\partial x_1}\mathrm{d}\vartheta$. As in Step 1, Δf_2 is approximated by a fuzzy logic system, and following similar manipulation as (13), we get

$$
\begin{aligned}
& \hat{\boldsymbol{\theta}}^{2\mathrm{T}}\hat{\boldsymbol{\xi}}^2 - \boldsymbol{\theta}^{2*\mathrm{T}}\boldsymbol{\xi}^{2*} \le \\
& \tilde{\boldsymbol{\theta}}^{2\mathrm{T}}(\hat{\boldsymbol{\xi}}^2 - \hat{\xi}'_{c^2}\hat{\boldsymbol{c}}^2 - \hat{\xi}'_{\sigma^2}\hat{\boldsymbol{\sigma}}^2) + \hat{\boldsymbol{\theta}}^{2\mathrm{T}}\hat{\xi}'_{c^2}\tilde{\boldsymbol{c}}^2 + \hat{\boldsymbol{\theta}}^{2\mathrm{T}}\hat{\xi}'_{\sigma^2}\tilde{\boldsymbol{\sigma}}^2 + \\
& \|\hat{\boldsymbol{\theta}}^{2\mathrm{T}}\hat{\xi}'_{c^2}\|_1\bar{c}^2 + \|\hat{\boldsymbol{\theta}}^{2\mathrm{T}}\hat{\xi}'_{\sigma^2}\|_1\bar{\sigma}^2 + \|\hat{\xi}'_{c^2}\hat{\boldsymbol{c}}^2 + \hat{\xi}'_{\sigma^2}\hat{\boldsymbol{\sigma}}^2\|_1\bar{\theta}^2 + \|\boldsymbol{\theta}^{2*}\|_1
\end{aligned} \tag{21}
$$

δ_2^*, $\hat{\xi}'_{c^2}$, $\hat{\xi}'_{\sigma^2}$, and ω_2 are defined similarly to δ_1^*, $\hat{\xi}'_{c^1}$, $\hat{\xi}'_{\sigma^1}$, and ω_1 respectively, with subscript 2 instead of 1. Design the virtual control and parameter updating laws in this step as

$$
\alpha_2 = -q_2z_2 - z_1 - \hat{\boldsymbol{\theta}}^{2\mathrm{T}}\hat{\boldsymbol{\xi}}^2 - \omega_2\tanh\left(\frac{z_2\omega_2}{\pi_2}\right) - \hat{\delta}_2\tanh\frac{z_2\hat{\delta}_2}{\tau_2} \tag{22}
$$

$$
\begin{aligned}
&\dot{\hat{\boldsymbol{\theta}}}^2 = \mathrm{Proj}[\Gamma_{\theta^2}z_2(\hat{\boldsymbol{\xi}}^2 - \hat{\xi}'_{c^2}\hat{\boldsymbol{c}}^2 - \hat{\xi}'_{\sigma^2}\hat{\boldsymbol{\sigma}}^2) - R_{\theta^2}\hat{\boldsymbol{\theta}}^2] \\
&\dot{\hat{\boldsymbol{c}}}^2 = \mathrm{Proj}[\Gamma_{c^2}z_2\hat{\xi}'^{\mathrm{T}}_{c^2}\hat{\boldsymbol{\theta}}^2 - R_{c^2}\hat{\boldsymbol{c}}^2] \\
&\dot{\hat{\boldsymbol{\sigma}}}^2 = \mathrm{Proj}[\Gamma_{\sigma^2}z_2\hat{\xi}'^{\mathrm{T}}_{\sigma^2}\hat{\boldsymbol{\theta}}^2 - R_{\sigma^2}\hat{\boldsymbol{\sigma}}^2] \\
&\dot{\hat{\delta}}_2 = \gamma_2z_2 - r_2\hat{\delta}_2
\end{aligned} \tag{23}
$$

Then, (20) can be rewritten as

$$
\dot{V}_2 \le \sum_{j=1}^{2}[-q_jz_j^2 - \frac{\lambda_{\theta^j}^{\min}}{2}\tilde{\boldsymbol{\theta}}^{j\mathrm{T}}\Gamma_{\theta^j}^{-1}\tilde{\boldsymbol{\theta}}^j - \frac{\lambda_{c^j}^{\min}}{2}\tilde{\boldsymbol{c}}^{j\mathrm{T}}\Gamma_{c^j}^{-1}\tilde{\boldsymbol{c}}^j - \frac{\lambda_{\sigma^j}^{\min}}{2}\tilde{\boldsymbol{\sigma}}^{j\mathrm{T}}\Gamma_{\sigma^j}^{-1}\tilde{\boldsymbol{\sigma}}^j - \frac{1}{2\gamma_j}\tilde{\delta}_j^2 + \kappa(\pi_j+\tau_j) + \frac{1}{2}\boldsymbol{\theta}^{j*\mathrm{T}}\Gamma_{\theta^j}^{-1}R_{\theta^j}\tilde{\boldsymbol{\theta}}^{j*} + \frac{1}{2}\boldsymbol{c}^{j*\mathrm{T}}\Gamma_{c^j}^{-1}R_{c^j}\boldsymbol{c}^{j*} + \frac{1}{2}\boldsymbol{\sigma}^{j*\mathrm{T}}\Gamma_{\sigma^j}^{-1}R_{\sigma^j}\boldsymbol{\sigma}^{j*} + \frac{1}{2\gamma_j}\delta_j^{*2}] + z_2z_3 \tag{24}
$$

Step $\boldsymbol{i}$ ($\boldsymbol{3 \le i \le (n-1)}$). Let $z_i = x_i - \alpha_{i-1}$, and design

$$
\alpha_i = -q_iz_i - z_{i-1} - \hat{\boldsymbol{\theta}}^{i\mathrm{T}}\hat{\boldsymbol{\xi}}^i - \omega_i\tanh\frac{z_i\omega_i}{\pi_i} - \hat{\delta}_i\tanh\frac{z_i\hat{\delta}_i}{\tau_i} \tag{25}
$$

$$
\begin{aligned}
&\dot{\hat{\boldsymbol{\theta}}}^i = \mathrm{Proj}[\Gamma_{\theta^i}z_i(\hat{\boldsymbol{\xi}}^i - \hat{\xi}'_{c^i}\hat{\boldsymbol{c}}^i - \hat{\xi}'_{\sigma^i}\hat{\boldsymbol{\sigma}}^i) - R_{\theta^i}\hat{\boldsymbol{\theta}}^i] \\
&\dot{\hat{\boldsymbol{c}}}^i = \mathrm{Proj}[\Gamma_{c^i}z_i\hat{\xi}'^{\mathrm{T}}_{c^i}\hat{\boldsymbol{\theta}}^i - R_{c^i}\hat{\boldsymbol{c}}^i] \\
&\dot{\hat{\boldsymbol{\sigma}}}^i = \mathrm{Proj}[\Gamma_{\sigma^i}z_i\hat{\xi}'^{\mathrm{T}}_{\sigma^i}\hat{\boldsymbol{\theta}}^i - R_{\sigma^i}\hat{\boldsymbol{\sigma}}^i] \\
&\dot{\hat{\delta}}_i = \gamma_iz_i - r_i\hat{\delta}_i
\end{aligned} \tag{26}
$$

Then, define Lyapunov function as

$$
V_i = V_{i-1} + \int_0^{z_i}\rho P_i(\bar{\boldsymbol{x}}_{i-1},\rho+\alpha_{i-1})\mathrm{d}\rho + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{i\mathrm{T}}\Gamma_{\theta^i}^{-1}\tilde{\boldsymbol{\theta}}^i + \frac{1}{2}\tilde{\boldsymbol{c}}^{i\mathrm{T}}\Gamma_{c^i}^{-1}\tilde{\boldsymbol{c}}^i + \frac{1}{2}\tilde{\boldsymbol{\sigma}}^{i\mathrm{T}}\Gamma_{\sigma^i}^{-1}\tilde{\boldsymbol{\sigma}}^i + \frac{1}{2\gamma_i}\tilde{\delta}_i^2 \tag{27}
$$

and its derivative satisfies the following inequality

$$
\dot{V}_i \le \sum_{j=1}^{i}[-q_jz_j^2 - \frac{\lambda_{\theta^j}^{\min}}{2}\tilde{\boldsymbol{\theta}}^{j\mathrm{T}}\Gamma_{\theta^j}^{-1}\tilde{\boldsymbol{\theta}}^j - \frac{\lambda_{c^j}^{\min}}{2}\tilde{\boldsymbol{c}}^{j\mathrm{T}}\Gamma_{c^j}^{-1}\tilde{\boldsymbol{c}}^j - \frac{\lambda_{\sigma^j}^{\min}}{2}\tilde{\boldsymbol{\sigma}}^{j\mathrm{T}}\Gamma_{\sigma^j}^{-1}\tilde{\boldsymbol{\sigma}}^j - \frac{1}{2\gamma_j}\tilde{\delta}_j^2 + \kappa(\pi_j+\tau_j) + \frac{1}{2}\boldsymbol{\theta}^{j*\mathrm{T}}\Gamma_{\theta^j}^{-1}R_{\theta^j}\tilde{\boldsymbol{\theta}}^{j*} + \frac{1}{2}\boldsymbol{c}^{j*\mathrm{T}}\Gamma_{c^j}^{-1}R_{c^j}\boldsymbol{c}^{j*} + \frac{1}{2}\boldsymbol{\sigma}^{j*\mathrm{T}}\Gamma_{\sigma^j}^{-1}R_{\sigma^j}\boldsymbol{\sigma}^{j*} + \frac{1}{2\gamma_j}\delta_j^{*2}] + z_iz_{i+1} \tag{28}
$$

Step $\boldsymbol{n}$. Let $z_n = x_n - \alpha_{n-1}$, $\dot{z}_n$ can be written as

$$
\dot{z}_n = f_n(\boldsymbol{x}) + g_n(\boldsymbol{x})[m(t)+\phi(\boldsymbol{x})]u - g_n(\boldsymbol{x})[m(t)+\phi(\boldsymbol{x})]\times b\tanh(\frac{u}{b}) + g_n(\boldsymbol{x})[m(t)+\phi(\boldsymbol{x})]\frac{\psi(\boldsymbol{x})}{m(t)+\phi(\boldsymbol{x})} - \dot{\alpha}_{n-1} \tag{29}
$$

Choose $P_n(m,\bar{\boldsymbol{x}}_{n-1},\rho+\alpha_{n-1}) = g_n^{-1}(\bar{\boldsymbol{x}}_{n-1},\rho+\alpha_{n-1})[m(t)+\phi(\bar{\boldsymbol{x}}_{n-1},\rho+\alpha_{n-1})]^{-1}$ and the Lyapunov function candidate

$$
V_n = V_{n-1} + \int_0^{z_n}\rho P_n(m,\bar{\boldsymbol{x}}_{n-1},\rho+\alpha_{n-1})\mathrm{d}\rho + \frac{1}{2}\tilde{\boldsymbol{\theta}}^{n\mathrm{T}}\Gamma_{\theta^n}^{-1}\tilde{\boldsymbol{\theta}}^n + \frac{1}{2}\tilde{\boldsymbol{c}}^{n\mathrm{T}}\Gamma_{c^n}^{-1}\tilde{\boldsymbol{c}}^n + \frac{1}{2}\tilde{\boldsymbol{\sigma}}^{n\mathrm{T}}\Gamma_{\sigma^n}^{-1}\tilde{\boldsymbol{\sigma}}^n + \frac{1}{2\gamma_n}\tilde{\delta}_n^2 + \frac{1}{2\gamma_b}\tilde{b}^2 \tag{30}
$$

where Γ_{θ^n}, Γ_{c^n}, and Γ_{σ^n} are positive definite matrices, γ_n and γ_b are positive constants. $\tilde{b} = \hat{b} - b$ with $\hat{b}$ the estimate of b. Define

$$\begin{aligned}\Delta f_n = &g_n^{-1}(\boldsymbol{x})[m(t)+\phi(\boldsymbol{x})]^{-1}f_n(\boldsymbol{x}) - \\ &\dot{\alpha}_{n-1}\int_0^1 P_n(\vartheta z_n+\alpha_{n-1})\mathrm{d}\vartheta + \\ &z_n\sum_{j=1}^{n-1}\dot{x}_j\int_0^1 \vartheta\frac{\partial P_n(\vartheta z_n+\alpha_{n-1})}{\partial x_j}\mathrm{d}\vartheta + \\ &z_n\dot{m}\int_0^1 \vartheta\frac{\partial P_n(\vartheta z_n+\alpha_{n-1})}{\partial m}\mathrm{d}\vartheta + b\tanh(\frac{u}{b})\end{aligned}$$

Then we can get

$$\begin{aligned}\dot{V}_n = &\dot{V}_{n-1} + z_n[u+\Delta f_n + \frac{\psi(\boldsymbol{x})}{m(t)+\phi(\boldsymbol{x})}] + \tilde{\boldsymbol{\theta}}^{n\mathrm{T}}\Gamma_{\theta^n}^{-1}\dot{\hat{\boldsymbol{\theta}}}^n + \\ &\tilde{\boldsymbol{c}}^{n\mathrm{T}}\Gamma_{c^n}^{-1}\dot{\hat{\boldsymbol{c}}}^n + \tilde{\boldsymbol{\sigma}}^{n\mathrm{T}}\Gamma_{\sigma^n}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^n + \frac{1}{\gamma_n}\tilde{\delta}_n\dot{\hat{\delta}}_n + \frac{1}{\gamma_b}\tilde{b}\dot{\hat{b}} \le \\ &\dot{V}_{n-1} + z_n\{u+\Delta f_n + \mathrm{sgn}(z_n)b[1-\tanh(1)]\} + \\ &\tilde{\boldsymbol{\theta}}^{n\mathrm{T}}\Gamma_{\theta^n}^{-1}\dot{\hat{\boldsymbol{\theta}}}^n + \tilde{\boldsymbol{c}}^{n\mathrm{T}}\Gamma_{c^n}^{-1}\dot{\hat{\boldsymbol{c}}}^n + \tilde{\boldsymbol{\sigma}}^{n\mathrm{T}}\Gamma_{\sigma^n}^{-1}\dot{\hat{\boldsymbol{\sigma}}}^n + \\ &\frac{1}{\gamma_n}\tilde{\delta}_n\dot{\hat{\delta}}_n + \frac{1}{\gamma_b}\tilde{b}\dot{\hat{b}}\end{aligned} \tag{31}$$

where the boundary of $\psi(\boldsymbol{x})$ described in (6) has been used. By choosing control signal

$$\begin{aligned}u = &-q_n z_n - z_{n-1} - \hat{\boldsymbol{\theta}}^{n\mathrm{T}}\hat{\boldsymbol{\xi}}^n - \omega_n\tanh\frac{z_n\omega_n}{\pi_n} - \\ &\hat{\delta}_n\tanh\frac{z_n\hat{\delta}_n}{\tau_n} - \mathrm{sgn}(z_n)\hat{b}[1-\tanh(1)]\end{aligned} \tag{32}$$

with q_n, π_n, and τ_n being some positive constants, $\hat{\delta}_n$ is the estimate of $\delta_n^* = \varepsilon_n + \|\boldsymbol{\theta}^{n*}\|_1$, $\omega_n = \|\hat{\boldsymbol{\theta}}^{n\mathrm{T}}\hat{\xi}'_{c^n}\|_1\bar{c}^n + \|\hat{\boldsymbol{\theta}}^{n\mathrm{T}}\hat{\xi}'_{\sigma^n}\|_1\bar{\sigma}^n + \|\hat{\xi}'_{c^n}\hat{\boldsymbol{c}}^n + \hat{\xi}'_{\sigma^n}\hat{\boldsymbol{\sigma}}^n\|_1\bar{\theta}^n$, and the adaptive laws are

$$\begin{aligned}&\dot{\hat{\boldsymbol{\theta}}}^n = \mathrm{Proj}[\Gamma_{\theta^n}z_n(\hat{\boldsymbol{\xi}}^n - \hat{\xi}'_{c^n}\hat{\boldsymbol{c}}^n - \hat{\xi}'_{\sigma^n}\hat{\boldsymbol{\sigma}}^n) - R_{\theta^n}\hat{\boldsymbol{\theta}}^n] \\ &\dot{\hat{\boldsymbol{c}}}^n = \mathrm{Proj}[\Gamma_{c^n}z_n\hat{\xi}'^{\mathrm{T}}_{c^n}\hat{\boldsymbol{\theta}}^n - R_{c^n}\hat{\boldsymbol{c}}^n] \\ &\dot{\hat{\boldsymbol{\sigma}}}^n = \mathrm{Proj}[\Gamma_{\sigma^n}z_n\hat{\xi}'^{\mathrm{T}}_{\sigma^n}\hat{\boldsymbol{\theta}}^n - R_{\sigma^n}\hat{\boldsymbol{\sigma}}^n] \\ &\dot{\hat{\delta}}_n = \gamma_n z_n - r_n\hat{\delta}_n \\ &\dot{\hat{b}} = \gamma_b[1-\tanh(1)]|z_n| - r_b\hat{b}\end{aligned} \tag{33}$$

with R_{θ^n}, R_{c^n}, and R_{σ^n} being positive matrices with proper dimensions, r_n and r_b are positive constants, and taking (28) into account with $i = n-1$, (31) can be rewritten as

$$\begin{aligned}\dot{V}_n \le &\sum_{j=1}^n(-q_j z_j^2 - \frac{\lambda_{\theta^j}^{\min}}{2}\tilde{\boldsymbol{\theta}}^{j\mathrm{T}}\Gamma_{\theta^j}^{-1}\tilde{\boldsymbol{\theta}}^j - \frac{\lambda_{c^j}^{\min}}{2}\tilde{\boldsymbol{c}}^{j\mathrm{T}}\Gamma_{c^j}^{-1}\tilde{\boldsymbol{c}}^j - \\ &\frac{\lambda_{\sigma^j}^{\min}}{2}\tilde{\boldsymbol{\sigma}}^{j\mathrm{T}}\Gamma_{\sigma^j}^{-1}\tilde{\boldsymbol{\sigma}}^j - \frac{1}{2\gamma_j}\tilde{\delta}_j^2 - \frac{1}{2\gamma_b}\tilde{b}^2) + \sum_{j=1}^n[\kappa(\pi_j+\tau_j) + \\ &\frac{1}{2}\boldsymbol{\theta}^{j*\mathrm{T}}\Gamma_{\theta^j}^{-1}R_{\theta^j}\tilde{\boldsymbol{\theta}}^{j*} + \frac{1}{2}\boldsymbol{c}^{j*\mathrm{T}}\Gamma_{c^j}^{-1}R_{c^j}\boldsymbol{c}^{j*} + \\ &\frac{1}{2}\boldsymbol{\sigma}^{j*\mathrm{T}}\Gamma_{\sigma^j}^{-1}R_{\sigma^j}\boldsymbol{\sigma}^{j*} + \frac{1}{2\gamma_j}\delta_j^{*2} + \frac{1}{2\gamma_b}b^2]\end{aligned} \tag{34}$$

with $\lambda_{\theta^j}^{\min}$, $\lambda_{c^j}^{\min}$, and $\lambda_{\sigma^j}^{\min}$ being the minimal eigenvalues of R_{θ^j}, R_{c^j}, and R_{σ^j}, $1 \le j \le n$, respectively. At this juncture, we are ready to give the main result of the paper.

2.3 Main result

The main result is summarized in the following theorem.

Theorem 1. Consider the unknown nonlinear system (1) which satisfies Assumptions 1∼3, the designed control law (32) and the adaptive laws (33), together with the intermediate variables (15), (22), (25) and the parameter updating laws (16), (23), (26) in the design steps can ensure all signals in the closed-loop system remain bounded. Furthermore, for any given value $\epsilon_0 > 0$, the tracking error z_1 satisfies $\lim_{t\to\infty}\| z_1 \|^2 \le \epsilon_0^2$.

Proof. Let $\underline{g}_i = g_{il}$ and $\bar{g}_i = g_{iu}$ for $1 \le i \le n-1$, $\underline{g}_n = \underline{m}g_{nl}$ and $\bar{g}_n = \bar{m}g_{nu}$, then from Assumptions 1 and 3, one can get $\bar{g}_i^{-1} \le g_i^{-1}(\bar{\boldsymbol{x}}_i) \le \underline{g}_i^{-1}$ for $1 \le i \le n-1$ and $\bar{g}_n^{-1} \le g_n^{-1}(\boldsymbol{x})(m(t)+\phi(\boldsymbol{x}))^{-1} \le \underline{g}_n^{-1}$. Then, from Lemma 2, it follows

$$-\frac{1}{2\underline{g}_i}z_i^2 \le -\int_0^{z_i}\rho P_i(\bar{\boldsymbol{x}}_{i-1}, \rho+\alpha_{i-1})\mathrm{d}\rho, \quad 1 \le i \le n \tag{35}$$

It can be concluded from (34) and (35) that

$$\begin{aligned}\dot{V}_n \le &\sum_{j=1}^n[-2\underline{g}_j q_j\int_0^{z_j}\rho P_j(\bar{\boldsymbol{x}}_{j-1}, \rho+\alpha_{j-1})\mathrm{d}\rho - \frac{\lambda_{\theta^j}^{\min}}{2}\tilde{\boldsymbol{\theta}}^{j\mathrm{T}} \times \\ &\Gamma_{\theta^j}^{-1}\tilde{\boldsymbol{\theta}}^j - \frac{\lambda_{c^j}^{\min}}{2}\tilde{\boldsymbol{c}}^{j\mathrm{T}}\Gamma_{c^j}^{-1}\tilde{\boldsymbol{c}}^j - \frac{\lambda_{\sigma^j}^{\min}}{2}\tilde{\boldsymbol{\sigma}}^{j\mathrm{T}}\Gamma_{\sigma^j}^{-1}\tilde{\boldsymbol{\sigma}}^j - \frac{1}{2\gamma_j}\tilde{\delta}_j^2 - \\ &\frac{1}{2\gamma_b}\tilde{b}^2] + \sum_{j=1}^n[\kappa(\pi_j+\tau_j) + \frac{1}{2}\boldsymbol{\theta}^{j*\mathrm{T}}\Gamma_{\theta^j}^{-1}R_{\theta^j}\tilde{\boldsymbol{\theta}}^{j*} + \\ &\frac{1}{2}\boldsymbol{c}^{j*\mathrm{T}}\Gamma_{c^j}^{-1}R_{c^j}\boldsymbol{c}^{j*} + \frac{1}{2}\boldsymbol{\sigma}^{j*\mathrm{T}}\Gamma_{\sigma^j}^{-1}R_{\sigma^j}\boldsymbol{\sigma}^{j*} + \\ &\frac{1}{2\gamma_j}\delta_j^{*2} + \frac{1}{2\gamma_b}b^2] \le -\mu V_n + \beta\end{aligned} \tag{36}$$

where $\mu = \min\{2\underline{g}_j q_j,\ \lambda_{\theta^j}^{\min},\ \lambda_{c^j}^{\min},\ \lambda_{\sigma^j}^{\min}\}$ and $\beta = \sum_{j=1}^n[\kappa(\pi_j + \tau_j) + \frac{1}{2}\boldsymbol{\theta}^{j*\mathrm{T}}\Gamma_{\theta^j}^{-1}R_{\theta^j}\tilde{\boldsymbol{\theta}}^{j*} + \frac{1}{2}\boldsymbol{c}^{j*\mathrm{T}}\Gamma_{c^j}^{-1}R_{c^j}\boldsymbol{c}^{j*} + \frac{1}{2}\boldsymbol{\sigma}^{j*\mathrm{T}}\Gamma_{\sigma^j}^{-1}R_{\sigma^j}\boldsymbol{\sigma}^{j*} + \frac{1}{2\gamma_j}\delta_j^{*2} + \frac{1}{2\gamma_b}b^2]$. Then for $t > 0$,

$$V_n \le [V_n(0) - \frac{\beta}{\mu}\mathrm{e}^{-\mu t}] + \frac{\beta}{\mu} \tag{37}$$

From Assumption 2, b is a nonnegative bounded constant, besides, π_j, τ_j, $\Gamma_{\theta^j}^{-1}R_{\theta^j}$, $\Gamma_{c^j}^{-1}R_{c^j}$, $\Gamma_{\sigma^j}^{-1}R_{\sigma^j}$, γ_j and γ_b are all determined by the designer, so β is bounded and can be designed as small as possible to obtain the desired tracking performance. It can be seen from (37) that z_i, $\tilde{\boldsymbol{\theta}}^i$, $\tilde{\boldsymbol{c}}^i$, $\tilde{\boldsymbol{\sigma}}^i$, $\tilde{\delta}_i$ and $\tilde{b}$ are bounded by the set $\Omega_s = \{(z_i, \tilde{\boldsymbol{\theta}}^i, \tilde{\boldsymbol{c}}^i, \tilde{\boldsymbol{\sigma}}^i, \tilde{\delta}_i, \tilde{b})|V_n \le \max(V_n(0), \frac{\beta}{\mu})\}$. Thus, it can be deduced that x_i $\hat{\boldsymbol{\theta}}^i$, $\hat{\boldsymbol{c}}^i$, $\hat{\boldsymbol{\sigma}}^i$, $\hat{\delta}_i$ and $\hat{b}$ remain bounded for bounded $V_n(0)$.

Note that μ and β can be tuned by choosing different design parameters, then one can always select appropriate parameters such that for any $\epsilon_0 > 0$, the inequality $\beta/\mu \le \epsilon_0^2/(2\bar{g}_1)$ is true. Thus, according to Assumption 3 and Lemma 2, the following inequalities can be obtained.

$$\frac{1}{2\bar{g}_1}\| z_1^2 \| \le \int_0^{z_1}\rho P_1(\rho + y_r)\mathrm{d}\rho \le V_1 \tag{38}$$

From this, we can further get that

$$\lim_{t\to\infty}\| z_1 \|^2 \le \lim_{t\to\infty}2\bar{g}_1 V_1 \le 2\bar{g}_1\frac{\beta}{\mu} \le \epsilon_0^2 \tag{39}$$

This proves that the tracking error can be made as small as possible by appropriately choosing the design parameters. So far, Theorem 1 has been proved. □

Remark 2. The control design presented in this paper, compared with the results in [8−12], has the following advantages:

1) Unlike [8−12] where η is considered only as a disturbance-like term, we further treat η as the sum of a nonlinear term and a disturbance-like term $\psi(\boldsymbol{x})$, where the nonlinear term can be approximated by fuzzy logic system together with the unknown system functions, and the upper bound of $\psi(\boldsymbol{x})$ is smaller than that of η, so the control effort for dealing with the disturbance-like term will be smaller than that of controller, which is designed by treating η only as a disturbance-like term.

2) The robust control against the disturbance-like term can be designed with neither the bound of $m(t)+\phi(\boldsymbol{x})$ nor the bound of b, while the existing methods require at least one of them.

3) The considered system (1) is more general than that in [8−12], since there are unknown nonlinear functions in the dynamics of each x_i, $1 \leq i \leq n$. Furthermore, a new dead-zone model is proposed, which is time-varying and perturbed. The new dead-zone is more complicated, for which the existing methods are not applicable.

3 Simulation example

We consider a dead-zone nonlinear system as follows:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\ \dot{x}_2 &= f_2(\boldsymbol{x}) + g_2(\boldsymbol{x})\mathscr{D}(u) \\ y &= x_1 \end{aligned} \tag{40}$$

where $f_1(x_1) = 0.5x_1^2$, $f_2(\boldsymbol{x}) = x_1x_2 - 2$, $g_1(x_1) = 1 + 0.1x_1^2$, $g_2(\boldsymbol{x}) = 2 + \cos(x_1x_2)$. $\mathscr{D}(u)$ is defined as (2) with $m(t) = 1.25\mathrm{e}^{(-0.01t)}$, $\phi(\boldsymbol{x}) = 0.1\sin(x_1)$, and $b = 10$. The reference signals are generated from the following system:

$$\begin{aligned} \dot{x}_{r1} &= x_{r2} \\ \dot{x}_{r2} &= -x_{r1} + 0.001(1 - x_{r1}^2)x_{r2} \\ y_r &= x_{r1}, \qquad i = 1, 2 \end{aligned} \tag{41}$$

The initial conditions are chosen as $\boldsymbol{x}_r(0) = (1.5, 0.8)^{\mathrm{T}}$, $\boldsymbol{x}(0) = (0.5, 2)^{\mathrm{T}}$. Two fuzzy logic systems with 11 fuzzy rules for each one are used as approximators in the backstepping design. The initial estimate values are $\hat{\boldsymbol{\theta}}^1(0) = \hat{\boldsymbol{\theta}}^2(0) = \boldsymbol{0} \in \mathbf{R}^{11}$, $\hat{\boldsymbol{c}}^1(0) = \hat{\boldsymbol{c}}^2(0) = (-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10)^{\mathrm{T}}$, $\hat{\boldsymbol{\sigma}}^1(0) = \hat{\boldsymbol{\sigma}}^2(0) = 0.5\boldsymbol{I}_{11}$ with $\boldsymbol{I}_{11}$ a unit column vector in $\mathbf{R}^{11}$, $\hat{\delta}_1(0) = \hat{\delta}_2(0) = 0$, $\hat{b}(0) = 1$. The design parameters are chosen as $\bar{\theta}^i = 1$, $\bar{c}^i = 10$, $\bar{\sigma}^i = 0.5$, $q_i = 1.5$, $\Gamma_{\theta^i} = 1.5I_{11\times11}$, $\Gamma_{c^i} = 1.5I_{11\times11}$, $\Gamma_{\sigma^i} = 1.5I_{11\times11}$, $R_{\theta^i} = 0.1I_{11\times11}$, $R_{c^i} = 0.1I_{11\times11}$, $R_{\sigma^i} = 0.1I_{11\times11}$, where $I_{11\times11}$ is the unit matrix, $\gamma_i = 1.5$, $r_i = 0.1$, $\pi_i = 0.5$, $\tau_i = 0.5$, for $i = 1, 2$, $\gamma_b = 1.5$, $r_b = 0.1$.

The simulation results are shown in Figs. 1 ∼ 3 where the output tracking, the control output of the controller, and the control input of the plant are plotted, respectively. In order to show that the proposed scheme is less conservative by specially treating the dead-zone nonlinearity, we also design a controller where η is viewed only as a disturbance-like term. The input and output of the dead-zone actuator are plotted in Figs. 4 and 5, respectively, it is obvious that the control is more conservative than that illustrated in Figs. 2 and 3.

Remark 3. As there are three parameter vectors need to be updated online in each approximator, computation burden may be heavy when the controlled system is of higher

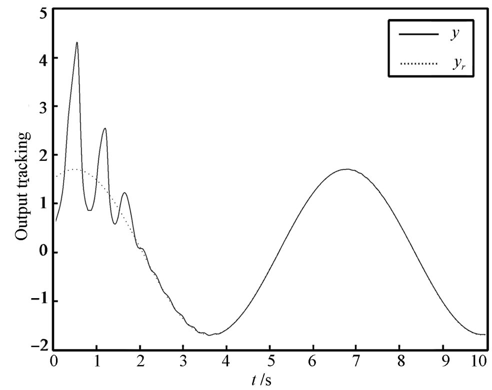

Fig. 1 The output tracking curves of the dead-zone control

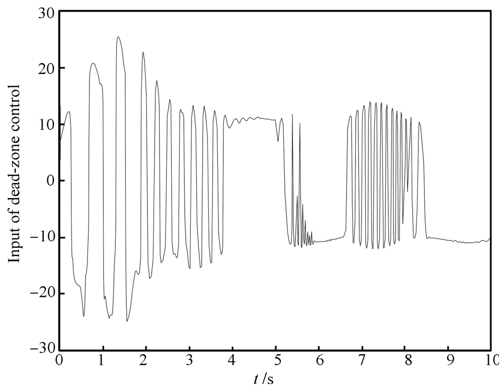

Fig. 2 The control input of the dead-zone actuator u

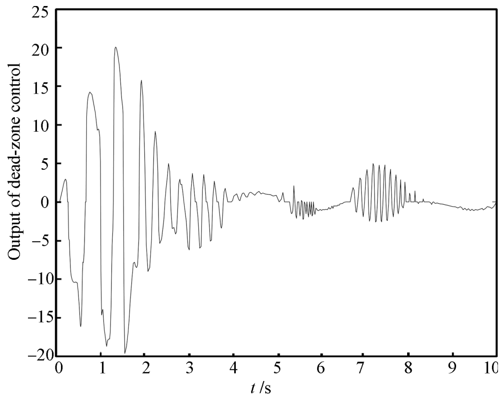

Fig. 3 The output of the dead-zone actuator $\mathscr{D}(u)$

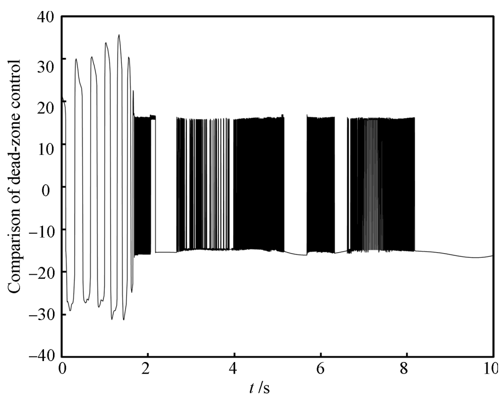

Fig. 4 The input of the dead-zone actuator u from the controller which is designed by viewing η as a disturbance-like term

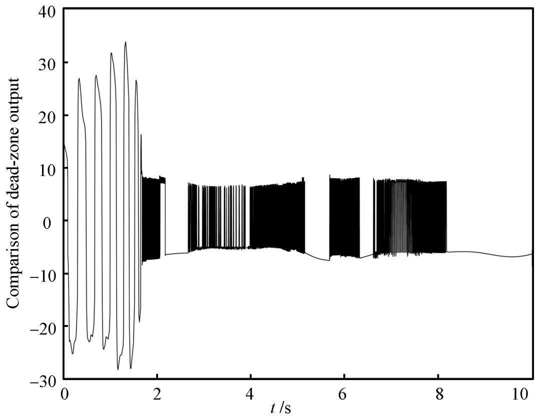

Fig. 5 The output of the dead-zone actuator $\mathscr{D}(u)$ driven by the controller which is designed by viewing η as a disturbance-like term

order and the fuzzy approximators have lots of rules. Generally speaking, more rules lead to a better approximation to unknown function, as well as heavier computation burden. So, there is always a tradeoff between number of fuzzy rules and control performance in existing method. In this paper, however, additional control is employed to compensate approximation error, which leads to a good control performance with few rules in the approximators. Also, because few rules are needed in our design, the online computation burden is not heavy for general systems and the controller can be achieved timely for application. We have tested the time needed for realizing the controller of example in Matlab, it is 0.16 s. The computer used is Pentium 4 with 2.93 GHz CPU and 512 MB RAM.

4 Conclusion

In this paper, a class of unknown nonlinear systems with time-varying and perturbed dead-zone inputs has been successfully controlled by an adaptive fuzzy control scheme. Since the system considered are not restricted to be feedback linearizable, backstepping technique is employed to obtain the controller step by step. In each step, a nonlinearly parameterized fuzzy logic system is used to approximate the packaged unknown function because there is no much a priori knowledge about the fuzzy membership functions. Adaptive laws are given based on Lyapunov stability to update the parameters online, so that the tracking error can be made as small as possible. The dead-zone width is estimated explicitly, thus the control scheme has the capability to adapt to the width of the dead-zone actuator. By specially treating the dead-zone characteristic as a perturbed linear-like term, a nonlinear term and a disturbance-like term, the robustness of the system is achieved by less control effort. It is proved in theory and shown in simulation that the closed-loop system is stable and the output tracks the given reference signal satisfactorily.

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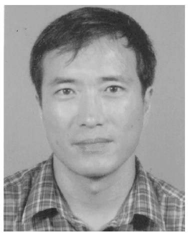

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