Conservatism Reduction in Two-level Control for Stabilization of Discrete Large Scale Systems

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Abstract The aim of this paper is to propose a new algorithm for multilevel stabilization of large scale systems. In two-level stabilization method, a set of local stabilizers for the individual subsystems in a completely decentralized environment is designed. The solution of the control problem involves designing of a global controller on a higher hierarchical level that provides corrective signals to account for interconnections effect. The principle feature of this paper is to reduce conservativeness in global controller design. Here, the key point is to reduce the effect of interactions instead of neutralizing them. In fact, unlike prior methods, our idea does not ignore the possible beneficial aspects of the interactions and does not try to neutralize them.

Key words Large scale systems, two-level control, stability, conservatism reduction

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There are different methods for control of large scale systems such as two-level method, decentralized, and centralized control and etc. In this paper we have focused on the two-level method. The idea of this method was originally introduced by Siljak[1] and is shown in Fig. 1.

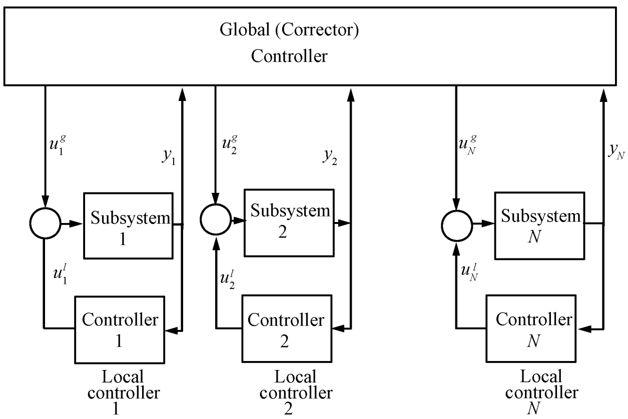

Fig. 1 A multi-level structure

In this method, each control signal consists of two segments: local and global controller signals. Local controllers are used to control each subsystem, ignoring the interactions. A global controller may be applied to minimize the effect of interactions and improve the performance of the overall system.

Other common methods in the field of large scale systems have some control problems which are mentioned in the following four points, but the multilevel control does not have these disadvantages.

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1) A decentralized approach will be effective if the large scale system is composed of weakly coupled subsystems[2−4], or a decomposition and rearrangement of variables are used to achieve weak coupling as is the case of sparse systems[5]. In situations when the subsystems are strongly connected and cannot be simply reconstituted by reducing the strength of the couplings, the assumption of weak coupling may produce gross inaccuracies in the obtained results.

2) Since a large-scale system will invariably be an interconnection of several subunits, one of the important phenomena that must be accounted for in the design of controllers and estimators is the occurrence of structural perturbations, i.e., changes in the interconnection pattern within the system during operation[6]. When a system is expected to undergo structural perturbations, the classical control techniques do not provide a satisfactory solution of the control problem and may results in a closed-loop system which is unstable.

3) Dealing with large systems, centralized controller design methods are either uneconomical because of excessive computation time or impossible due to excessive computer storage requirement.

4) Hierarchical coordinating methods, though conceptually very simple, require iterative solution procedures, which often lead to convergence difficulties[7].

Most of the researches adopt the philosophy that interactions are non-beneficial. Any interaction would deteriorate the system objectives. Therefore, they tend to design a global controller to neutralize all such interactions[8−9]. Indeed it is assumed that:

$$H - BM = 0 \Rightarrow M = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}H$$

where H, B, and M represent interaction matrix, system input, and global controller, respectively. However, selecting this structure for global controller will lead to the following problems:

1) The ideal effect of two level control (neutralizing the effect of interactions) cannot be obtained unless the rank of the composite matrix $[B|H]$ is equal to the rank of the matrix B itself. In case of inequality, some performance criteria may not be satisfied.

2) Even in case of rank equality, is it necessary to neutralize the effect of interactions completely? Is there a way not to neutralize the interactions? Are all of them non-beneficial? It might be an interaction which can help the system toward its objectives. And maybe we can achieve the goals and desired performances or also improve them by assuming another structure for matrix M[10].

3) Also not trying to neutralize all of the interactions would lead to less conservativeness in global controller design.

As we mentioned, although the idea of multi-level control appeared very early, no significant development has been made in this research area for a long time. Recently, Duan et al. presented a series of significant results on cooperative control of linear and nonlinear systems[11−14], and showed that unstable subsystems can form a stable large-scale system through appropriate interaction and cooperation of feedback controllers.

In this paper, two new algorithms to design a global controller for large scale system are developed. We shall make use of a different idea to develop an alternative multilevel scheme, which, unlike prior methods, does not ignore the possible beneficial aspects of the interconnections and does not try to neutralize them. Therefore, the design will not be much conservative. Furthermore, all of these problems which are explained earlier will be solved by using these algorithms. Also, our newly proposed idea can be possibly used for innovation in control design for systems which involve subsystems interaction.

This paper is organized as follows. In Section 1, system description, required definition and lemmas are described. Algorithms for designing two-level stabilizer are introduced in Section 2. The application of design methods in three-region energy resources system and simulation results are presented in Section 3. Finally, concluding remarks follow in Section 4.

1 System description and preliminaries

Let a discrete large scale system that contains two subsystems be described as follow:

$$x_{i+1} = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} u_i \tag{1}$$

where (A_1, B_1) and (A_2, B_2) are respectively state-space matrices of subsystems (1) and (2), A_{12} and A_{21} are their interaction matrices.

Throughout this paper, following assumptions are made.

Assumption 1. Both subsystems are controllable.

Assumption 2. Each subsystem is stabilized with a local controller (i.e., A_1 and A_2 are stable). We will stabilize the system using a state-feedback (i.e., $u_i = k_g x$).

We will refer to several existing methods to design local stabilizers and do not go for further discussion on this subject.

The objective of this work is to design the two-level stabilizer such that:

Objective 1. Overall closed loop system is stable. According to Assumptions 1 and 2, we know two above subsystems are stable but the system (1) may be unstable because of being interacted.

Objective 2. Unlike to prior methods, we do not want the global controller neutralizes the effect of interactions and instead we consider the possible beneficial aspects of the interconnections effect.

Definition 1. Polynomial:

$$P(z) = d_0 + d_1 z + d_2 z^2 + \cdots + d_n z^n \tag{2}$$

is Schur stable, if and only if[15]

$$\begin{aligned} P(1) &> 0 \\ (-1)^n P(-1) &> 0 \\ \det(F(d)) &> 0 \end{aligned} \tag{3}$$

where $F(d)$ is a $(n-1)\times(n-1)$ matrix such as:

$$F(d) = \begin{bmatrix} d_n & d_{n-1} & \cdots & d_3 & d_2 - d_0 \\ 0 & d_n & \cdots & d_4 - d_0 & d_3 - d_1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -d_0 & \cdots & d_n - d_{n-4} & d_{n-1} - d_{n-3} \\ -d_0 & -d_1 & \cdots & -d_{n-3} & d_n - d_{n-2} \end{bmatrix} \tag{4}$$

Lemma 1. Given any real matrices A, B, C, and D of appropriate dimensions, then the following inequality holds

$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det(A) \times \det(D - CA^{-1} B) = \det(D) \times \det(A - BD^{-1} C) \tag{5}$$

Lemma 2. Given a real matrix A, if A is Schur stable then from [14]

$$\det(I - A) > 0 \tag{6}$$

Lemma 3. Let $A = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix}$. If there are two matrices A'_{12} and A'_{21} which decompose A_{12} and A_{21} as follows:

$$A_{12} = A'_{12}(I - A_2) \tag{7}$$

$$A_{21} = A'_{21}(I - A_1) \tag{8}$$

then, the below equality holds

$$\det(I - A) = \det(I - A_1) \times \det(I - A_2) \times \det(I - A'_{21}A'_{12}) \tag{9}$$

Proof. Considering lemma 1, the following equation is obtained:

$$\det(I - A) = \det(I - A_1) \times \det(I - A_2 - A_{21}(I - A_1)^{-1} A_{12})$$

by substituting (7) and (8) in above equation, we have:

$$\det(I - A) = \det(I - A_1)\times \det(I - A_2 - A'_{21}(I - A_1)(I - A_1)^{-1} A'_{12}(I - A_2))$$

from simple calculation:

$$\det(I - A) = \det(I - A_1) \times \det(I - A_2) \times \det(I - A'_{21}A'_{12})$$

□

2 Main results

According to Lemma 3, if large scale system (1) is Schur stable, then $\det(I - A) > 0$. From Assumption 2, the two subsystems A_1 and A_2 are stable therefore $\det(I - A_1) > 0$ and $\det(I - A_2) > 0$. Thus, it implies $\det(I - A'_{21}A'_{12}) > 0$ to guarantee stability of (1) logically. This condition is a necessary not a sufficient condition.

Now by replacing nonzero interactions (nonzero elements of A_{12} and A_{21} matrices) with parameters such as $a_1, a_2, \cdots$, and using (7) and (8), we have the following equation:

$$\varphi(a_i) = \det(I - A'_{21}(a_i)A'_{12}(a_i)) > 0, \quad i = 1, 2, \cdots \tag{10}$$

By solving (10), some constraints on interactions can be obtained. In this step, in order to satisfy attained constraints and finally stabilize closed loop system, we have to design a suitable global controller.

The global controller is obtained such as:

$$BK_g = F, \quad F = \tilde{H} - H \tag{11}$$

where K_g is the global controller gain matrix, $B = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$ is the global input matrix, $H = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$ represents interaction matrix, and $\tilde{H}$ is the modified interaction matrix. In fact, $\tilde{H}$ is obtained by replacing the nonzero elements of H with values calculated from (10). We can obtain the global controller from a set of LMIs as follows:

$$\begin{bmatrix} \beta S & (BK_g - F)^{\mathrm{T}} \\ (BK_g - F) & I \end{bmatrix} > 0 \tag{12}$$

$$\begin{bmatrix} I & S \\ S & I \end{bmatrix} > 0 \tag{13}$$

where β is a sufficiently small positive scalar and S is a positive definite matrix.

As a conclusion, we can use the following algorithm for designing the global controller.

Algorithm 1. Design of the global controller for time-discontinuous large scale systems.

Step 1. Replace the nonzero elements of A_{12} and A_{21} matrices with some parameters such as $a_1, a_2, \cdots$;

Step 2. Calculate A'_{12} and A'_{21} matrices using (7) and (8), in form of a function of $a_1, a_2, \cdots$;

Step 3. Identify $a_1, a_2, \cdots$ in accordance with (10);

Step 4. Compute the global controller from (11);

Step 5. Make closed loop system by replacing (11) with (1) and then test its stability.

Remark 1. Lemma 3 introduces a necessary but not a sufficient condition; in fact if $\det(I - A) > 0$, we cannot conclude stability, because this determinant is greater than zero when all system eigenvalues are in the unit circle or an even number of eigenvalues are out of the unit circle. Therefore, since Algorithm 1 is the basis of Lemma 3, it is always impossible to attain solution from this algorithm and we must apply it when large scale system has odd number of unstable poles before designing global controller.

Remark 2. A'_{12} and A'_{21} exist if $(I - A_1)$ and $(I - A_2)$ are nonsingular.

Remark 3. Algorithm 1 has been proposed for the large scale systems with two subsystems. This algorithm is also applicable for systems with more than two subsystems provided the main system is considered to have a special structure. For instance, in case of the system with following three subsystems or any similar system with larger number of subsystems.

$$x_{i+1} = \begin{bmatrix} A_1 & A_{12} & 0 \\ A_{21} & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} u_i \tag{14}$$

In fact, if the main system has two dependent and some independent subsystems, for such a case, the above algorithm is applicable.

Remark 4. Let us now consider the system

$$x_{i+1} = \begin{bmatrix} A_1 & A_{12} & A_{13} \\ A_{21} & A_2 & A_{23} \\ A_{31} & A_{32} & A_3 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} u_i \tag{15}$$

To stabilize it, one can use Algorithm 2. The aim is to design a global controller to stabilize the closed-loop system.

Algorithm 2.

Step 1. Consider subsystem

$$\tilde{x}_{i+1} = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix} \tilde{x}_i + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \tilde{u}_i \tag{16}$$

From Algorithm 1, we find the necessary conditions for the stability of subsystem (16).

Step 2. Calculate the modified interaction matrices $\tilde{H}_{12}$ and $\tilde{H}_{21}$ for subsystem (16) and create below system.

$$x_{i+1} = \begin{bmatrix} A_1 & \tilde{H}_{12} & A_{13} \\ \tilde{H}_{21} & A_2 & A_{23} \\ A_{31} & A_{32} & A_3 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} u_i \tag{17}$$

Assume that system (17) is composed of two independent subsystems like:

$$x^1_{i+1} = \begin{bmatrix} A_1 & \tilde{H}_{12} \\ \tilde{H}_{21} & A_2 \end{bmatrix} x^1_i + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} u^1_i \tag{18}$$

$$x^2_{i+1} = A_3 x^2_i + B_3 u^2_i \tag{19}$$

Systems (18) and (19) are stable, then any instability in (17) is because of A_{13}, A_{23}, A_{31} and A_{32} interaction matrices. Go to Step 3.

Step 3. Calculate stability conditions on A_{13}, A_{23}, A_{31} and A_{32} using Algorithm 1.

Step 4. Design the global controller from modified interactions as follows:

$$\begin{aligned} & BK_g = F \\ & F = \tilde{H} - H = \\ & \quad \begin{bmatrix} 0 & \tilde{H}_{12} & \tilde{H}_{13} \\ \tilde{H}_{21} & 0 & \tilde{H}_{23} \\ \tilde{H}_{31} & \tilde{H}_{32} & 0 \end{bmatrix} - \begin{bmatrix} 0 & A_{12} & A_{13} \\ A_{21} & 0 & A_{23} \\ A_{31} & A_{32} & 0 \end{bmatrix} \end{aligned} \tag{20}$$

Now, test the stability of closed-loop global system.

3 Simulation results

In this section the algorithm 1 is simulated to demonstrate its performance. Consider a three-region energy resources system as shown in Fig. 2[16].

This system can be described by following discrete state space:

$$x_{i+1} = \begin{bmatrix} A_1 & A_{12} & 0 \\ A_{21} & A_2 & 0 \\ 0 & 0 & A_3 \end{bmatrix} x_i + \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix} u_i \tag{21}$$

where

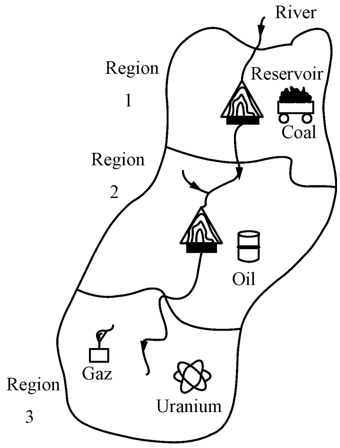


Fig. 2 A three-region energy resources system

$$A_1 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0.5 & 0.75 \\ 0.75 & 0.5 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 1 & 0.5 & 0.25 & 0 \\ 0 & 0.5 & 0 & 0.2 \\ 0.5 & 0 & 0.5 & 0.25 \\ 0 & 0.2 & 0.25 & 0.25 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.5 & 0.25 & 0.2 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$
$$A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 1.5 & 0 & 0 & 0 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{22}$$

It is composed of three coupled subsystems. Here, the aim is to stabilize it using proposed Algorithm 1.

Step 1. By computing eigenvalues of each subsystem, we can observe that all of them are unstable. First, we should set interactions to zero and stabilize every subsystem with local controllers. This part is not presented in Algorithm 1 because of stability Assumption 2.

Stabilizing subsystems.

The open loop eigenvalues of Subsystems are:

$$\begin{aligned} &\lambda_i^1 = 2.1622,\ 0.1689 \pm 0.3406\text{j} \\ &\lambda_i^2 = 1.2207,\ -0.0324,\ 0.5309 \pm 0.1364\text{j} \\ &\lambda_i^3 = 1.3618,\ 0.3975,\ 0.0092 \end{aligned} \tag{23}$$

where λ_i^j represents eigenvalues of j-th subsystem. Since they are unstable, we can stabilize them with static state feedback:

$$\begin{aligned} &u_1 = K_1 x = [0.7419\ 0.7715\ 1.2425]x \\ &u_2 = K_2 x = [0.7381\ 0.5462\ 0.3468\ 0.22]x \\ &u_3 = K_3 x = [1.1280\ 0.5911\ 0.5055]x \end{aligned} \tag{24}$$

Now substitute A_1, A_2 and A_3 for each subsystem closed-loop matrices $A_{1_{cl}}, A_{2_{cl}}$ and $A_{3_{cl}}$ and compose below matrix:

$$\begin{bmatrix} A_{1_{cl}} & A_{12} & 0 \\ A_{21} & A_{2_{cl}} & 0 \\ 0 & 0 & A_{3_{cl}} \end{bmatrix} \tag{25}$$

where $A_{j_{cl}}$ is the closed-loop matrix of each subsystem i.e., $A_{j_{cl}} = A_j - B_j K_j$ and $|\lambda_i(A_{j_{cl}})| < 1$.

The eigenvalues of the new system (25) are:

$$\begin{aligned} \lambda_i = {} & 1.5889,\ -0.7902,\ 0.7795,\ -0.3920,\ 0.2960 \\ & 0.2539,\ 0.1364,\ 0.6284,\ -0.0098,\ 0.0348 \end{aligned}$$

Clearly, it is unstable, therefore we can infer that the source of instability is interactions. Since there is no connection between subsystem (3) and set of subsystems (1) and (2), this subsystem does not affect the overall stability and we can use Algorithm 1.

Step 2. Replace the nonzero elements of A_{12} and A_{21} matrices with some parameters such as $a_1, a_2, \cdots$, and we get

$$A_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & a_1 & 0 & 0 \\ a_2 & 0 & 0 & 0 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 0 & a_3 \\ 0 & a_4 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{26}$$

Step 3. Calculate matrices A'_{12} and A'_{21} using (7) and (8), in form of a function of $a_1, a_2, \cdots$, and we get

$$A'_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -2.3193a_1 & 0.2588a_1 & -0.8022a_1 & -0.039a_1 \\ 2.9984a_2 & 1.234a_2 & 0.3745a_2 & -0.0179a_2 \end{bmatrix} \tag{27}$$

$$A'_{21} = \begin{bmatrix} 0.0066a_3 & 0.3263a_3 & 0.6066a_3 \\ 0.0099a_4 & 1.5105a_4 & 0.9098a_4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{28}$$

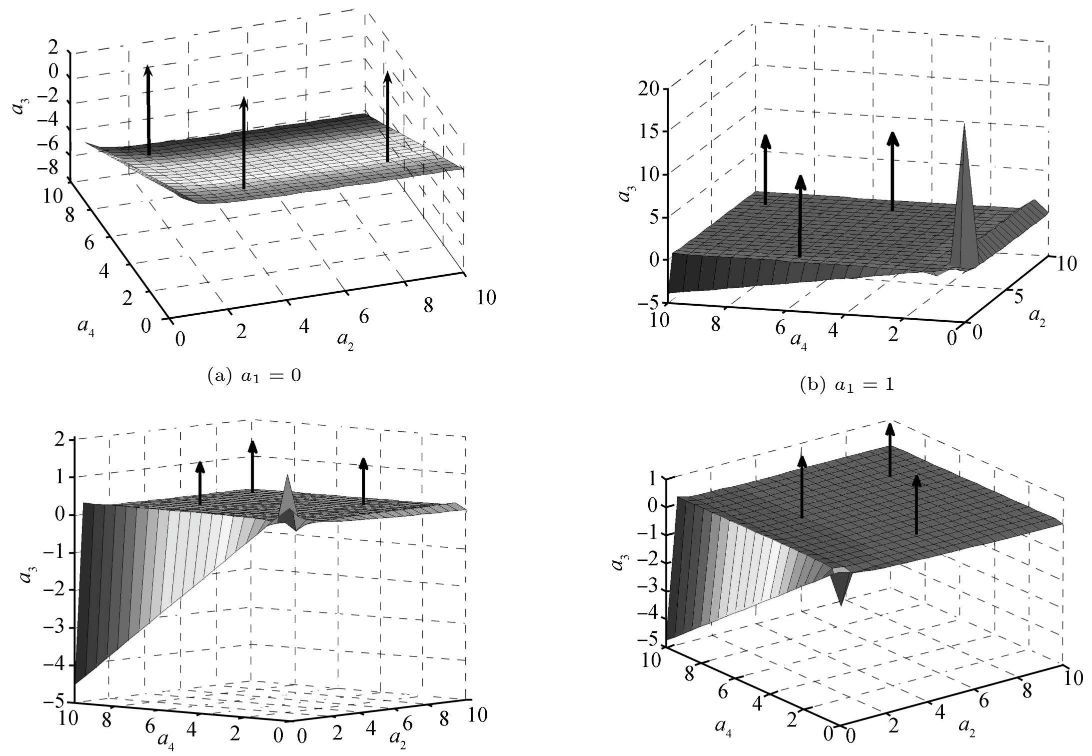
Step 4. Identify a_1, a_2, $\cdots$ using (10). To satisfy (10), first we should calculate:

$$\begin{aligned} &\det(I - A'_{21}A'_{12}) = \\ &\quad 1 - 0.3909a_1a_4 - 1.1227a_2a_4 - 0.7566a_1a_3 - \\ &\quad 1.8188a_2a_3 + 4.4128a_1a_2a_3a_4 + 0.0001a_1^2a_4a_3 + \\ &\quad 0.0002a_2^2a_3a_4 \end{aligned} \tag{29}$$

There are different choices to satisfy (10). One of them, e.g., is $a_1 = a_2 = a_4 = 0.5$ and $a_3 = 1$. Then, we can form $\tilde{H}$ as:

$$\tilde{H} = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0.5&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0 \\ 0&0.5&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \end{bmatrix} \tag{30}$$

Step 5. Compute the global controller from (11), where



(a) $a_1 = 0$

(b) $a_1 = 1$

(c) $a_1 = 2$

(d) $a_1 = 3$

Fig. 3 Different choices of a_2, a_3, a_4 for the given a_1

$$F = \tilde{H} - H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{31}$$

Step 6. We can now form closed-loop system and then test its stability.

Note that all of its eigenvalues are in unit circle.

As we expected this controller reduces conservativeness. Other previous methods ideally lead to a controller with 4 nonzero elements in matrix F as it is illustrated in the following matrix:

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{32}$$

which leads to neutralizing all the interactions. It is worth mentioning that selected values for a_i, $i = 1, \cdots, 4$ are not optimal. As it is mentioned earlier there are different choices for $a_1, a_2, \cdots$. Fig. 3 shows possible values of a_2, a_3, and a_4 by initializing parameter a_1 with several values.

4 Conclusion

In this paper, we addressed the stability of two-level control for discrete time linear large scale systems. We have developed two new algorithms to design a global controller for these systems, which led to conservatism reduction. As it is stated, there are several solutions for those algorithms and an optimal one might be found among them. We exploited one of these solutions and applied to a three-region energy resources system. The effectiveness of the design was shown in simulation results. Our future work intends to consider conservatism reduction in H_∞ control of large scale systems.

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