Control of Spatially Interconnected Systems with Random Communication Losses

LI Hui[1, 2] WU Qing-He[1, 2] HUANG Huang[1, 2]

Abstract This paper deals with analysis and synthesis problems of spatially interconnected systems where communicated information may get lost between subsystems. Spatial shift operator and temporal forward shift operator are introduced to model the interconnected systems as discrete time-space multidimensional linear systems with Markovian jumping parameters which reflect the state of communication channels. To ensure the whole system′s well-posedness and mean square stability for a given packet loss rate, a condition is derived through analysis. Then a procedure of designing distributed dynamic output feedback controllers is proposed. The controllers have the same structure as the plants and are solved within the linear matrix inequality (LMI) framework. Finally, we apply these results to study the effect of communication losses on the multiple vehicle platoon control system, which further illustrates the effectiveness of the proposed model and method.

Key words Spatially interconnected system, communication loss, Markovian jump linear system, linear matrix inequality (LMI)
DOI 10.3724/SP.J.1004.2010.00258

Many large-scale systems consist of similar units which interact with their nearest neighbors, such as automated vehicle platoon, airplane formation flight, and multi-robot formation system. Researchers have studied the control of these systems in different manners, such as centralized control[1], decentralized control[2], and distributed control[3]. These studies have assumed perfect communications between subsystems, i.e. each subsystem could get information about the state of either the global leader or its neighbors.

In most cases, the information is transmitted through a network. The use of network may lead to intermittent losses or delay of communicated information, which will deteriorate the control performance or even destabilize the system. For example, Tanner et al.[4] showed that the formation string stability would be lost if the followers could not get information of their leader. So it is necessary to study the stability problem of interconnected systems under the effect of communication packet losses between subsystems.

Teo et al.[5] have looked into the problem of multiple vehicle control over a lossy data link. They presented an estimation method to handle the dropouts and analyzed the stability of the closed loop system. However, their method was based on a centralized control scheme. As a result, the system had a large number of inputs and outputs, which imposed high costs of computation. Furthermore, the centralized scheme was technically more sensitive to model transformation: if the number of subsystems was changed, the controller would be transformed into a completely different one.

Typically, the subsystems often have their own sensors and actuators. So, taking advantage of this characteristic, we design distributed controllers and present a comparison with the former study[5]. In this framework, each subsystem uses its own controller, which can reduce calculation burden. The controllers have the same structure as the plants, i.e., controllers also interact with their neighbors, which are called structured distributed controllers. This scheme respects the interconnection topology and is easy to reconfigure.

Manuscript received November 14, 2008; accepted April 13, 2009
Supported by Innovation Foundation of Graduate Students in Beijing Institute of Technology
1. School of Automation, Beijing Institute of Technology, Beijing 100081, P. R. China 2. Key Laboratory of Complex System Intelligent Control and Decision (Beijing Institute of Technology), Ministry of Education, Beijing 100081, P. R. China

There are already some results for this kind of structured distributed control of large-scale spatially interconnected systems. In the light of conditions developed in [6] for the well-posedness, stability, and performance of these systems, the authors used models with identical units, which are well coupled with their nearest neighbors. The results were later extended to parameter-varying systems[7] and heterogeneous systems[8]. In [9], the results of [6, 8] were further extended to a larger class of interconnection topology with both ideal and non-ideal interconnections considered over an arbitrary graph. It was later applied to the analysis and synthesis problems of spatially interconnected systems with small communication delays between subsystems[10]. However, relatively little attention has been paid to the communication packet losses problem in the structured distributed control community, with the exception of [11]. Although [11] is based on the method of [9] and has discussed distributed control of interconnected system over failing channels, but the result of [11] is a litter conservative: the designed controllers guaranteed a given performance level just for arbitrary failures model. In addition, the degree of influence of different packet loss rates on control performance could not be manifested from their simulation results.

On the other hand, the field of networked control systems (NCSs) has rich literatures on studying the effect of packet losses[12−14]. But what have been considered are the control problems with lossy data links between sensors and controllers, or between controllers and actuators. Furthermore, the results are derived in the context of a single plant, hence cannot be applied directly to large-scale distributed control systems.

In this paper, we analyze the effect of random information losses between subsystems on the stability of spatially interconnected systems, due to the unreliable communication network, as shown in Fig. 1. we find a group of structured distributed controllers such that the whole system is mean square stable for a given packet loss rate. We consider spatially interconnected discrete time systems and model the interconnected systems with random packet losses as discrete time-space multidimensional systems with Markovian jumping parameters. Analysis conditions and controller synthesis method are developed to ensure the mean square stability of the whole interconnected system. The results are stated in terms of linear matrix inequalities (LMIs) and are thus tractable for computation.

The paper is organized as follows: In Section 1, the model of spatially interconnected systems with random communication dropouts between subsystems is presented. Then, the results of discrete time Markovian jump linear systems (MJLSs) are extended to our spatially interconnected systems in Section 2. An analysis condition is developed to ensure the well-posedness and mean square stability of the whole system. It is noteworthy that the existing outcomes on MJLS are just dynamic in time domain, while our results are based on interconnected systems not only in temporal direction but also in spatial direction. Section 3 gives an LMI condition for controller synthesis. An example with simulation results is presented in Section 4 and the conclusion is given in Section 5.

1 Spatially interconnected systems with random connection

The system we considered in this paper is shown as Fig. 1. It consists of several similar subsystems that communicate and interact with their neighbors through network. Compared with the interconnection over arbitrary graph, this structure is more regular. It can reduce the number of connections and lighten the communication burden, while ensuring sufficient information exchange. This model of interconnected systems is employed in many cases as well[2, 15].

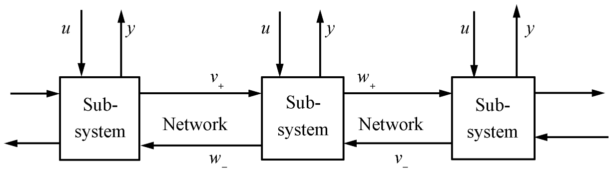

Fig. 1 The model of spatially interconnected systems

The signals we deal with in the system are functions indexed by $L+1$ independent variables $x = x(k, s_1, s_2, \cdots, s_L)$, where $k \in \mathbf{Z}^+$ denotes the temporal variable, and $s_i \in \mathbf{Z}$ expresses the spatial variables, L denotes the spatial dimension. For simplicity, we confine ourselves to one-dimensional systems, i.e., $L = 1$. Our approach can be extended directly to the cases of higher dimensions[6].

The basic building block (shown in Fig. 2) is a finite dimensional linear discrete time system governed by the following state-space equations

$$\begin{bmatrix} x_T(k+1,s) \\ w(k,s) \\ y(k,s) \end{bmatrix} = \begin{bmatrix} A_{TT} & A_{TS} & B_T \\ A_{ST} & A_{SS} & B_S \\ C_T & C_S & 0 \end{bmatrix} \begin{bmatrix} x_T(k,s) \\ v(k,s) \\ u(k,s) \end{bmatrix} \tag{1}$$

where $x_T(k,s) \in \mathbf{R}^{m_0}$ are the state variables, $u(k,s) \in \mathbf{R}^p$ are control inputs, and $y(k,s) \in \mathbf{R}^q$ are the measured output signals. $v(k,s) = \begin{bmatrix} v_+(k,s) \\ v_-(k,s) \end{bmatrix}$ and $w(k,s) = \begin{bmatrix} w_+(k,s) \\ w_-(k,s) \end{bmatrix}$ are the interconnected variables between subsystems. We assume that $v_+(k,s)$ and $w_+(k,s)$ are of the same size as m_+ and $v_-(k,s)$ and $w_-(k,s)$ are of the same size as m_-.

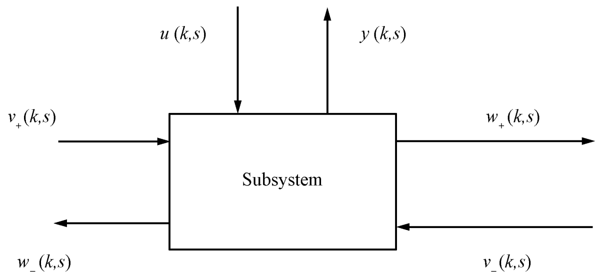

Fig. 2 A basic subsystem block

For a signal $x(k,s)$, define a spatially shift operator S as

$$\begin{aligned} Sx(k,s) &= x(k,s+1) \\ S^{-1}x(k,s) &= x(k,s-1) \end{aligned} \tag{2}$$

and a temporal forward shift operator T as

$$Tx(k,s) = x(k+1,s) \tag{3}$$

Since there may be communication losses between subsystems, the connections between subsystems are modeled as

$$\begin{aligned} v_+(k,s+1) &= \gamma(k) w_+(k,s) \\ v_-(k,s-1) &= \delta(k) w_-(k,s) \end{aligned} \tag{4}$$

Here, $\gamma(k)$ and $\delta(k)$ are Bernoulli processes respectively and are independent of each other, governed by $P(\gamma(k) = 0) = P(\delta(k) = 0) = p$ and $P(\gamma(k) = 1) = P(\delta(k) = 1) = 1 - p$. $\gamma(k) = 1$ or $\delta(k) = 1$ implies that the packet is transmitted perfectly, while $\gamma(k) = 0$ or $\delta(k) = 0$ means a packet is lost. Thus, p represents the communication packet loss rate.

Using the definition of spatially shift operator S, (4) can be rewritten as

$$\begin{aligned} v_+(k,s) &= \gamma(k) S^{-1} w_+(k,s) \\ v_-(k,s) &= \delta(k) S w_-(k,s) \end{aligned} \tag{5}$$

So, the plant model (1) becomes

$$\begin{aligned} Tx_T(k,s) &= A_{TT}x_T(k,s) + A_{TS}^{1}\gamma(k)S^{-1}w_+(k,s) + \\ &\quad A_{TS}^{-1}\delta(k)Sw_-(k,s) + B_T u(k,s) \\ w_+(k,s) &= A_{ST}^{1}x_T(k,s) + A_{SS}^{1,1}\gamma(k)S^{-1}w_+(k,s) + \\ &\quad A_{SS}^{1,-1}\delta(k)Sw_-(k,s) + B_S^{1}u(k,s) \\ w_-(k,s) &= A_{ST}^{-1}x_T(k,s) + A_{SS}^{-1,1}\gamma(k)S^{-1}w_+(k,s) + \\ &\quad A_{SS}^{-1,-1}\delta(k)Sw_-(k,s) + B_S^{-1}u(k,s) \\ y(k,s) &= C_T x_T(k,s) + C_S^{1}\gamma(k)S^{-1}w_+(k,s) + \\ &\quad C_S^{-1}\delta(k)Sw_-(k,s) \end{aligned} \tag{6}$$

Let $x_{S_1}(k,s) = w_+(k,s-1)$, $x_{S_{-1}}(k,s) = w_-(k,s+1)$, and group the spatial variables together as $x_S(k,s) = \left[x_{S_1}^{\mathrm{T}}(k,s), x_{S_{-1}}^{\mathrm{T}}(k,s)\right]^{\mathrm{T}}$. Define structured operator $\triangle_S = \mathrm{diag}\left\{SI_{m_+}, S^{-1}I_{m_-}\right\}$. Then, model (6) can be rewritten as

$$\begin{bmatrix} Tx_T(k,s) \\ \triangle_S x_S(k,s) \\ y(k,s) \end{bmatrix} = \begin{bmatrix} A_{TT} & A_{TS}(\theta(k)) & B_T \\ A_{ST} & A_{SS}(\theta(k)) & B_S \\ C_T & C_S(\theta(k)) & 0 \end{bmatrix} \begin{bmatrix} x_T(k,s) \\ x_S(k,s) \\ u(k,s) \end{bmatrix} \tag{7}$$

where $\theta(k) = (\gamma(k), \delta(k))$ are the time-varying parameters of the system matrices which reflect the network communication situations. We can find that the interconnected systems have four modes:

1) $\theta(k) = 0$, i.e., $\gamma(k) = 0$, $\delta(k) = 0$. The packets are lost from both sides.

2) $\theta(k) = 1$, i.e., $\gamma(k) = 0$, $\delta(k) = 1$. The packet from the left side is dropped, but the right side packet is received.

3) $\theta(k) = 2$, i.e., $\gamma(k) = 1$, $\delta(k) = 0$. The packet from the right side is dropped, but the left side packet is received.

4) $\theta(k) = 3$, i.e., $\gamma(k) = 1$, $\delta(k) = 1$. The packets are received from both sides.

With the probability distributions of $\gamma(k)$ and $\delta(k)$, the probabilities of being in the four modes are as follows, respectively,

$$\begin{aligned} P(\theta(k)=0) &= P(\gamma(k)=0,\delta(k)=0)=p^2=p_0 \\ P(\theta(k)=1) &= P(\gamma(k)=0,\delta(k)=1)=p(1-p)=p_1 \\ P(\theta(k)=2) &= P(\gamma(k)=1,\delta(k)=0)=(1-p)p=p_2 \\ P(\theta(k)=3) &= P(\gamma(k)=1,\delta(k)=1)=(1-p)^2=p_3 \end{aligned} \tag{8}$$

It follows that the model (7) is a multidimensional discrete time-space MJLS. We will analyze the well-posedness and stability of such system in the next section.

2 Analysis of Markovian jump linear interconnected systems

2.1 Well-posedness

A system is well-posed if it is physically realizable. The reader can refer to [16] for a thorough discussion of well-posedness.

From the model (7), we can eliminate the interconnection variables, and express the system as

$$\begin{aligned} Tx_T(k,s) &= \tilde{A}(\theta(k))x_T(k,s)+\tilde{B}(\theta(k))u(k,s) \\ y(k,s) &= \tilde{C}(\theta(k))x_T(k,s)+\tilde{D}(\theta(k))u(k,s) \end{aligned} \tag{9}$$

where

$$\begin{bmatrix} \tilde{A}(\theta(k)) & \tilde{B}(\theta(k)) \\ \tilde{C}(\theta(k)) & \tilde{D}(\theta(k)) \end{bmatrix} = \begin{bmatrix} A_{TT} & B_T \\ C_T & D \end{bmatrix} + \begin{bmatrix} A_{TS}(\theta(k)) \\ C_S(\theta(k)) \end{bmatrix} \times (\triangle_S - A_{SS}(\theta(k)))^{-1} \begin{bmatrix} A_{ST} & B_S \end{bmatrix} \tag{10}$$

with the assumption that $\triangle_S - A_{SS}(\theta(k))$ is invertible for whatever $\theta(k)$. It is important to note that this assumption is equivalent to the well-poseness of interconnected systems[6].

2.2 Stability

For discrete-time MJLS

$$\begin{aligned} x(k+1) &= A(\theta(k))x(k)+B(\theta(k))u(k) \\ y(k) &= C(\theta(k))x(k) \\ x(0) &= x_0, \theta(0)=\theta_0 \end{aligned} \tag{11}$$

where Markovian chain $\theta(k)$ takes values in a finite set $\Omega = \{0, 1, 2, \cdots, N\}$ and has a transition probability matrix $P = [p_{ij}]$, with $p_{ij} = P(\theta(k+1) = j|\theta(k) = i), i, j \in \Omega$, subject to $p_{ij} \geq 0$ and $\sum_{j=1}^{N} p_{ij} = 1, \forall i \in \Omega$, we discuss the following forms of stability[17]:

Definition 1. The MJLS given by (11) with $u = 0$ is:

1) mean square stable (MSS) if for every initial state (x_0, θ_0), $\lim_{k\to\propto} \mathrm{E}(\|x(k)\|^2) = 0$;

2) stochastically stable (SS) if for every initial state (x_0, θ_0), $\mathrm{E}(\sum_{k=0}^{\propto} \|x(k)\|^2) <\propto$;

3) exponentially mean square stable (EMSS) if for every initial state (x_0, θ_0), there exist constants $0 < \alpha < 1$ and $\beta > 0$ such that $\forall k \geq 0, \mathrm{E}(\|x(k)\|^2) < \beta\alpha^k \|x_0\|^2$;

4) almost surely stable if for every initial state (x_0, θ_0), $P(\lim_{k\to\propto} \|x(k)\| = 0) = 1$.

It was shown in [18] that for MJLS (11), the first three definitions of stability are actually equivalent and anyone of them implies almost surely stability. So we would like to study the MSS of the system sequentially.

2.3 Analysis condition for well-posedness and stability of interconnected systems

Define $\triangle_m = \mathrm{diag}\{TI_{m_0}, \triangle_S\}$, and $\boldsymbol{x}(k,s) = \left[x_T^{\mathrm{T}}(k,s), x_S^{\mathrm{T}}(k,s)\right]^{\mathrm{T}}$. We get a more briefer expression of model(7)

$$\begin{bmatrix} \triangle_m \boldsymbol{x}(k,s) \\ y(k,s) \end{bmatrix} = \begin{bmatrix} A(\theta(k)) & B(\theta(k)) \\ C(\theta(k)) & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(k,s) \\ u(k,s) \end{bmatrix} \tag{12}$$

where

$$\begin{aligned} A(\theta(k)) &= \begin{bmatrix} A_{TT} & A_{TS}(\theta(k)) \\ A_{ST} & A_{SS}(\theta(k)) \end{bmatrix} \\ B(\theta(k)) &= \begin{bmatrix} B_T \\ B_S \end{bmatrix} \\ C(\theta(k)) &= \begin{bmatrix} C_T & C_S(\theta(k)) \end{bmatrix} \end{aligned} \tag{13}$$

For notation simplicity, when $\theta(k) = j$, i.e., the system is in mode $j \in \Omega = \{0, 1, 2, 3\}$, we use the following notations: $A(\theta(k)) = A_j, B(\theta(k)) = B_j$, and $C(\theta(k)) = C_j$. The system is denoted as $M = \{A_j, B_j, C_j\}$.

Define the following set of scaling matrices

$$\mathbf{G} = \{G = G^{\mathrm{T}}, G = \mathrm{diag}\{G_0, G_1, G_{-1}\}, \det(G) \neq 0, G_0 > 0\} \tag{14}$$

Assume that the probability of the plant being in mode j at time $k+1$ is independent of the plant′s mode at time k, i.e., $p_{ij} = p_j$ for all $i, j \in \{0, 1, 2, 3\}$. We have the following theorem, whose proof can be consulted from Appendix.

Theorem 1. Assume that $p_{ij} = p_j$, $\forall i, j \in \{0, 1, 2, 3\}$, the spatially interconnected systems which are composed of the subsystems (12) with random communication losses between adjacent units are well-posed and MSS, if there exists a matrix $G \in \mathbf{G}$, such that

$$G - \sum_{j=0}^{3} p_j A_j^{\mathrm{T}} G A_j > 0 \tag{15}$$

Remark 1. This analysis condition is independent of the number of blocks in interconnected systems. The size of the resulting condition (15) is only a function of the size of the basic building block (12). This is propitious to system reconfigurability: elements can be added or removed without affecting the well-posedness and stability of the whole interconnected system.

Remark 2. In the absence of spatial dynamics, condition (15) simply reduces to the MSS condition of discrete-time Markovian jump linear system[17−18]. Here, matrix G is structured, which may have some conservativeness. Note that only G_0 is required to be positive definite. This is because the system is causal in temporal direction but not causal in spatial direction.

3 Controller synthesis

In this section, Theorem 1 is used to derive an LMI condition for controller synthesis. We assume that the controller has the same structure and the same packet loss rate as the plant, i.e., if the plant cannot receive the packet from its neighbors, neither can the controller.

Consider a plant $M^G = \{A_j^G, B_j^G, C_j^G\}$ which has a state space realization as model (12) and (13). The controller to be designed is captured by

$$\begin{bmatrix} \triangle_m^K x^K(k,s) \\ u(k,s) \end{bmatrix} = \begin{bmatrix} A^K(\theta(k)) & B^K(\theta(k)) \\ C^K(\theta(k)) & 0 \end{bmatrix} \begin{bmatrix} x^K(k,s) \\ y(k,s) \end{bmatrix} \tag{16}$$

where $x^K(k,s) \in \mathbf{R}^{m^K}$ are the controller states. Again, for $\theta(k) = j \in \{0,1,2,3\}$, we use A_j^K, B_j^K, and C_j^K to denote the matrices of the controller in four modes, respectively.

So the closed loop system matrices are

$$A_{cl,j}^C = \begin{bmatrix} A_j^G & B_j^G C_j^K \\ B_j^K C_j^G & A_j^K \end{bmatrix}, \quad j = 0,1,2,3 \tag{17}$$

The transition probabilities are the same as the plant′s, i.e., $p_{cl,j} = p_j$. Note that the system matrices (17) are not in the standard form as given in (12). For $x_{cl}(k,s) = \left[x_T(k,s)^{\mathrm{T}}, x_S(k,s)^{\mathrm{T}}, x_T^K(k,s)^{\mathrm{T}}, x_S^K(k,s)^{\mathrm{T}}\right]^{\mathrm{T}}$, the temporal and spatial variables are not grouped together as they are in (12). Define a permutation matrix P as follows

$$P = \begin{bmatrix} I_{m_0^G} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{m_0^K} & 0 & 0 \\ 0 & I_{m_+^G} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{m_+^K} & 0 \\ 0 & 0 & I_{m_-^G} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{m_-^K} \end{bmatrix} \tag{18}$$

Note that $PP^{\mathrm{T}} = P^{\mathrm{T}}P = I$, and that

$$P\mathrm{diag}\{\triangle_m^G, \triangle_m^K\}P^{\mathrm{T}} = \mathrm{diag}\{TI_{m_0^G+m_0^K},\ SI_{m_+^G+m_+^K}, S^{-1}I_{m_-^G+m_-^K}\} \tag{19}$$

The closed loop system matrices which have the same structure as the model (12) are expressed as

$$A_{cl,j} = PA_{cl,j}^C P^{\mathrm{T}},\ \ B_{cl,j} = PB_{cl,j}^C,\ \ C_{cl,j} = C_{cl,j}^C P^{\mathrm{T}} \tag{20}$$

Then, by Theorem 1, we can conclude that the closed loop interconnected systems are MSS if there exists a matrix $G \in \mathbf{G}$, such that

$$G - \sum_{j=0}^{3} p_{cl,j} A_{cl,j}^{\mathrm{T}} G A_{cl,j} > 0 \tag{21}$$

Since $\det(G) \neq 0$, let $Z = G^{-1}$. By pre- and post-multiplying this condition (21) with Z, and using Schur complements lemma four times, we have the following theorem.

Theorem 2. If $p_{cl,ij} = p_{cl,j}$ for all $i,j \in \{0,1,2,3\}$, then the closed loop interconnected systems are well-posed and MSS if there exists a matrix $Z = \mathrm{diag}\{Z_0, Z_1, Z_{-1}\}$, where $Z_i = \begin{bmatrix} Z_i^G & Z_i^{GK} \\ (Z_i^{GK})^{\mathrm{T}} & Z_i^K \end{bmatrix}$, and $Z_0 > 0$, such that

$$\begin{bmatrix} Z & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} \\ \sqrt{p_0}A_{cl,0}Z & Z & 0 & 0 & 0 \\ \sqrt{p_1}A_{cl,1}Z & 0 & Z & 0 & 0 \\ \sqrt{p_2}A_{cl,2}Z & 0 & 0 & Z & 0 \\ \sqrt{p_3}A_{cl,3}Z & 0 & 0 & 0 & Z \end{bmatrix} > 0 \tag{22}$$

$(\cdot)^{\mathrm{T}}$ denotes matrix entries which can be inferred from the symmetry of the matrix.

This theorem gives a bilinear matrix inequality condition for the closed loop system to be mean square stable. It is linear in the controller parameters which are embedded in $A_{cl,j}$ for a fixed scaling matrix Z, or in Z for fixed controller matrices. Our goal is to design controllers to make the whole interconnected system MSS. The approach of variable substitution is used to obtain an equivalent LMI condition which is more convenient for controller synthesis[16, 19].

Let P be the permutation matrix defined in (18). Denote $\bar{Z} = P^{\mathrm{T}}ZP$, Pre- and post-multiply condition (22) with P^{T} and P, respectively. The inequality (22) becomes

$$\begin{bmatrix} \bar{Z} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} \\ \sqrt{p_0}A_{cl,0}^C\bar{Z} & \bar{Z} & 0 & 0 & 0 \\ \sqrt{p_1}A_{cl,1}^C\bar{Z} & 0 & \bar{Z} & 0 & 0 \\ \sqrt{p_2}A_{cl,2}^C\bar{Z} & 0 & 0 & \bar{Z} & 0 \\ \sqrt{p_3}A_{cl,3}^C\bar{Z} & 0 & 0 & 0 & \bar{Z} \end{bmatrix} > 0 \tag{23}$$

Recall the structure of scaling matrix Z, it follows that the matrix $\bar{Z}$ inherits the following structure

$$\bar{Z} = \begin{bmatrix} Z^G & Z^{GK} \\ (Z^{GK})^{\mathrm{T}} & Z^K \end{bmatrix} \tag{24}$$

where

$$\begin{aligned} Z^G &= \mathrm{diag}\{Z_0^G, Z_1^G, Z_{-1}^G\}, \quad Z_0^G > 0 \\ Z^{GK} &= \mathrm{diag}\{Z_0^{GK}, Z_1^{GK}, Z_{-1}^{GK}\} \\ Z^K &= \mathrm{diag}\{Z_0^K, Z_1^K, Z_{-1}^K\}, \quad Z_0^K > 0 \end{aligned} \tag{25}$$

Define the following set of scaling matrices

$$\mathbf{X}^G = \{X^G : X^G = \mathrm{diag}\{X_0^G, X_1^G, X_{-1}^G\},\ X_i^G \in \mathbf{R}_S^{m_i^G \times m_i^G}, X_0^G > 0\} \tag{26}$$

Now we are in the position to state the main result for controller synthesis.

Theorem 3. There exists $\bar{Z}$ as shown in (24), and A_j^K, B_j^K, C_j^K for $j = 0,1,2,3$ such that the inequality (23) holds if and only if there exist matrices R^G and Z^G in $\mathbf{X}^G$, and matrices L_j, F_j, and W_j for $j = 0,1,2,3$, such that the inequality (30) holds.

Proof. Since Z is invertible, so is $\bar{Z}$.

For $\bar{Z} = \begin{bmatrix} Z^G & Z^{GK} \\ (Z^{GK})^{\mathrm{T}} & Z^K \end{bmatrix}$, denote $\bar{R} = \bar{Z}^{-1} = \begin{bmatrix} R^G & R^{GK} \\ (R^{GK})^{\mathrm{T}} & R^K \end{bmatrix}$, where R^G and Z^G in $\mathbf{X}^G$. Then $\bar{Z}\bar{R} = I$. So

$$\bar{Z}\begin{bmatrix} R^G \\ (R^{GK})^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} \tag{27}$$

and

$$\bar{Z}\begin{bmatrix} R^G & I \\ (R^{GK})^{\mathrm{T}} & 0 \end{bmatrix} = \begin{bmatrix} I & Z^G \\ 0 & (Z^{GK})^{\mathrm{T}} \end{bmatrix} \tag{28}$$

Define $T_1 = \begin{bmatrix} R^G & I \\ (R^{GK})^{\mathrm{T}} & 0 \end{bmatrix}$ and $T_2 = \begin{bmatrix} I & Z^G \\ 0 & (Z^{GK})^{\mathrm{T}} \end{bmatrix}$. Then

$$\bar{Z}T_1 = T_2 \tag{29}$$

$$\begin{bmatrix} \begin{bmatrix} R^G & I \\ I & Z^G \end{bmatrix} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} \\ \sqrt{p_0}\begin{bmatrix} YA_0^G + L_0C_0^G & W_0 \\ A_0^G & A_0^GX + B_0^GF_0 \end{bmatrix} & \begin{bmatrix} R^G & I \\ I & Z^G \end{bmatrix} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} \\ \sqrt{p_1}\begin{bmatrix} YA_1^G + L_1C_1^G & W_1 \\ A_1^G & A_1^GX + B_1^GF_1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} R^G & I \\ I & Z^G \end{bmatrix} & (\cdot)^{\mathrm{T}} & (\cdot)^{\mathrm{T}} \\ \sqrt{p_2}\begin{bmatrix} YA_2^G + L_2C_2^G & W_2 \\ A_2^G & A_2^GX + B_2^GF_2 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} R^G & I \\ I & Z^G \end{bmatrix} & (\cdot)^{\mathrm{T}} \\ \sqrt{p_3}\begin{bmatrix} YA_3^G + L_3C_3^G & W_3 \\ A_3^G & A_3^GX + B_3^GF_3 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} R^G & I \\ I & Z^G \end{bmatrix} \end{bmatrix} > 0 \tag{30}$$

and

$$T_1^{\mathrm{T}}A_{cl,j}^C\bar{Z}T_1 = T_1^{\mathrm{T}}A_{cl,j}^CT_2 = \begin{bmatrix} R^GA_j^G + R^{GK}B_j^KC_j^G & R^GA_j^GZ^G + R^{GK}B_j^KC_j^GZ^G + R^GB_j^GC_j^K(Z^{GK})^{\mathrm{T}} + R^{GK}A_j^K(Z^{GK})^{\mathrm{T}} \\ A_j^G & A_j^GZ^G + B_j^GC_j^K(Z^{GK})^{\mathrm{T}} \end{bmatrix} \tag{31}$$

$$T_1^{\mathrm{T}}\bar{Z}T_1 = T_1^{\mathrm{T}}T_2 = \begin{bmatrix} R^G & I \\ I & Z^G \end{bmatrix} \tag{32}$$

For $j = 0, 1, 2, 3$, denote

$$\begin{aligned} F_j &= C_j^K(Z^{GK})^{\mathrm{T}} \\ L_j &= R^{GK}B_j^K \\ W_j &= R^GA_j^GZ^G + R^GB_j^GF_j + \\ &\quad L_jC_j^GZ^G + R^{GK}A_j^K(Z^{GK})^{\mathrm{T}} \end{aligned} \tag{33}$$

It follows that, given matrices R^G and Z^G in $\mathbf{X}^G$, and non-singular matrices R^{GK} and Z^{GK}, we can acquire the controller matrices A_j^K, B_j^K, and C_j^K exclusively from F_j, L_j, and W_j.

On the other side, pre- and post-multiplying inequality (23) by $\mathrm{diag}\{T_1^{\mathrm{T}}, T_1^{\mathrm{T}}, T_1^{\mathrm{T}}, T_1^{\mathrm{T}}, T_1^{\mathrm{T}}\}$, and $\mathrm{diag}\{T_1, T_1, T_1, T_1, T_1\}$, we can get the matrix inequality (30) with denotations (33). □

Remark 3. The inequality (30) is an LMI on matrix variables F_j, L_j, W_j, R^G, and Z^G. We can get solutions if the LMI is feasible. However, when we construct controllers, the non-singular matrices R^{GK} and Z^{GK} should be acquired at first. The following lemma shows the existence condition for R^{GK} and Z^{GK}. The reader can refer to [6] for the proof.

Lemma 1. Given in X_1, Y_1 in $\mathbf{R}_S^{n\times n}$, let $k = \mathrm{rank}(I - X_1Y_1)$. Then there exist X_2, Y_2 in $\mathbf{R}^{n\times k}$, and X_3, Y_3 in $\mathbf{R}_S^{k\times k}$ such that

$$\begin{bmatrix} Y_1 & Y_2 \\ (Y_2)^{\mathrm{T}} & Y_3 \end{bmatrix}^{-1} = \begin{bmatrix} X_1 & X_2 \\ (X_2)^{\mathrm{T}} & X_3 \end{bmatrix} \tag{34}$$

Let $k = m_0^K + m_+^K + m_-^K = \mathrm{rank}(I - R^GZ^G)$, and make a full-rank factorization on matrix $(I - R^GZ^G)$. Then, we can obtain R^{GK} and Z^{GK} from $R^{GK}(Z^{GK})^{\mathrm{T}} = I - R^GZ^G$.

Remark 4. When the LMI (30) has feasible solutions, we can construct a mean square stabilizing controller from the above procedure. The Matlab LMI toolbox makes this feasibility problem computationally tractable.

4 An illustrative example

In this section, we study an example of multiple vehicle platoon system which suffers from the communication losses when information is transmitted between vehicles. The technique presented above is used.

Consider a group of three vehicles platoon as shown in Fig. 3. In this framework, we view vehicles as masses and the connections through network between them as virtual springs. This model is adopted from [20]. The i-th vehicle′s dynamics is governed by

$$\begin{aligned} m_i\ddot{x}_i &= k_{i+1}(x_{i+1} - x_i) - k_{i-1}(x_i - x_{i-1}) + u_i \\ y_i &= x_i \end{aligned} \tag{35}$$

where x_i is the position of i-th vehicle from its equilibrium position, u_i is its control input, y_i is the measurement signal, m_i is the mass of the vehicle, k_{i+1} and k_{i-1} are pre- and post-virtual spring coefficients respectively. They are just used to reflect the influence coefficients of other vehicles on vehicle i. Here, we assume that $m_i = m$, and $k_i = k$ for $\forall i$. The goal of platoon is to keep the vehicle moving with desired constant spacing behind its preceding one. This is equivalent to the stability with respect to the zero equilibrium of the plant′s error dynamical model (35).

$$\begin{bmatrix} \dot{x}_2(t,s) \\ \dot{x}_1(t,s) \\ x_{S_1}(t,s+1) \\ x_{S_{-1}}(t,s-1) \\ y(t,s) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{2k}{m} & \frac{k}{m}\gamma(t) & \frac{k}{m}\delta(t) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_2(t,s) \\ x_1(t,s) \\ x_{S_1}(t,s) \\ x_{S_{-1}}(t,s) \end{bmatrix} + \begin{bmatrix} \frac{1}{m} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t,s) \quad (36)$$

$$A_0^K = \begin{bmatrix} -0.3103 & -0.5163 & -0.0009 & -0.0009 \\ 0.1780 & -0.2241 & -0.0002 & -0.0002 \\ 0.0001 & -0.0010 & 0.0000 & 0.0000 \\ 0.0001 & -0.0010 & 0.0000 & 0.0000 \end{bmatrix}, B_0^K = \begin{bmatrix} 0.6776 \\ -0.9069 \\ -0.9999 \\ -0.9999 \end{bmatrix}, C_0^K = \begin{bmatrix} 2.3314 & 0.6029 & 0.0021 & 0.0021 \end{bmatrix}$$

$$A_1^K = \begin{bmatrix} -0.3110 & -0.5197 & 0.0003 & -0.0011 \\ 0.1782 & -0.2241 & 0.0001 & -0.0003 \\ 0.0001 & -0.0010 & 0.0000 & 0.0000 \\ 0.0001 & -0.0010 & 0.0000 & 0.0000 \end{bmatrix}, B_1^K = \begin{bmatrix} 0.6756 \\ -0.9065 \\ -0.9999 \\ -0.9999 \end{bmatrix}, C_1^K = \begin{bmatrix} 2.3326 & 0.6051 & 0.0026 & 0.9993 \end{bmatrix}$$

$$A_2^K = \begin{bmatrix} -0.3110 & -0.5197 & 0.0003 & -0.0011 \\ 0.1782 & -0.2241 & 0.0001 & -0.0003 \\ 0.0001 & -0.0010 & 0.0000 & 0.0000 \\ 0.0001 & -0.0010 & 0.0000 & 0.0000 \end{bmatrix}, B_2^K = \begin{bmatrix} 0.6756 \\ -0.9065 \\ -0.9999 \\ -0.9999 \end{bmatrix}, C_2^K = \begin{bmatrix} 2.3326 & 0.6051 & 0.9993 & 0.0026 \end{bmatrix}$$

$$A_3^K = \begin{bmatrix} -0.3116 & -0.5231 & -0.0001 & 0.0001 \\ 0.1786 & -0.2242 & -0.0000 & -0.0000 \\ 0.0001 & -0.0010 & 0.0000 & 0.0000 \\ 0.0001 & -0.0010 & 0.0000 & 0.0000 \end{bmatrix}, B_3^K = \begin{bmatrix} 0.6737 \\ -0.9062 \\ -0.9999 \\ -0.9999 \end{bmatrix}, C_3^K = \begin{bmatrix} 2.3337 & 0.6074 & 0.9998 & 0.9998 \end{bmatrix} \quad (37)$$

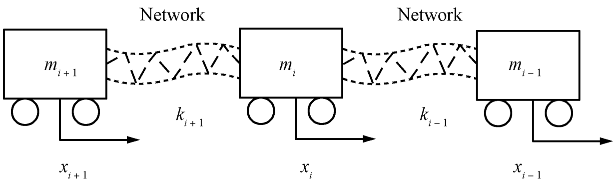

Fig. 3 Vehicle platoon model

Since the vehicle state information x_i is transmitted through network for compact hardwiring, vehicle i may not get its neighbors′ states sometimes when there are communication losses. We want to design controllers to ensure that the whole formation system is still stable under this situation. Here, by stability we mean mean square stability.

Defining $x_1(t,s) = x_i, x_2(t,s) = \dot{x}_1(t,s)$, $x_{S_{-1}}(t,s) = x_{i+1}, x_{S_1}(t,s) = x_{i-1}$, and $u(t,s) = u_i$, we can get the realization of system as (36).

In (36), $\gamma(t)$ and $\delta(t)$ describe the network packet loss situations which have been explained in detail in Section 1.

Assume that $k = m$ and the sampling time is 0.5 s. Let the package loss rate $p = 0.2$. Checking the feasibility of LMI (30) and designing controllers using Theorem 3, we have the mean square stabilizing controllers (37).

Note that the controllers′ structure satisfies

$$\begin{aligned} A_0^K &\approx A_0^G + B_0^K C_0^G - B_0^G C_0^K \\ A_1^K &\approx A_1^G + B_1^K C_1^G - B_1^G C_1^K \\ A_2^K &\approx A_2^G + B_2^K C_2^G - B_2^G C_2^K \\ A_3^K &\approx A_3^G + B_3^K C_3^G - B_3^G C_3^K \end{aligned} \quad (38)$$

In other words, the controllers can play the role of observers based on $x^K(k)$ being estimates of $-x(k)$, the states of the augmented plant. Furthermore, B_j^K are the observer gains and C_j^K are the feedback gains. Finally, we can find that $C_j^K \approx \begin{bmatrix} 2.33 & 0.61 & \gamma(k) & \delta(k) \end{bmatrix}$, with $\gamma(k)$ and $\delta(k)$ reflecting the situation of communication channels. Thus, once the estimates of the states are obtained, we do not have the problem of varying the feedback gain based on the loss or arrival of the information from its neighboring subsystems.

Let the initial errors of each vehicle be $x_1 = 5\,\text{m}, x_2 = 20\,\text{m}$, and $x_3 = -10\,\text{m}$, respectively. We can get the simulation result as shown in Fig. 4, which indicates that the platoon system is MSS with the designed controllers. Compared with the result of packet loss rate $p = 0.7$ (See Fig. 5), it shows that the higher the packet loss rate, the poorer the control performance.

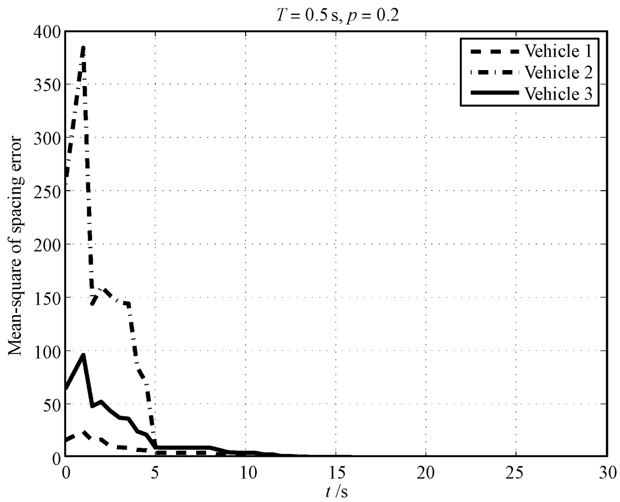

Fig. 4 Simulation results with $p = 0.2$

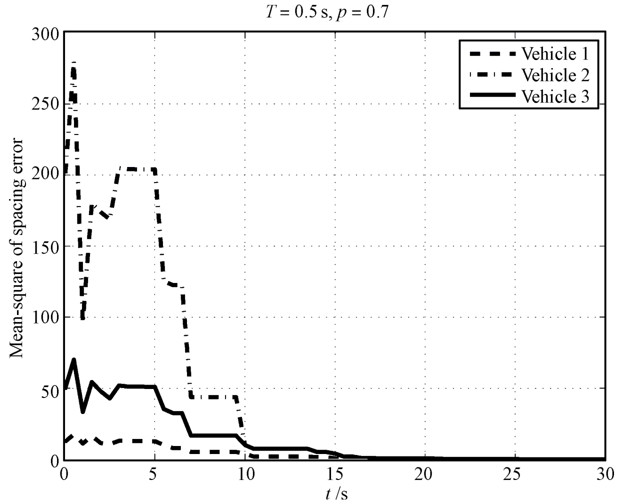

Fig. 5 Simulation results with $p = 0.7$

5 Conclusions

In this paper, we considered the distributed control of spatially interconnected systems with the effect of random communication packet losses between subsystems. We modeled the spatially interconnected systems with uncertain stochastic connection as discrete time-space MJLSs, and got the mean square stable condition for such systems. An LMI condition was developed for the existence of the mean square stabilizing controllers, and the designing procedure was presented too.

An example of multiple vehicle platoon was given to show the effectiveness of our method. However, we can see that mean square stability is a weak condition. In particular, it is possible to find stabilizing controllers even for very high packet loss rates, but it is obvious that the performance with different packet loss rates is quite different. So these results need to be extended to measure performance versus different packet loss rates, which is our next work. On the other hand, this work is about non-ideal coupling with the nearest neighbors, and we will try to further extend the results to arbitrary interconnections.

Appendix

Proof of Theorem 1.

1) Well-posedness

Define the following set

$$\mathbf{S}\triangle = \{\triangle : \triangle = \mathrm{diag}\{\delta_1 I_{m_1}, \delta_2 I_{m_2}, \cdots, \delta_L I_{m_L}\}, |\delta_i| = 1\} \tag{A1}$$

It is obvious that operator $\triangle_S = \mathrm{diag}\{SI_{m_1}, S^{-1}I_{m_{-1}}\} \in \mathbf{S}\triangle$.

Referring to (10), the well-posedness of the system is equivalent to the invertibility of $(\triangle_S - A_{SS}(\theta(k)))$ for whatever $\theta(k)$. So it needs to be shown that $(\triangle_S - A_{SS,j})$ is invertible for $j = 0, 1, 2, 3$ if there exist a matrix $G \in \mathbf{G}$, such that $G - \sum_{j=0}^{3} p_j A_j^{\mathrm{T}} G A_j > 0$.

Since $(\triangle_S - A_{SS,j}) = \triangle_S \left(I - \triangle_S^{-1} A_{SS,j}\right)$, the invertibility of $(\triangle_S - A_{SS,j})$ is equivalent to the invertibility of $\left(I - \triangle_S^{-1} A_{SS,j}\right)$. We use the method of reduction to absurdity to complete the proof.

Assume that, for $j = 0, 1, 2, 3$, $\left(I - \triangle_S^{-1} A_{SS,j}\right)$ is not invertible, then there exists $x_j \neq 0$, such that $x_j = \triangle_S^{-1} A_{SS,j} x_j$.

As $G - \sum_{j=0}^{3} p_j A_j^{\mathrm{T}} G A_j > 0$, it follows that

$$\begin{bmatrix} G_1 & \\ & G_{-1} \end{bmatrix} - \sum_{j=0}^{3} p_j A_{SS,j}^{\mathrm{T}} \begin{bmatrix} G_1 & \\ & G_{-1} \end{bmatrix} A_{SS,j} > 0 \tag{A2}$$

and then

$$x_j^{\mathrm{T}} A_{SS,j}^{\mathrm{T}} \left(\triangle_S^{-1}\right)^{\mathrm{T}} \begin{bmatrix} G_1 & \\ & G_{-1} \end{bmatrix} \triangle_S^{-1} A_{SS,j} x_j - \sum_{j=0}^{3} p_j x_j^{\mathrm{T}} A_{SS,j}^{\mathrm{T}} \begin{bmatrix} G_1 & \\ & G_{-1} \end{bmatrix} A_{SS,j} x_j > 0 \tag{A3}$$

Since the matrix parameter $A_{SS,j}$ varies with time according to some probability distribution, we make mathematical expectation on (A3),

$$\sum_{j=0}^{3} p_j x_j^{\mathrm{T}} A_{SS,j}^{\mathrm{T}} \begin{bmatrix} G_1 & \\ & G_{-1} \end{bmatrix} \left(\left(\triangle_S^{-1}\right)^{\mathrm{T}} \triangle_S^{-1} - I \right) A_{SS,j} x_j > 0 \tag{A4}$$

However, as $\triangle_S \in \mathbf{S}\triangle$, i.e., $\left(\triangle_S^{-1}\right)^{\mathrm{T}} \triangle_S^{-1} - I = 0$, a contradiction is encountered. So the assumption that $\left(I - \triangle_S^{-1} A_{SS,j}\right)$ is not invertible for $j = 0, 1, 2, 3$ is false, which implies that $(\triangle_S - A_{SS,j})$ is invertible for $j = 0, 1, 2, 3$, as required.

2) Mean square stability

Suppose that there exists a matrix $G \in \mathbf{G}$ such that inequality (15) holds. Let $x = \left[x_T(k,s)^{\mathrm{T}},\ x_{S_1}(k,s)^{\mathrm{T}}, x_{S_{-1}}(k,s)^{\mathrm{T}}\right]^{\mathrm{T}}$ and $x^1 = \left[x_T(k+1,s)^{\mathrm{T}}, x_{S_1}(k,s+1)^{\mathrm{T}}, x_{S_{-1}}(k,s-1)^{\mathrm{T}}\right]^{\mathrm{T}}$. Multiplying inequality (15) with x^{T} on the left and x on the right, and making mathematical expectation, we can obtain the inequality (A5), where the dynamical characteristic of the system (7) is exploited.

$$\sum_{j=0}^{3} p_j \left(x^{\mathrm{T}} \mathrm{diag}\{G_0, G_1, G_{-1}\} x - x^{1\mathrm{T}} \mathrm{diag}\{G_0, G_1, G_{-1}\} x^1 \right) > 0 \tag{A5}$$

Summing up (A5) over all spatial coordinates, where the well-posedness is used to guarantee that the finite sums exist, we have

$$\begin{aligned} \sum_{j=0}^{3} p_j \big(& \langle x_T(k,s), G_0 x_T(k,s) \rangle_{l_2} + \\ & \langle x_{S_1}(k,s), G_1 x_{S_1}(k,s) \rangle_{l_2} + \\ & \langle x_{S_{-1}}(k,s), G_{-1} x_{S_{-1}}(k,s) \rangle_{l_2} - \\ & \langle T x_T(k,s), G_0 (T x_T(k,s)) \rangle_{l_2} - \\ & \langle S x_{S_1}(k,s), G_1 S x_{S_1}(k,s) \rangle_{l_2} - \\ & \langle S^{-1} x_{S_{-1}}(k,s), G_{-1} S^{-1} x_{S_{-1}}(k,s) \rangle_{l_2} \big) > 0 \end{aligned} \tag{A6}$$

Note that

$$\begin{aligned} & \langle S x_{S_1}(k,s), G_1 (S x_{S_1}(k,s)) \rangle_{l_2} = \\ & \quad \langle x_{S_1}(k,s), S^{-1} G_1 (S x_{S_1}(k,s) \rangle_{l_2} = \\ & \quad \langle x_{S_1}(k,s), G_1 x_{S_1}(k,s) \rangle_{l_2} \end{aligned} \tag{A7}$$

So does

$$\begin{aligned} & \langle S^{-1} x_{S_{-1}}(k,s), G_{-1} (S^{-1} x_{S_{-1}}(k,s) \rangle_{l_2} = \\ & \quad \langle x_{S_{-1}}(k,s), G_{-1} x_{S_{-1}}(k,s) \rangle_{l_2} \end{aligned} \tag{A8}$$

We have

$$\sum_{j=0}^{3} p_j \left(\langle x_T(k,s), G_0 x_T(k,s)\rangle_{l_2} - \langle Tx_T(k,s), G_0(Tx_T(k,s))\rangle_{l_2}\right) > 0 \quad \text{(A9)}$$

It follows that there exists $\varepsilon > 0$, such that

$$\sum_{j=0}^{3} p_j \left(\langle Tx_T, G_0(Tx_T)\rangle_{l_2} - \langle x_T, G_0 x_T\rangle_{l_2}\right) < -\varepsilon \langle x_T, x_T\rangle_{l_2} \quad \text{(A10)}$$

Denote

$$V\left(x_T(k,s)\right) = \langle x_T(k,s), G_0 x_T(k,s)\rangle_{l_2} \quad \text{(A11)}$$

as the Lyapunov function.

Then

$$\begin{aligned} &\mathrm{E}_{\theta(k)}\left[V(x_T(k+1,s)|x_T(k,s))\right] = \\ &\sum_{j=0}^{3} p_j \langle Tx_T(k,s), G_0(Tx_T(k,s))\rangle_{l_2} < \\ &V\left(x_T(k,s)\right) - \varepsilon\langle x_T(k,s), x_T(k,s)\rangle_{l_2} \le \\ &\left[1 - \frac{\varepsilon}{\lambda_{\max}(G_0)}\right] V(x_T(k,s)) = \\ &\alpha V(x_T(k,s)) \end{aligned} \quad \text{(A12)}$$

where $\mathrm{E}_{\theta(k)}[\cdot]$ denotes the mathematical expectation taken over $\theta(k)$. We can choose ε small enough so that

$$\alpha = 1 - \frac{\varepsilon}{\lambda_{\max}(G_0)} > 0 \quad \text{(A13)}$$

Inequality (A12) follows since

$$\langle z, Gz\rangle_{l_2} \le \lambda_{\max}(G)\cdot\langle z, z\rangle_{l_2}, \forall z \quad \text{(A14)}$$

We will show that the following lemma is true for $\forall n \in \mathbf{N}$ by induction.

Lemma A1.

$$P_n: \mathrm{E}_{\theta(k),\cdots,\theta(k+n-1)}\left[V(x_T(k+n,s)|x_T(k,s))\right] < \alpha^n V\left(x_T(k,s)\right), \forall k \quad \text{(A15)}$$

P_1 follows directly from inequality (A12). Assume that P_n is true, then

$$\begin{aligned} &\mathrm{E}_{\theta(k),\cdots,\theta(k+n)}\left[V(x_T(k+n+1,s)|x_T(k,s))\right] = \\ &\sum_{j=0}^{3} p_j \mathrm{E}_{\theta(k+1),\cdots,\theta(k+n)}\left[V(x_T(k+n+1,s)|x_T(k+1,s))\right] < \\ &\sum_{j=0}^{3} \alpha^n p_j V\left(x_T(k+1,s)\right) = \\ &\alpha^n \mathrm{E}_{\theta(k)}\left[V(x_T(k+1,s)|x_T(k,s))\right] < \\ &\alpha^{n+1} V(x_T(k,s)) \end{aligned} \quad \text{(A16)}$$

That means P_{n+1} holds as well. Therefore,

$$\begin{aligned} &\mathrm{E}_{\theta(k),\cdots,\theta(N-1)}\left[\sum_{k=0}^{n} V(x_T(k,s)|x_T(0,s))\right] < \\ &(1+\alpha+\cdots+\alpha^n)\,V\left(x_T(0,s)\right) = \\ &\frac{1-\alpha^n}{1-\alpha} V(x_T(0,s)) \end{aligned} \quad \text{(A17)}$$

Take the limit as n tends to infinity gives inequality (A19) below

$$\begin{aligned} &\lim_{n\to\infty} \mathrm{E}_{\theta(0),\cdots,\theta(n-1)}\left[\sum_{k=0}^{n} \langle x_T(k,s), x_T(k,s)\rangle_{l_2} | x_T(0,s)\right] \le \\ &\lim_{n\to\infty} \frac{1}{\lambda_{\min}(G_0)} \times \\ &\quad \mathrm{E}_{\theta(0),\cdots,\theta(n-1)}\left[\sum_{k=0}^{n} \langle x_T(k,s), G_0 x_T(k,s)\rangle_{l_2} | x_T(0,s)\right] \le \end{aligned} \quad \text{(A18)}$$

$$\frac{1}{\lambda_{\min}(G_0)} \frac{1}{1-\alpha} \langle x_T(0,s), G_0 x_T(0,s)\rangle_{l_2} = \quad \text{(A19)}$$

$$\frac{\lambda_{\max}(G_0)}{\varepsilon\lambda_{\min}(G_0)} V(x_T(0,s)) < \infty \quad \text{(A20)}$$

Inequality (A18) follows because $\lambda_{\min}(G_0)\langle z, z\rangle_{l_2} \le \langle z, G_0 z\rangle_{l_2}$, for $\forall z$. Equality (A20) holds by definition of α(A13), and the final cost is finite because $G_0 > 0$ implies $\lambda_{\min}(G_0) > 0$ and $\lambda_{\max}(G_0) > 0$.

Then, we can conclude that the system is stochastically stable and hence MSS. □

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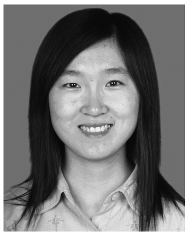

LI Hui Ph. D. candidate at the School of Automation, Beijing Institute of Technology. Her research interest covers interconnected systems, multi-robot formation system, and Markovian jump systems. Corresponding author of this paper.
E-mail: huili03855@bit.edu.cn

WU Qing-He Professor at Beijing Institute of Technology. His research interest covers H_∞ control, robust control, and multidimensional systems.
E-mail: qinghew@bit.edu.cn

HUANG Huang Ph. D. candidate at the School of Automation, Beijing Institute of Technology. Her research interest covers network-based distributed control, multi-agent cooperation, and spatially interconnected systems.
E-mail: hhuang33@gmail.com