

Multiple Model-based Adaptive Reconfiguration Control for Actuator Fault

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Abstract In this paper, an active fault tolerant control strategy is developed to compensate for the effect of actuator fault in the presence of non-measurable rate on the actuator second-order dynamics. The proposed control scheme is a combination between multiple model and adaptive reconfiguration control. By means of the designed method, the system output can track that of the reference model asymptotically, and the simulation results have illustrated the effectiveness of the proposed algorithms.

Key words Actuator fault, multiple model, fault accommodation, flight control

Flight control systems are generally subject to various faults caused by actuators, sensors, and unexpected parameter changes in the system. A fault in a dynamical system is a deviation of the system structure or the system parameters from the nominal situation. Examples are the freezing of an actuator, the loss of a sensor, or the disconnection of a system component due to wear or damages. These faults may cause serious performance deterioration and may lead to instability, possibly resulting in catastrophic accidents^[1-4].

Reconfiguration is likely to be a feature of future generations of flight control systems^[5-8]. The main motivation for reconfiguration is greater survivability. A large number of techniques for reconfiguration have been proposed and some of those have actually been flight tested. A common feature of these schemes is that they were developed for linearized models of aircraft dynamics and are well suited for actuator faults which have moderate effect on the closed-loop dynamics. A neural network-based adaptive control approach was developed and evaluated on a simulation of F/A-18 aircraft. However, neural networks require on-line adjustment of a large number of parameters (weights), which may require very complex tuning in different flight regimes. In [9], interactive multiple model (IMM)-based fault detection and diagnosis (FDD) approach was proposed, and an integrated FDD and reconfigurable control was designed for discrete-time system. The method from [10] is based on multiple model fault detection and identification (FDI) and fast switching among multiple controllers based on the on-line information obtained from the FDI subsystem. However, because a finite number of models were used and if none of the models coincided with the actual damage, the resulting control system could only assure that the output errors were bounded, but not that they tended to zero asymptotically. It was also shown that adaptive control using a single model may not be adequate for achieving this task in the presence of critical actuator faults^[11]. This is due to the fact that in a particular flight

regime, aircraft dynamics immediately after the fault may be very far from its nominal (no-fault) dynamics. Hence, single model-based adaptive controller may be too slow to bring the closed-loop system close to the new operating regime, which may result in unacceptably large transients. Moreover, most of the available fault reconfiguration techniques are not explicitly designed for the case of higher-order actuator dynamics, and in many situations only the output of the actuator is available for measurement, i.e., its rate is not measurable, which makes the related reconfiguration problem highly challenging.

From these points of view, in this paper, a new adaptive reconfiguration algorithm is developed based on multiple models. Fault is parameterized by second-order actuator dynamics and estimated accurately. In a multiple model scheme, a bank of parallel models (observers) are constructed, each of which is based on a model that describes the system in the presence of a particular actuator fault. The simulation results have proved that the proposed adaptive reconfiguration algorithm guarantees the convergence of the tracking error.

The rest of the paper is organized as follows. In Section 1, the system under consideration is described and some preliminary definitions are stated. Moreover, the adaptive laws for unknown parameters are designed. In Section 2, a multiple model-based adaptive reconfigurable control algorithm is proposed and multiple model scheme and switching mechanism are presented consecutively. Section 3 provides simulation results showing the effectiveness of the developed control scheme, and finally some conclusions are drawn in Section 4.

1 Preliminaries and problem formulation

1.1 Problem statement

Consider the following linearized aircraft models:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (1)$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \quad (2)$$

$$\dot{u}_{1i} = u_{2i} \quad (3)$$

$$\dot{u}_{2i} = -\lambda_{1i}u_{1i} - \lambda_{2i}u_{2i} + \lambda_{1i}u_{ci} \quad (4)$$

$$i = 1, 2, \dots, m$$

where $\mathbf{x} \in \mathbf{R}^n$ and $\mathbf{y} \in \mathbf{R}^p$ denote the state vector and output vector, respectively. $\mathbf{u} \in \mathbf{R}^m$ is the input vector whose components may fail during the system operation. u_{1i} is actuator position and u_{2i} is actuator rate. $\lambda_{1i} \gg \lambda_{2i}$, $\lambda_{1i} \gg 1$, $\lambda_{1i}/\lambda_{2i} \geq 20$, and the pair (A, B) is known and controllable.

1.2 Actuator fault modeling

Many aircraft accidents were caused by operational faults in the control surfaces, such as rudder and elevator. Because actuators are the link between the control commands issued by the controller and the physical actions performed for the system, the probability of occurrence of faults in actuators is higher and more severe compared with other components. Typical actuator faults are classed into two categories^[5]: 1) total LOE: the case of total Loss-of-effectiveness (LOE) includes Lock-in-place (LIP), float, and Hard-over-fault (HOF); 2) partial LOE.

The case of LIP faults, the effector "freezes" at a certain condition and does not respond to subsequent commands. HOF is characterized by the effector moving to the upper or lower position limit regardless of the command. The

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speed of response is limited by the effector rate limit. Float fault occurs when the effector “floats” with zero moment and does not contribute to the control authority. Loss of effectiveness is characterized by lowering the effector gain with respect to its nominal value.

The parameterization of different types of actuator faults is

$$u_i(t) = \begin{cases} u_{ci}(t), & k_i(t) = 1, \forall t > t_0, \text{ No fault} \\ k_i(t)u_c(t), & 0 < \epsilon \leq k_i(t) < 1, \forall t \geq t_{fi}, \text{ LOE} \\ 0, & k_i(t) = 0, \forall t \geq t_{fi}, \text{ Float} \\ u_{ci}(t_{fi}), & k_i(t) = 0, \forall t \geq t_{fi}, \text{ LIP} \\ u_{im} \text{ or } u_{iM}, & k_i(t) = 0, \forall t \geq t_{fi}, \text{ HOF} \end{cases}$$

Using only two parameters, i.e., σ_i and k_i , a second-order actuator fault model which describes both partial and total LOE is proposed in the form^[12]:

$$\dot{u}_{1i} = \sigma_i u_{2i} \quad (5)$$

$$\dot{u}_{2i} = -[\lambda_{2i} + (1 - \sigma_i)\beta_i]u_{2i} + \sigma_i \lambda_{1i}(k_i u_{ci} - u_{1i}) \quad (6)$$

$$i = 1, 2, \dots, m$$

where t_{Fi} denotes the time instant of fault of the i -th actuator, $\mathbf{u}_c \in \mathbf{R}^m$ is the output of the controller, and $u_i = k_i u_{ci}$, u_{im} , and u_{iM} are the upper or lower position limit. $k_i \in (\epsilon, 1]$ denotes the actuator effectiveness coefficient and models partial LOE fault, and $\epsilon \ll 1$. σ_i is the actuator mobility coefficient. If $\sigma_i = 1$, the actuator is operational, while during float, LIP, and HOF, $\sigma_i = 0$, and $\sigma_i = k_i = 1$ for no fault cases.

Assumption 1. λ_{2i} is sufficiently large to assure fast convergence of u_{2i} to zero in the case of fault, so that β_i in (6) can be set to zero.

Rewrite the actuator fault model as

$$\dot{u}_{1i} = u_{2i} \quad (7)$$

$$\dot{u}_{2i} = -\lambda_{2i}u_{2i} + \sigma_i \lambda_{1i}(k_i u_{ci} - u_{1i}) \quad (8)$$

where only u_{1i} is measurable, i.e., \dot{u}_{1i} is not measurable.

Remark 1. In [12], the rate was measurable and adaptive control reconfiguration was relatively easy to implement, but when it was not measurable, the reconfiguration problem became more challenging.

In order to arrive at stable adjustment laws for the estimates of σ_i and k_i , the following filtered variables are introduced:

$$\begin{aligned} u_{1i}^F &= \frac{1}{s + \lambda_{F_i}} u_{1i} \\ u_{2i}^F &= \frac{1}{s + \lambda_{F_i}} u_{2i} \\ u_{ci}^F &= \frac{1}{s + \lambda_{F_i}} u_{ci} \end{aligned} \quad (9)$$

where $\lambda_{F_i} > 0$ denotes time constants of the filters. Then, by adding the term λ_{F_i} to both sides of (8) and considering (9), we have

$$u_{2i} = (\lambda_{F_i} - \lambda_{2i})u_{2i}^F - \sigma_i \lambda_{1i}(u_{1i}^F - u_{ci}^F) \quad (10)$$

with modulo exponentially decaying initial conditions. Further,

$$u_{2i}^F = \frac{1}{s + \lambda_{F_i}} u_{2i} = \frac{1}{s + \lambda_{F_i}} s u_{1i} = u_{1i} - \lambda_{F_i} u_{1i}^F \quad (11)$$

also with modulo exponentially decaying initial conditions. Hence, u_{2i} can be expressed in terms of measurable of obtainable signals.

1.3 Estimating fault parameters

After the fault, the values σ_i and k_i are all unknown, a separate adaptive observer is run for each actuator, and estimates of σ_i and k_i are generated on-line. In order to estimate them, a series of adaptive observers are constructed as

$$\begin{aligned} \dot{\hat{u}}_{1i} &= -\tau_i(\hat{u}_{1i} - u_{1i}) + (\lambda_{F_i} - \lambda_{2i})u_{2i}^F + \\ &\lambda_{1i}\hat{\sigma}_i(\hat{k}_i u_{ci}^F - u_{1i}^F) + \lambda_{1i}\xi_i \end{aligned} \quad (12)$$

where $\tau_i = \lambda_{1i}/\lambda_{2i}$ and signal ξ_i are to be designed to assure the stability of the overall system. Then, we have the following error dynamics:

$$\dot{\hat{e}}_{ui} = -\tau_i \hat{e}_{ui} + \lambda_{1i} \phi_{\sigma i} (\hat{k}_i u_{ci}^F - u_{1i}^F) + \sigma_i \lambda_{1i} \phi_{k i} u_{ci}^F + \lambda_{1i} \xi_i \quad (13)$$

where $\hat{e}_{ui} = \hat{u}_{1i} - u_{1i}$ denotes the actuator position error, $\phi_{\sigma i} = \hat{\sigma}_i - \sigma_i$ and $\phi_{k i} = \hat{k}_i - k_i$ are parameter errors. Using the singular perturbation arguments, we have

$$\hat{e}_{ui} \cong \lambda_{1i} (\phi_{\sigma i} \theta_{\sigma i}^F + \sigma_i \phi_{k i} \theta_{k i}^F + \xi_i) \quad (14)$$

where $\theta_{\sigma i}^F = \hat{k}_i u_{ci}^F - u_{1i}^F$ and $\theta_{k i}^F = u_{ci}^F$.

Theorem 1. The following adaptive laws assure that $\hat{e}_{ui} \in L^\infty \cap L^2$:

$$\dot{\hat{\sigma}}_i = \text{Proj}_{[0,1]} \{-\gamma_{\sigma i} \lambda_{2i} \hat{e}_{ui} (\hat{k}_i u_{ci} - u_{1i})\}, \quad \hat{\sigma}_i(0) = 1 \quad (15)$$

$$\dot{\hat{k}}_i = \text{Proj}_{[\epsilon,1]} \{-\gamma_{k i} \lambda_{2i} \hat{e}_{ui} u_{ci}\}, \quad \hat{k}_i(0) = 1 \quad (16)$$

$$\dot{\xi}_i = -\lambda_{2i} \xi_i - \theta_{\sigma i}^F \dot{\hat{\sigma}}_i - \theta_{k i}^F \dot{\hat{k}}_i, \quad \xi_i(0) = 0 \quad (17)$$

where $\theta_{\sigma i} = \hat{k}_i u_{ci} - u_{1i}$ and $\theta_{k i} = u_{ci}$. The projection operator is used to keep the parameter estimates within the parameter bounds.

Proof. Consider the following tentative Lyapunov function:

$$V(\hat{e}_{ui}, \phi_{\sigma i}, \phi_{k i}) = \frac{1}{2} [\hat{e}_{ui}^2 + \frac{\phi_{\sigma i}^2}{\gamma_{\sigma i}} + \sigma_i \frac{\phi_{k i}^2}{\gamma_{k i}}]$$

From the property of the adaptive algorithms that $\dot{\zeta} = \text{Proj}_{[-\bar{\zeta}, \bar{\zeta}]} \{-e\omega\}$, then $\zeta \dot{\zeta} \leq -e\zeta\omega$, and σ_i and k_i are constant for $t \geq t_{Fi}$, so we have that $\dot{\phi}_{\sigma i} = \dot{\hat{\sigma}}_i$ and $\dot{\phi}_{k i} = \dot{\hat{k}}_i$, which implies that $\phi_{\sigma i} \dot{\phi}_{\sigma i} \leq -\hat{e}_{ui} \phi_{\sigma i} \omega_{\sigma i}$. Considering these facts, we can obtain the first derivative of V along the trajectory of the system as

$$\begin{aligned} \dot{V} &= -\tau_i \hat{e}_{ui}^2 + \lambda_{1i} (\phi_{\sigma i} \omega_{\sigma i} + \sigma_i \phi_{k i} \omega_{k i}) + \\ &\frac{\phi_{\sigma i} \dot{\phi}_{\sigma i}}{\gamma_{\sigma i}} + \sigma_i \frac{\phi_{k i} \dot{\phi}_{k i}}{\gamma_{k i}} \leq -\tau_i \hat{e}_{ui}^2 \leq 0 \end{aligned}$$

So each \hat{e}_{ui} is bounded, $\phi_{\sigma i}$ and $\phi_{k i}$ are bounded due to the use of the projection algorithm. Upon integrating \dot{V} from 0 to ∞ , we obtain

$$V(0) - V(\infty) \geq \tau_i \int_0^\infty \hat{e}_{ui}^2(\tau) d\tau$$

Since the term on the left-hand side is bounded, it follows that $\hat{e}_{ui} \in L^\infty \cap L^2$. \square

2 Adaptive control reconfiguration

In this section, a reconfigurable controller will be designed to compensate for the effect of both total and partial LOE effectively. First, we rewrite (7) and (8) as

$$\ddot{u}_{1i} = \dot{u}_{2i} = -\lambda_{2i} u_{2i} + \sigma_i \lambda_{1i} (k_i u_{ci} - u_{1i}) \quad (18)$$

then add the term $\lambda_{1i}u_{1i}$ to both sides of (18) and have

$$\ddot{u}_{1i} + \lambda_{2i}\dot{u}_{1i} + \lambda_{1i}u_{1i} = \lambda_{1i}(\sigma_i k_i u_{ci} + (1 - \sigma_i)u_{1i}) \quad (19)$$

Equation (19) is divided by λ_{1i} to obtain

$$\frac{1}{\lambda_{1i}}\ddot{u}_{1i} + \frac{\lambda_{2i}}{\lambda_{1i}}\dot{u}_{1i} + u_{1i} = \sigma_i k_i u_{ci} + (1 - \sigma_i)u_{1i} \quad (20)$$

From the fact that $\lambda_{1i} \gg \lambda_{2i}$, $\lambda_{1i} \gg 1$, and using the singular perturbation arguments, we have the approximate lower order actuator fault model of the form:

$$u_{1i} \cong \sigma_i k_i u_{ci} + (1 - \sigma_i)\bar{u}_{1i} \quad (21)$$

Then, we rewrite system (1) as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{K}\sigma\mathbf{u}_c + \mathbf{B}(\mathbf{I} - \sigma)\bar{\mathbf{u}} \quad (22)$$

where

$$\begin{aligned} \mathbf{K} &= \text{diag}\{k_1 \quad k_2 \quad \cdots \quad k_m\} \\ \sigma &= \text{diag}\{\sigma_1 \quad \sigma_2 \quad \cdots \quad \sigma_m\} \\ \bar{\mathbf{u}} &= [\bar{u}_1 \quad \bar{u}_2 \quad \cdots \quad \bar{u}_m]^T \\ & \quad i = 1, 2, \dots, m \end{aligned}$$

and \bar{u}_i denotes the current position of any failed actuator.

Remark 2. With each of the actuator model, there are the following uncertainties: 1) unknown time of fault t_{Fi} ; 2) unknown LOE coefficient k_i ; 3) unknown value at which the actuator freezes.

Assumption 2. As in [2], we assume that system (1) is constructed such that for any up to $m - 1$ actuator faults, the remaining actuators can still achieve a desired control objective, when implemented with known parameters, which is the standard assumption for reconfiguration control.

2.1 Control objective

The transfer function of plant (1) with actuator fault is

$$\mathbf{y}(t) = W_p(s)\mathbf{u}_{cad}^*(t) + \bar{\mathbf{y}}(t) \quad (23)$$

where $W_p(s)$ is a $p \times p$ transfer matrix associated with the normal actuators, $\mathbf{u}_{cad}^*(t)$ is the nominal control vector, and $\bar{\mathbf{y}}(t)$ is the output with failed actuator.

The reference model to be track is of the form:

$$\mathbf{y}_m(t) = W_m(s)\mathbf{r} \quad (24)$$

where $W_m(s) = k_m Z_m(s)/R_m(s)$, $k_m \in \mathbf{R}$ is nonzero, and $R_m(s)$ and $Z_m(s)$ are monic coprime polynomials of relative degree n^* , $\mathbf{r} \in \mathbf{R}^m$ is a bounded external reference input signal vector, and all closed-loop systems are bounded.

The control objectives are to design a adaptive control controller $\mathbf{u}_{cad}(t)$ for (23) such that the output error asymptotically converges to zero, i.e., $\lim_{t \rightarrow \infty} \mathbf{e}_y(t) = \lim_{t \rightarrow \infty} (\mathbf{y}(t) - \mathbf{y}_m(t)) = \mathbf{0}$, and all closed-loop signals are bounded.

For the synthesis of the controller, we consider the following input and output filters^[11]:

$$\dot{\mathbf{w}}_1 = \mathbf{F}\mathbf{w}_1 + \mathbf{L}\mathbf{u}_{cad} \quad (25)$$

$$\dot{\mathbf{w}}_2 = \mathbf{F}\mathbf{w}_2 + \mathbf{L}\mathbf{y} \quad (26)$$

where $\mathbf{w}_1 = \Lambda(s)[\mathbf{u}_0](t)$, $\mathbf{w}_2 = \Lambda(s)[\mathbf{y}](t)$, $\Lambda(s) = A_0(s)/n(s)$, $A_0(s) = [I, sI, \dots, s^{l-2}]^T$, $n(s)$ is a monic stable polynomial of degrees $l - 1$, the pair (F, L) is an asymptotically stable and controllable. l denotes a known upper bound on the observability indices of $W_p(s)$.

Define $\mathbf{w}(t) = [\mathbf{w}_1^T(t), \mathbf{w}_2^T(t), \mathbf{y}^T(t), \mathbf{r}^T(t), 1]^T$, $\vartheta(t) = [\vartheta_1^T(t), \vartheta_2^T(t), \vartheta_{20}(t), \vartheta_3(t), \boldsymbol{\vartheta}_4(t)]^T$, $\vartheta_1 = [\vartheta_{11}, \dots, \vartheta_{1(l-1)}]^T$, $\vartheta_2 = [\vartheta_{21}, \dots, \vartheta_{2(l-1)}]^T$, $\vartheta_3, \vartheta_{ij} \in \mathbf{R}^{p \times p}$, $i = 1, 2, j = 1, 2, \dots, l - 1$, $\boldsymbol{\vartheta}_4 \in \mathbf{R}^p$ is used to cancel the effect of the failed actuators and $\boldsymbol{\vartheta}_4 = \mathbf{0}$ when there is no actuator fault. Then, we can obtain the following controller form:

$$\mathbf{u}_{cad}^*(t) = \vartheta^T(t)\mathbf{w}(t) \quad (27)$$

Remark 3. When the actuator fault pattern changes, to follow the plant output, the parameters of ϑ are also changed and unknown, hence, (27) becomes

$$\mathbf{u}_{cad}(t) = \hat{\vartheta}^T(t)\mathbf{w}(t) \quad (28)$$

where $\hat{\vartheta}(t) = [\hat{\vartheta}_1^T(t), \hat{\vartheta}_2^T(t), \hat{\vartheta}_{20}(t), \hat{\vartheta}_3(t), \hat{\boldsymbol{\vartheta}}_4(t)]^T$ is the estimate of $\vartheta(t)$.

2.2 Residual generation

The residuals play a vital role in fault detection in ensuring the safety of a flight control system. They represent the difference between the measurements and their normal-model values and can be used as signatures for detecting and isolating various faults.

Substituting (23) into (27), we have

$$\begin{aligned} \mathbf{u}_{cad}^* &= \vartheta_1^T \mathbf{w}_1 + \vartheta_2^T \mathbf{w}_2 + \vartheta_{20} [W_p(s)\mathbf{u}_0 + \bar{\mathbf{y}}(t)] + \\ & \quad \vartheta_3 \mathbf{r} + \boldsymbol{\vartheta}_4 = \\ & \quad \vartheta_1^T \Lambda(s)\mathbf{u}_{cad}^*(t) + \vartheta_2^T \Lambda(s)W_p(s)\mathbf{u}_{cad}^*(t) + \\ & \quad \vartheta_2^T \Lambda(s)\bar{\mathbf{y}}(t) + \vartheta_{20} W_p(s)\mathbf{u}_{cad}^*(t) + \\ & \quad \vartheta_{20} \bar{\mathbf{y}}(t) + \vartheta_3 \mathbf{r}(t) + \boldsymbol{\vartheta}_4 = \\ & \quad (I - \vartheta_1^T \Lambda(s) - \vartheta_2^T \Lambda(s)W_p(s) - \vartheta_{20} W_p(s))^{-1} \times \\ & \quad (\vartheta_2^T \Lambda(s)\bar{\mathbf{y}}(t) + \vartheta_{20} \bar{\mathbf{y}}(t) + \vartheta_3 \mathbf{r}(t) + \boldsymbol{\vartheta}_4) \end{aligned} \quad (29)$$

There exist $\vartheta_1, \vartheta_2, \vartheta_{20}$, and ϑ_3 such that

$$\begin{aligned} & \vartheta_3 W_m^{-1}(s)W_p(s) = \\ & I - \vartheta_1^T \Lambda(s) - \vartheta_2^T \Lambda(s)W_p(s) - \vartheta_{20} W_p(s) \end{aligned} \quad (30)$$

Substituting (29) and (30) into (23), we have

$$\begin{aligned} \mathbf{y}(t) &= W_p(s)(\vartheta_3 W_m^{-1}(s)W_p(s))^{-1} \times \\ & (\vartheta_2^T \Lambda(s)\bar{\mathbf{y}}(t) + \vartheta_{20} \bar{\mathbf{y}}(t) + \vartheta_3 \mathbf{r}(t) + \boldsymbol{\vartheta}_4) + \bar{\mathbf{y}}(t) = \\ & W_m(s)\mathbf{r}(t) + W_m(s)K_a \times \\ & (\vartheta_2^T \Lambda(s)\bar{\mathbf{y}}(t) + \vartheta_{20} \bar{\mathbf{y}}(t) + \vartheta_3 \varrho(s)\bar{\mathbf{y}}(t) + \boldsymbol{\vartheta}_4) = \\ & W_m(s)\mathbf{r}(t) + \mathbf{f}_a(t) \end{aligned} \quad (31)$$

where $\varrho(s)$ is a known modified left interactor matrix and for all fault patterns $\lim_{s \rightarrow \infty} \varrho(s)W_p(s) = K_a$, $\vartheta_3^{-1} = K_a$, and $W_m(s) = \varrho^{-1}(s)$. Defining $\mathbf{f}_a(t)$ as the exponentially decaying term, we have

$$\begin{aligned} \mathbf{f}_a(t) &\triangleq W_m(s)(K_a \vartheta_2^T \Lambda(s)\bar{\mathbf{y}}(t) + \vartheta_{20} \bar{\mathbf{y}}(t) + \\ & \quad \vartheta_3 \varrho(s)\bar{\mathbf{y}}(t) + \boldsymbol{\vartheta}_4(t)) \end{aligned} \quad (32)$$

which satisfies $\lim_{t \rightarrow \infty} \mathbf{f}_a(t) = \mathbf{0}$. So, ignoring $\mathbf{f}_a(t)$ in (31), we have

$$\mathbf{y}(t) = W_m(s)\mathbf{r}(t) \quad (33)$$

Operating both sides of (30) on \mathbf{u}_{cad} and considering (23),

we have

$$\begin{aligned} \mathbf{u}_{cad} = & \vartheta_3 W_m^{-1}(s)[\mathbf{y} - \bar{\mathbf{y}}](t) + \\ & \vartheta_1^T \Lambda(s) \mathbf{u}_{cad} + (\vartheta_2^T \Lambda(s) - \vartheta_{20})[\mathbf{y} - \bar{\mathbf{y}}](t) = \\ & \vartheta_1^T \mathbf{w}_1(t) + \vartheta_2^T \mathbf{w}_2(t) + \vartheta_{20} \mathbf{y}(t) - \vartheta_2^T \Lambda(s) \bar{\mathbf{y}}(t) + \\ & \vartheta_{20} \bar{\mathbf{y}}(t) + \vartheta_3 W_m^{-1}(s) \mathbf{y}(t) - \vartheta_3 W_m^{-1}(s) \bar{\mathbf{y}}(t) \end{aligned} \quad (34)$$

Substituting (28) into (34), we have

$$\begin{aligned} \tilde{\vartheta} \mathbf{w}(t) = & -\vartheta_3 \mathbf{r}(t) - \vartheta_4 - \vartheta_2^T \Lambda(s) \bar{\mathbf{y}}(t) - \vartheta_{20} \bar{\mathbf{y}}(t) + \\ & \vartheta_3 W_m^{-1}(s) \mathbf{y}(t) - \vartheta_3 W_m^{-1}(s) \bar{\mathbf{y}}(t) \end{aligned} \quad (35)$$

From (35), we can obtain

$$\begin{aligned} \varrho(s)[\mathbf{y} - \mathbf{y}_m](t) = & \vartheta_3^{-1} \tilde{\vartheta}^T \mathbf{w}(t) + \vartheta_3^{-1} (\vartheta_2^T \Lambda(s) \bar{\mathbf{y}}(t) + \\ & \vartheta_{20} \bar{\mathbf{y}}(t) + \vartheta_3 W_m^{-1}(s) \bar{\mathbf{y}}(t) + \vartheta_4) = \\ & \vartheta_3^{-1} \tilde{\vartheta}^T \mathbf{w}(t) + \varrho(s) \mathbf{f}_a(t) \end{aligned} \quad (36)$$

where $\mathbf{e}_y(t) = \mathbf{y}(t) - \mathbf{y}_m(t)$ and $\tilde{\vartheta} = \hat{\vartheta} - \vartheta$. Let $\Theta = \vartheta_3^{-1} = K_a$, and $f(s)$ be a stable polynomial with appropriate degrees. Then,

$$\begin{aligned} \varrho(s) \left(\frac{1}{f(s)} \right) [\mathbf{y} - \mathbf{y}_m](t) = \\ \Theta \left(\frac{1}{f(s)} \right) \mathbf{u}_{cad} - \vartheta^T \left(\frac{1}{f(s)} \right) \mathbf{w}(t) \end{aligned} \quad (37)$$

Define the normalized estimation error as

$$\boldsymbol{\varepsilon}(t) = \frac{\varrho(s) \left(\frac{1}{f(s)} \right) [\mathbf{y} - \mathbf{y}_m](t) + \hat{\Theta} \boldsymbol{\xi}(t)}{\Xi^2(t)} \quad (38)$$

where $\hat{\Theta}$ is the estimate of Θ , and

$$\boldsymbol{\xi} = \vartheta^T(t) \left(\frac{1}{f(s)} \right) \mathbf{w}(t) - \left(\frac{1}{f(s)} \right) \mathbf{u}_{cad}^* \quad (39)$$

$$\begin{aligned} \Xi^2(t) = & 1 + \boldsymbol{\xi}^T(t) \boldsymbol{\xi}(t) + \\ & \left(\left(\frac{1}{f(s)} \right) \mathbf{w}(t) \right)^T \left(\frac{1}{f(s)} \right) \mathbf{w}(t) \end{aligned} \quad (40)$$

Ignoring some exponentially decaying terms, with $\tilde{\Theta} = \hat{\Theta} - \Theta = \vartheta_3^{-1} - \vartheta_3^{-1}$, we can obtain

$$\boldsymbol{\varepsilon}(t) = \frac{\Theta \tilde{\vartheta}^T(t) \left(\frac{1}{f(s)} \right) \mathbf{w}(t) + \tilde{\Theta}(t) \boldsymbol{\xi}(t)}{\Xi^2(t)} \quad (41)$$

Then, the adaptive laws are constructed as

$$\dot{\vartheta}^T(t) = -S_a \boldsymbol{\varepsilon}(t) \left[\left(\frac{1}{f(s)} \right) \mathbf{w}(t) \right]^T \quad (42)$$

$$\dot{\Theta}(t) = -\Gamma \boldsymbol{\varepsilon}(t) \boldsymbol{\xi}^T(t) \quad (43)$$

where $\Gamma = \Gamma^T > 0$ and $\Gamma_a = \vartheta_3^{-1} S_a^{-1} = \Gamma_a^T > 0$.

Theorem 2. For plant (23), under the control laws (42) and (43), all closed-loop system signals are bounded. Moreover, for any bounded initial conditions and bounded reference input \mathbf{r} , \mathbf{e}_y tends to zero within a finite time $T > t_0$.

Proof. Define the following Lyapunov function:

$$V = \frac{1}{2} \text{tr}[\tilde{\vartheta} \Gamma_a \tilde{\vartheta}^T] + \frac{1}{2} \text{tr}[\tilde{\Theta}^T \Gamma^{-1} \tilde{\Theta}]$$

Taking the derivative of V and considering (41) ~ (43), we can obtain

$$\begin{aligned} \dot{V} = & \text{tr}[\tilde{\vartheta} \Gamma_a \dot{\tilde{\vartheta}}^T] + \frac{1}{2} \text{tr}[\tilde{\vartheta} \frac{d\Gamma_a}{dt} \tilde{\vartheta}^T] + \text{tr}[\Theta^T \Gamma^{-1} \dot{\Theta}] = \\ & \text{tr}[\tilde{\vartheta} \Gamma_a \dot{\tilde{\vartheta}}^T] + \text{tr}[\Theta^T \Gamma^{-1} \dot{\Theta}] = \\ & - \text{tr}[\tilde{\vartheta} \Gamma_a S_a \boldsymbol{\varepsilon}(t) \left[\left(\frac{1}{f(s)} \right) \mathbf{w}(t) \right]^T] - \text{tr}[\Theta^T \Gamma^{-1} \Gamma \boldsymbol{\varepsilon}(t) \boldsymbol{\xi}(t)^T] = \\ & - (\tilde{\vartheta} K_a \boldsymbol{\varepsilon}(t))^T \left(\frac{1}{f(s)} \right) \mathbf{w}(t) - (\tilde{\Theta}^T \boldsymbol{\varepsilon}(t))^T \boldsymbol{\xi}(t) = \\ & - \boldsymbol{\varepsilon}(t)^T (\Theta \tilde{\vartheta}(t))^T \left(\frac{1}{f(s)} \right) \mathbf{w}(t) + \tilde{\Theta} \boldsymbol{\xi}(t) = \\ & - \boldsymbol{\varepsilon}(t)^T \boldsymbol{\varepsilon}(t) \Xi^2(t) \leq 0 \end{aligned}$$

Hence, $V \in L^\infty$, which implies that $\boldsymbol{\varepsilon}(t) \Xi(t) \in L^2 \cap L^\infty$, and thus $\dot{\vartheta}, \dot{\Theta} \in L^2 \cap L^\infty$. \square

2.3 Multiple model switching

As shown in Fig. 1, faults may cause the plant dynamics to switch abruptly from some nominal point P_0 in the parametric space to the point P_{fault} corresponding to the failed plant^[13]. It was shown that adaptive control using a single model may not be adequate for achieving this task in the presence of faults of critical control effectors. This is due to the fact that in a particular flight regime, the fault can be such that the corresponding parameter jumps are large, and the time interval needed for a single adaptive controller to adapt to the new operating regime may be large. Over this interval, the performance can deteriorate substantially and may be unacceptable in practice. Hence, single model-based adaptive controller may be too slow to bring the closed-loop system close to the new operating regime, which may result in unacceptably large transients. On the other hand, a well-known problem in adaptive control is the poor transient response which is observed when adaptation is initiated. In such a case, placing several models in the parametric set, switching to the model close to the dynamics of the failed plant, and adapting from there can result in fast and accurate control reconfiguration.

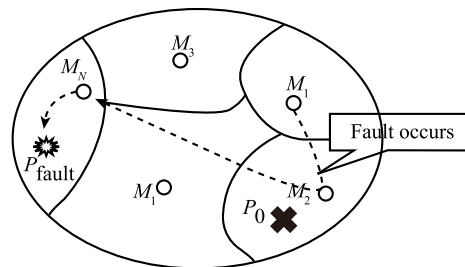


Fig. 1 Concept of multiple model adaptive control

The proposed scheme (see Fig. 2) consists of N identification models (observers) M_j ($j = 1, 2, \dots, N$), with identical structures, but different initial estimates of the plant parameters are used in the parameter space to describe the different fault scenarios. Corresponding to each M_j is a controller C_j and all the identification models operate in parallel. Only one of the controllers, which is determined by the switching index, is connected to the plant at every instant^[14–15]. The idea behind the above scheme is, based

on a switching index, to find a controller such that the output error $\mathbf{e}_y(t)$ converges to zero asymptotically for an arbitrary initial condition and a bounded reference control input.

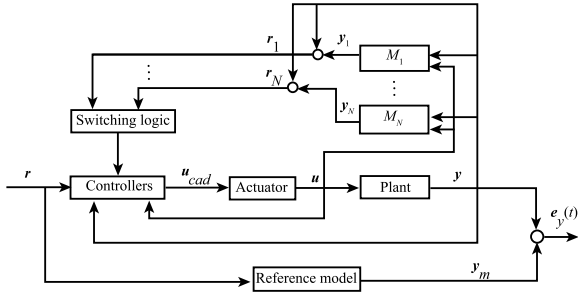


Fig. 2 Multiple model scheme

A bank of identification models (observers) are chosen as the following form:

$$\dot{\hat{\mathbf{x}}}_j = A\hat{\mathbf{x}}_j + \sum_{i=1}^m b_{ji}[\hat{\sigma}_{ji}\hat{k}_{ji}u_{cji} + (1 - \hat{\sigma}_{ji})\hat{u}_{ji}] + L(\mathbf{y} - \hat{\mathbf{y}}_j) \quad (44)$$

$$\hat{\mathbf{y}}_j = C\hat{\mathbf{x}}_j, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, N \quad (45)$$

Define the state error as $\hat{\mathbf{e}}_{xj} = \hat{\mathbf{x}}_j - \mathbf{x}$. Then,

$$\dot{\hat{\mathbf{e}}}_{xj} = (A - LC)\hat{\mathbf{e}}_{xj} + \sum_{i=1}^m b_{ji}[\phi_{\sigma ji}\phi_{kji}u_{cji} + (1 - \hat{\sigma}_{ji})\phi_{ujj}] \quad (46)$$

where the definition of $\phi_{\sigma ji}$ is as same as that defined in Subsection 1.3, and $\phi_{ujj} = \hat{u}_{ji} - \bar{u}_{ji}$, \hat{u}_{ji} is estimate of \bar{u}_{ji} , and

$$\dot{\hat{u}}_{ji} = \text{Proj}_{[u_{im}, u_{iM}]} \{-\gamma_{ujj}\mathbf{e}_{xj}^T P b_{ji}\} \quad (47)$$

where $\gamma_{ujj} > 0$ is the weighting coefficient. P is a symmetric positive solution of Lyapunov matrix equation $(A - LC)^T P + P(A - LC) = -Q$, $Q > 0$, and $PB = C$.

Switching among the controllers is based on the following performance indices:

$$J_j(t) = c_1 \|\mathbf{r}_j\|^2 + c_2 \int_{t_0}^t \exp(-\lambda_J(\tau - t_0)) \|\mathbf{r}_j\|^2 d\tau \quad (48)$$

where $\mathbf{r}_j = \hat{\mathbf{y}}_j - \mathbf{y}$, $c_1, c_2 > 0$ can be chosen to yield a desired combination of instantaneous and long-term accuracy measures. The forgetting factor $\lambda_J > 0$ determines the memory of the index in rapidly switching environments and ensures boundedness of $J_j(t)$ for bounded $\hat{\mathbf{e}}_{xj}$. The scheme is implemented by calculating and comparing the above indices every t_s instant to find their minimum. Once the minimum is found, the scheme switches to (or stays at) the corresponding controller.

In residual vectors, some are more sensitive in model matching than others, they should be given a larger weighted coefficient to enhance sensitivity, i.e.,

$$\mathbf{r}_j^*(t) = W_j \mathbf{r}_j(t) \quad (49)$$

where W_j is a diagonal weighting matrix. Then, (44) becomes

$$J_j(t) = c_1 \|\mathbf{r}_j^*\|^2 + c_2 \int_{t_0}^t \exp(-\lambda_J(\tau - t_0)) \|\mathbf{r}_j^*\|^2 d\tau \quad (50)$$

when the "best" model is found, $\lim_{t \rightarrow \infty} \mathbf{r}_j(t) = \mathbf{0}$, then $\lim_{t \rightarrow \infty} J_j(t) = 0$.

Remark 4. The switching mechanism's unique feature, sharply distinguishing it from other logics which might be used for the same purpose, is that: 1) The controller selection is made by continuously comparing in real-time suitably defined norm-squared output estimations errors or "performance signals" J_j ; 2) When placing a candidate controller in the feedback-loop, its corresponding performance signal is the smallest.

Theorem 3. The above switching scheme assures the stability of system (40) and (41), and guarantees that $\lim_{t \rightarrow \infty} J_j(t) = 0$ and in the case of j -th fault of actuator, $\lim_{t \rightarrow \infty} [\hat{\mathbf{u}}_j - \bar{\mathbf{u}}_j] = \mathbf{0}$.

Proof. Consider a tentative Lyapunov function of the form:

$$V_j(\hat{\mathbf{e}}_{xj}, \phi_{ujj}) = -\frac{1}{2} [\hat{\mathbf{e}}_{xj}^T P \hat{\mathbf{e}}_{xj} + \frac{\phi_{ujj}^2}{\gamma_j}]$$

The derivative of $V_j(\hat{\mathbf{e}}_{xj}, \phi_{ujj})$ along the trajectories of the j -th model is

$$\dot{V}_j(\hat{\mathbf{e}}_{xj}, \phi_{ujj}) \leq -\frac{1}{2} \lambda_m \|\hat{\mathbf{e}}_{xj}\|^2 \leq 0$$

where the properties of adaptive algorithm with projections were used, i.e., $\phi_{ujj} \dot{\phi}_{ujj} \leq -\gamma_j \phi_{ujj} (\hat{\mathbf{e}}_{xj}^T P b_j + \dot{u}_j)$. The above results imply that $\hat{\mathbf{e}}_{xj} \in L^\infty \cap L^2$, $\hat{\mathbf{e}}_{xj} \in L^\infty$, $\lim_{t \rightarrow \infty} J_j(t) = 0$. Since all other observer will have forcing terms, the scheme will switch to the j -th observer. In such a case, $\hat{\mathbf{e}}_{xj}$ will be bounded (ϕ_{ujj} is bounded due to the use of the projection algorithm), which implies the boundedness of $\hat{\mathbf{x}}_j$ and, consequently, of $\mathbf{x}(t)$. The latter implies that u_{cj} is bounded, which in turn implies that all signals are bounded, and that $\lim_{t \rightarrow \infty} \hat{\mathbf{e}}_{xj}(t) = \mathbf{0}$, hence, $\lim_{t \rightarrow \infty} \hat{\mathbf{e}}_x(t) = \mathbf{0}$. \square

3 Numerical simulations

As an application, the lateral dynamic model of a Boeing 747 airplane^[2] is used to illustrate the effectiveness of the developed algorithm. In horizontal flight at 40 000 ft and nominal forward speed 774 ft/s the linearized model of the lateral dynamics of Boeing 747 with two augmented actuation vectors can be described as

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$

$$\mathbf{y}(t) = C\mathbf{x}(t)$$

$$A = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ -3.05 & 0.388 & -0.465 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.00729 & 0.01 & 0.005 \\ -0.475 & -0.5 & -0.3 \\ 0.153 & 0.2 & 0.1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = [0 \quad 1 \quad 0 \quad 0]$$

where $\mathbf{x}(t) = [\beta, r, p, \phi]^T$ is the state vector with the sideslip angle β , the yaw rate r , the roll rate p , and the roll angle ϕ . \mathbf{y} is the system output which is the yaw rate and $\mathbf{u} = [\delta_{r1}, \delta_{r2}, \delta_{r3}]^T$ is the control input vector with three signals to represent three rudder servos.

The reference model is constructed as

$$\dot{\mathbf{x}}_m(t) = A_m \mathbf{x}_m(t) + B_m \mathbf{u}(t)$$

$$y_m(t) = C_m x_m(t)$$

$$A_m = \begin{bmatrix} -0.003 & 0.039 & 0 & -0.322 \\ -0.065 & -0.319 & 7.74 & 0 \\ 0.02 & -0.101 & -0.429 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_m = \begin{bmatrix} 0.01 & 1 & 1 \\ -0.18 & -0.04 & -0.04 \\ -1.16 & 0.598 & 0.598 \\ 0 & 0 & 0 \end{bmatrix}$$

and $C_m = C$.

In simulation, we consider the actuator fault pattern as LIP fault:

$$u_2(t) = \delta_{r2} = 0.05, \quad t \in [5, 10]$$

and the parameter is $\Gamma = I$, the reference input is $r(t) = \sin(0.1t)$. The fault and its estimation and the tracking error are shown in Figs. 3 and 4, respectively.

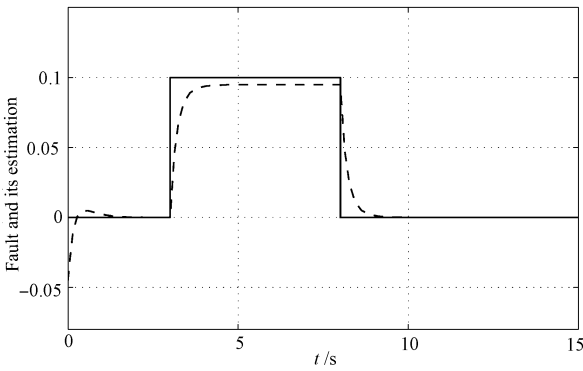


Fig. 3 Fault (solid) and its estimation (dotted)

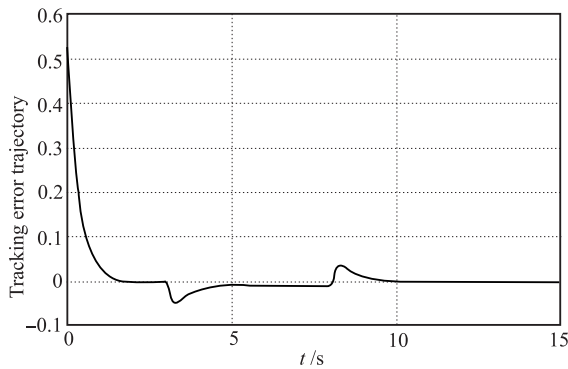


Fig. 4 The tracking error $e_y(t)$

LOE fault:

$$K = 0.8 \text{ for } t = [0, 2]$$

$$r = 4\sin(3t)$$

The simulation results are shown in Figs. 5 ~ 8.

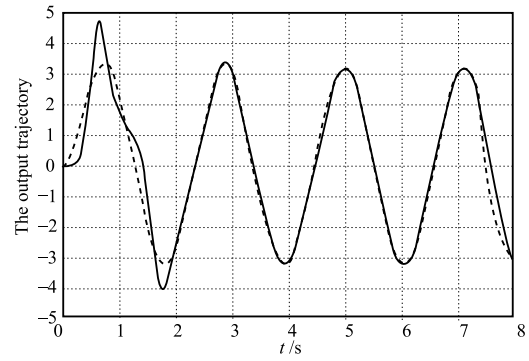


Fig. 5 The outputs $y(t)$ (solid) and $y_m(t)$ (dashed) (with fault accommodation)

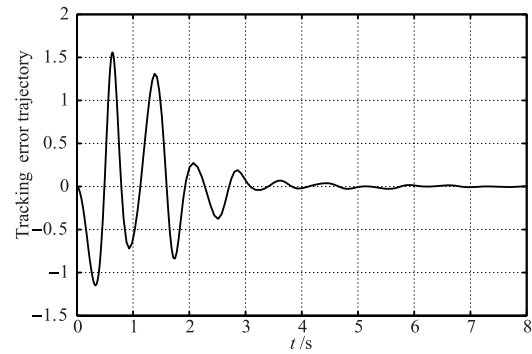


Fig. 6 The tracking error $e_y(t)$ (with fault accommodation)

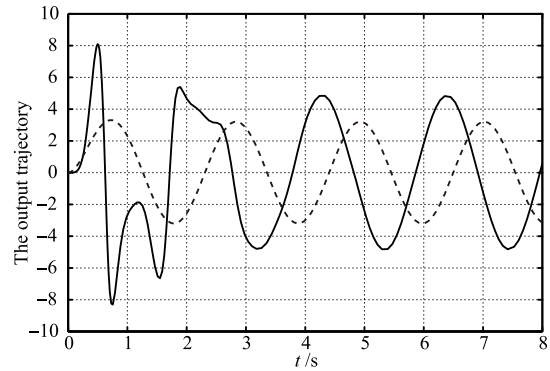


Fig. 7 The outputs $y(t)$ (solid) and $y_m(t)$ (dashed) (without fault accommodation)

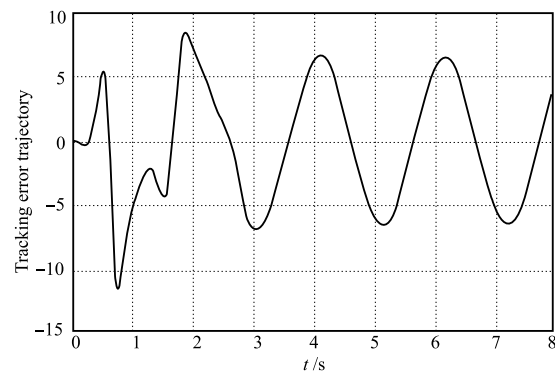


Fig. 8 The tracking error $e_y(t)$ (without fault accommodation)

From these figures, we can see that the fault can be estimated accurately and when one of the actuators fails, there is a transient response in the tracking error, which tends to zero rapidly. All signals in the reconfiguration control system are bounded, and the stability and convergence are ensured.

4 Conclusion

In this paper, we considered an reconfigurable control for linear time-invariant plants with unknown actuator fault. The actuator model is described by second-order dynamics which is in the presence of non-measurable rate. The design is based on a multiple model adaptive control approach with appropriate switching logic. The combined design achieves the control objective of asymptotic output tracking while ensuring closed-loop stability. This provides an improvement to the existing results in the fault tolerant control literature.

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