# Non-synchronized Observer-based Control of Discrete-time Piecewise Affine Systems: an LMI Approach

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**Abstract** This paper presents a novel observer-based control scheme for discrete-time piecewise affine systems based on a piecewise-quadratic Lyapunov function. The key issue addressed in this paper is that the currently active region of the system is unknown, and can not be inferred from the measured outputs. By approximating polytopic operating regions by ellipsoids and using the singular value decomposition technique to treat the constraint of matrix equality, the suggested control method can be formulated as linear matrix inequalities (LMIs), and solved much more efficiently than existing methods which could be only cast as bilinear matrix inequalities. A numerical example is also given to verify the proposed approach.

**Key words** Piecewise affine system, piecewise Lyapunov function, controller design, observer, linear matrix inequality (LMI)

Piecewise affine (PWA) systems have been receiving increasing attention by the control community in recent years because of their wide scopes of applications. In fact, several types of nonlinearities, such as relays, saturation, and dead zone, are naturally written as PWA systems. In addition, many other classes of nonlinear systems can also be approximated by the PWA systems[1−2]. Furthermore, some hybrid systems, such as mixed logical dynamical (MLD) systems, linear hybrid automata, etc., are equivalent to PWA systems[3−4].

One of the most important open questions in control theory and applications is the output feedback stabilization problem. During the last decade, several output feedback controller design methods have been developed for PWA systems based on piecewise-quadratic Lyapunov function (PWQLF). In the continuous-time case, Rodrigues[5] discussed dynamic output feedback controller design for the systems which may involve multiple equilibria. In [6], a static output feedback (SOF) control law was suggested for a special class of PWA systems, and the results were applied to the chaos stabilization. A robust dynamic output feedback controller was designed for the systems with norm-bounded uncertainties and external disturbance in [7]. In the discrete-time case, an observer-based controller was designed for piecewise linear (PWL) (without affine terms) systems in [8], and the controller and observer gains can be obtained according to the so-called weaker separation principle. Then, the results are extended to the output regulation problem of PWA systems in [9]. The problem of SOF control for PWL systems is investigated in [10], and the extension of the method is also given in order to incorporate $H_\infty$ performance.

There is a common restriction in aforementioned papers, i.e. both the plant and the controller should always switch to the same region at the same time. However, there is no guarantee for this restriction in practice, because PWA systems are mostly partitioned based on state space and the currently active region can not be inferred from the measured outputs[11]. In [12], Rodrigues made a pioneering contribution to the non-synchronized output feedback controller synthesis for continuous-time PWA systems. The issue that the plant and the controller may stay in different regions from time to time was explicitly considered. Recently, a non-synchronized dynamic output feedback controller was designed for the discrete-time PWA systems in [13]. Unfortunately, all the results in [12−13] can only be cast as bilinear matrix inequalities (BMIs), which are non-convex, NP-hard, and very expensive to solve globally[14]. Motivated by this situation, this paper presents a novel non-synchronized observer-based control scheme for discrete-time PWA systems whose regions can be approximated by ellipsoids. During the synthesis procedure, the PWQLF technique is used and the region information is taken into account. It will be shown that the resulting closed-loop system is piecewise-quadratically (PWQ) stable, and the controller gains can be obtained by solving a set of linear matrix inequalities (LMIs)[15], which are numerically feasible with commercially available software.

The paper is organized as follows. Section 1 provides the problem statement. Main results are presented in Section 2. Then, simulation results are shown in Section 3. Finally, Section 4 concludes the paper.

**Notations.** $\mathbf{R}^n$ denotes the $n$ dimensional Euclidean space, $\mathbf{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices, a real symmetric matrix $P>0$ denotes $P$ being a positive definite matrix, the superscript "T" represents the transpose, "$*$" is used as an ellipsis for terms that are induced by symmetry, $I$ means an identity matrix of appropriate dimension, $||\boldsymbol{x}||$ represents Euclidean norm of vector $\boldsymbol{x}$, and diag$\{\cdot\}$ stands for a block-diagonal matrix.

## 1 Problem statement

Consider the discrete-time PWA system of the form

$$\begin{array}{l}\boldsymbol{x}(t+1)=A_i\boldsymbol{x}(t)+B_i\boldsymbol{u}(t)+\boldsymbol{b}_i\\ \boldsymbol{y}(t)=C\boldsymbol{x}(t)\\ \text{for } \boldsymbol{x}(t)\in S_i,\ i\in\wp\end{array}\qquad(1)$$

where $\{S_i\}_{i\in\wp}\subseteq\mathbf{R}^n$ denotes a partition of the state space into a number of closed polyhedral regions, $\wp$ is the index set of these regions, $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the state, $\boldsymbol{u}(t)\in\mathbf{R}^m$ is the input, $\boldsymbol{y}(t)\in\mathbf{R}^p$ is the measured output, $\boldsymbol{b}_i$ is the affine term, and $(A_i,B_i,\boldsymbol{b}_i,C_i)$ is the $i$-th local model.

$\wp$ is partitioned as $\wp=\wp_0\cup\wp_1$, where $\wp_0$ is the index set of the regions that contain the origin, i.e. $\boldsymbol{b}_i=\mathbf{0}$ for $i\in\wp_0$, and $\wp_1$ is the index set of the regions otherwise. For future use, define a set $\Omega$ that represents all possible transitions from one region to itself or another region, that is

$$\Omega=\{(i,j)\mid \boldsymbol{x}(t)\in S_i,\ \boldsymbol{x}(t+1)\in S_j\}$$

**Remark 1.** The set $\Omega$ can be determined by the reachability analysis for MLD systems[16−17]. If it is possible for the transitions happen between all regions, then $\Omega$ can be defined as $\Omega=\{(i,j)\mid i,j\in\wp\}$.

There are three basic assumptions in this paper.

**Assumption 1.** When the system transits from the region $S_i$ to $S_j$ at the time $t$, the dynamics of the system is governed by the dynamics of the local model of $S_i$ at that time.

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**Assumption 2.** Matrix $C$ is full row rank, i.e. $\mathrm{rank}(C) = p$. For convenience, the singular value decomposition of $C$ is presented as

$$C = U \begin{bmatrix} S & 0 \end{bmatrix} V^{\mathrm{T}}$$

where $U \in \mathbf{R}^{p\times p}$ and $V \in \mathbf{R}^{n\times n}$ are unitary matrices, and $S \in \mathbf{R}^{p\times p}$ is a diagonal matrix with positive diagonal elements in decreasing order.

**Assumption 3.** Matrix $E_i$ and scalar $f_i$ exist such that

$$S_i \subseteq \varepsilon_i \text{ where } \varepsilon_i = \{\boldsymbol{x}(t) \mid ||\boldsymbol{E}_i\boldsymbol{x}(t) + f_i|| \le 1\}$$

There are many methods[15, 18] to compute this ellipsoidal outer approximation. The approximation is especially useful when $S_i$ is a slab, because in this case an ellipsoid can be found to cover $S_i$ exactly. In other words, if $S_i = \{\boldsymbol{x}(t) \mid d_i^1 \le \boldsymbol{\eta}_i\boldsymbol{x}(t) \le d_i^2\}$, one can take $\boldsymbol{E}_i = 2\boldsymbol{\eta}_i/(d_i^2 - d_i^1)$ and $f_i = -(d_i^2 + d_i^1)/(d_i^2 - d_i^1)$ with the result that $S_i \subseteq \varepsilon_i$ and $\varepsilon_i \subseteq S_i$ [19−20]. The ellipsoid $\varepsilon_i$ can also be described in the form of

$$\begin{bmatrix} \boldsymbol{x}(t) \\ 1 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{E}_i^{\mathrm{T}}\boldsymbol{E}_i & * \\ f_i^{\mathrm{T}}\boldsymbol{E}_i & f_i^{\mathrm{T}}f_i - 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ 1 \end{bmatrix} \le 0 \qquad (2)$$

The following observer-based control law is proposed to stabilize the system (1):

$$\begin{array}{l} \hat{\boldsymbol{x}}(t+1) = A_k\hat{\boldsymbol{x}}(t) + B_k\boldsymbol{u}(t) + \boldsymbol{b}_k + L_k\left(\hat{\boldsymbol{y}}(t) - \boldsymbol{y}(t)\right) \\ \hat{\boldsymbol{y}}(t) = C\hat{\boldsymbol{x}}(t) \\ \boldsymbol{u}(t) = K_k\hat{\boldsymbol{x}}(t) \\ \text{for } \hat{\boldsymbol{x}}(t) \in S_k,\ k \in \wp \end{array} \qquad (3)$$

where $\hat{\boldsymbol{x}}(t) \in \mathbf{R}^n$ is the estimation of $\boldsymbol{x}(t)$ and $\hat{\boldsymbol{y}}(t) \in \mathbf{R}^p$ is the observer output. The control gain $K_k$ and the observer gain $L_k$ will be determined in the framework of LMI theory. For $\hat{\boldsymbol{x}}(t) \in S_k$, we have

$$\begin{bmatrix} \hat{\boldsymbol{x}}(t) \\ 1 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{E}_k & * \\ f_k^{\mathrm{T}}\boldsymbol{E}_k & f_k^{\mathrm{T}}f_k - 1 \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{x}}(t) \\ 1 \end{bmatrix} \le 0 \qquad (4)$$

We also define a set $\widehat{\Omega}$ that represents all possible transitions of the estimated state from one region to itself or another region, that is

$$\widehat{\Omega} = \{(k,l) \mid \hat{\boldsymbol{x}}(t) \in S_k,\ \hat{\boldsymbol{x}}(t+1) \in S_l\}$$

It should be noted that since the estimated state can not be computed *a* priori, the set $\widehat{\Omega}$ should be defined by the set of all possible transitions between regions, that is $\widehat{\Omega} = \{(k,l) \mid k,l \in \wp\}$[11].

## 2 Main results

We consider the general case $\boldsymbol{x}(t) \in S_i$ and $\hat{\boldsymbol{x}}(t) \in S_k$ in this section. Defining $\boldsymbol{e}(t) = \hat{\boldsymbol{x}}(t) - \boldsymbol{x}(t)$, the closed-loop system admits the realization

$$\begin{array}{l} \boldsymbol{\xi}(t+1) = \tilde{A}_{ik}\boldsymbol{\xi}(t) + \tilde{\boldsymbol{b}}_{ik} \\ \text{for } \boldsymbol{x}(t) \in S_i,\ \hat{\boldsymbol{x}}(t) \in S_k \end{array} \qquad (5)$$

where

$$\tilde{A}_{ik} = \begin{bmatrix} A_i + B_iK_k & B_iK_k \\ \Delta_{1ik} & A_k + L_kC + \Delta_{2ik} \end{bmatrix}$$

$$\tilde{\boldsymbol{b}}_{ik} = \begin{bmatrix} \boldsymbol{b}_i \\ \boldsymbol{b}_k - \boldsymbol{b}_i \end{bmatrix},\ \boldsymbol{\xi}(t) = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{e}(t) \end{bmatrix}$$

$$\Delta_{1ik} = A_k - A_i + (B_k - B_i)K_k$$
$$\Delta_{2ik} = (B_k - B_i)K_k$$

Consider a PWQLF $V(t) = \boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{P}_{ik}\boldsymbol{\xi}(t)$ with $\tilde{P}_{ik} = \tilde{P}_{ik}^{\mathrm{T}} > 0$. Its difference along the solution of (5) is negative if

$$\Delta V(t+1) = \boldsymbol{\xi}^{\mathrm{T}}(t+1)\tilde{P}_{jl}\boldsymbol{\xi}(t+1) - \boldsymbol{\xi}^{\mathrm{T}}(t)\tilde{P}_{ik}\boldsymbol{\xi}(t) < 0 \qquad (6)$$

**Definition 1.** The closed-loop system (5) is said to be PWQ stable if the condition (6) is satisfied for all $(i,j) \in \Omega$ and $(k,l) \in \widehat{\Omega}$.

In the following, our interest is to seek conditions in the form of LMIs for guaranteeing (6). Substituting the state space (5) in (6) leads to

$$\Theta_{ikjl} = \begin{bmatrix} \boldsymbol{\xi}(t) \\ 1 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \tilde{A}_{ik}^{\mathrm{T}}\tilde{P}_{jl}\tilde{A}_{ik} - \tilde{P}_{ik} & * \\ \tilde{\boldsymbol{b}}_{ik}^{\mathrm{T}}\tilde{P}_{jl}\tilde{A}_{ik} & \tilde{\boldsymbol{b}}_{ik}^{\mathrm{T}}\tilde{P}_{jl}\tilde{\boldsymbol{b}}_{ik} \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}(t) \\ 1 \end{bmatrix} < 0 \qquad (7)$$

To treat the affine term $\tilde{\boldsymbol{b}}_{ik}$, the region information (4) of the controller is taken into account. By the S-procedure[15], (7) is satisfied if there exists $\lambda_{ik} > 0$ such that

$$\Theta_{ikjl} - \lambda_{ik}\begin{bmatrix} \hat{\boldsymbol{x}}(t) \\ 1 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{E}_k & * \\ f_k^{\mathrm{T}}\boldsymbol{E}_k & f_k^{\mathrm{T}}f_k - 1 \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{x}}(t) \\ 1 \end{bmatrix} < 0 \qquad (8)$$

which is equivalent to

$$\Theta_{ikjl} - \lambda_{ik}\begin{bmatrix} \boldsymbol{x}(t) + \boldsymbol{e}(t) \\ 1 \end{bmatrix}^{\mathrm{T}} \times \begin{bmatrix} \boldsymbol{E}_k^{\mathrm{T}}\boldsymbol{E}_k & * \\ f_k^{\mathrm{T}}\boldsymbol{E}_k & f_k^{\mathrm{T}}f_k - 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) + \boldsymbol{e}(t) \\ 1 \end{bmatrix} < 0 \qquad (9)$$

Notice that (9) can be further written as

$$\Theta_{ikjl} - \lambda_{ik}\begin{bmatrix} \boldsymbol{\xi}(t) \\ 1 \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \tilde{\boldsymbol{E}}_k^{\mathrm{T}}\tilde{\boldsymbol{E}}_k & * \\ f_k^{\mathrm{T}}\tilde{\boldsymbol{E}}_k & f_k^{\mathrm{T}}f_k - 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}(t) \\ 1 \end{bmatrix} < 0 \qquad (10)$$

where $\tilde{\boldsymbol{E}}_k = \begin{bmatrix} \boldsymbol{E}_k & \boldsymbol{E}_k \end{bmatrix}$.

The sufficient condition of (10) is that

$$\begin{bmatrix} \tilde{A}_{ik}^{\mathrm{T}}\tilde{P}_{jl}\tilde{A}_{ik} - \tilde{P}_{ik} & * \\ \tilde{\boldsymbol{b}}_{ik}^{\mathrm{T}}\tilde{P}_{jl}\tilde{A}_{ik} & \tilde{\boldsymbol{b}}_{ik}^{\mathrm{T}}\tilde{P}_{jl}\tilde{\boldsymbol{b}}_{ik} \end{bmatrix} - \begin{bmatrix} \lambda_{ik}\tilde{\boldsymbol{E}}_k^{\mathrm{T}}\tilde{\boldsymbol{E}}_k & * \\ \lambda_{ik}f_k^{\mathrm{T}}\tilde{\boldsymbol{E}}_k & \lambda_{ik}\left(f_k^{\mathrm{T}}f_k - 1\right) \end{bmatrix} < 0 \qquad (11)$$

By substituting $\tilde{Q}_{ik} = \tilde{P}_{ik}^{-1}$ and $\tilde{Q}_{jl} = \tilde{P}_{jl}^{-1}$, (11) can be rearranged as

$$\begin{bmatrix} \tilde{Q}_{ik}^{-1} + \lambda_{ik}\tilde{\boldsymbol{E}}_k^{\mathrm{T}}\tilde{\boldsymbol{E}}_k & * \\ \lambda_{ik}f_k^{\mathrm{T}}\tilde{\boldsymbol{E}}_k & \lambda_{ik}\left(f_k^{\mathrm{T}}f_k - 1\right) \end{bmatrix} - \begin{bmatrix} \tilde{A}_{ik}^{\mathrm{T}} \\ \tilde{\boldsymbol{b}}_{ik}^{\mathrm{T}} \end{bmatrix} \tilde{Q}_{jl}^{-1} \begin{bmatrix} \tilde{A}_{ik} & \tilde{\boldsymbol{b}}_{ik} \end{bmatrix} > 0 \qquad (12)$$

Applying the Schur complement[15] to (12) results in

$$\begin{bmatrix} \tilde{Q}_{ik}^{-1} + \lambda_{ik}\tilde{\boldsymbol{E}}_k^{\mathrm{T}}\tilde{\boldsymbol{E}}_k & * & * \\ \lambda_{ik}f_k^{\mathrm{T}}\tilde{\boldsymbol{E}}_k & \lambda_{ik}\left(f_k^{\mathrm{T}}f_k - 1\right) & * \\ \tilde{A}_{ik} & \tilde{\boldsymbol{b}}_{ik} & \tilde{Q}_{jl} \end{bmatrix} > 0 \qquad (13)$$

By pre- and post-multiplying (13) via diag$\left\{I, \begin{bmatrix} 0 & * \\ I & 0 \end{bmatrix}\right\}$, we get

$$\begin{bmatrix} \tilde{Q}_{ik}^{-1} + \lambda_{ik}\tilde{\boldsymbol{E}}_k^{\rm T}\tilde{\boldsymbol{E}}_k & * & * \\ \tilde{A}_{ik} & \tilde{Q}_{jl} & * \\ \lambda_{ik} f_k^{\rm T}\tilde{\boldsymbol{E}}_k & \tilde{\boldsymbol{b}}_{ik}^{\rm T} & \lambda_{ik}\left(f_k^{\rm T} f_k - 1\right) \end{bmatrix} > 0 \quad (14)$$

which is equivalent, by the Schur complement, to

$$\begin{bmatrix} \tilde{Q}_{ik}^{-1} + \lambda_{ik}\tilde{\boldsymbol{E}}_k^{\rm T}\tilde{\boldsymbol{E}}_k & * \\ \tilde{A}_{ik} & \tilde{Q}_{jl} \end{bmatrix} - \begin{bmatrix} \lambda_{ik}\tilde{\boldsymbol{E}}_k^{\rm T} f_k \\ \tilde{\boldsymbol{b}}_{ik} \end{bmatrix} \lambda_{ik}^{-1}\left(f_k^{\rm T} f_k - 1\right)^{-1} \begin{bmatrix} \lambda_{ik} f_k^{\rm T}\tilde{\boldsymbol{E}}_k & \tilde{\boldsymbol{b}}_{ik}^{\rm T} \end{bmatrix} > 0 \quad (15)$$

The second term in (15) can be written as

$$\begin{bmatrix} \lambda_{ik}\tilde{\boldsymbol{E}}_k^{\rm T} f_k \left(f_k^{\rm T} f_k - 1\right)^{-1} f_k^{\rm T}\tilde{\boldsymbol{E}}_k & * \\ \tilde{\boldsymbol{b}}_{ik}\left(f_k^{\rm T} f_k - 1\right)^{-1} f_k^{\rm T}\tilde{\boldsymbol{E}}_k & \lambda_{ik}^{-1}\tilde{\boldsymbol{b}}_{ik}\left(f_k^{\rm T} f_k - 1\right)^{-1}\tilde{\boldsymbol{b}}_{ik}^{\rm T} \end{bmatrix} \quad (16)$$

The matrix inversion lemma $(A+BCD)^{-1} = A^{-1} - A^{-1}B\left(C^{-1} + DA^{-1}B\right)^{-1} DA^{-1}$[21] states that

$$\begin{aligned} \left(I - f_k^{\rm T} f_k\right)^{-1} f_k^{\rm T} &= f_k^{\rm T}\left(I - f_k f_k^{\rm T}\right)^{-1} \\ \left(I - f_k^{\rm T} f_k\right)^{-1} &= I + f_k^{\rm T}\left(I - f_k f_k^{\rm T}\right)^{-1} f_k \end{aligned} \quad (17)$$

According to (17), (16) can be transformed into

$$\begin{bmatrix} \lambda_{ik}\tilde{\boldsymbol{E}}_k^{\rm T}\tilde{\boldsymbol{E}}_k & * \\ 0 & -\lambda_{ik}^{-1}\tilde{\boldsymbol{b}}_{ik}\tilde{\boldsymbol{b}}_{ik}^{\rm T} \end{bmatrix} + \begin{bmatrix} \tilde{\boldsymbol{E}}_k^{\rm T} \\ \lambda_{ik}^{-1}\tilde{\boldsymbol{b}}_{ik} f_k^{\rm T} \end{bmatrix} \lambda_{ik}\left(f_k f_k^{\rm T} - I\right)^{-1} \begin{bmatrix} \tilde{\boldsymbol{E}}_k & \lambda_{ik}^{-1} f_k \tilde{\boldsymbol{b}}_{ik}^{\rm T} \end{bmatrix} \quad (18)$$

By replacing the second term of (15) by (18), (15) becomes

$$\begin{bmatrix} \tilde{Q}_{ik}^{-1} & * \\ \tilde{A}_{ik} & \tilde{Q}_{jl} + \alpha_{ik}\tilde{\boldsymbol{b}}_{ik}\tilde{\boldsymbol{b}}_{ik}^{\rm T} \end{bmatrix} - \begin{bmatrix} \tilde{\boldsymbol{E}}_k^{\rm T} \\ \alpha_{ik}\tilde{\boldsymbol{b}}_{ik} f_k^{\rm T} \end{bmatrix} \alpha_{ik}^{-1}\left(f_k f_k^{\rm T} - I\right)^{-1} \begin{bmatrix} \tilde{\boldsymbol{E}}_k & \alpha_{ik} f_k \tilde{\boldsymbol{b}}_{ik}^{\rm T} \end{bmatrix} > 0 \quad (19)$$

where $\alpha_{ik} = \lambda_{ik}^{-1}$.

Applying the Schur complement to (19) leads to

$$\begin{bmatrix} \tilde{Q}_{ik}^{-1} & * & * \\ \tilde{A}_{ik} & \tilde{Q}_{jl} + \alpha_{ik}\tilde{\boldsymbol{b}}_{ik}\tilde{\boldsymbol{b}}_{ik}^{\rm T} & * \\ \tilde{\boldsymbol{E}}_k & \alpha_{ik} f_k \tilde{\boldsymbol{b}}_{ik}^{\rm T} & \alpha_{ik}\left(f_k f_k^{\rm T} - I\right) \end{bmatrix} > 0 \quad (20)$$

Suppose that $\tilde{R}_k = \tilde{R}_k^{\rm T} > 0$. By pre- and post-multiplying (20) via diag$\{\ \tilde{R}_k \quad I \quad I\ \}$, we get

$$\begin{bmatrix} \tilde{R}_k\tilde{Q}_{ik}^{-1}\tilde{R}_k & * & * \\ \tilde{A}_{ik}\tilde{R}_k & \tilde{Q}_{jl} + \alpha_{ik}\tilde{\boldsymbol{b}}_{ik}\tilde{\boldsymbol{b}}_{ik}^{\rm T} & * \\ \tilde{\boldsymbol{E}}_k\tilde{R}_k & \alpha_{ik} f_k \tilde{\boldsymbol{b}}_{ik}^{\rm T} & \alpha_{ik}\left(f_k f_k^{\rm T} - I\right) \end{bmatrix} > 0 \quad (21)$$

Due to $\tilde{Q}_{ik} > 0$, we have $\left(\tilde{R}_k - \tilde{Q}_{ik}\right)^{\rm T}\tilde{Q}_{ik}^{-1}\left(\tilde{R}_k - \tilde{Q}_{ik}\right) \geq 0$, which is equivalent to $\tilde{R}_k\tilde{Q}_{ik}^{-1}\tilde{R}_k \geq 2\tilde{R}_k - \tilde{Q}_{ik}$. Thus, (21) can be guaranteed by

$$\begin{bmatrix} 2\tilde{R}_k - \tilde{Q}_{ik} & * & * \\ \tilde{A}_{ik}\tilde{R}_k & \tilde{Q}_{jl} + \alpha_{ik}\tilde{\boldsymbol{b}}_{ik}\tilde{\boldsymbol{b}}_{ik}^{\rm T} & * \\ \tilde{\boldsymbol{E}}_k\tilde{R}_k & \alpha_{ik} f_k \tilde{\boldsymbol{b}}_{ik}^{\rm T} & \alpha_{ik}\left(f_k f_k^{\rm T} - I\right) \end{bmatrix} > 0 \quad (22)$$

Let $\tilde{Q}_{ik} = \begin{bmatrix} Q_{1ik} & * \\ G_{ik} & Q_{2ik} \end{bmatrix}$ and $\tilde{R}_k = \begin{bmatrix} R_k & * \\ 0 & R_k \end{bmatrix}$. Then, with the substitution of $W_{1k} = K_k R_k$, $J_k C = C R_k$, and $W_{2k} = L_k J_k$, the LMI (23) can be achieved from (22).

In (23), $\tilde{\Delta}_{1ik} = (A_k - A_i) R_k + (B_k - B_i) W_{1k}$ and $\tilde{\Delta}_{2ik} = (B_k - B_i) W_{1k}$.

**Remark 2.** When $f_k f_k^{\rm T} - I < 0$, (23) is no longer feasible, $f_k f_k^{\rm T} - I < 0$ means that the origin of the controller lies inside the ellipsoid $\varepsilon_k$, i.e. $k \in \wp_0$[19]. In this case, the region information (2) of the controlled system can be taken into account if $i \in \wp_1$. Then, the condition (7) is satisfied if

$$\Theta_{ikjl} - \lambda_{ik}\begin{bmatrix} \boldsymbol{x}(t) \\ 1 \end{bmatrix}^{\rm T}\begin{bmatrix} \boldsymbol{E}_i^{\rm T}\boldsymbol{E}_i & * \\ f_i^{\rm T}\boldsymbol{E}_i & f_i^{\rm T} f_i - 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{x}(t) \\ 1 \end{bmatrix} \leq 0 \quad (24)$$

The inequality (24) can be further written as

$$\Theta_{ikjl} - \lambda_{ik}\begin{bmatrix} \boldsymbol{\xi}(t) \\ 1 \end{bmatrix}^{\rm T}\begin{bmatrix} \bar{\boldsymbol{E}}_i^{\rm T}\bar{\boldsymbol{E}}_i & * \\ f_i^{\rm T}\bar{\boldsymbol{E}}_i & f_i^{\rm T} f_i - 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{\xi}(t) \\ 1 \end{bmatrix} \leq 0 \quad (25)$$

where $\bar{\boldsymbol{E}}_i = \begin{bmatrix} \boldsymbol{E}_i & 0 \end{bmatrix}$.

Similar to the derivation of (10) $\Leftarrow$ (23), it is derived that (25) can be guaranteed by the LMI (26).

According to Remark 2, it is seen that both the LMIs (23) and (26) are not feasible if $i \in \wp_0$ and $k \in \wp_0$. In this case, the condition (7) is equivalent to

$$\begin{bmatrix} 2R_k - Q_{1ik} & * & * & * & * \\ -G_{ik} & 2R_k - Q_{2ik} & * & * & * \\ A_i R_k + B_i W_{1k} & B_i W_{1k} & Q_{1jl} + \alpha_{ik}\boldsymbol{b}_i\boldsymbol{b}_i^{\rm T} & * & * \\ \tilde{\Delta}_{1ik} & A_k R_k + W_{2k} C + \tilde{\Delta}_{2ik} & G_{jl} + \alpha_{ik}\left(\boldsymbol{b}_k - \boldsymbol{b}_i\right)\boldsymbol{b}_i^{\rm T} & Q_{2jl} + \alpha_{ik}\left(\boldsymbol{b}_k - \boldsymbol{b}_i\right)\left(\boldsymbol{b}_k - \boldsymbol{b}_i\right)^{\rm T} & * \\ \boldsymbol{E}_k R_k & \boldsymbol{E}_k R_k & \alpha_{ik} f_k \boldsymbol{b}_i^{\rm T} & \alpha_{ik} f_k\left(\boldsymbol{b}_k - \boldsymbol{b}_i\right)^{\rm T} & \alpha_{ik}\left(f_k f_k^{\rm T} - I\right) \end{bmatrix} > 0 \quad (23)$$

$$\begin{bmatrix} 2R_k - Q_{1ik} & * & * & * & * \\ -G_{ik} & 2R_k - Q_{2ik} & * & * & * \\ A_i R_k + B_i W_{1k} & B_i W_{1k} & Q_{1jl} + \alpha_{ik}\boldsymbol{b}_i\boldsymbol{b}_i^{\rm T} & * & * \\ \tilde{\Delta}_{1ik} & A_k R_k + W_{2k} C + \tilde{\Delta}_{2ik} & G_{jl} - \alpha_{ik}\boldsymbol{b}_i\boldsymbol{b}_i^{\rm T} & Q_{2jl} + \alpha_{ik}\boldsymbol{b}_i\boldsymbol{b}_i^{\rm T} & * \\ \boldsymbol{E}_i R_k & 0 & \alpha_{ik} f_i \boldsymbol{b}_i^{\rm T} & -\alpha_{ik} f_i \boldsymbol{b}_i^{\rm T} & \alpha_{ik}\left(f_i f_i^{\rm T} - I\right) \end{bmatrix} > 0 \quad (26)$$

$$\boldsymbol{\xi}^{\mathrm{T}}(t)\left(\tilde{A}_{ik}^{\mathrm{T}}\tilde{P}_{jl}\tilde{A}_{ik}-\tilde{P}_{ik}\right)\boldsymbol{\xi}(t)\leq 0 \tag{27}$$

Neither the region information (2) of the controlled system nor the region information (4) of the controller is incorporated in (27). Then, the following sufficient condition (28) can be derived.

$$\begin{bmatrix} 2R_k-Q_{1ik} & * & * & * \\ -G_{ik} & 2R_k-Q_{2ik} & * & * \\ A_iR_k+B_iW_{1k} & B_iW_{1k} & Q_{1jl} & * \\ \tilde{\Delta}_{1ik} & A_kR_k+W_{2k}C+\tilde{\Delta}_{2ik} & G_{jl} & Q_{2jl}\end{bmatrix}>0 \tag{28}$$

The above discussion leads to the following results.

**Theorem 1.** For all $(i,j)\in\Omega$ and $(k,l)\in\widehat{\Omega}$, if there exist symmetric positive definite matrices $Q_{1ik}\in\mathbf{R}^{n\times n}$, $Q_{1jl}\in\mathbf{R}^{n\times n}$, $Q_{2ik}\in\mathbf{R}^{n\times n}$, $Q_{2jl}\in\mathbf{R}^{n\times n}$, $R_{1k}\in\mathbf{R}^{p\times p}$, $R_{2k}\in\mathbf{R}^{(n-p)\times(n-p)}$, matrices $G_{ik}\in\mathbf{R}^{n\times n}$, $G_{jl}\in\mathbf{R}^{n\times n}$, $W_{1k}\in\mathbf{R}^{m\times n}$, $W_{2k}\in\mathbf{R}^{n\times p}$, and positive scalars $\alpha_{ik}$ satisfying (23) for $k\in\wp_1$, (26) for $(i\in\wp_1)\cap(k\in\wp_0)$ and (28) for $(i\in\wp_0)\cap(k\in\wp_0)$, where $R_k=V\begin{bmatrix}R_{1k} & * \\ 0 & R_{2k}\end{bmatrix}V^{\mathrm{T}}$, the discrete-time PWA system (1) is PWQ stabilizable by the observer-based control law (3), the gains of which are computed by $K_k=W_{1k}R_k^{-1}$ and $L_k=W_{2k}USR_{1k}^{-1}S^{-1}U^{\mathrm{T}}$.

**Proof.** From Lemma A1 (see Appendix), it is seen that the conditions $R_k=V\begin{bmatrix}R_{1k} & * \\ 0 & R_{2k}\end{bmatrix}V^{\mathrm{T}}$ and $J_k=USR_{1k}S^{-1}U^{\mathrm{T}}$ imply the condition $CR_k=J_kC$. By combining $CR_k=J_kC$ with $L_k=W_{2k}J_k^{-1}$ and $K_k=W_{1k}R_k^{-1}$, the matrix inequality (22) is achieved from LMI (23). According to the above discussion, it is known that the condition (6) for $k\in\wp_1$ can be guaranteed by (22). In a similar way, the condition (6) for $(i\in\wp_1)\cap(k\in\wp_0)$ and $(i\in\wp_0)\cap(k\in\wp_0)$ can be guaranteed by LMIs (26) and (28), respectively. Thus, the system (1) is PWQ stabilizable by the control law (3). □

**Remark 3.** It should be pointed out that a PWQLF has been adopted in this paper. If we consider a global-quadratic Lyapunov function, i.e. $\tilde{Q}_{ik}=\tilde{Q}=\begin{bmatrix}Q_1 & * \\ G & Q_2\end{bmatrix}$, the gains of the control law (3) can be obtained by solving the following LMIs:

$$\Phi_{1ik}>0 \text{ for } k\in\wp_1 \tag{29}$$

$$\Phi_{2ik}>0 \text{ for } (i\in\wp_1)\cap(k\in\wp_0) \tag{30}$$

$$\Phi_{3ik}>0 \text{ for } (i\in\wp_0)\cap(k\in\wp_0) \tag{31}$$

where $\Phi_{1ik}$, $\Phi_{2ik}$, and $\Phi_{3ik}$ denote the matrices in (23), (26), and (28) with $Q_{1ik}=Q_{1jl}=Q_1$, $Q_{2ik}=Q_{2jl}=Q_2$, and $G_{ik}=G_{jl}=G$, respectively.

**Remark 4.** The conditions in Theorem 1 are less conservative than (29) $\sim$ (31). In fact, if (29) $\sim$ (31) have a feasible solution ($Q_{1,f}$, $Q_{2,f}$, $R_{k,f}$, $G_f$, $W_{1k,f}$, $W_{2k,f}$, $\alpha_{ik,f}$), Theorem 1 has the feasible solution ($Q_{1ik,f}$, $Q_{1jl,f}$, $Q_{2ik,f}$, $Q_{2jl,f}$, $R_{k,f}$, $G_{ik,f}$, $G_{jl,f}$, $W_{1k,f}$,$W_{2k,f}$, $\alpha_{ik,f}$) with $Q_{1ik,f}=Q_{1jl,f}=Q_{1,f}$, $Q_{2ik,f}=Q_{2jl,f}=Q_{2,f}$, and $G_{ik,f}=G_{jl,f}=G_f$, but not vice versa.

If system (1) has no affine terms, i.e. $\wp=\wp_0$, it degenerates to the following PWL system.

$$\begin{aligned}&\boldsymbol{x}(t+1)=A_i\boldsymbol{x}(t)+B_i\boldsymbol{u}(t)\\&\boldsymbol{y}(t)=C\boldsymbol{x}(t)\\&\text{for } \boldsymbol{x}(t)\in S_i,\ i\in\wp\end{aligned} \tag{32}$$

For this PWL system, we can design an observer-based control law of the form

$$\begin{aligned}&\hat{\boldsymbol{x}}(t+1)=A_k\hat{\boldsymbol{x}}(t)+B_k\boldsymbol{u}(t)+L_k\left(\hat{\boldsymbol{y}}(t)-\boldsymbol{y}(t)\right)\\&\hat{\boldsymbol{y}}(t)=C\hat{\boldsymbol{x}}(t)\\&\boldsymbol{u}(t)=K_k\hat{\boldsymbol{x}}(t)\\&\text{for } \hat{\boldsymbol{x}}(t)\in S_k,\ k\in\wp\end{aligned} \tag{33}$$

In addition, the synthesis results for the non-synchronized output feedback controller (33) can be extracted from Theorem 1.

**Corollary 1.** For all $(i,j)\in\Omega$ and $(k,l)\in\widehat{\Omega}$, if there exist symmetric positive definite matrices $Q_{1ik}\in\mathbf{R}^{n\times n}$, $Q_{1jl}\in\mathbf{R}^{n\times n}$, $Q_{2ik}\in\mathbf{R}^{n\times n}$, $Q_{2jl}\in\mathbf{R}^{n\times n}$, $R_{1k}\in\mathbf{R}^{p\times p}$, $R_{2k}\in\mathbf{R}^{(n-p)\times(n-p)}$, and matrices $G_{ik}\in\mathbf{R}^{n\times n}$, $G_{jl}\in\mathbf{R}^{n\times n}$, $W_{1k}\in\mathbf{R}^{m\times n}$, $W_{2k}\in\mathbf{R}^{n\times p}$ satisfying (28) where $R_k=V\begin{bmatrix}R_{1k} & * \\ 0 & R_{2k}\end{bmatrix}V^{\mathrm{T}}$, the discrete-time PWL system (32) is PWQ stabilizable by the observer-based control law (33), the gains of which are computed by $K_k=W_{1k}R_k^{-1}$ and $L_k=W_{2k}USR_{1k}^{-1}S^{-1}U^{\mathrm{T}}$.

**Proof.** The proof is the same as that for the case $(i\in\wp_0)\cap(k\in\wp_0)$ in Theorem 1. □

## 3 Example

The proposed non-synchronized output feedback control scheme will be applied to stabilize a piecewise affine chaotic system in this section. The dynamical behavior of the system is described as follows[6, 22]:

$$\begin{aligned}&\dot{x}_1=-9.2156\,[x_1-x_2+g(x_1)]+u\\&\dot{x}_2=x_1-x_2+x_3\\&\dot{x}_3=-15.9946x_2\end{aligned} \tag{34}$$

where $g(x_1)$ is a PWA function described as follows

$$g(x_1)=\begin{cases}-0.75735x_1-0.4917, & 1\leq x_1\leq 4\\ -1.24905x_1, & -1<x_1<1\\ -0.75735x_1+0.4917, & -4\leq x_1\leq -1\end{cases}$$

The autonomous chaotic behavior with the initial condition $\boldsymbol{x}(0)=[\ -1\quad -0.5\quad 0\ ]^{\mathrm{T}}$ is shown in Fig. 1. Discretizing (34) with the sampling period $T_s=0.01\,\mathrm{s}$ leads to

$$\begin{aligned}&\boldsymbol{x}(t+1)=A_i\boldsymbol{x}(t)+B\boldsymbol{u}(t)+\boldsymbol{b}_i\\&\boldsymbol{y}(t)=C\boldsymbol{x}(t)\\&\text{for } \boldsymbol{x}(t)\in S_i,\ i=1,2,3\end{aligned}$$

with

$$A_1=A_3=\begin{bmatrix}0.9776 & 0.0922 & 0\\ 0.0100 & 0.9900 & 0.0100\\ 0 & -0.1599 & 1.0000\end{bmatrix}$$

$$A_2=\begin{bmatrix}1.0230 & 0.0922 & 0\\ 0.0100 & 0.9900 & 0.0100\\ 0 & -0.1599 & 1.0000\end{bmatrix}$$

$$\boldsymbol{B}=\begin{bmatrix}0.01\\0\\0\end{bmatrix},\ \boldsymbol{b}_1=-\boldsymbol{b}_3=\begin{bmatrix}0.0453\\0\\0\end{bmatrix},\ \boldsymbol{b}_2=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\boldsymbol{C}=[\ 1\quad 1\quad 0\ ]$$

Here, each of the three polyhedral operating regions can be precisely represented by an ellipsoid, i.e.,

$$\varepsilon_i=\{\boldsymbol{x}(t)\mid\ ||\boldsymbol{E}_i\boldsymbol{x}(t)+f_i||\leq 1\}=S_i,\ i=1,2,3$$

with $\boldsymbol{E}_1 = \boldsymbol{E}_3 = \left[\begin{array}{ccc} 0.6667 & 0 & 0 \end{array}\right]$, $\boldsymbol{E}_2 = \left[\begin{array}{ccc} 1 & 0 & 0 \end{array}\right]$, $f_1 = -f_3 = -1.6667$, and $f_2 = 0$.

According to theorem 1, we design an observer-based control law (3) with the following gains.

$$\boldsymbol{K}_1 = \boldsymbol{K}_3 = \left[\begin{array}{ccc} -21.7239 & -12.0737 & -0.5445 \end{array}\right]$$

$$\boldsymbol{K}_2 = \left[\begin{array}{ccc} -23.1988 & -11.7606 & -0.4953 \end{array}\right]$$

$$\boldsymbol{L}_1 = \boldsymbol{L}_3 = \left[\begin{array}{c} -0.5811 \\ -0.5070 \\ -0.0189 \end{array}\right], \quad \boldsymbol{L}_2 = \left[\begin{array}{c} -0.5779 \\ -0.5169 \\ -0.0293 \end{array}\right]$$

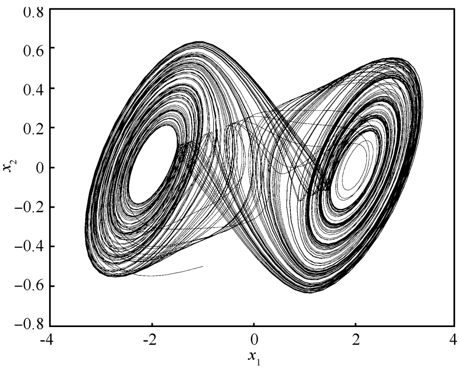



Fig. 1 Chaotic attractor of the autonomous system

A number of simulation studies with this designed controller have been carried out and the results show that the system can be stabilized to the origin successfully. Two typical cases with the initial condition $\boldsymbol{x}(0) = \left[\begin{array}{ccc} -1 & -0.5 & 0 \end{array}\right]^{\mathrm{T}}$ are recorded in Figs. 2 and 3, respectively. In the first case, it is tested that whether the system can be stabilized if the system state is in chaos. During the initial 10 s, no control input is added, so the system is in autonomous mode and performs a chaotic behavior. At $t = 10$ s, the system state stays in the region $S_1$, and the controller state is set to $\hat{\boldsymbol{x}}(0) = \left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right]^{\mathrm{T}}$ and thus stays in the region $S_2$. After $t \geq 10$ s, the controller estimates the system state and calculates the control input. Moreover, the calculated control input is added to the system. Though the system and the controller can not be guaranteed to switch to the same region at the same time, the system can be stabilized several seconds later. In the second case, it is tested that whether the system can be stabilized if the control input is directly added after $t \geq 0$ s. The initial system state stays in the region $S_3$, and the initial controller state is also set to $\hat{\boldsymbol{x}}(0) = \left[\begin{array}{ccc} 0 & 0 & 0 \end{array}\right]^{\mathrm{T}}$ and stays in the region $S_2$. Though the system and the controller can not be guaranteed to switch to the same region at the same time, the system can be stabilized several seconds later.

## 4 Conclusion

In the framework of LMI theory, this paper designs an observer-based controller for discrete-time PWA systems based on a PWQLF. The controller does not need to know in which region the system state is, so the transitions of the controller and the system are not required to be synchronized. The designed controller guarantees the closed-loop system to be PWQ stable. Application to chaos stabilization is presented to demonstrate the controller performance.

## Appendix

**Lemma A1**[23]**.** For a given $C \in \mathbf{R}^{p\times n}$ with rank $(C) = P$, assume that $R_k \in \mathbf{R}^{n\times n}$ is a symmetric matrix, then there exists a matrix $J_k \in \mathbf{R}^{p\times p}$ such that $CR_k = J_kC$ if and only if

$$R_k = V\left[\begin{array}{cc} R_{1k} & * \\ 0 & R_{2k} \end{array}\right]V^{\mathrm{T}}$$

where $R_{1k} \in \mathbf{R}^{p\times p}$ and $R_{2k} \in \mathbf{R}^{(n-p)\times(n-p)}$.

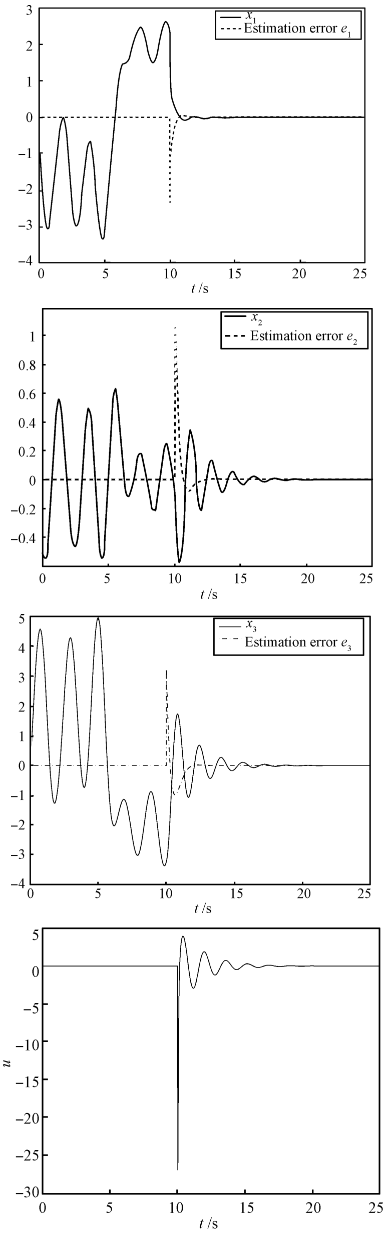
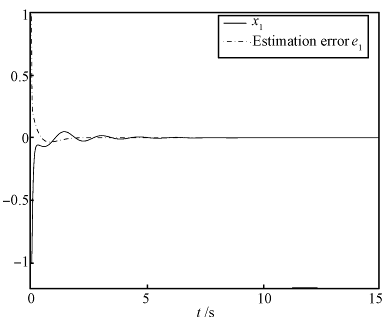



Fig. 2 Time responses of closed-loop control system: the first case



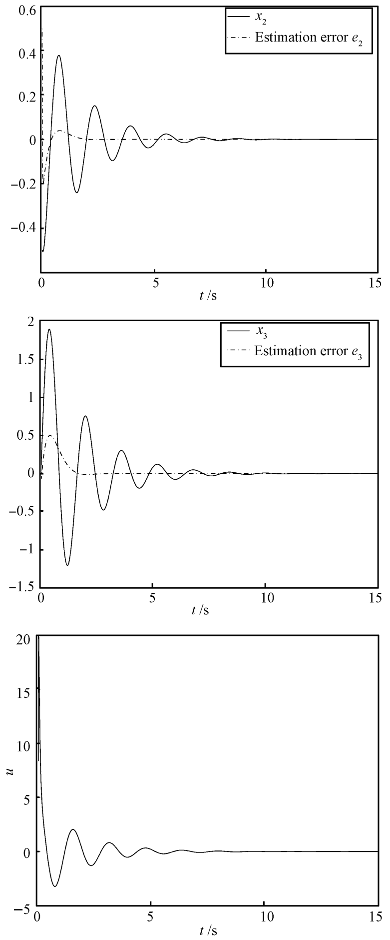



Fig. 3 Time responses of closed-loop control system: the second case

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