Non-fragile H_∞ Filter Design for Delta Operator Formulated Systems with Circular Region Pole Constraints: an LMI Optimization Approach

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Abstract The problem of non-fragile H_∞ filtering for a class of linear systems described by delta operator with circular region pole constraints is investigated. The purpose of the paper is to design a filter such that the error filtering system not only satisfies the prescribed circular pole constraints or D-stability constraint, but also meets the prescribed H_∞ norm constraint on the transfer function from the disturbance input to the estimation error. The filter gain to be designed is assumed to have multiplicative gain variations. A sufficient condition for the existence of such a filter is obtained by using appropriate Lyapunov function and linear matrix inequality (LMI) technique. A numerical example is provided to demonstrate the effectiveness and less conservativeness of the proposed design.

Key words Delta operator systems, non-fragile, H_∞ filtering, multiplicative gain variations, linear matrix inequality (LMI)

While robustness relates to uncertainties in the plant, fragility relates to the inaccuracies or uncertainties in the implementation of a designed filter or controller[1]. In actual engineering systems, the filters and the controllers realized by microprocessors/microcontrollers do have some uncertainties due to limitation in available microprocessor/microcontroller memory, effects of finite word length of the digital processor, quantization of A/D and D/A converters, and so on[2−3]. It has been shown that optimum and robust controllers designed by modern robust control design techniques could be very sensitive or fragile with respect to error/uncertainty in controller parameters[4]. Therefore, the design of non-fragile (or resilient) controller and filter has recently received increasing attention, mainly in additive gain variations[5−7], and the multiplicative cases are investigated by Yang[1, 8]. In 2002, Yang[5] investigated a non-fragile nonlinear H_∞ control with additive controller gain variations. In 2006, Yang[6] studied the problem of non-fragile filter design for continuous-time systems, in which the filters to be designed were assumed to be with additive gain variations. In 2008, Che[7] investigated the non-fragile H_∞ filtering problem affected by finite word length (FWL) for linear discrete-time systems. A robust non-fragile Kalman filtering problem for uncertain linear systems with estimator gain uncertainty was addressed by Yang[1], where the multiplicative uncertainty model was used to describe degradations of sensors. In 2001, Yang[8] presented the non-fragile H_∞ output feedback controller design with multiplicative controller gain variations using Riccati equations method. However, most of the existing results of analysis and synthesis for non-fragile H_∞ filter or controller have been obtained separately for continuous-time and discrete-time systems.

Meanwhile, there has also been a increasing interest in constructing delta operator instead of traditional Z-transform in sampling the continuous systems. The delta operator as a new discretization method can solve the unstable problem caused by the design method of the Z-transform[9]. Two major advantages are known for the use of delta operator parametrization: a theoretically unified formulation of continuous-time and discrete-time systems[10−11], and better numerical properties in FWL implementations compared with traditional Z-transform at a high sampling period[12]. On the other hand, as is well known, the estimation dynamics of a linear system is closely related to the location of its poles. By constraining the filter′s poles to lie inside a prescribed region in the complex plane, the designed filter would have the expected transient performance[13]. Hence, by combining the delta operator theory and pole-placement method with non-fragile filter theory, the unstable and fragile problem of filtering error system can be solved, and well transient performance can be obtained at the same time. In the past few years, the robust filter problem for delta operator systems has been studied by a number of researchers[14−15], but all the studies are based on an implicit assumption that the filter is implemented exactly. Although the robust non-fragile H_∞ state feedback controller for a class of uncertain systems was designed based on delta operator, where the controller and the controlled object parameters were assumed to have additive norm-bounded variations by Lin[9], the non-fragile filter problem for delta operator systems remains to be resolved.

Motivated by above points, a non-fragile H_∞ filter with the considerations of the multiplicative gain variations is designed for a class of linear systems described by delta operator with circular region pole constraints. The rest of paper is organized as follows. At first, we introduce the delta operator model to overcome unstable problem caused by using traditional Z-transform at high sampling rates. Next, a sufficient condition for the existence of such a non-fragile H_∞ filter is obtained by appropriate Lyapunov function and LMI technique. Less conservativeness can be introduced by considering a more general type of filter gain uncertainties. Then, a convex optimization problem is formulated, and the optimal solutions to the non-fragile H_∞ filter problem with pole location for the domain considered is also provided. Finally, a numerical example is given to illustrate the effectiveness of the developed techniques.

The notations used throughout this paper are fairly standard. $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, $\mathbf{R}^{m\times n}$ is the set of all $m \times n$ real matrices. We use "$*$" as an ellipsis for the terms that are introduced by symmetry.

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1 Problem formulation

Consider the following linear continuous system:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= A\boldsymbol{x}(t) + B\boldsymbol{w}(t) \\ \boldsymbol{y}(t) &= C\boldsymbol{x}(t) + D\boldsymbol{w}(t) \\ \boldsymbol{z}(t) &= L\boldsymbol{x}(t) \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ is the state, $\boldsymbol{w}(t) \in \mathbf{R}^r$ is the disturbance input that belongs to $l_2[0,\infty)$, $\boldsymbol{z}(t) \in \mathbf{R}^q$ is the regulated output, and $\boldsymbol{y}(t) \in \mathbf{R}^p$ is the measured output, respectively. The system matrices A, B, C, D, and L are known constant matrices of appropriate dimensions.

Then, the delta operator system can be given as follows

$$\begin{cases} \delta\boldsymbol{x}(k) = A_\delta\boldsymbol{x}(k) + B_\delta\boldsymbol{w}(k) \\ \boldsymbol{y}(k) = C\boldsymbol{x}(k) + D\boldsymbol{w}(k) \\ \boldsymbol{z}(k) = L\boldsymbol{x}(k) \\ \boldsymbol{x}(k) = \mathbf{0}, \quad k \le 0 \\ A_\delta = \dfrac{(A_z - I)}{h}, A_z = \mathrm{e}^{Ah}, B_\delta = \dfrac{B_z}{h}, B_z = \int_0^h \mathrm{e}^{A\tau} B \mathrm{d}\tau \\ \delta\boldsymbol{x}(k) = \dfrac{(\boldsymbol{x}(k+1) - \boldsymbol{x}(k))}{h} \end{cases} \tag{2}$$

Throughout the paper, I denotes an identity matrix of appropriate dimension and h denotes the sampling period. A_δ and B_δ are the corresponding delta operator system matrices, A_z and B_z are the z-domain discrete system matrices. C, D, and L are the same as the z-domain discrete system matrices, respectively. And δ is the delta operator defined by

$$\delta\boldsymbol{x}(t) \triangleq \begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}(t), & h = 0 \\ \dfrac{\boldsymbol{x}(t+h) - \boldsymbol{x}(t)}{h}, & h \neq 0 \end{cases} \tag{3}$$

On the other hand, it is obvious that

$$\lim_{h\to 0} A_\delta = \lim_{h\to 0} \frac{(\mathrm{e}^{Ah} - I)}{h} = A, \ \lim_{h\to 0} B_\delta = B \tag{4}$$

Consequently, when $h \to 0$, the δ-domain discrete system changes to a continuous system.

We are interested in designing an delta operator filter

$$\begin{aligned} \delta\bar{\boldsymbol{x}}(k) &= A_{\delta F}\bar{\boldsymbol{x}}(k) + B_{\delta F}\boldsymbol{y}(k) \\ \bar{\boldsymbol{z}}(k) &= C_{\delta F}\bar{\boldsymbol{x}}(k) \end{aligned} \tag{5}$$

where $\bar{\boldsymbol{x}}(k) \in \mathbf{R}^n$ is the filter state, $A_{\delta F}$, $B_{\delta F}$, and $C_{\delta F}$ are the parameters of the filter with multiplicative gain variations described by

$$\begin{aligned} A_{\delta F} &= A_{\delta F1}(I + \Gamma_1) \\ B_{\delta F} &= B_{\delta F1}(I + \Gamma_2) \\ C_{\delta F} &= C_{\delta F1}(I + \Gamma_3) \end{aligned} \tag{6}$$

where $A_{\delta F1}$, $B_{\delta F1}$, and $C_{\delta F1}$ are the filter parameters to be designed. Γ_1, Γ_2, and Γ_3 represent the gain variations with the following form

$$\Gamma_1 = H_1\Re_1(k)E_1, \ \Gamma_2 = H_2\Re_2(k)E_2, \ \Gamma_3 = H_3\Re_3(k)E_3 \tag{7}$$

where H_i and E_i $(i = 1, 2, 3)$ are known constant real matrices with appropriate dimensions, $\Re_i(k)$ denotes time-varying parameter uncertainties, and is assumed to be of diagonal form

$$\Re_i(k) = \mathrm{diag}\{\Re_{i1}(k), \cdots, \Re_{ir}(k)\} \tag{8}$$

where $\Re_{il} \in \mathbf{R}^{p_l \times q_l}, l = 1, \cdots, r$ are unknown real time-varying matrices satisfying

$$\Re_{il}^{\mathrm{T}}(k)\Re_{il}(k) \le I, \quad k = 0, 1, 2, \cdots$$

Remark 1. Lee[16] used the above uncertain model to describe system uncertainties. In this paper, we use it to describe the filter gain variations, which is more general than the uncertain model in the study by Yang[1], i.e., the less conservatism is introduced. This fact will be illustrated by numerical examples in the last section.

Consider the linear transformation on the filter state

$$\hat{\boldsymbol{x}}(t) = M\bar{\boldsymbol{x}}(t) \tag{9}$$

where M is an invertible matrix to make the design easy, which can be given out during the design of the filter. We have a new representation form of the filter as follows

$$\begin{aligned} \delta\hat{\boldsymbol{x}}(k) &= MA_{\delta F}M^{-1}\hat{\boldsymbol{x}}(k) + MB_{\delta F}\boldsymbol{y}(k) \\ \hat{\boldsymbol{z}}(k) &= C_{\delta F}M^{-1}\hat{\boldsymbol{x}}(k) \end{aligned} \tag{10}$$

Applying filter (10) to system (2), we obtain the filtering error system

$$\begin{aligned} \delta\boldsymbol{\xi}(k) &= \bar{A}_\delta\boldsymbol{\xi}(k) + \bar{B}_\delta\boldsymbol{w}(k) \\ \boldsymbol{e}(k) &= \bar{C}_\delta\boldsymbol{\xi}(k) \end{aligned} \tag{11}$$

where $\boldsymbol{\xi}(k) = \begin{bmatrix} \boldsymbol{x}(t) \\ \hat{\boldsymbol{x}}(t) \end{bmatrix}$, $\boldsymbol{e}(k) = \boldsymbol{z}(k) - \hat{\boldsymbol{z}}(k)$ is the estimation error, $\bar{A}_\delta = \begin{bmatrix} A_\delta & 0 \\ MB_{\delta F}C & MA_{\delta F}M^{-1} \end{bmatrix}$, $\bar{B}_\delta = \begin{bmatrix} B_\delta \\ MB_{\delta F}D \end{bmatrix}$, and $\bar{C}_\delta = \begin{bmatrix} L & -C_{\delta F}M^{-1} \end{bmatrix}$.

The transfer function matrix of the filtering error system (11) from $\boldsymbol{w}(k)$ to $\boldsymbol{e}(k)$ is given by

$$G(z) = \bar{C}_\delta(zI - \bar{A}_\delta)^{-1}\bar{B}_\delta \tag{12}$$

Our objective is to develop a filter of the form (5)(or (10)) such that for all admissible filter gain variations (6), the filtering error system (11) satisfies the following requirements:

1) While there is no exogenous disturbance, i.e., $\boldsymbol{w}(k) = \mathbf{0}$, the filtering error system (11) is asymptotically stable, and all filtering error system′s poles lie in the region $D(a, r)$ in the complex plane with the center at $(a, \mathrm{j}0)$ and the radius r, and have

$$\lambda(\bar{A}_\delta) \subset D(a, r), \ |a| + r < \frac{2}{h}, \ r < \frac{1}{h} \tag{13}$$

where $\lambda(\bar{A}_\delta)$ denotes the eigenvalue of $\bar{A}_\delta$.

2) The filtering error system (11) satisfies a prescribed H_∞ performance γ, i.e., the transfer function matrix $G(z)$ satisfies

$$\|G(z)\|_\infty < \gamma \tag{14}$$

Now, we first provide some important lemmas, which will be useful in the derivation of our main results.

Lemma 1[17]. All the poles of matrix $\bar{A}_\delta \in \mathbf{R}^{n\times n}$ are located in a given circular region $D(a, r)$, i.e., $\lambda(\bar{A}_\delta) \subset D(a, r)$, if and only if there exists matrix $X > 0$ such that

1)

$$(I + hA_a)^{\mathrm{T}}\frac{X}{h}(I + hA_a) - \frac{X}{h} < 0 \tag{15}$$

2)

$$\begin{bmatrix} -rX & * \\ X\bar{A}_\delta + \beta X & -rX \end{bmatrix} < 0 \tag{16}$$

where $A_a = \dfrac{\bar{A}_\delta - aI - (1/h)I}{rh}$, $\beta = r - a - 1/h$, and the above two matrix inequalities are equivalent.

The following lemmas are introduced to handle the parameter uncertainties.

Lemma 2[16]**.** Let F, E, and $\Re$ be real matrices of appropriate dimensions with $\Re = \mathrm{diag}\{\Re_1, \cdots, \Re_r\}$, $\Re_i^{\mathrm{T}}\Re_i \leq I, i = 1, \cdots, r$. Then, for any real matrix $\Lambda = \mathrm{diag}\{\lambda_1 I, \cdots, \lambda_r I\} > 0$, the following inequality holds

$$F\Re E + E^{\mathrm{T}}\Re^{\mathrm{T}}F^{\mathrm{T}} \leq F\Lambda F^{\mathrm{T}} + E^{\mathrm{T}}\Lambda^{-1}E \tag{17}$$

Lemma 3[18]**.** If $N = N^{\mathrm{T}}$, H and E are real matrices of appropriate dimensions, with $F(k)$ satisfying $F^{\mathrm{T}}(k)F(k) \leq I$, then

$$N + HF(k)E + E^{\mathrm{T}}F^{\mathrm{T}}(k)H^{\mathrm{T}} < 0 \tag{18}$$

if and only if there exists a constant $\epsilon > 0$ such that

$$N + \frac{1}{\epsilon}HH^{\mathrm{T}} + \epsilon E^{\mathrm{T}}E < 0 \tag{19}$$

2 Main result

2.1 H_∞ filtering for delta operator formulated systems

Here, we have not included any appendant objective in the synthesis. Before continuing with the solution to the synthesis problem, we present the following theorem that guarantees that the filtering error system (11) is asymptotically stable and has H_∞ performance criteria at the same time.

Theorem 1. Given scalar $\gamma > 0$ and the sampling period h, if there exist some matrices $X = X^{\mathrm{T}} > 0$, $\bar{A}_\delta$, $\bar{B}_\delta$, $\bar{C}_\delta$ such that

$$\begin{bmatrix} hX\bar{A}_\delta + h\bar{A}_\delta^{\mathrm{T}}X & * & * & * \\ h\bar{B}_\delta^{\mathrm{T}}X & -\gamma^2h^2I & * & * \\ hX\bar{A}_\delta & hX\bar{B}_\delta & -X & * \\ h\bar{C}_\delta & 0 & 0 & -I \end{bmatrix} < 0 \tag{20}$$

holds, then the filtering error system (11) is asymptotically stable and satisfies H_∞ performance constraint.

Proof. Due to the limit of the space, the proof is omitted. □

2.2 Non-fragile H_∞ filter with D-stability constraints

In this subsection, we develop the non-fragile H_∞ filter with D-stability constraints based on LMI technique.

Definition 1. For a prescribed $\gamma > 0$ and the sampling period h, assume that there exist $X = X^{\mathrm{T}} > 0$ and filter parameters $\bar{A}_\delta$, $\bar{B}_\delta$, and $\bar{C}_\delta$ satisfying (16) and (20) at the same time. Then, the filtering error system (11) is D-stable and satisfies H_∞ norm constraint simultaneously.

It is obvious that (16) and (20) are not LMIs due to the products of the variable X with the filtering error system matrices $\bar{A}_\delta$ and $\bar{B}_\delta$, respectively. As a result, the LMI software fails to solve (16) and (20). Hence, based on LMI technique, we give the sufficient condition for the existence of the non-fragile H_∞ filter such that the filtering error system (11) is D-stable and satisfies H_∞ norm constraint simultaneously as the following theorem.

Theorem 2. For a prescribed $\gamma > 0$, β defined in (16) and the sampling period h, we assume that there exist $S = S^{\mathrm{T}} > 0$, $R = R^{\mathrm{T}} > 0$, $\hat{A}_{\delta F}$, $\hat{B}_{\delta F}$, $\hat{C}_{\delta F}$ and $\Lambda_i = \mathrm{diag}\{\lambda_{i1}I, \cdots, \lambda_{ir}I\} (i = 1, 2, 3)$, such that

$$\Omega_1 = \begin{bmatrix} \Xi_{01} & * \\ \Xi_{02} & \Xi_{03} \end{bmatrix} < 0, \ \Omega_2 = \begin{bmatrix} \Xi_1 & * \\ \Xi_2 & \Xi_3 \end{bmatrix} < 0 \tag{21}$$

where

$$\Xi_{01} = \begin{bmatrix} -rS & * & * & * \\ -rS & -rR & * & * \\ \emptyset_{21} & \emptyset_{21} & -rS & * \\ \varphi_{31} & \varphi_{32} & -rS & -rR \end{bmatrix}$$

$$\Xi_{02} = \begin{bmatrix} 0 & 0 & 0 & H_1^{\mathrm{T}}\hat{A}_{\delta F}^{\mathrm{T}} \\ \Lambda_1E_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & H_2^{\mathrm{T}}\hat{B}_{\delta F}^{\mathrm{T}} \\ \Lambda_2E_2C & \Lambda_2E_2C & 0 & 0 \end{bmatrix}$$

$$\Xi_{03} = \{-\Lambda_1I, -\Lambda_1I, -\Lambda_2I, -\Lambda_2I\}$$

$$\Xi_1 = \begin{bmatrix} \emptyset_{22} & * & * & * & * & * \\ \Upsilon_{31} & \Upsilon_{32} & * & * & * & * \\ hB_\delta^{\mathrm{T}}S & \Upsilon_{33} & -\gamma^2h^2I & * & * & * \\ hSA_\delta & hSA_\delta & hSB_\delta & -S & * & * \\ \Upsilon_{34} & \Upsilon_{35} & \Upsilon_{36} & -I & -R & * \\ \Upsilon_{37} & hL & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\Xi_2 = \begin{bmatrix} 0 & H_2^{\mathrm{T}}\hat{B}_{\delta F}^{\mathrm{T}} & 0 & 0 & H_2^{\mathrm{T}}\hat{B}_{\delta F}^{\mathrm{T}} & 0 \\ h\Lambda_2E_2C & h\Lambda_2E_2C & h\Lambda_2E_2D & 0 & 0 & 0 \\ 0 & H_1^{\mathrm{T}}\hat{A}_{\delta F}^{\mathrm{T}} & 0 & 0 & H_1^{\mathrm{T}}\hat{A}_{\delta F}^{\mathrm{T}} & 0 \\ h\Lambda_1E_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -H_3^{\mathrm{T}}\hat{C}_{\delta F}^{\mathrm{T}} \\ h\Lambda_3E_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_3 = \{-\Lambda_2I, -\Lambda_2I, -\Lambda_1I, -\Lambda_1I, -\Lambda_3I, -\Lambda_3I\}$$

with $\emptyset_{21} = SA_\delta + \beta S$, $\emptyset_{22} = hSA_\delta + hA_\delta^{\mathrm{T}}S$, $\varphi_{31} = RA_\delta + \hat{B}_{\delta F}C + \hat{A}_{\delta F} + \beta S$, $\varphi_{32} = RA_\delta + \hat{B}_{\delta F}C + \beta R$, $\Upsilon_{31} = hA_\delta^{\mathrm{T}}S + hRA_\delta + h\hat{B}_{\delta F}C + h\hat{A}_{\delta F}$, $\Upsilon_{32} = hA_\delta^{\mathrm{T}}R + hRA_\delta + h\hat{B}_{\delta F}C + hC^{\mathrm{T}}\hat{B}_{\delta F}^{\mathrm{T}}$, $\Upsilon_{33} = hB_\delta^{\mathrm{T}}R + hD^{\mathrm{T}}\hat{B}_{\delta F}^{\mathrm{T}}$, $\Upsilon_{34} = h(RA_\delta + \hat{B}_{\delta F}C + \hat{A}_{\delta F})$, $\Upsilon_{35} = hRA_\delta + h\hat{B}_{\delta F}C$, $\Upsilon_{36} = hRB_\delta + h\hat{B}_{\delta F}D$, and $\Upsilon_{37} = hL - h\hat{C}_{\delta F}$. Then, the filtering error system (11) is D-stable, i.e., all its poles lie in the region $D(a, r)$ and satisfies H_∞ norm constraint simultaneously.

Moreover, if there exist solutions to these inequalities, the non-fragile filter can be given by

$$A_{\delta F1} = (S - R)^{-1}\hat{A}_{\delta F}, B_{\delta F1} = (S - R)^{-1}\hat{B}_{\delta F}, C_{\delta F1} = \hat{C}_{\delta F} \tag{22}$$

Proof. Using the idea of Gahinet[19], we partition X and X^{-1} as

$$X = \begin{bmatrix} R & X_{12} \\ X_{12}^{\mathrm{T}} & X_{22} \end{bmatrix}, \ X^{-1} = \begin{bmatrix} S^{-1} & Y_{12} \\ Y_{12}^{\mathrm{T}} & Y_{22} \end{bmatrix} \tag{23}$$

where X and X^{-1} have appropriate dimensions, and $R > 0$, $S > 0$, $S = S^{\mathrm{T}} \in \mathbf{R}^{n\times n}$, $R = R^{\mathrm{T}} \in \mathbf{R}^{n\times n}$, $X_{12} \in \mathbf{R}^{n\times n}$, $X_{22} \in \mathbf{R}^{n\times n}$, $Y_{12} \in \mathbf{R}^{n\times n}$, and $Y_{22} \in \mathbf{R}^{n\times n}$. We can assume without loss of generality that X_{12} and Y_{12} have full row rank (see [19] for details).

Construct the following matrices

$$J_1 = \begin{bmatrix} S^{-1} & I \\ Y_{12}^{\mathrm{T}} & 0 \end{bmatrix}, \quad J_2 = \begin{bmatrix} I & R \\ 0 & X_{12}^{\mathrm{T}} \end{bmatrix} \tag{24}$$

From $XX^{-1} = I$, we can get

$$\begin{bmatrix} S & S \\ S & R \end{bmatrix} > 0, \quad I - S^{-1}R = Y_{12}X_{12}^{\mathrm{T}} \tag{25}$$

where (25) can also be inferred from $\Omega_1 < 0$, and from (25), we infer $S - R < 0$ such that $I - S^{-1}R$ is nonsingular.

Then, after some manipulation including applying Lemma 2, we can obtain (21). Due to the limit of the space, the detail is omitted. □

Remark 2. It is noted that the conditions in Theorem 2 are LMI conditions with respect to the scalar γ. Hence, we can find a minimum γ using convex optimization algorithms. Then, the problem of H_∞ filter with circular pole constraints design can be converted to the following optimization problem:

$$\begin{array}{ll} \min\limits_{S,R,\hat{A}_{\delta F},\hat{B}_{\delta F},\hat{C}_{\delta F},\Lambda_1,\Lambda_2,\Lambda_3,\theta} & \theta \\ \text{s.t.} & (21), \theta = \gamma \end{array} \tag{26}$$

The minimal disturbance attenuation $\gamma^* = \theta^*$, θ^* is the optimization value of θ, and the designed filter's parameters can be obtained by (22).

Remark 3. In the above theorem, when the D-stable is not considered, i.e., $D(a,r) = D(-1/h, 1/h)$, the problem is reduced to non-fragile H_∞ filter design without circular pole constraints. Then, the problem of non-fragile H_∞ filter without circular pole constraints design can be resolved by solving LMIs $\Omega_2 < 0$ and (25).

Remark 4. The following theorem presents a sufficient condition for the solvability of the non-fragile H_∞ filtering problem with the filter gain variations (27), when the filter gain variations model is the same as the model in [1], i.e.,

$$\Gamma_1 = H_1\Re_1(k)E_1,\ \ \Gamma_2 = H_2\Re_2(k)E_2,\ \ \Gamma_3 = H_3\Re_3(k)E_3 \tag{27}$$

where $H_i, E_i\ (i = 1,2,3)$ are known constant matrices of appropriate dimensions, and $\Re_i (i = 1,2,3)$ are real uncertain matrices with

$$\Re_i^{\rm T}(k)\Re_i(k) \le I, \quad i = 1,2,3 \tag{28}$$

where $\Re_i(k)$ is without the constraint (8).

Theorem 3. For a prescribed $\gamma > 0$ and the sampling period h, we assume that there exist $S = S^{\rm T} > 0$, $R = R^{\rm T} > 0$, $\hat{A}_{\delta F}$, $\hat{B}_{\delta F}$, $\hat{C}_{\delta F}$, and scalars $\lambda_i\ (i = 1,2,3)$ such that

$$\begin{bmatrix} \Xi_{01} & * \\ \Xi_{12} & \Xi_{13} \end{bmatrix} < 0,\ \begin{bmatrix} \Xi_1 & * \\ \Xi_{22} & \Xi_{23} \end{bmatrix} < 0 \tag{29}$$

where

$$\Xi_{12} = \begin{bmatrix} 0 & 0 & 0 & H_1^{\rm T}\hat{A}_{\delta F}^{\rm T} \\ \lambda_1 E_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & H_2^{\rm T}\hat{B}_{\delta F}^{\rm T} \\ \lambda_2 E_2 C & \lambda_2 E_2 C & 0 & 0 \end{bmatrix}$$

$$\Xi_{13} = \{-\lambda_1 I, -\lambda_1 I, -\lambda_2 I, -\lambda_2 I\}$$

$$\Xi_{22} = \begin{bmatrix} 0 & H_2^{\rm T}\hat{B}_{\delta F}^{\rm T} & 0 & 0 & H_2^{\rm T}\hat{B}_{\delta F}^{\rm T} & 0 \\ h\lambda_2 E_2 C & h\lambda_2 E_2 C & h\lambda_2 E_2 D & 0 & 0 & 0 \\ 0 & H_1^{\rm T}\hat{A}_{\delta F}^{\rm T} & 0 & 0 & H_1^{\rm T}\hat{A}_{\delta F}^{\rm T} & 0 \\ h\lambda_1 E_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -H_3^{\rm T}\hat{C}_{\delta F}^{\rm T} \\ h\lambda_3 E_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_{23} = \{-\lambda_2 I, -\lambda_2 I, -\lambda_1 I, -\lambda_1 I, -\lambda_3 I, -\lambda_3 I\}$$

with Ξ_{01}, Ξ_1 defined in (21) and β defined in (16), respectively. Then, the filtering error system (11) with the filter uncertainties as (27) is D-stable and satisfies H_∞ norm constraint simultaneously. The filter parameters can also be given by (22).

Proof. By using the proposed design method as Theorem 2, and applying Lemma 3 to handle the parameter uncertainties (27), in which we only substitute three scalars. $i\,(i = 1,2,3)$ for the matrices $\Lambda_i = {\rm diag}\{\lambda_{i1}I, \cdots, \lambda_{ir}I\}\,(i = 1,2,3)$, then the non-fragile H_∞ filter with the filter uncertainties as (27) is resolved. □

Remark 5. From the proof of Theorem 3, it follows that the proposed uncertainty (7) is less conservative than the normal norm-bound parameter uncertainties (27), i.e., when the case $r = 1$ as indicated in the constraint (8), the proposed uncertainty (7) is reduced to the normal norm-bound parameter uncertainties (27).

Remark 6. When the filter parameter uncertainties are not considered, i.e., $\Gamma_1 = 0$, $\Gamma_2 = 0$, $\Gamma_3 = 0$, the problem reduces to a standard H_∞ filter design with circular pole constraints. Hence, (21) or (29) is reduced to the following LMIs:

$$\Xi_{01} < 0,\ \ \Xi_1 < 0 \tag{30}$$

2.3 Comparison with the existing design method

In this subsection, we compare our results, which do not consider the condition of D-stable, with Yang[6] (continuous system) and Che[7] (discrete system), respectively. Here we denote

$$\delta \boldsymbol{x}(t) = \begin{cases} \dot{\boldsymbol{x}}(t), & \text{continuous case} \\ \boldsymbol{x}(t+1), & \text{disctrete case} \end{cases}$$

i.e., the signification of δ in this subsection is different from the one in other sections. And the filter and filtering error system are similar to those in Section 1.

Then, by using the proposed design method as Theorem 2, we can easily obtain the following lemma for continuous system and discrete system.

Lemma 4. For a prescribed $\gamma > 0$ and Ξ_3 defined in (21), the filtering error system is asymptotically stable and satisfies H_∞ norm constraint, if there exist $S = S^{\rm T} > 0$, $R = R^{\rm T} > 0$, $\hat{A}_{\delta F}$, $\hat{B}_{\delta F}$, $\hat{C}_{\delta F}$ and $\Lambda_i = {\rm diag}\{\lambda_{i1}I, \cdots, \lambda_{ir}I\}\,(i = 1,2,3)$ such that

1) For continuous system:

$$\begin{bmatrix} \Xi_{31} & * \\ \Xi_{32} & \Xi_3 \end{bmatrix} < 0 \tag{31}$$

where

$$\Xi_{31} = \begin{bmatrix} SA + A^{\rm T}S & * & * & * \\ \Upsilon_{51} & \Upsilon_{52} & * & * \\ B^{\rm T}S & \Upsilon_{53} & -\gamma^2 I & * \\ \Upsilon_{54} & L & 0 & -I \end{bmatrix}$$

$$\Xi_{32} = \begin{bmatrix} 0 & H_2^{\rm T}\hat{B}_{\delta F}^{\rm T} & 0 & 0 \\ \Lambda_2 E_2 C & \Lambda_2 E_2 C & \Lambda_2 E_2 D & 0 \\ 0 & H_1^{\rm T}\hat{A}_{\delta F}^{\rm T} & 0 & 0 \\ \Lambda_1 E_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -H_3^{\rm T}\hat{C}_{\delta F}^{\rm T} \\ \Lambda_3 E_3 & 0 & 0 & 0 \end{bmatrix}$$

and $\Upsilon_{51} = A^{\rm T}S + R^{\rm T}A + \hat{B}_{\delta F}C + \hat{A}_{\delta F}$, $\Upsilon_{52} = A^{\rm T}R + R^{\rm T}A + \hat{B}_{\delta F}C + C^{\rm T}\hat{B}_{\delta F}^{\rm T}$, $\Upsilon_{53} = B^{\rm T}R + D^{\rm T}\hat{B}_{\delta F}^{\rm T}$, $\Upsilon_{54} = L - \hat{C}_{\delta F}$.

2) For discrete system:

$$\begin{bmatrix} \Xi_{41} & * \\ \Xi_{42} & \Xi_3 \end{bmatrix} < 0 \tag{32}$$

where

$$\Xi_{41}=\begin{bmatrix} -S & * & * & * & * & * \\ -S & -R & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ SA_z & SA_z & SB_z & -S & * & * \\ \Upsilon_{61} & \Upsilon_{62} & \Upsilon_{63} & -S & -R & * \\ \Upsilon_{64} & L & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\Xi_{42}=\begin{bmatrix} 0 & 0 & 0 & 0 & H_2^{\rm T}\hat{B}_{\delta F}^{\rm T} & 0 \\ \Lambda_2 E_2 C & \Lambda_2 E_2 C & \Lambda_2 E_2 D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & H_1^{\rm T}\hat{A}_{\delta F}^{\rm T} & 0 \\ \Lambda_1 E_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -H_3^{\rm T}\hat{C}_{\delta F}^{\rm T} \\ \Lambda_3 E_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Upsilon_{61}=RA_z+\hat{B}_{\delta F}C+\hat{A}_{\delta F}$, $\Upsilon_{62}=RA_z+\hat{B}_{\delta F}C$, $\Upsilon_{63}=RB_z+\hat{B}_{\delta F}D$, and $\Upsilon_{64}=L-\hat{C}_{\delta F}$. Furthermore, the inequality (25) should also be satisfied for the continuous case and the discrete case. The designed filter′s parameters can also be obtained by (22).

3 Numerical simulations

In this section, numerical simulations are carried out to confirm validity and advantages of the proposed method, and to show the characteristics of discrete-time systems and delta operator systems in sampling the continuous-time systems.

3.1 Simulation for the proposed method

Consider a continuous-time system in s-domain:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \begin{bmatrix} -0.7 & 0.4 & 0.6 \\ -0.4 & -0.5 & 0.4 \\ -0.6 & -0.4 & -0.5 \end{bmatrix}\boldsymbol{x}(t)+\begin{bmatrix} 0.05 & 0 \\ 0.05 & 0 \\ 0.06 & 0 \end{bmatrix}\boldsymbol{w}(t) \\ \boldsymbol{y}(t) &= \begin{bmatrix} 3 & -2 & -1 \end{bmatrix}\boldsymbol{x}(t)+\begin{bmatrix} 1 & 0.9 \end{bmatrix}\boldsymbol{w}(t) \\ \boldsymbol{z}(t) &= \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\boldsymbol{x}(t) \end{aligned}$$

and the part of filter gain variations as follows

$$H_1=\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix},\quad E_1=0.02\times\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\boldsymbol{H}_2=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\boldsymbol{E}_2=0.02\times\begin{bmatrix} 1 & 2 & 1 & 2 & 1 & 2 \end{bmatrix}^{\rm T}$$

$$H_3=\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}, E_3=0.02\times\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & 2 & 1 \\ 4 & 1 & -1 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\Re_i(k)=\begin{bmatrix} \Re_{i1}(k) & 0 \\ 0 & \Re_{i2}(k) \end{bmatrix},\quad i=1,2,3$$

where $\Re_{i1}(k),\Re_{i2}(k)\in\mathbf{R}^{3\times 3}$.

By using shift operator and delta operator in sampling the continuous-time system, we get the relevant different discrete-time systems in z-domain and δ-domain.

1) When $h=0.1\,$s, there exist

$$A_z=\begin{bmatrix} 0.9300 & 0.0365 & 0.0572 \\ -0.0388 & 0.9497 & 0.0369 \\ -0.0557 & -0.0391 & 0.9488 \end{bmatrix}, B_z=\begin{bmatrix} 0.0051 & 0 \\ 0.0049 & 0 \\ 0.0056 & 0 \end{bmatrix}$$

$$A_\delta=\begin{bmatrix} -0.7005 & 0.3650 & 0.5720 \\ -0.3876 & -0.5029 & 0.3687 \\ -0.5569 & -0.3914 & -0.5123 \end{bmatrix}, B_\delta=\begin{bmatrix} 0.0509 & 0 \\ 0.0489 & 0 \\ 0.0561 & 0 \end{bmatrix}$$

2) When $h=0.1\,$ms, there exist

$$A_z=\begin{bmatrix} 0.9999 & 0.0000 & 0.0001 \\ -0.0000 & 0.9999 & 0.0000 \\ -0.0001 & -0.0000 & 0.9999 \end{bmatrix}$$
$$B_z=10^{-5}\times\begin{bmatrix} 0.5000 & 0 \\ 0.5000 & 0 \\ 0.5000 & 0 \end{bmatrix}$$

$$A_\delta=\begin{bmatrix} -0.7000 & 0.4000 & 0.6000 \\ -0.4000 & -0.5000 & 0.4000 \\ -0.6000 & -0.4000 & -0.5000 \end{bmatrix}, B_\delta=\begin{bmatrix} 0.0500 & 0 \\ 0.0500 & 0 \\ 0.0600 & 0 \end{bmatrix}$$

3) When $h=1\,$s, there exist

$$A_z=\begin{bmatrix} 0.3662 & 0.1311 & 0.3362 \\ -0.2599 & 0.5177 & 0.1526 \\ -0.2504 & -0.2814 & 0.4640 \end{bmatrix}, B_z=\begin{bmatrix} 0.0511 & 0 \\ 0.0366 & 0 \\ 0.0269 & 0 \end{bmatrix}$$

$$A_\delta=\begin{bmatrix} -0.6338 & 0.1311 & 0.3362 \\ -0.2599 & -0.4823 & 0.1526 \\ -0.2504 & -0.2814 & -0.5360 \end{bmatrix}, B_\delta=\begin{bmatrix} 0.0511 & 0 \\ 0.0366 & 0 \\ 0.0269 & 0 \end{bmatrix}$$

From the results, we find that when $h=0.0001\,$s, the delta operator system is reduced to continuous system, and we can also find the delta operator model has the advantage of better numerical properties at high sampling rates, when $h=1\,$s, the delta operator system is reduced to one of the discrete systems.

For different sampling periods and the given circular region $D(a,r)$, we can obtain the filter gain matrices and the filtering error system′s poles by solving the optimization problem (26). Then, we have the following results.

1) When $h=0.1\,$s and the region is given as $D(-5,5)$, we obtain the filter gain matrices

$$A_{\delta F1}=\begin{bmatrix} -1.0123 & -0.2165 & 0.4245 \\ -0.6543 & -0.6154 & 0.0019 \\ -0.5664 & -0.4715 & -0.7116 \end{bmatrix}$$

$$\boldsymbol{B}_{\delta F1}=\begin{bmatrix} 0.0814 \\ 0.0681 \\ 0.0590 \end{bmatrix}$$
$$\boldsymbol{C}_{\delta F1}=\begin{bmatrix} 1.2048 & 1.4689 & 1.7028 \end{bmatrix}$$

with the optimal H_∞ performance $\gamma^*=0.2583$ and the filtering error system′s poles as follows: $-0.5901+0.7676\,$j$, -0.5901-0.7676\,$j$, -1.1522+0.4842\,$j$, -1.1522-0.4842\,$j$, -0.5356, -0.2674$.

2) When $h=0.1\,$ms and the region is given as $D(-4\,000, 4\,000)$, we obtain the filter gain matrices

$$A_{\delta F1}=\begin{bmatrix} -0.7538 & 0.0251 & 0.4866 \\ -0.6492 & -0.3805 & 0.1313 \\ -0.7318 & -0.4542 & -0.4089 \end{bmatrix}$$

$$\boldsymbol{B}_{\delta F1}=\begin{bmatrix} 0.0585 \\ 0.0557 \\ 0.0579 \end{bmatrix}$$
$$\boldsymbol{C}_{\delta F1}=\begin{bmatrix} -0.5454 & 3.5431 & 4.1607 \end{bmatrix}$$

with the optimal H_∞ performance $\gamma^* = 6.6361$ and the filtering error system's poles as follows: $-0.5762 + 0.8167\,\mathrm{j}, -0.5762 - 0.8167\,\mathrm{j}, -0.7737 + 0.7246\,\mathrm{j}, -0.7737 - 0.7246\,\mathrm{j}, -0.5476, -0.1686$.

3) When $h = 1\,\mathrm{s}$ and the region is given as $D(-0.8, 0.8)$, we obtain the filter gain matrices

$$A_{\delta F1} = \begin{bmatrix} -0.7071 & -0.0154 & 0.5073 \\ -0.3306 & -0.5223 & 0.2429 \\ -0.3423 & -0.2010 & -0.5094 \end{bmatrix}$$

$$\boldsymbol{B}_{\delta F1} = \begin{bmatrix} 0.0365 \\ 0.0247 \\ 0.0158 \end{bmatrix}$$

$$\boldsymbol{C}_{\delta F1} = \begin{bmatrix} 1.3030 & 1.6271 & 2.3019 \end{bmatrix}$$

with the optimal H_∞ performance $\gamma^* = 0.3421$ and the filtering error system's poles as follows: $-0.7290 + 0.5046\,\mathrm{j}, -0.7290 - 0.5046\,\mathrm{j}, -0.6152 + 0.4096\,\mathrm{j}, -0.6152 - 0.4096\,\mathrm{j}, -0.4217, -0.4069$.

For comparison, we compute non-fragile H_∞ performance with different filter gain uncertainties (7) and (27). The optimal γ under different sampling periods are given in Table 1.

Table 1 The optimal γ with different uncertainties

h	Theorem 2	Theorem 3
$h = 0.1\,\mathrm{s}$	0.2583	0.2620
$h = 0.1\,\mathrm{ms}$	6.6361	6.7184
$h = 1\,\mathrm{s}$	0.3421	0.3477

From Table 1, the optimal γ by Theorem 2 is smaller than those by Theorem 3 under different sampling periods. Obviously, Theorem 2 is less conservative than Theorem 3.

Furthermore, to demonstrate the advantages of the designed filter, we make a comparison between non-fragile H_∞ filter and standard H_∞ one in the presence of filter gain variations (7). For different sampling periods, the optimal γ by different methods is given in Table 2.

Table 2 Comparison of non-fragile H_∞ filter with standard H_∞ filter for the optimal γ

h	Theorem 2	Remark 6	Remark 6 with (7)
$h = 0.1\,\mathrm{s}$	0.2583	0.2277	28.9412
$h = 0.1\,\mathrm{ms}$	6.6361	6.1812	68.2714
$h = 1\,\mathrm{s}$	0.3421	0.2190	375.7355

From Table 2, the optimal γ by Remark 6 is obviously smaller than those by Theorem 2 for different sampling periods, respectively. However, when the standard filter is with the uncertainties described by (7), the optimal H_∞ performance of the standard filter is seriously deteriorative.

On the other hand, to further demonstrate the advantage of the non-fragile filter, we assume the disturbance input $\boldsymbol{w}(k) = \begin{bmatrix} w_1(k) & w_2(k) \end{bmatrix}^{\mathrm{T}}$ as the following:

$$w_1(k) = w_2(k) = \begin{cases} 0.5, & 10 \le k \le 11 \\ -0.5, & 40 \le k \le 41 \\ 0, & \text{otherwise} \end{cases} \tag{33}$$

Figs. 1 $\sim$ 3 show the responses of estimation error $\boldsymbol{e}(k)$ under disturbance $\boldsymbol{w}(k)$. From Fig. 1, Fig. 2 and Fig. 3, we can easily find that the performance of the standard filter is serious deteriorative when the filter is with multiplicative gain variations, while the proposed designed filter is performed well. In turn, this illuminates the effectiveness of the non-fragile filter design.

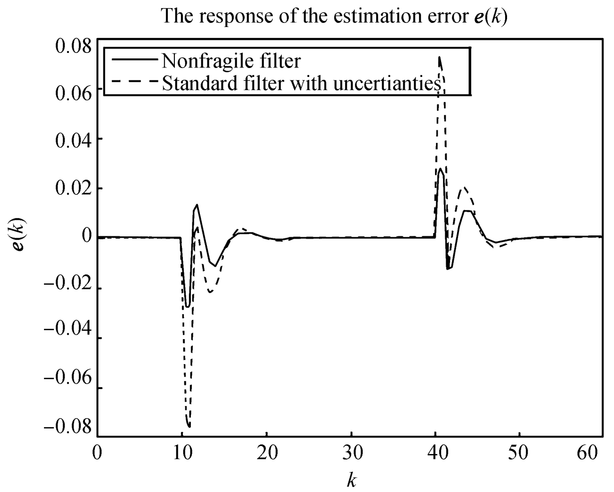

Fig. 1 Comparison of non-fragile H_∞ filter with standard H_∞ filter when $h = 0.1\,\mathrm{s}$

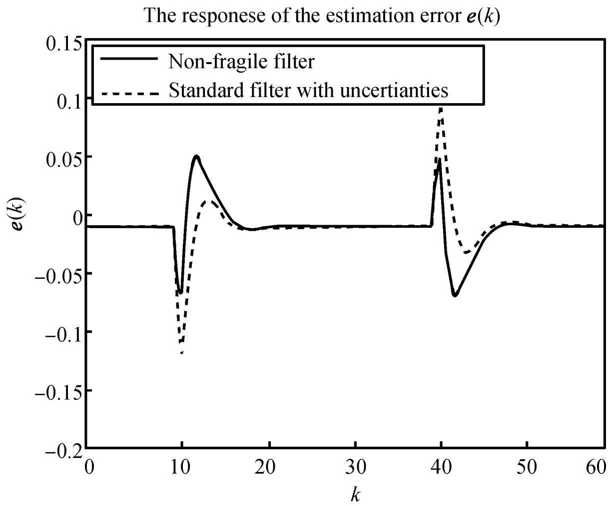

Fig. 2 Comparison of non-fragile H_∞ filter with standard H_∞ filter when $h = 0.1\,\mathrm{ms}$

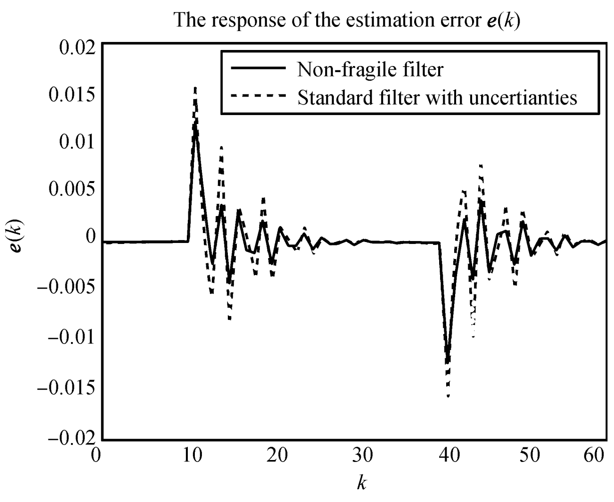

Fig. 3 Comparison of non-fragile H_∞ filter with standard H_∞ filter when $h = 1\,\mathrm{s}$

3.2 Comparison with the existing works

In this subsection, the results are given to provide a comparison between the non-fragile H_∞ filter designed by the proposed method (Theorem 2) and the non-fragile H_∞ filter designed by the existing method (Lemma 4). The H_∞ performance indexes are shown in Table 3.

Table 3 Comparison of γ with different methods

Design	Methods	γ with different conditions		
		$h = 0.01\,\mathrm{ms}$	$h = 0.1\,\mathrm{s}$	$h = 1\,\mathrm{s}$
Theorem 2	Delta Domain	20.9919	0.2585	0.3142
Lemma 4	z Domain	Infeasible	0.2461	0.2323

We can obtain the optimal performance index $\gamma = 0.2353$ for continuous system. From Table 4, it is easy to see that the delta operator can solve the unstable problem caused by using traditional Z-transform for sampling continuous system at high sampling period though our results are not better than the results of the existing works at low sampling period. On the other hand, our proposed method can unify the related continuous-time and discrete-time systems into the delta operator systems framework. Therefore, the delta operator is widely applied in many fields of engineering such as high-speed digital signal processing, system modeling and computer control based on fast sampled data.

4 Conclusion

The problem of a non-fragile H_∞ filter design for a class of linear systems described by delta operator with circular pole constraints is investigated, where the filter to be designed is assumed to be with multiplicative gain variations. It is worth pointing out that the filtering problems of continuous-time and discrete-time systems are investigated in the unified form by using delta operator. A sufficient condition for the existence of the filter to meet a prescribed H_∞ performance and to be D-stable is presented by LMI, and the explicit expression of the desired filter is also developed. In addition, the proposed uncertainties are less conservative than the normal norm-bound parameter uncertainties.

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