Decentralized Excitation Control of Multi-machine Multi-load Power Systems Using Hamiltonian Function Method

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Abstract Using the Hamiltonian function method, we investigate the excitation control of multi-machine multi-load power systems presented by nonlinear differential algebraic equations. First, the power system is reformulated as a novel Hamiltonian realization structure via pre-feedback state control. Then, based on the dissipative Hamiltonian realization of the system, a decentralized nonlinear excitation control scheme is constructed. The stability of the closed loop system is analyzed as well. The proposed strategy takes advantage of the intrinsic properties especially including the internal power balance of the power system. Simulation illustrates the effectiveness of the control strategy.

Key words Nonlinear differential algebraic systems, multi-machine multi-load power systems, dissipative Hamiltonian realization, decentralized stabilization

Generally, multi-machine multi-load power systems can be modelled as a set of nonlinear differential algebraic equations in the form of $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{u})$ and $\boldsymbol{0} = \boldsymbol{\sigma}(\boldsymbol{x}, \boldsymbol{z})$, where the differential equations describe the dynamics of dynamic components (e.g. generators, control devices, dynamic loads, and power electronics equipments) and the algebraic equations express the characteristics of static components and the network structure[1−4]. This kind of nonlinear differential algebraic system model has been widely adopted in numerical simulation, stability analysis, and stability region estimation of power systems because it can represent more realistic components and facilitate the utility of the sparse matrix techniques in numerical calculation[3, 5−10]. However, as for the stability and dynamic performance enhancement control, the undergoing power systems are generally simplified to the differential equation system model obtained from the differential algebraic power system model under the constant impedance load assumption[11−15]. This will definitely put stringent limitations on the control effect because almost all the real loads are nonlinear.

Recently, many state feedback linearization techniques were extended to nonlinear differential algebraic systems (NDAS) and gained some applications to the excitation control of differential algebraic power systems immediately[16−18]. In [16], some algorithms were provided to regularize, linearize, and stabilize NDAS via coordinate transformation and state feedback. In [17], Wang applied the ideas from differential geometric control theory to NDAS and proposed a systematic state feedback linearization strategy to produce stabilization and adaptive stabilization control laws for structure preserving power systems with nonlinear loads. Reference [18] used the same technique to design a nonlinear static var compensator controller for a single-machine infinite bus system with nonlinear loads.

It is well-known that the central idea of linearization is to obtain a feedback equivalent linear system by cancelling the internal nonlinearities of the considered systems, which may destruct the original structural properties of the system that are useful in the controller design procedure. The Hamiltonian function method employs a different design principle that can effectively utilize the internal structural characteristics of the systems in the procedure of controller design, hence it has attracted great attention in nonlinear system synthesis and has gained great achievements in the power system stabilization and performance enhancement[19−25].

One of the key steps in the Hamiltonian function method is to draw an equivalent representation of the system under consideration as a dissipative Hamiltonian system that interacts with external environments through port power conjugated variables and satisfies the energy balance equation[26−27]. For multi-machine multi-load power systems, the algebraic equations describe the internal energy balance in the systems and do not affect the energy balance between the system and external world. Motivated by this intuition, we discuss the excitation control problem of differential algebraic power systems via Hamiltonian function method in this paper. First, we express the power system under consideration as a dissipative Hamiltonian system, which can effectively use the internal energy balance property of the power system. Then, based on the achieved dissipative Hamiltonian realization, we investigate the excitation control of the power systems and propose a decentralized nonlinear excitation controller. Simulation on a six-machine eight-load power system shows the effectiveness of the control scheme.

The rest of the paper is organized as follows. In Section 1, the differential algebraic power system model is presented as a dissipative Hamiltonian system. In Section 2, a nonlinear excitation controller is constructed based on Hamiltonian function method. In Section 3, we simulate a six-machine eight-load power system to illustrate the effectiveness of the proposed control scheme. Finally, in Section 4, the results obtained in this paper are summarized and the conclusion is drawn.

1 Dissipative Hamiltonian realization of multi-machine multi-load power systems

1.1 Dynamics of multi-machine multi-load power systems

Consider a power system with n machines and m loads connected by lossless transmission lines. Each generator

Received April 14, 2008; in revised form October 24, 2008
Supported by National Natural Science Foundation of China (60674039, 60704004) and Innovation Fund for Outstanding Scholar of Henan Province (084200510009)
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DOI: 10.3724/SP.J.1004.2009.00919

is represented by an internal bus and a terminal bus. Let $J_I = 1, 2, \cdots, n$ and $J_T = 1, 2, \cdots, n$ be the generator internal and terminal buses, respectively. $J_I = n + 1$ refers to the reference bus with the voltage and phase assumed to be $V_{n+1}\angle\theta_{n+1} = 1\angle 0$. The load buses are denoted by $J_L = n + 2, n + 3, \cdots, n + m + 1$. Suppose that all the load buses are PQ buses, i.e., each load corresponds to a constant active P_d and reactive Q_d power demand. The dynamics of the multi-machine multi-load power system can be represented by the following nonlinear differential algebraic equations:

$$\begin{cases} \dot{\delta}_i = \omega_0(\omega_i - 1) \\ \dot{\omega}_i = -\dfrac{D_i}{M_i}(\omega_i - 1) + \dfrac{1}{M_i}(P_{mi} - P_{ei}) \\ \dot{E}'_{qi} = \phi_i + \dfrac{E_{fdi}}{T'_{d0i}} \end{cases} \tag{1}$$

for generator $i = 1, \cdots, n$, where

$$P_{ei} = \frac{E'_{qi}V_i \sin(\delta_i - \theta_i)}{x'_{di}}$$

$$\phi_i = \frac{x_{di}}{x'_{di}T'_{d0i}}E'_{qi} + \frac{x_{di} - x'_{di}}{x'_{di}T'_{d0i}}V_i \cos(\delta_i - \theta_i)$$

At the i-th terminal generator bus, $i = 1, \cdots, n$,

$$0 = P_{Ti}(\boldsymbol{\delta}, \boldsymbol{E}'_q, \boldsymbol{\theta}, \boldsymbol{V}, \boldsymbol{\varphi}) = \frac{E'_{qi}V_i \sin(\theta_i - \delta_i)}{x'_{di}} + \sum_{j=1, j\neq i}^{n+1} B_{ij}V_iV_j \sin(\theta_i - \theta_j) + \sum_{k=n+2}^{n+m+1} B_{ik}V_iV_k \sin(\theta_i - \varphi_k) \tag{2}$$

$$0 = Q_{Ti}(\boldsymbol{\delta}, \boldsymbol{E}'_q, \boldsymbol{\theta}, \boldsymbol{V}, \boldsymbol{\varphi}) = \frac{V_i^2}{x'_{di}} - \frac{E'_{qi}V_i \cos(\theta_i - \delta_i)}{x'_{di}} - B_{ii}V_i^2 - \sum_{j=1, j\neq i}^{n+1} B_{ij}V_iV_j \cos(\theta_i - \theta_j) - \sum_{k=n+2}^{n+m+1} B_{ik}V_iV_k \cos(\theta_i - \varphi_k) \tag{3}$$

At the k-th load bus, $k = n + 2, \cdots, n + m + 1$,

$$0 = -P_{dk} + P_{Lk}(\boldsymbol{\delta}, \boldsymbol{E}'_q, \boldsymbol{\theta}, \boldsymbol{V}, \boldsymbol{\varphi}) = -P_{dk} + \sum_{i=1}^{n+1} B_{ki}V_kV_i \sin(\varphi_k - \theta_i) + \sum_{l=n+2, l\neq k}^{n+m+1} B_{kl}V_kV_l \sin(\varphi_k - \varphi_l) \tag{4}$$

$$0 = -Q_{dk} + Q_{Lk}(\boldsymbol{\delta}, \boldsymbol{E}'_q, \boldsymbol{\theta}, \boldsymbol{V}, \boldsymbol{\varphi}) = -Q_{dk} - B_{kk}V_k^2 - \sum_{i=1}^{n+1} B_{ki}V_kV_i \cos(\varphi_k - \theta_i) - \sum_{l=n+2, l\neq k}^{n+m+1} B_{kl}V_kV_l \cos(\varphi_k - \varphi_l) \tag{5}$$

where $\boldsymbol{\delta} = (\delta_1, \cdots, \delta_n)^{\mathrm{T}}$, $\boldsymbol{\omega} = (\omega_1, \cdots, \omega_n)^{\mathrm{T}}$, $\boldsymbol{E}'_q = (E'_{q1}, \cdots, E'_{qn})^{\mathrm{T}}$, $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_n)^{\mathrm{T}}$, $\boldsymbol{V} = (V_1, V_2, \cdots, V_n, V_{n+1}, \cdots, V_{n+m+1})^{\mathrm{T}}$, and $\boldsymbol{\varphi} = (\varphi_{n+2}, \cdots, \varphi_{n+m+1})^{\mathrm{T}}$. δ_i is the power angle of the i-th generator, in radian; ω_i is the rotor speed of the i-th generator, $\omega_0 = 2\pi f_0$, in rad/s; E'_{qi} is the q-axis internal transient voltage of the i-th generator, in per unit; E_{fdi} is the voltage of the field circuit of the i-th generator, the control input, in per unit; P_{mi} is the mechanical power, assumed to be constant, in per unit; P_{ei} is the active electrical power, in per unit; V_i is the terminal voltage of the i-th generator, in per unit; θ_i is the terminal voltage angle of the i-th generator, in radian; V_k is the voltage of the k-th load bus, in per unit; φ_k is the voltage angle of the k-th load bus, in radian; x'_{di} is the d-axis transient reactance of the i-th generator, in per unit; x_{di} is the d-axis reactance, in per unit; M_i is the inertia coefficient of the i-th generator, in seconds; D_i is the damping constant, in per unit; T'_{d0i} is the d-axis transient open-circuit time constant, in seconds; and B_{ij} is the susceptance of the linear connecting bus i and j, in per unit.

Equations (1) ~ (5) constitute the nonlinear differential algebraic equation model of multi-machine multi-load power systems.

Choose the state variables $\boldsymbol{x} = (\boldsymbol{x}_1^{\mathrm{T}}, \boldsymbol{x}_2^{\mathrm{T}}, \cdots, \boldsymbol{x}_n^{\mathrm{T}})^{\mathrm{T}}$ with $\boldsymbol{x}_i = (\delta_i, \omega_i, E'_{qi})$, and algebraic variables $\boldsymbol{z} = (\boldsymbol{z}_{g1}^{\mathrm{T}}, \cdots, \boldsymbol{z}_{gn}^{\mathrm{T}}, \boldsymbol{z}_{l1}^{\mathrm{T}}, \cdots, \boldsymbol{z}_{l,n+m+1}^{\mathrm{T}})$ with $\boldsymbol{z}_{gi} = (\theta_i, v_i)$ and $\boldsymbol{z}_{lk} = (\varphi_k, v_k)$, where $v_i = \ln V_i$, $v_k = \ln V_k$ (note that $V_i > 0$ and $V_k > 0$). Denote control parameters by $u_i = E_{fdi}$. Then, the multi-machine multi-load power system can be written in the following NDAS

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{z}) + g(\boldsymbol{x}, \boldsymbol{z})\boldsymbol{u} \\ \boldsymbol{0} = \boldsymbol{\sigma}(\boldsymbol{x}, \boldsymbol{z}) \end{cases} \tag{6}$$

where the smooth vector functions of state and algebraic variables $\boldsymbol{f} = [\boldsymbol{f}_1^{\mathrm{T}}, \cdots, \boldsymbol{f}_n^{\mathrm{T}}]^{\mathrm{T}}$ with $f_i = \left[\omega_0(\omega_i - 1), -\dfrac{D_i}{M_i}(\omega_i - 1) - \dfrac{1}{M_i}(P_{ei} - P_{mi}), \phi_i\right]^{\mathrm{T}}$; $g = \mathrm{diag}\{\boldsymbol{g}_1, \cdots, \boldsymbol{g}_n\}$ with $\boldsymbol{g}_i = \left[0, 0, \dfrac{1}{T'_{d0i}}\right]^{\mathrm{T}}$; and $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_i^{\mathrm{T}}, \boldsymbol{\sigma}_k^{\mathrm{T}}]^{\mathrm{T}}$ with $\boldsymbol{\sigma}_i = [P_{Ti}, Q_{Ti}]^{\mathrm{T}}$ and $\boldsymbol{\sigma}_k = [-P_{dk} + P_{Lk}, -Q_{dk} + Q_{Lk}]^{\mathrm{T}}$.

In order to guarantee that the above NDAS has a unique solution without impulses (or jumps), we assume that system (6) is of index one in a neighborhood Ω of the equilibrium point $(\boldsymbol{x}_e, \boldsymbol{z}_e)$, i.e., $\mathrm{rank}\dfrac{\boldsymbol{\sigma}(\boldsymbol{x}, \boldsymbol{z})}{\boldsymbol{z}} = 2(n + m)$, $\forall(\boldsymbol{x}, \boldsymbol{z}) \in \Omega$ and the initial condition $(\boldsymbol{x}(0), \boldsymbol{z}(0))$ satisfies $\boldsymbol{\sigma}(\boldsymbol{x}(0), \boldsymbol{z}(0)) = \boldsymbol{0}$.

1.2 Dissipative Hamiltonian realization of multi-machine multi-load power systems

To employ the Hamiltonian function method, it is essential to express the system under consideration as a dissipative Hamiltonian system. First, for NDAS in the form of (6), its Hamiltonian realization is defined as[28]:

Definition 1. Suppose there exists a continuous differentiable function $H(\boldsymbol{x}, \boldsymbol{z})$ such that NDAS (6) can be represented as

$$\begin{cases} \dot{\boldsymbol{x}} = (J(\boldsymbol{x}, \boldsymbol{z}) - R(\boldsymbol{x}, \boldsymbol{z}))\nabla_{\boldsymbol{x}} H(\boldsymbol{x}, \boldsymbol{z}) + g(\boldsymbol{x}, \boldsymbol{z})\boldsymbol{u} \\ \boldsymbol{0} = \nabla_{\boldsymbol{z}} H(\boldsymbol{x}, \boldsymbol{z}) \end{cases} \tag{7}$$

where $\nabla_{\boldsymbol{x}} H(\boldsymbol{x}, \boldsymbol{z})$ and $\nabla_{\boldsymbol{z}} H(\boldsymbol{x}, \boldsymbol{z})$ are gradient vectors of $H(\boldsymbol{x}, \boldsymbol{z})$ with respect to $\boldsymbol{x}$ and $\boldsymbol{z}$, respectively. If, pointwisely, $J(\boldsymbol{x}, \boldsymbol{z})$ is skew-symmetric and $R(\boldsymbol{x}, \boldsymbol{z})$ is positive semi-definite, then (7) is called a dissipative Hamiltonian realization of NDAS (6) and $H(\boldsymbol{x}, \boldsymbol{z})$ is the corresponding Hamiltonian function.

According to Definition 1, if there exists a dissipative Hamiltonian realization of NDAS (6), the Hamiltonian function $H(\boldsymbol{x},\boldsymbol{z})$ must satisfy

$$\mathbf{0} = \nabla_{\boldsymbol{z}} H(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) \tag{8}$$

along the system trajectories. This equation implies that the system energy does not depend on the algebraic variables along the system trajectories constrained by the algebraic equation although the Hamiltonian function may explicitly contain the algebraic variables.

Now, consider the dissipative Hamiltonian realization of the multi-machine multi-load power systems. We have the following result.

Theorem 1. The multi-machine multi-load power system has a dissipative Hamiltonian realization under the following pre-feedback control

$$u_i = \mu_i + \bar{u}_i, \quad i = 1, \cdots, n \tag{9}$$

where μ_i is the new reference input and $\bar{u}_i$ is a constant control input defined by

$$\bar{u}_i = -\frac{x_{di}}{x'_{di}} E'_{qie} - \frac{x_{di} - x'_{di}}{x'_{di}} V_{ie} \cos(\delta_{ie} - \theta_{ie}) \tag{10}$$

in which $(V_{ie},\ \theta_{ie})$ is implicitly determined by the algebraic equations (2) ~ (5) and generator dynamic variables $(\delta_{ie}, 1, E'_{qie})$.

Proof. Note that the feedback control is inserted to operate the power system at a desired equilibrium point. Substituting (9) into the differential algebraic power system (1) ~ (5), we have

$$\begin{cases} \dot{\delta}_i = \omega_0(\omega_i - 1) \\ \dot{\omega}_i = -\dfrac{D_i}{M_i}(\omega_i - 1) - \dfrac{1}{M_i}(P_{ei} - P_{mi}) \\ \dot{E}'_{qi} = \phi_i + \dfrac{\bar{u}_i}{T'_{d0i}} + \dfrac{\mu_i}{T'_{d0i}} \end{cases} \tag{11}$$

where the power flow equations of the system under the pre-feedback control are the same as (2) ~ (5). Define

$$H(\boldsymbol{\delta}, \boldsymbol{E}'_q, \boldsymbol{\theta}, \boldsymbol{V}, \boldsymbol{\varphi}) = \sum_{i=1}^{n} \frac{1}{2} M_i \omega_0 (\omega_i - 1)^2 + P(\boldsymbol{\delta}, \boldsymbol{E}'_q, \boldsymbol{\theta}, \boldsymbol{V}, \boldsymbol{\varphi}) \tag{12}$$

where $P(\cdot)$ is the potential energy and $H(\cdot)$ is the total energy for the structure preserving multi-machine power system. And

$$\begin{aligned} P(\boldsymbol{\delta}, \boldsymbol{E}'_q, \boldsymbol{\theta}, \boldsymbol{V}, \boldsymbol{\varphi}) = & -\sum_{i=1}^{n} P_{mi}\delta_i - \sum_{k=n+2}^{n+m+1} (P_{dj}\varphi_j + Q_{dj}v_j) - \\ & \sum_{i=1}^{n} \frac{E'_{qi} e^{v_i} \cos(\delta_i - \theta_i)}{x'_{di}} - \sum_{i=1}^{n} \frac{E'^2_{qi} x_{di}}{2x'_{di}(x_{di} - x'_{di})} + \\ & \sum_{i=1}^{n} \frac{e^{2v_i}}{2} \left(\frac{1}{x'_{di}} - B_{ii}\right) - \sum_{i<j}^{n+1} B_{ij} e^{v_i + v_j} \cos(\theta_i - \theta_j) - \\ & \sum_{i=1}^{n+1} \sum_{k=n+2}^{n+m+1} B_{ik} e^{v_i + v_k} \cos(\theta_i - \varphi_k) - \\ & \sum_{k<l}^{n+m+1} B_{kl} e^{v_k + v_l} \cos(\varphi_k - \varphi_l) - \\ & \sum_{k=n+2}^{n+m+1} \frac{1}{2} B_{kk} e^{2v_k} - \sum_{i=1}^{n} \frac{E'_{qi}\bar{u}_i}{x_{di} - x'_{di}} \end{aligned} \tag{13}$$

Under the pre-feedback control (9), the gradient of $H(\cdot)$ becomes

$$\nabla_{\boldsymbol{x}_i} H = \begin{bmatrix} -P_{mi} + P_{ei} \\ M_i\omega_0(\omega_i - 1) \\ -\dfrac{T'_{d0i}}{x_{di} - x'_{di}} \left(\phi_i + \dfrac{\bar{u}_i}{T'_{d0i}}\right) \end{bmatrix}$$

$$\nabla_{\boldsymbol{z}_i} H = \begin{bmatrix} P_{Ti} \\ Q_{Ti} \end{bmatrix}, \quad \nabla_{\boldsymbol{z}_k} H = \begin{bmatrix} -P_{dk} + P_{Lk} \\ -Q_{dk} + Q_{Lk} \end{bmatrix}$$

Therefore, the multi-machine multi-load power system (2) ~ (5) can be expressed as

$$\begin{cases} \dot{\boldsymbol{x}} = (J - R)\nabla_{\boldsymbol{x}} H + g\boldsymbol{\mu} \\ \mathbf{0} = \nabla_{\boldsymbol{z}} H \end{cases} \tag{14}$$

where $J = \mathrm{diag}\{J_1, \cdots, J_n\}$, $R = \mathrm{diag}\{R_1, \cdots, R_n\}$, $\boldsymbol{\mu} = (\mu_1, \mu_2, \cdots, \mu_n)^{\mathrm{T}}$, and

$$J_i = \begin{bmatrix} 0 & \dfrac{1}{M_i} & 0 \\ -\dfrac{1}{M_i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad R_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \dfrac{D_i}{M_i^2} & 0 \\ 0 & 0 & \dfrac{x_{di} - x'_{di}}{T'_{d0i}} \end{bmatrix}$$

Obviously, J is skew symmetric as well as J_i. Noticing that $x_d = x_l + x_a$ and $x'_d = x_l + x_f/x_a$, we can directly get that $x_d > x'_d$ and $R_i \geq 0$ as well as R. Here, x_l is the armature leakage reactance, x_a is the armature reaction reactance, and x_f is the reactance corresponding to the flue path around the field winding. So (14) is a dissipative Hamiltonian realization of the multi-machine multi-load power system. □

2 Nonlinear excitation design of multi-machine multi-load power systems

In this section, we will adopt the following excitation control law

$$\mu_i = -K_i \boldsymbol{g}_i^{\mathrm{T}} \nabla_{\boldsymbol{x}_i} H(\boldsymbol{x},\boldsymbol{z}) = \frac{K_i}{x_{di} - x'_{di}} \left(\phi_i + \frac{\bar{u}_i}{T'_{d0}}\right), \ K_i > 0 \tag{15}$$

to stabilize the power systems around the desired operating point $(\boldsymbol{x}_e, \boldsymbol{z}_e)$. The resulting closed-loop system of (14) then becomes

$$\begin{cases} \dot{\boldsymbol{x}} = (J - R - gKg^{\mathrm{T}})\nabla_{\boldsymbol{x}} H \\ \mathbf{0} = \nabla_{\boldsymbol{z}} H \end{cases} \tag{16}$$

where $K = \mathrm{diag}\{K_1, \cdots, K_n\}$. Obviously, the closed loop system is still a dissipative Hamiltonian system.

To show that (15) is a stabilization control law, it is sufficient to demonstrate that the close-loop system is stable. As is well known in the energy-based approach, the Hamiltonian function is a natural candidate of Lyapunov function in stability analysis. Since the derivative of Hamiltonian function is always semi-negative definite, we need the following LaSalle′s invariance principle for NDAS[29] to assist the stability analysis.

Lemma 1. Consider the following NDAS

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}_1(\boldsymbol{x},\boldsymbol{z}) \\ \mathbf{0} = \boldsymbol{f}_2(\boldsymbol{x},\boldsymbol{z}) \end{cases} \tag{17}$$

where $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ are continuous differentiable vector fields. Suppose $(\boldsymbol{x}_e, \boldsymbol{z}_e)$ is an isolated equilibrium point. Let

$V(\boldsymbol{x},\boldsymbol{z}): D \rightarrow \mathbf{R}^+$ be a smooth function that is positive definite in a neighborhood D of $(\boldsymbol{x}_e,\boldsymbol{z}_e)$, such that $\dot{V}(\boldsymbol{x},\boldsymbol{z}) \leq 0$. Let $S = \{(\boldsymbol{x},\boldsymbol{z}) \in D | \dot{V}(\boldsymbol{x},\boldsymbol{z}) = 0\}$. Then, $(\boldsymbol{x}_e,\boldsymbol{z}_e)$ is locally asymptotically stable if no solution can stay forever in S except the trivial solution $\boldsymbol{x}(t) \equiv \boldsymbol{x}_e$ and $\boldsymbol{z}(t) \equiv \boldsymbol{z}_e$.

Based on Lemma 1, a stability criterion for a dissipative Hamiltonian realizable NDAS can be given as follows.

Theorem 2. Suppose (17) has a Hamiltonian as

$$\begin{cases} \dot{\boldsymbol{x}} = (J(\boldsymbol{x},\boldsymbol{z}) - R(\boldsymbol{x},\boldsymbol{z}))\nabla_{\boldsymbol{x}} H(\boldsymbol{x},\boldsymbol{z}) \\ \mathbf{0} = \nabla_{\boldsymbol{z}} H(\boldsymbol{x},\boldsymbol{z}) \end{cases} \tag{18}$$

The equilibrium $(\boldsymbol{x}_e,\boldsymbol{z}_e)$ is locally asymptotically stable if

1) $(\boldsymbol{x}_e,\boldsymbol{z}_e)$ is a strict local minimum of Hamiltonian function $H(\boldsymbol{x},\boldsymbol{z})$;

2) $\dot{H}(\boldsymbol{x},\boldsymbol{z}) \leq 0$ holds in some neighborhood of $(\boldsymbol{x}_e,\boldsymbol{z}_e)$;

3) No trajectory stays forever in the set $S = \{(\boldsymbol{x},\boldsymbol{z}) \in D | \dot{H}(\boldsymbol{x},\boldsymbol{z}) = 0\}$ except the trivial one $\boldsymbol{x}(t) \equiv \boldsymbol{x}_e$ and $\boldsymbol{z}(t) \equiv \boldsymbol{z}_e$.

Proof. Let $\bar{H} = H(\boldsymbol{x},\boldsymbol{z}) - H(\boldsymbol{x}_e,\boldsymbol{z}_e)$. Obviously, it is a proper Lyapunov function because $\bar{H}(\boldsymbol{x},\boldsymbol{z}) > 0$ and its time derivative $\dot{\bar{H}}(\boldsymbol{x},\boldsymbol{z}) \leq 0$. Combined with condition 3) and Lemma 1, the equilibrium is locally asymptotically stable. □

According to Theorem 2, in order to show the asymptotically stability of the closed-loop system (16), we need first to verify if $H(\boldsymbol{x},\boldsymbol{z})$ achieves a strict local minimum at the desired equilibrium point. This property is actually an extreme value problem constrained by algebraic equation $\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) = \mathbf{0}$ because each trajectory of the differential algebraic power system must meet this equation. By taking into consideration of the dissipative Hamiltonian realization formulation of power system, the strict minimum property is determined by the Hamiltonian function (12) without algebraic equation constraints[28, 30]. Unfortunately, having tried many means, we find it almost impossible to examine the definiteness of the Hessian matrix directly because of the high complexity of the Hamiltonian function (12). Representation of the Hamiltonian function seems necessary. Noting that the potential energy stored in a lossless transmission line is equal to half of the reactive power loss in the line[4], we can rewrite the Hamiltonian function (12) as

$$\begin{aligned} H(\boldsymbol{x},\boldsymbol{z}) = & \sum_{i=1}^{n} \frac{1}{2} M_i \omega_0 (\omega_i - 1)^2 - \sum_{i=1}^{n} P_{mi}\delta_i - \\ & \sum_{j=n+2}^{n+m+1} P_{dj}\varphi_j - \sum_{j=n+2}^{n+m+1} Q_{dj} v_j + \\ & \sum_{i=1}^{n} \frac{e^{2v_i}}{2x'_{di}} - \sum_{i=1}^{n} \frac{V_i E'_{qi}\cos(\delta_i - \theta_i)}{x'_{di}} + \\ & \sum_{i<j}^{n} B_{ij}\left[\frac{V_i^2 + V_j^2}{2} - V_i V_j \cos(\theta_i - \theta_j)\right] + \\ & \sum_{i=1}^{n} B_{i,n+1}\left[\frac{V_i^2}{2} - V_i\cos\theta_i\right] + \\ & \sum_{i=1}^{n}\sum_{k=n+2}^{n+m+1} B_{ik}\left[\frac{V_i^2 + V_k^2}{2} - V_i V_k \cos(\theta_i - \varphi_k)\right] + \\ & \sum_{k=n+2}^{n+m+1} B_{k,n+1}\left[\frac{V_k^2}{2} - V_k\cos\varphi_k\right] + \\ & \sum_{k<l}^{n+m+1} B_{kl}\left[\frac{V_k^2 + V_l^2}{2} - V_k V_l \cos(\varphi_k - \varphi_l)\right] + \\ & \sum_{i=1}^{n}\left[\frac{x_{di}E'^2_{qi}}{2x'_{di}(x_{di} - x'_{di})} - \frac{E'_{qi}\bar{u}_i}{x_{di} - x'_{di}}\right] \end{aligned} \tag{19}$$

It is easy to verify that

$$H_\alpha(\boldsymbol{x},\boldsymbol{z}) + c \leq H(\boldsymbol{x},\boldsymbol{z}) \leq H_\beta(\boldsymbol{x},\boldsymbol{z}) + c \tag{20}$$

where

$$\begin{aligned} H_\alpha(\boldsymbol{x},\boldsymbol{z}) = & \sum_{i=1}^{n} \frac{1}{2} M_i \omega_0 (\omega_i - 1)^2 - \sum_{i=1}^{n} P_{mi}\delta_i - \\ & \sum_{j=n+2}^{n+m+1} P_{dj}\varphi_j - \sum_{j=n+2}^{n+m+1} Q_{dj} v_j + \\ & \sum_{i<j}^{n} \frac{1}{2} B_{ij}(V_i - V_j)^2 + \sum_{i=1}^{n} \frac{1}{2} B_{i,n+1}(V_i - 1)^2 + \\ & \sum_{i=1}^{n}\sum_{k=n+2}^{n+m+1} \frac{1}{2} B_{ik}(V_i - V_k)^2 + \\ & \sum_{k=n+2}^{n+m+1} \frac{1}{2} B_{k,n+1}(V_k - 1)^2 + \\ & \sum_{k<l}^{n+m+1} \frac{1}{2} B_{kl}(V_k - V_l)^2 + \sum_{i=1}^{n} \frac{1}{x'_{di}}(V_i - E'_{qi})^2 + \\ & \sum_{i=1}^{n} \frac{1}{2(x_{di} - x'_{di})}(E'_{qi} - \bar{u}_i)^2 \end{aligned} \tag{21}$$

$$\begin{aligned} H_\beta(\boldsymbol{x},\boldsymbol{z}) = & \sum_{i=1}^{n} \frac{1}{2} M_i \omega_0 (\omega_i - 1)^2 - \sum_{i=1}^{n} P_{mi}\delta_i - \\ & \sum_{j=n+2}^{n+m+1} P_{dj}\varphi_j - \sum_{j=n+2}^{n+m+1} Q_{dj} v_j + \\ & \sum_{i<j}^{n} \frac{1}{2} B_{ij}(V_i + V_j)^2 + \sum_{i=1}^{n} \frac{1}{2} B_{i,n+1}(V_i + 1)^2 + \\ & \sum_{i=1}^{n}\sum_{k=n+2}^{n+m+1} \frac{1}{2} B_{ik}(V_i + V_k)^2 + \\ & \sum_{k=n+2}^{n+m+1} \frac{1}{2} B_{k,n+1}(V_k + 1)^2 + \\ & \sum_{k<l}^{n+m+1} \frac{1}{2} B_{kl}(V_k + V_l)^2 + \sum_{i=1}^{n} \frac{1}{x'_{di}}(V_i + E'_{qi})^2 + \\ & \sum_{i=1}^{n} \frac{1}{2(x_{di} - x'_{di})}(E'_{qi} - \bar{u}_i)^2 \end{aligned} \tag{22}$$

$$c = -\sum_{i=1}^{n} \frac{1}{2} B_{i,n+1} - \sum_{k=n+2}^{n+m+1} \frac{1}{2} B_{k,n+1} - \sum_{i=1}^{n} \frac{\bar{u}_i^2}{2(x_{di} - x'_{di})} \tag{23}$$

Because $\delta_i \in [-\pi,\pi]$ and $\varphi_k \in [-\pi,\pi]$, $H_\alpha(\boldsymbol{x},\boldsymbol{z})$ is bounded from below. From (20), the Hamiltonian function $H(\boldsymbol{x},\boldsymbol{z})$ is also bounded from below and for $l > 0$, the set $\{(\boldsymbol{x},\boldsymbol{z}) | H(\boldsymbol{x},\boldsymbol{z}) \leq l\}$ is compact. Noticing that every equilibrium point of the power system is also an extremum point of $H(\boldsymbol{x},\boldsymbol{z})$ and for a real power system, there exists only one desired operating point in the interested region, similar to [22], where we can know that $H(\boldsymbol{x},\boldsymbol{z})$ has a strict local minimum at the desired operating point.

Integrating the above discussions, we arrive at the following result.

Theorem 3. The differential algebraic power system (1) ~ (5) can be stabilized around the prescribed operating point $(\boldsymbol{x}_e,\boldsymbol{z}_e)$ by the following nonlinear feedback excitation control law

$$u_i = \bar{u}_i + \frac{K_i}{x_{di} - x'_{di}}\left(\phi_i + \frac{\bar{u}_i}{T'_{d0i}}\right) \tag{24}$$

Proof. It has been shown that the Hamiltonian function (12) has a strict local minimum at the desired equilibrium point. Furthermore, the derivative of $H(\boldsymbol{x},\boldsymbol{z})$ along the trajectories of closed loop system (16) satisfies

$$\begin{aligned}\dot{H} =& (\nabla_{\boldsymbol{x}} H)^{\mathrm{T}}(J - R - gKg^{\mathrm{T}})\nabla_{\boldsymbol{x}} H = \\ &-\sum_{i=1}^{n}\left[D_i\omega_0^2(\omega_i - 1)^2 + \frac{T'^2_{d0i}}{(x_{di}-x'_{di})^2}\times\right.\\ &\left.\left(\frac{x_{di}-x'_{di}}{T'_{d0i}} + \frac{K_i}{T'^2_{d0i}}\right)\left(\phi_i + \frac{\bar{u}_i}{T'_{d0i}}\right)^2\right] \le 0\end{aligned} \tag{25}$$

Observing that

$$\begin{aligned}S =& \{(\boldsymbol{x},\boldsymbol{z}) : \dot{H}(\boldsymbol{x},\boldsymbol{z}) = 0, \boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{0}\} = \\ &\{(\boldsymbol{x},\boldsymbol{z}) : \omega_i = 1, \phi_i + \frac{\bar{u}_i}{T'_{d0i}} = 0, \\ &\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{0}, i = 1,\cdots,n\} = \\ &\{(\boldsymbol{x},\boldsymbol{z}) : \omega_i = 1, P_{mi} = P_{ei}, \phi_i + \frac{\bar{u}_i}{T'_{d0i}} = 0, \\ &\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{0}, i = 1,\cdots,n\}\end{aligned} \tag{26}$$

and taking into consideration that $\omega_i = 1$, $P_{mi} = P_{ei}$, $\phi_i + \bar{u}_i/T'_{d0i} = 0$, and $\boldsymbol{\sigma}(\boldsymbol{x},\boldsymbol{z}) = \boldsymbol{0}$ are the equations the desired equilibrium point should satisfy, we can know that S contains no trajectories other than the desired equilibrium point. According to Theorem 2, the closed loop system is asymptotically stable. □

Before the end of this section, let us discuss the realization problem of the controller (24). It is known that E'_{qi} is unmeasurable, and δ_i and θ_i are difficult to get. So, in order to complete the feedback control, we must replace these signals by measurable variables such as P_{ei}, Q_{ei}, and V_i. The reactive power of generator can be presented as

$$Q_{ei} = \frac{V_i^2}{x'_{di}} - \frac{E'_{qi}V_i\cos(\theta_i - \delta_i)}{x'_{di}} \tag{27}$$

With the active power P_{ei}, we can get that

$$E'_{qi} = \frac{\sqrt{P_{ei}^2 x'^2_{di} + (V_i^2 - Q_{ei}x'_{di})^2}}{V_i} \tag{28}$$

$$V_i\cos(\theta_i - \delta_i) = \frac{V_i(V_i^2 - Q_{ei}x'_{di})}{\sqrt{P_{ei}^2 x'^2_{di} + (V_i^2 - Q_{ei}x'_{di})^2}} \tag{29}$$

Thus, we can complete the nonlinear control. Furthermore, it can be seen that the proposed excitation controller (24) is a decentralized one, that is, only the local information is used for the feedback control.

3 Simulation

We choose a six-machine eight-load power network system[11, 31] as a paradigm to demonstrate the effectiveness of the proposed control strategy where No. 6 machine is a synchronous condenser and No. 1 generator represents an equivalent large power system, which is used as the reference here. As to the generator data, load parameters, and other data we refer to [31]. The simulation is accomplished by the PSASP package [1].

In order to compare the effectiveness of different control strategies, we simulate the system under the following control configurations:

Case 1. Generators No. 2 ~ No. 5 are equipped with the nonlinear optimal excitation controller (NOEC)[11].

Case 2. Generators No. 2 ~ No. 5 are equipped with nonlinear decentralized excitation controller (NDEC) proposed in this paper, where the feedback gain $K_i = 0.02$ for $i = 2,\cdots,5$.

In the two cases, a three-phase temporary short-circuit fault is assumed to occur at the middle of the transmission line between buses 11 and 12 during the time period 0.1 ~ 0.39 s. The simulation results are depicted in Fig. 1 ~ Fig. 6, where Fig. 1 and Fig. 2 show the responses of the rotor angles under the above two control configurations ($\delta_{i1} = \delta_i - \delta_1$, $(i = 2,\cdots,5)$), Fig. 3 and Fig. 4 show the angle speed responses, and Fig. 5 and Fig. 6 show the voltage responses of bus 11 and bus 18, respectively.

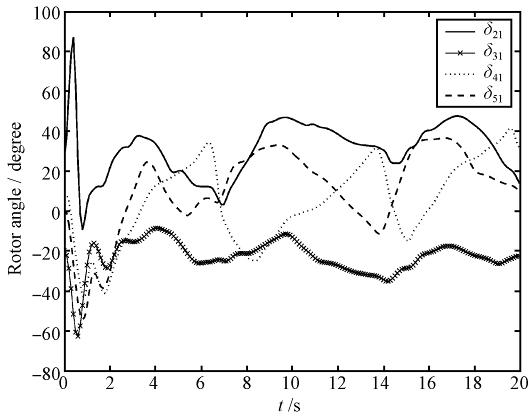

Fig. 1 Responses of the generator angles under NOEC

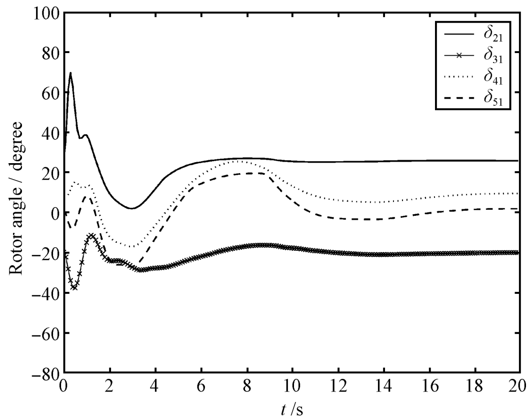

Fig. 2 Responses of the generator angles under NDEC

[1]PSASP is a professional testing software for power systems designed by the China Electrical Power Research Institute, Beijing, China.

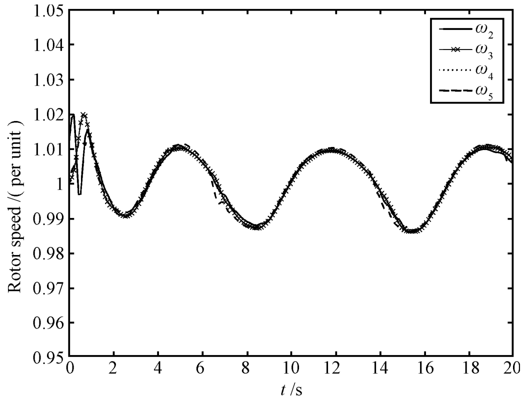

Fig. 3 Responses of the angle speeds under NOEC

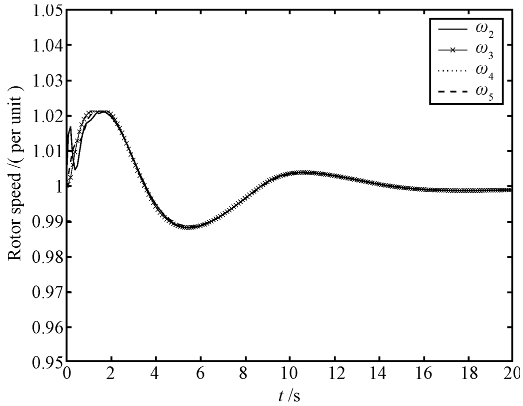

Fig. 4 Responses of the angle speeds under NDEC

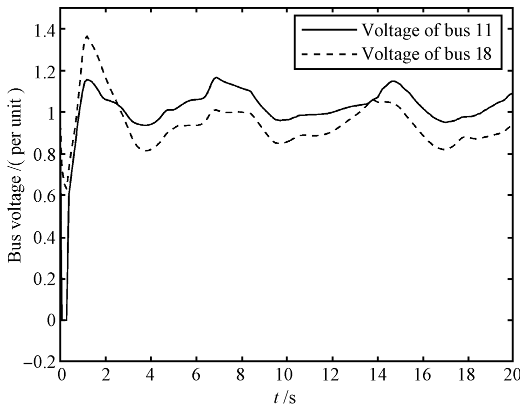

Fig. 5 Responses of the bus voltages under NOEC

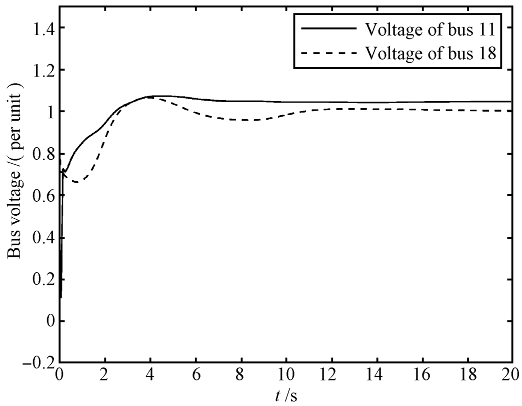

Fig. 6 Responses of the bus voltages under NDEC

From the simulation results, it can be seen that the proposed nonlinear decentralized excitation scheme makes the system respond much faster than the NOEC controller when the fault occurs. The overshot is also smaller. So the proposed nonlinear excitation control law outperforms the nonlinear optimal excitation controller in transient stability enhancement and dynamic performance improvement of power systems.

4 Conclusion

The stabilization problem of the multi-machine multi-load power systems is investigated within a novel dissipative Hamiltonian realization framework for NDAS. We generalize the existing Hamiltonian function method and propose a nonlinear decentralized excitation control strategy by utilizing the internal structure properties and the internal energy balance of the power systems. Simulations on a six-machine eight-nonlinear-load power system show that the control scheme proposed in this paper is more effective compared with the widely used power system stabilizer scheme.

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