Global $\mathcal{K}$-exponential Tracking Control of Nonholonomic Systems in Chained-form by Output Feedback

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Abstract The output tracking control problem of nonholonomic chained form systems is studied in this paper and global $\mathcal{K}$-exponential output trackers are presented without persistent excitation or not-converging to zero on reference trajectories. First, a time-varying coordinate transformation is introduced to avoid manipulating exponentially converging signals. Then, with the help of theory of cascaded systems and linear perturbed systems, global $\mathcal{K}$-exponential output trackers are successfully obtained. A new feature of the proposed controller is that the output tracking control problem of nonholonomic chained form systems is also resolvable without the popular condition of persistent excitation or not converging to zero on reference signals in the previous works. The proposed method is demonstrated and discussed by means of nonholonomic mobile robots and cars with one trailer.

Key words Trajectory tracking, nonholonomic systems, cascaded systems, exponential stability, persistent excitation

Due to Brockett′s famous necessary condition[1], there does not exist smooth or even continuous time-invariant state feedback law for the stabilization problems of nonholonomic system. Thus, the stabilization problems of such systems become a focus of research[2−9], which still remain to be a very interesting topic today. Compared with the stabilization problems, the tracking problems — sometimes called stabilization of trajectories[10] — have received less considerations.

Results about the tracking problems of nonholonomic system can be classified as local results and global results[11]. Methods related to local results are mainly local linearization or input-output linearization. Controllers obtained using local linearization are only effective in local area while those obtained using input-output linearization have some singularity due to the methods adopted. There are mainly two methods to deal with the global tracking problems: the method based on backstepping and the method based on cascade-design. The main differences of the above two methods lie in dealing with coupling terms between subsystems: controllers based on the cascade-design method are much simpler than those based on backstepping because the coupling terms are neglected when some conditions are satisfied without affecting performance of the entire systems. Under the requirement that the reference target′s velocity must satisfy some persistent conditions, the global tracking control laws based on output were presented in [12] by using method of backstepping and Lyapunov function. Although global $\mathcal{K}$-exponential tracking controllers were successfully brought out in [13−15] by using cascaded-design method for low-dimensional and general chained-form systems, persistent excitations were also required for reference target systems.

For ease of application, recent years have also seen increasing interests in controllers dealing with stabilization and tracking problems simultaneously. Inspired by Samson′s[16] stabilization method, Lee[17] and Do[18−19] have designed a time-varying "universal" controller to solve both stabilization and tracking problems under some persistent excitations. However, as pointed in [17], there was no straight way to extend their methods to n-dimensional chained-from systems and the converging speed of tracking-error systems was not exponential.

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To avoid using velocity measurements in mechanical systems, it is more preferred to design stabilizers or trackers based on output feedback. Both Jiang[12] and Lefeber[13] have presented global feedback trackers based on output provided that the reference trajectory satisfies some persistent excitations or does not converge to zero. The theme of this paper is to construct output-feedback trackers for the chained-form nonholonomic system based on our previous results[20−21]. Tools from cascaded systems and linear perturbed systems were utilized to construct global $\mathcal{K}$-exponential output trackers. The obtained conclusions enlarged the previous results[11, 13, 20−21] and showed that the popular condition of persistent excitations or not-converging to zero is not necessary.

1 Problem statement

After a suitable change of coordinates and state feedback, many nonholonomic systems can be transformed into the following chained form[22]:

$$\begin{aligned} \dot{x}_1 &= u_1 \\ \dot{x}_2 &= x_3 u_1 \\ &\ \vdots \\ \dot{x}_{n-1} &= x_n u_1 \\ \dot{x}_n &= u_2 \\ \boldsymbol{y} &= [x_1, x_2]^{\mathrm{T}} \end{aligned} \tag{1}$$

where $\boldsymbol{u} = [u_1, u_2]^{\mathrm{T}}$ is the control input, $\boldsymbol{x} = [x_1, \cdots, x_n]^{\mathrm{T}}$ is the system state, and $\boldsymbol{y}$ is the system output.

A reference signal $\boldsymbol{x}_d(t) = [x_{1d}(t), \cdots, x_{nd}(t)]^{\mathrm{T}}$ is produced by

$$\begin{aligned} \dot{x}_{1d} &= u_{1d} \\ \dot{x}_{2d} &= x_{3d} u_{1d} \\ &\ \vdots \\ \dot{x}_{n-1,d} &= x_{nd} u_{1d} \\ \dot{x}_{nd} &= u_{2d} \end{aligned} \tag{2}$$

where $\boldsymbol{u}_d = [u_{1d}, u_{2d}]^{\mathrm{T}}$ denotes the reference input.

Problem of global output-feedback tracking. Given a vector-valued reference signal $\boldsymbol{x}_d(t) = [x_{1d}(t), \cdots, x_{nd}(t)]^{\mathrm{T}}$ defined by (2) and tracking error $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_d(t)$, the global output tracking problem is to design a $\mathbf{C}^0$

time-varying dynamic output feedback law as

$$\begin{aligned}\dot{\boldsymbol{\chi}} &= \boldsymbol{\nu}_0(t, \boldsymbol{y}, \boldsymbol{\chi})\\ \boldsymbol{u} &= \boldsymbol{\mu}_0(t, \boldsymbol{y}, \boldsymbol{\chi})\end{aligned} \tag{3}$$

such that the closed-loop trajectories ($\boldsymbol{e}(t)$, $\boldsymbol{\chi}(t)$) are globally uniformly bounded and $\lim_{t\to\infty} |\boldsymbol{e}(t)| = 0$ holds for arbitrary initial tracking error $\boldsymbol{e}(0)$ in $\mathbf{R}^n$.

In this paper, the global output-feedback tracking problem is considered under the following assumption.

Assumption 1. $u_{1d} = \mathrm{e}^{-\lambda(t-t_0)}D(t)$, λ is a nonnegative constant and there exists a nonzero constant D such that the bounded continuous $D(t)$ satisfies

$$\lim_{t\to\infty} D(t) = D, \quad \int_0^\infty \|D(t) - D\| \,\mathrm{d}t < +\infty \tag{4}$$

Remark 1. Compared with [11−15] related to the same problem, we can see that the above assumption has relaxed the condition imposed on the reference input u_{1d}. The signal that is persistent excited or not-converging to zero satisfies Assumption 1 when $\lambda = 0$ while the signal u_{1d} in Assumption 1 does not satisfy the condition of persistent excitation or not-converging to zero when $\lambda \neq 0$.

First, we give some preliminary results that will be used in the proof of our main results.

Definition 1[13, 23]**.** Consider a nonlinear system $\dot{\boldsymbol{x}} = f(t, \boldsymbol{x})$. The equilibrium $\boldsymbol{x} = 0$ of the system is said to be globally $\mathcal{K}$-exponentially stable if there exist a class $\mathcal{K}$ function $\alpha(\cdot)$ and a positive constant γ such that for all $t_0 \geq 0$ and $\boldsymbol{x}(t_0) \in \mathbf{R}^n$, we have

$$\|\boldsymbol{x}(t)\| \leq \alpha(\|\boldsymbol{x}(t_0)\|)\mathrm{e}^{-\gamma(t-t_0)}$$

Lemma 1[9, 24−25]**.** Consider the linear time-varying system

$$\dot{\boldsymbol{x}} = (A_0 + A_1(t))\boldsymbol{x} \tag{5}$$

where $\boldsymbol{x} \in \mathbf{R}^n$ is the state vector of the system. Suppose $A_0 \in \mathbf{R}^{n\times n}$ is a Hurwitz matrix, and $A_1(t) \in \mathbf{R}^{n\times n}$ is smooth in t and satisfies:

1) $A_1(t) \to 0$ as $t \to \infty$;

2) $\int_0^\infty \|A_1(t)\| \,\mathrm{d}t < \infty$.

Then, the state $\boldsymbol{x}(t)$ of system (5) is globally uniformly exponentially stable.

Consider a time-varying system $\dot{\boldsymbol{z}} = f(t, \boldsymbol{z})$ given by

$$\begin{aligned}\dot{\boldsymbol{z}}_1 &= f_1(t, \boldsymbol{z}_1) + g(t, \boldsymbol{z}_1, \boldsymbol{z}_2)\boldsymbol{z}_2\\ \dot{\boldsymbol{z}}_2 &= f_2(t, \boldsymbol{z}_2)\end{aligned} \tag{6}$$

We call system (6) a cascaded system because it can be viewed as the following system

$$\Sigma_1 : \dot{\boldsymbol{z}}_1 = f_1(t, \boldsymbol{z}_1) \tag{7}$$

perturbed by the output of the system

$$\Sigma_2 : \dot{\boldsymbol{z}}_2 = f_2(t, \boldsymbol{z}_2) \tag{8}$$

Lemma 2[13−15, 26−27]**.** The cascaded time-varying system (6) is globally $\mathcal{K}$-exponentially stable if the following assumptions are satisfied:

1) Subsystem (7) is globally uniformly exponentially stable (GUES);

2) Function $g(t, \boldsymbol{z}_1, \boldsymbol{z}_2)$ satisfies the following condition for all $t \geq t_0$

$$\|g(t, \boldsymbol{z}_1, \boldsymbol{z}_2)\| \leq \theta_1(\|\boldsymbol{z}_2\|) + \theta_2(\|\boldsymbol{z}_2\|)\|\boldsymbol{z}_1\|$$

where $\theta_1(\cdot)$ and $\theta_2(\cdot)$ are continuous functions;

3) Subsystem (8) is globally $\mathcal{K}$-exponentially stable.

2 Observer-based tracking control law

2.1 Model transformation

Apply the following coordinate transformation for system (1)

$$\begin{aligned}y_1 &= x_1\\ y_2 &= x_2\mathrm{e}^{(n-2)\lambda(t-t_0)}\\ &\vdots\\ y_{n-1} &= x_{n-1}\mathrm{e}^{\lambda(t-t_0)}\\ y_n &= x_n\end{aligned} \tag{9}$$

Then, system (1) is transformed into (where $\sigma(t) = u_1\mathrm{e}^{\lambda(t-t_0)}$)

$$\begin{bmatrix}\dot{y}_2\\ \dot{y}_3\\ \vdots\\ \dot{y}_{n-1}\\ \dot{y}_n\end{bmatrix} = \begin{bmatrix}(n-2)\lambda & \sigma(t) & 0 & \cdots & 0\\ 0 & (n-3)\lambda & \sigma(t) & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda & \sigma(t)\\ 0 & 0 & \cdots & 0 & 0\end{bmatrix}\begin{bmatrix}y_2\\ y_3\\ \vdots\\ y_{n-1}\\ y_n\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \vdots\\ 0\\ u_2\end{bmatrix} \tag{10}$$

$$\dot{y}_1 = u_1 \tag{11}$$

Similarly, apply the following coordinate transformation to system (2)

$$\begin{aligned}y_{1d} &= x_{1d}\\ y_{2d} &= x_{2d}\mathrm{e}^{(n-2)\lambda(t-t_0)}\\ &\vdots\\ y_{n-1,d} &= x_{n-1,d}\mathrm{e}^{\lambda(t-t_0)}\\ y_{nd} &= x_{nd}\end{aligned} \tag{12}$$

System (2) is transformed into

$$\begin{bmatrix}\dot{y}_{2d}\\ \dot{y}_{3d}\\ \vdots\\ \dot{y}_{n-1,d}\\ \dot{y}_{nd}\end{bmatrix} = \begin{bmatrix}(n-2)\lambda & D(t) & 0 & \cdots & 0\\ 0 & (n-3)\lambda & D(t) & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda & D(t)\\ 0 & 0 & \cdots & 0 & 0\end{bmatrix}\begin{bmatrix}y_{2d}\\ y_{3d}\\ \vdots\\ y_{n-1,d}\\ y_{nd}\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \vdots\\ 0\\ u_{2d}\end{bmatrix} \tag{13}$$

$$\dot{y}_{1d} = u_{1d} \tag{14}$$

2.2 Design of a reduced-order observer

Motivated by the linear Luenberger observer design, we introduce the new variables

$$\begin{aligned}z_3 &= y_3 - h_1y_2\\ &\vdots\\ z_i &= y_i - h_{i-2}y_2\\ &\vdots\\ z_n &= y_n - h_{n-2}y_2\end{aligned} \tag{15}$$

where h_i are constant parameters to be determined later.

As can be directly checked, we have

$$\begin{bmatrix}\dot z_3\\ \dot z_4\\ \vdots\\ \dot z_{n-1}\\ \dot z_n\end{bmatrix}=\begin{bmatrix}(n-3)\lambda-h_1\sigma(t) & \sigma(t) & 0 & \cdots & 0\\ -h_2\sigma(t) & (n-4)\lambda & \sigma(t) & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ -h_{n-3}\sigma(t) & 0 & \cdots & \lambda & \sigma(t)\\ -h_{n-2}\sigma(t) & 0 & \cdots & 0 & 0\end{bmatrix}\begin{bmatrix}z_3\\ z_4\\ \vdots\\ z_{n-1}\\ z_n\end{bmatrix}+\begin{bmatrix}0\\0\\ \vdots\\ 0\\ u_2\end{bmatrix}+\begin{bmatrix}(h_2-h_1^2)\sigma(t)y_2-\lambda h_1y_2\\ (h_3-h_1h_2)\sigma(t)y_2-2\lambda h_2y_2\\ \vdots\\ (h_{n-2}-h_1h_{n-3})\sigma(t)y_2-(n-3)\lambda h_{n-3}y_2\\ (-h_1h_{n-2})\sigma(t)y_2-(n-2)\lambda h_{n-2}y_2\end{bmatrix} \tag{16}$$

Then, a time-varying observer is introduced which depends on the output y_2 and the input u_2

$$\begin{bmatrix}\dot{\hat z}_3\\ \dot{\hat z}_4\\ \vdots\\ \dot{\hat z}_{n-1}\\ \dot{\hat z}_n\end{bmatrix}=\begin{bmatrix}(n-3)\lambda-h_1\sigma(t) & \sigma(t) & 0 & \cdots & 0\\ -h_2\sigma(t) & (n-4)\lambda & \sigma(t) & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ -h_{n-3}\sigma(t) & 0 & \cdots & \lambda & \sigma(t)\\ -h_{n-2}\sigma(t) & 0 & \cdots & 0 & 0\end{bmatrix}\begin{bmatrix}\hat z_3\\ \hat z_4\\ \vdots\\ \hat z_{n-1}\\ \hat z_n\end{bmatrix}+\begin{bmatrix}0\\0\\ \vdots\\ 0\\ u_2\end{bmatrix}+\begin{bmatrix}(h_2-h_1^2)\sigma(t)y_2-\lambda h_1y_2\\ (h_3-h_1h_2)\sigma(t)y_2-2\lambda h_2y_2\\ \vdots\\ (h_{n-2}-h_1h_{n-3})\sigma(t)y_2-(n-3)\lambda h_{n-3}y_2\\ (-h_1h_{n-2})\sigma(t)y_2-(n-2)\lambda h_{n-2}y_2\end{bmatrix} \tag{17}$$

For notational simplicity, let us denote $\tilde{\boldsymbol{Z}}=[\tilde z_3,\cdots,\tilde z_n]^{\mathrm T}$ as the observer error with $\tilde z_i=z_i-\hat z_i$ for all $3\le i\le n$. It is easily seen that the $\tilde{\boldsymbol{Z}}$-dynamics satisfy

$$\begin{bmatrix}\dot{\tilde z}_3\\ \dot{\tilde z}_4\\ \vdots\\ \dot{\tilde z}_{n-1}\\ \dot{\tilde z}_n\end{bmatrix}=\begin{bmatrix}(n-3)\lambda-h_1\sigma(t) & \sigma(t) & 0 & \cdots & 0\\ -h_2\sigma(t) & (n-4)\lambda & \sigma(t) & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ -h_{n-3}\sigma(t) & 0 & \cdots & \lambda & \sigma(t)\\ -h_{n-2}\sigma(t) & 0 & \cdots & 0 & 0\end{bmatrix}\begin{bmatrix}\tilde z_3\\ \tilde z_4\\ \vdots\\ \tilde z_{n-1}\\ \tilde z_n\end{bmatrix} \tag{18}$$

Theorem 1. Considering the observer error system (18), if u_{1d} satisfies Assumption 1 then the following control input

$$u_1=u_{1d}-\mathrm{e}^{-(n-2)\lambda(t-t_0)}k_1w_1 \tag{19}$$

renders the observer error system (18) globally $\mathcal{K}$-exponentially stable, provided that $k_1>(n-2)\lambda$, $\boldsymbol{H}=[h_1,\cdots,h_{n-2}]^{\mathrm T}$ is a constant vector such that matrix $A_0-\boldsymbol{HDC}$ is Hurwitzian, where D is nonzero constant defined in Assumption 1 and $\boldsymbol{C}=[1,0,\cdots,0]^{\mathrm T}$, $w_1=(x_1-x_{1d})\times\mathrm{e}^{(n-2)\lambda(t-t_0)}$.

Proof. The observer error system (18) can also be written as

$$\begin{bmatrix}\dot{\tilde z}_3\\ \dot{\tilde z}_4\\ \vdots\\ \dot{\tilde z}_{n-1}\\ \dot{\tilde z}_n\end{bmatrix}=\begin{bmatrix}(n-3)\lambda-h_1D(t) & D(t) & 0 & \cdots & 0\\ -h_2D(t) & (n-4)\lambda & D(t) & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ -h_{n-3}D(t) & 0 & \cdots & \lambda & D(t)\\ -h_{n-2}D(t) & 0 & \cdots & 0 & 0\end{bmatrix}\begin{bmatrix}\tilde z_3\\ \tilde z_4\\ \vdots\\ \tilde z_{n-1}\\ \tilde z_n\end{bmatrix}+\begin{bmatrix}-h_1 & 1 & 0 & \cdots & 0\\ -h_2 & 0 & 1 & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ -h_{n-3} & 0 & 0 & \cdots & 1\\ -h_{n-2} & 0 & 0 & \cdots & 0\end{bmatrix}\begin{bmatrix}\tilde z_3\\ \tilde z_4\\ \vdots\\ \tilde z_{n-1}\\ \tilde z_n\end{bmatrix}(u_1-u_{1d})\mathrm{e}^{\lambda(t-t_0)} \tag{20}$$

and the state w_1 satisfies the following differential equation

$$\dot w_1=(n-2)\lambda w_1+\mathrm{e}^{(n-2)\lambda(t-t_0)}(u_1-u_{1d}) \tag{21}$$

Under the control law u_1 of (19), system (20) can be viewed as

$$\dot{\tilde{\boldsymbol{Z}}}=(A_0+\boldsymbol{HDC}+A_1(t))\tilde{\boldsymbol{Z}} \tag{22}$$

cascaded by system (21), where

$$A_0=\begin{bmatrix}(n-3)\lambda & D & 0 & \cdots & 0\\ 0 & (n-4)\lambda & D & \cdots & 0\\ \vdots & \vdots & \ddots & \ddots & \vdots\\ 0 & 0 & \cdots & \lambda & D\\ 0 & 0 & \cdots & 0 & 0\end{bmatrix}$$

$$A_1(t)=\begin{bmatrix}-h_1(D(t)-D) & D(t)-D & 0 & \cdots & 0\\ -h_2(D(t)-D) & 0 & D(t)-D & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ -h_{n-3}(D(t)-D) & 0 & \cdots & 0 & D(t)-D\\ -h_{n-2}(D(t)-D) & 0 & \cdots & 0 & 0\end{bmatrix}$$

and the cascaded term is

$$\begin{bmatrix}-h_1 & 1 & 0 & \cdots & 0\\ -h_2 & 0 & 1 & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ -h_{n-3} & 0 & 0 & \cdots & 1\\ -h_{n-2} & 0 & 0 & \cdots & 0\end{bmatrix}\begin{bmatrix}\tilde z_3\\ \tilde z_4\\ \vdots\\ \tilde z_{n-1}\\ \tilde z_n\end{bmatrix}(u_1-u_{1d})\mathrm{e}^{\lambda(t-t_0)} \tag{23}$$

Considering u_1 defined as (19), the cascade term (23) can also be written as $g(t,\tilde{\boldsymbol{Z}},w_1)w_1$, where $g(t,\tilde{\boldsymbol{Z}},w_1)$ is defined as

$$-k_1\mathrm{e}^{-(n-3)\lambda(t-t_0)}\begin{bmatrix}-h_1 & 1 & 0 & \cdots & 0\\ -h_2 & 0 & 1 & \cdots & 0\\ \vdots & & \ddots & \ddots & \vdots\\ -h_{n-3} & 0 & 0 & \cdots & 1\\ -h_{n-2} & 0 & 0 & \cdots & 0\end{bmatrix}\begin{bmatrix}\tilde z_3\\ \tilde z_4\\ \vdots\\ \tilde z_{n-1}\\ \tilde z_n\end{bmatrix} \tag{24}$$

Now, we check the three conditions of Lemma 2 for the cascaded system (21) and (22) as follows.

1) With substitution (19) into (21), it is easily to obtain that system (21) is globally exponentially stable.

2) Since $(A_0,\boldsymbol{C})$ is an observable pair, there exists $\boldsymbol{H}$ such that $(A_0+\boldsymbol{HDC})$ is Hurwitzian. And it follows that system (22) is globally exponentially stable under Assumption 1 and Lemma 1.

3) Because $g(t,\tilde{\boldsymbol{Z}},w_1)$ defined as (24) is a product of exponentially converging term and system′s state, it is easily to obtain that the second condition of Lemma 2 is also satisfied.

Therefore, by Lemma 2 we conclude that system (20) is globally $\mathcal{K}$-exponentially stable under the control law u_1 of (19) and constant vector $\boldsymbol{H}$ which makes $(A_0+\boldsymbol{HDC})$ Hurwitz. □

2.3 Output-feedback design procedure of $\boldsymbol{u}_2$

Before stating our main results the following variables for system (13) is introduced first

$$\begin{aligned} z_{3d} &= y_{3d} - h_1 y_{2d} \\ &\vdots \\ z_{id} &= y_{id} - h_{i-2} y_{2d} \\ &\vdots \\ z_{nd} &= y_{nd} - h_{n-2} y_{2d} \end{aligned} \tag{25}$$

Then, it is easily to get

$$\begin{bmatrix} \dot z_{3d} \\ \dot z_{4d} \\ \vdots \\ \dot z_{n-1,d} \\ \dot z_{nd} \end{bmatrix} = \begin{bmatrix} (n-3)\lambda - h_1 D(t) & D(t) & 0 & \cdots & 0 \\ -h_2 D(t) & (n-4)\lambda & D(t) & \cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ -h_{n-3} D(t) & 0 & \cdots & \lambda & D(t) \\ -h_{n-2} D(t) & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} z_{3d} \\ z_{4d} \\ \vdots \\ z_{n-1,d} \\ z_{nd} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ u_{2d} \end{bmatrix} + \begin{bmatrix} (h_2 - h_1^2) D(t) y_{2d} - \lambda h_1 y_{2d} \\ (h_3 - h_1 h_2) D(t) y_{2d} - 2\lambda h_2 y_{2d} \\ \vdots \\ (h_{n-2} - h_1 h_{n-3}) D(t) y_{2d} - (n-3)\lambda h_{n-3} y_{2d} \\ (-h_1 h_{n-2}) D(t) y_{2d} - (n-2)\lambda h_{n-2} y_{2d} \end{bmatrix}$$

For simplicity of notation, we denote

$$\begin{aligned} w_2 &= y_2 - y_{2d} \\ w_3 &= \hat z_3 - z_{3d} \\ &\vdots \\ w_4 &= \hat z_4 - z_{4d} \\ &\vdots \\ w_n &= \hat z_n - z_{nd} \end{aligned} \tag{26}$$

where w_1 is defined in Theorem 1 and $\Xi(t), \boldsymbol{\Sigma}(t)$ are as follows, respectively.

$$\Xi(t) = \begin{bmatrix} \xi_1(t) & D(t) & 0 & \cdots & \cdots & 0 \\ \xi_2(t) & -h_1 D(t) + (n-3)\lambda & D(t) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \xi_i(t) & -h_{i-2} D(t) & 0 & (n-i)\lambda & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & D(t) \\ \xi_{n-1}(t) & -h_{n-2} D(t) & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\Sigma(t) = \begin{bmatrix} \sigma(t)\tilde z_3 + (\sigma(t) - D(t))[\hat z_3 + h_1 y_2] \\ (\sigma(t) - D(t))[\hat z_4 - h_1 \hat z_3 + (h_2 - h_1^2) y_2] \\ \vdots \\ (\sigma(t) - D(t))[\hat z_{i+1} - h_{i-2} \hat z_3 + (h_{i-1} - h_1 h_{i-2}) y_2] \\ \vdots \\ (\sigma(t) - D(t))[-h_{n-2} \hat z_3 - h_1 h_{n-2} y_2] \end{bmatrix} \tag{27}$$

where $\xi_1(t) = h_1 D(t) + (n-2)\lambda$, $\xi_2(t) = (h_2 - h_1^2) D(t) - \lambda h_1$, $\xi_{i-1}(t) = (h_{i-1} - h_1 h_{i-2}) D(t) - (i-2)\lambda h_{i-2}$ $(3 \le i \le n-1)$, and $\xi_{n-1}(t) = -h_1 h_{n-2} D(t) - (n-2)\lambda h_{n-2}$.

We are ready to state and prove our main results on global output-feedback tracking.

Theorem 2. Considering the output-feedback tracking control problem of system (1) and system (2), assume that u_{1d} satisfies Assumption 1 and $x_{3d}, x_{4d}, \cdots, x_{nd}$ are bounded. Then, the output feedback control law given by

$$u_1 = u_{1d} - \exp(-(n-2)\lambda(t - t_0)) \cdot k_1 w_1 \tag{28}$$

$$u_2 = u_{2d} - \boldsymbol{K}[w_2\ w_3\ \cdots\ w_n]^{\rm T} \tag{29}$$

renders the closed system (1), (2), (17), (28), (29) globally $\mathcal{K}$-exponentially stable, where control law u_1 defined as (28) is the same as that obtained in Theorem 1 and $\boldsymbol{K}$ is constant vector such that matrix $\bar A_0 + \boldsymbol{BK}$ is Hurwitzian.

Proof. Based on new coordinate transformation (26), the new tracking error dynamics satisfy

$$\dot{\boldsymbol{W}}_1 = \Xi(t)\boldsymbol{W}_1 + \boldsymbol{\Sigma}(t) + \begin{bmatrix} 0 & \cdots & 0 & u_2 - u_{2d} \end{bmatrix}^{\rm T} \tag{30}$$

where $\boldsymbol{W}_1 = [w_2, w_3, \cdots, w_n]^{\rm T}$. And system (30) can be viewed as

$$\dot{\boldsymbol{W}}_1 = \Xi(t)\boldsymbol{W}_1 + \begin{bmatrix} 0 & \cdots & 0 & u_2 - u_{2d} \end{bmatrix}^{\rm T} \tag{31}$$

cascaded by system (20) and (21), and the cascaded term is $\boldsymbol{\Sigma}(t)$ which is defined in (27).

Under the control law u_2 of (29), system (31) can also be written as

$$\dot{\boldsymbol{W}}_1 = (\bar A_0 + \boldsymbol{BK} + \bar A_1(t))\boldsymbol{W}_1 \tag{32}$$

where

$$\bar A_0 = \begin{bmatrix} \xi_1 & D & 0 & \cdots & \cdots & 0 \\ \xi_2 & -h_1 D + (n-3)\lambda & D & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \xi_i & -h_{i-2} D & 0 & (n-i)\lambda & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & D \\ \xi_{n-1} & -h_{n-2} D & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{K} = \begin{bmatrix} -k_2 \\ \vdots \\ -k_i \\ \vdots \\ -k_{n-1} \\ -k_n \end{bmatrix}^{\rm T}$$

$$\bar A_1(t) = (D(t) - D) \begin{bmatrix} h_1 & 1 & 0 & \cdots & \cdots & 0 \\ (h_2 - h_1^2) & -h_1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ (h_{i-1} - h_1 h_{i-2}) & -h_{i-2} & 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 1 \\ -h_1 h_{n-2} & -h_{n-2} & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$\xi_1 = h_1 D + (n-2)\lambda$, $\xi_2 = (h_2 - h_1^2)D - \lambda h_1$, $\xi_i = (h_{i-1} - h_1 h_{i-2})D - (i-2)\lambda h_{i-2}$, and $\xi_{n-1} = -h_1 h_{n-2} D - (n-2)\lambda h_{n-2}$.

Since the cascaded term $\Sigma(t)$ defined as (27) is equivalent to

$$\begin{bmatrix} \hat{z}_3+\tilde{z}_3+h_1y_2 \\ -h_1\hat{z}_3+\hat{z}_4+(h_2-h_1^2)y_2 \\ \vdots \\ -h_{i-2}\hat{z}_3+\hat{z}_{i+1}+(h_{i-1}-h_1h_{i-2})y_2 \\ \vdots \\ -h_{n-2}\hat{z}_3-h_1h_{n-2}y_2 \end{bmatrix} \mathrm{e}^{\lambda(t-t_0)}(u_1-u_{1d})+\begin{bmatrix} D(t) \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}\tilde{z}_3$$

under the control law u_1 of (28), $\Sigma(t)$ can be written as

$$G(t,\boldsymbol{W}_1,\boldsymbol{W}_2)\boldsymbol{W}_2$$

where $G(t,\boldsymbol{W}_1,\boldsymbol{W}_2)$ is defined as

$$\begin{bmatrix} (\hat{z}_3+\tilde{z}_3+h_1y_2)\mathrm{e}^{-(n-3)\lambda(t-t_0)} & D(t) \\ (-h_1\hat{z}_3+\hat{z}_4+(h_2-h_1^2)y_2)\mathrm{e}^{-(n-3)\lambda(t-t_0)} & 0 \\ \vdots & \\ (-h_{i-2}\hat{z}_3+\hat{z}_{i+1}+(h_{i-1}-h_1h_{i-2})y_2)\mathrm{e}^{-(n-3)\lambda(t-t_0)} & 0 \\ \vdots & \\ (-h_{n-2}\hat{z}_3-h_1h_{n-2}y_2)\mathrm{e}^{-(n-3)\lambda(t-t_0)} & 0 \end{bmatrix}$$

and $\boldsymbol{W}_2=[-k_1w_1\ \tilde{z}_3]^{\mathrm{T}}$. It is easily to check that the above formula of $G(t,\boldsymbol{W}_1,\boldsymbol{W}_2)$ can be simplified as

$$\mathrm{e}^{-(n-3)\lambda(t-t_0)}\begin{bmatrix} w_3+\tilde{z}_3+h_1w_2 & 0 \\ -h_1w_3+w_4+(h_2-h_1^2)w_2 & 0 \\ \vdots & \vdots \\ -h_{i-2}w_3+w_{i+1}+(h_{i-1}-h_1h_{i-2})w_2 & 0 \\ \vdots & \vdots \\ -h_{n-2}w_3-h_1h_{n-2}w_2 & 0 \end{bmatrix}+$$

$$\mathrm{e}^{-(n-3)\lambda(t-t_0)}\begin{bmatrix} y_{3d} & \mathrm{e}^{(n-3)\lambda(t-t_0)}D(t) \\ -h_1y_{3d}+y_{4d} & 0 \\ \vdots & \vdots \\ -h_{i-2}y_{3d}+y_{(i+1)d} & 0 \\ \vdots & \vdots \\ -h_{n-2}y_{3d} & 0 \end{bmatrix}=$$

$$\mathrm{e}^{-(n-3)\lambda(t-t_0)}\begin{bmatrix} w_3+\tilde{z}_3+h_1w_2 & 0 \\ -h_1w_3+w_4+(h_2-h_1^2)w_2 & 0 \\ \vdots & \vdots \\ -h_{i-2}w_3+w_{i+1}+(h_{i-1}-h_1h_{i-2})w_2 & 0 \\ \vdots & \vdots \\ -h_{n-2}w_3-h_1h_{n-2}w_2 & 0 \end{bmatrix}+$$

$$\begin{bmatrix} x_{3d} & D(t) \\ -h_1x_{3d}+\mathrm{e}^{-\lambda(t-t_0)}x_{4d} & 0 \\ \vdots & \vdots \\ -h_{i-2}x_{3d}+\mathrm{e}^{-(i-2)\lambda(t-t_0)}x_{(i+1)d} & 0 \\ \vdots & \vdots \\ -h_{n-2}x_{3d} & 0 \end{bmatrix} \tag{33}$$

Now, the output tracking problem composed of systems (1) and (2) is converted into a stabilizing problem of system formed by (21) and (30) under the observer error system (20). In the following, we will check the three conditions of Lemma 2 for cascaded system (20), (21) and (32).

1) Since $(\bar{A}_0,\boldsymbol{B})$ is a controllable pair, there exists $\boldsymbol{K}$ such that $(\bar{A}_0+\boldsymbol{BK})$ is Hurwitzian. System (32) is globally exponentially stable under Assumption 1 and Lemma 1.

2) It has been proved that system (20) and (21) is globally $\mathcal{K}$-exponentially stable in Theorem 2.

3) Since $D(t)$ is bounded in Assumption 1, there exists constant M such that

$$|D(t)|\le M$$

Since $x_{3d},x_{4d},\cdots,x_{nd}$ are also bounded signals, the second condition of Lemma 2 is also satisfied from (33).

Therefore, by Lemma 2 it can be concluded that system formed by (1), (2), (17), (28), (29) is globally $\mathcal{K}$-exponentially stable. □

3 Applications

In this section, we apply our tracking control method to two benchmark mechanical systems under nonholonomic constrains: a unicycle mobile robot and an articulated vehicle.

3.1 Example 1: a unicycle mobile robot

A unicycle mobile robot is described by

$$\begin{aligned}\dot{x}&=v\cos\theta\\ \dot{y}&=v\sin\theta\\ \dot{\theta}&=\omega\end{aligned} \tag{34}$$

where (x,y) denotes the coordinates of the center of mass, θ is the angle between the heading direction and the x axis. Using the following well known coordinate transformation

$$\begin{aligned}x_1&=\theta\\ x_2&=x\sin\theta-y\cos\theta\\ x_3&=x\cos\theta+y\sin\theta\\ u_1&=\omega\\ u_2&=v-x_3\omega\end{aligned} \tag{35}$$

we easily obtain the chained form of system (34)

$$\begin{aligned}\dot{x}_1&=u_1\\ \dot{x}_2&=x_3u_1\\ \dot{x}_3&=u_2\end{aligned}$$

The following initial conditions of the target system and the dilated error system are selected in this simulation

$$\begin{bmatrix} x_{1d}(0)\\ x_{2d}(0)\\ x_{3d}(0)\end{bmatrix}=\begin{bmatrix} -2.9865\\ 1.3153\\ 0.18698\end{bmatrix},\quad \begin{bmatrix} w_1(0)\\ w_2(0)\\ w_3(0)\end{bmatrix}=\begin{bmatrix} 0.05\\ -0.1415\\ 1.2079\end{bmatrix}$$

Case A (Movement exponentially to a steady point). The tracking problem that cannot be manipulated using control law presented in previous papers[11−15, 28−30] is considered first. In this case, the reference velocities v_d and ω_d are assumed to be $\mathrm{e}^{-0.5t}$ and $\mathrm{e}^{-0.5t}+\mathrm{e}^{-1.5t}$, respectively. After simple computation, it is easy to get

$$\theta_d(t)=\theta_d(0)-2\mathrm{e}^{-0.5t}-\frac{2}{3}\mathrm{e}^{-1.5t}-\frac{8}{3}$$

$$|x_d(t)|=|x_d(0)+\int_0^t \mathrm{e}^{-0.5t}\cos\theta\,\mathrm{d}t|\le|x_d(0)|+\int_0^t|\mathrm{e}^{-0.5t}|\,\mathrm{d}t=|x_d(0)|+2(1-\mathrm{e}^{-0.5t})$$

$$|y_d(t)|=|y_d(0)+\int_0^t \mathrm{e}^{-0.5t}\sin\theta\,\mathrm{d}t|\le|y_d(0)|+\int_0^t|\mathrm{e}^{-0.5t}|\mathrm{d}t=|y_d(0)|+2(1-\mathrm{e}^{-0.5t})$$

And it is easy to see that Assumption 1 is satisfied in this case. The observer error is presented in Fig. 1. Fig. 2 shows the moving trajectory of the mobile robot. Tracking errors and control inputs are given in Figs. 3 and 4, respectively.

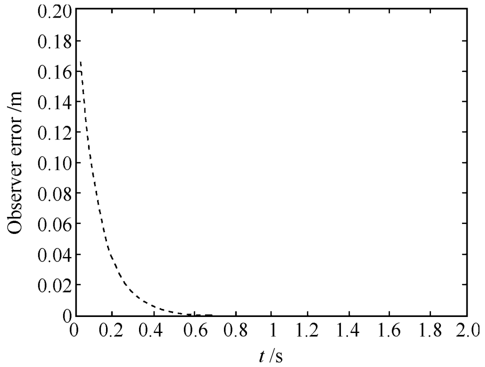

Fig. 1 Observer error with respect to time (Case A)

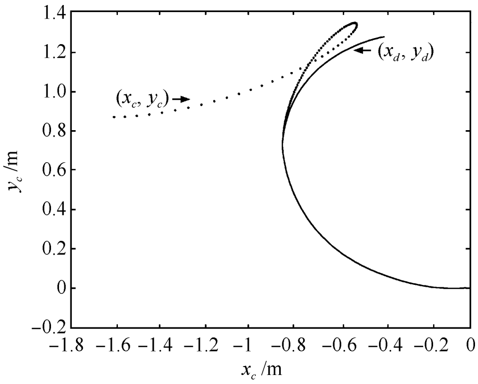

Fig. 2 Plot of $(x_c(t), y_c(t))$ (Case A)

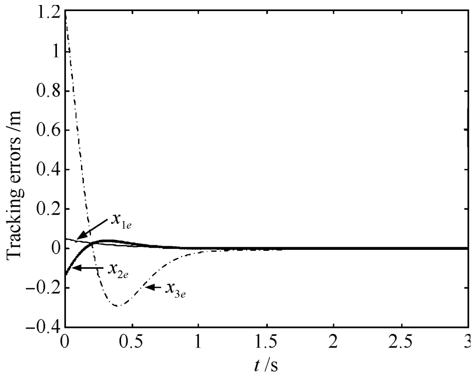

Fig. 3 Tracking errors with respect to time (Case A)

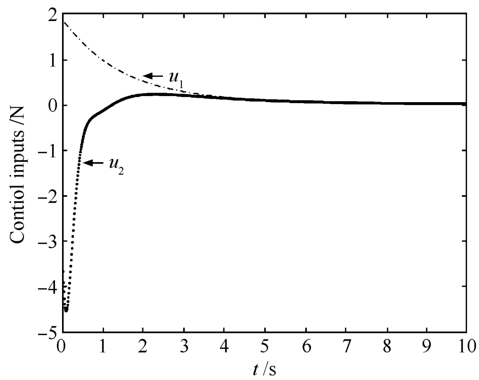

Fig. 4 Control signals with respect to time (Case A)

Case B (Movement along a circular path). Similar to previous papers[11−15] on this problem, the reference velocities v_d and ω_d are assumed to be $1 + t/(t + 10)$ and 1 in this case.

Through simple integration it can be obtained that

$$\begin{aligned}
&\theta_d(t) = \theta_d(0) + t \\
&|x_d(t)| = \left| x_d(0) + \int_0^t \left[1 + \frac{t}{t+10}\right] \cos(t + \theta_d(0))\,\mathrm{d}t \right| \leq \\
&\qquad \left| x_d(0) + 2\int_0^t \cos(t + \theta_d(0))\,\mathrm{d}t \right| = \\
&\qquad |x_d(0)| + 2(\sin(t + \theta_d(0)) - \sin\theta_d(0)) \\
&|y_d(t)| \leq |y_d(0)| + 2(\cos(t + \theta_d(0)) - \cos\theta_d(0))
\end{aligned}$$

and Assumption 1 is also satisfied in this case.

The observer error is presented in Fig. 5. Fig. 6 shows the moving trajectory of the mobile robot. Tracking errors and control inputs are given in Figs. 7 and 8, respectively.

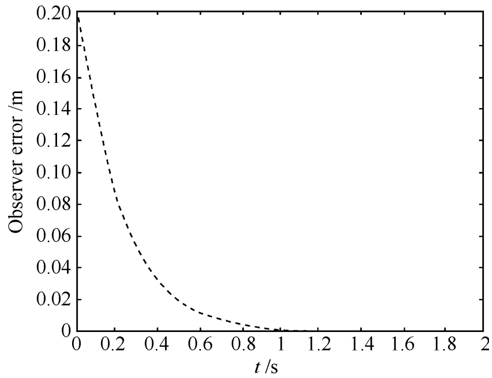

Fig. 5 Observer error with respect to time (Case B)

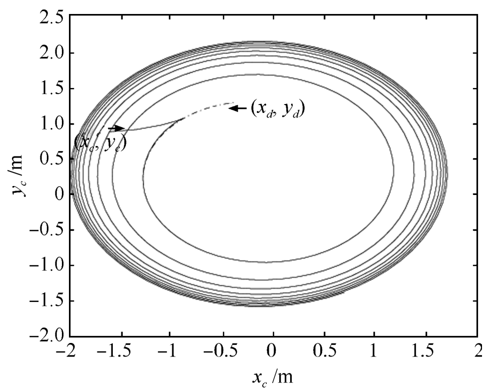

Fig. 6 Plot of $(x_c(t), y_c(t))$ (Case B)

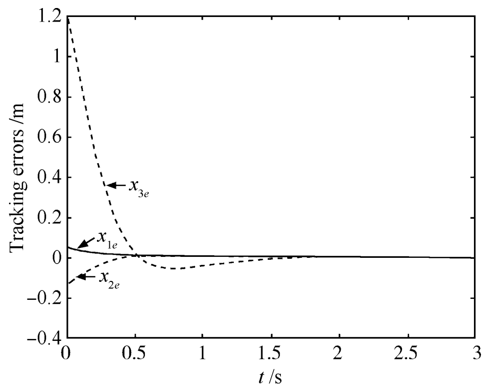

Fig. 7 Tracking errors with respect to time (Case B)

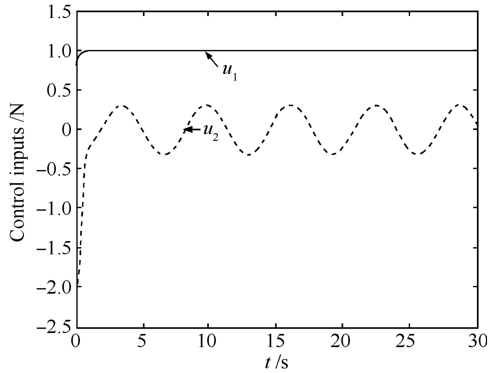

Fig. 8 Control signals with respect to time (Case B)

3.2 Example 2: an articulated vehicles

An articulated vehicle, which is a car pulling a single trailer, is described by

$$\begin{aligned}\dot{x}_c &= v\cos\theta_0\\ \dot{y}_c &= v\sin\theta_0\\ \dot{\phi} &= \omega\\ \dot{\theta}_0 &= \frac{1}{l}\tan\phi\\ \dot{\theta}_1 &= \frac{1}{d_1}\sin(\theta_0-\theta_1)\end{aligned}\tag{36}$$

where (x_c, y_c) denotes the coordinates of the center of mass of the tow car, $\theta_0(\theta_1)$ is the angle between the heading directions of the car (trailer) and the x axis, l is the wheelbase of the car, d_1 is the distance from the wheel of trailer to the wheel of the car, and v and ω represent the driving velocity and steering velocity of the tow car, respectively, which can be controlled independently (for a figure of the articulated vehicle, please refer to [11−12]).

Although system (36) is not in a chained form, it can be transformed into (1) with $n = 5$ via a change of coordinates and state feedback[22]. As done in [11], we still select parameter l and d_1 as 1, and initial condition as $x_{id}(0) = 0, 1 \le i \le 5$.

Jiang[11] solved the tracking control problem using the method of backstepping under the following condition

$$u_{1d} = 1, u_{2d} = 0,\ x_{1d}(t) = t,\ x_{id}(t) = 0\ (2 \le i \le 5)\tag{37}$$

However, the control law presented by [11] is complicated and semi-global. Lefeber[13] brought out a simpler global $\mathcal{K}$ exponential controller

$$u_1 = u_{1d} - k_1 x_{1e}\tag{38}$$

$$u_2 = u_{2d} - k_2\hat{x}_{2e} - k_3\hat{x}_{3e} - k_4\hat{x}_{4e} - k_5\hat{x}_{5e}\tag{39}$$

using the cascaded structure in the error system under the following condition

$$u_{1d} = 1, u_{2d} = 0, x_{id}(0) = 0,\quad \forall 1 \le i \le 5\tag{40}$$

where $\hat{x}_{ie} = \hat{x}_i - x_{id},\ i = 1, \cdots, 5$.

In order to show the differences with previous results[11−15, 28−30], the following two cases are considered.

Case C. First, we select $u_{1d} = 1$ and $u_{2d} = 0$ the same as those selected in [11, 13]. From (37), it is easy to testify that Assumption 1 is satisfied in this case.

Since A_0 in the design of reduced observer is

$$A_0 = \begin{bmatrix}(n-3)\lambda & D & 0\\ 0 & (n-4)\lambda & D\\ 0 & 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

we select $\boldsymbol{H} = [-6\ \ -11\ \ -6]^{\mathrm{T}}$ such that $A_0 + \boldsymbol{HC}$ is Hurwitzen. Hence, $\bar{A}_0$ in the design of output feedback controller is

$$\bar{A}_0 = \begin{bmatrix}h_1D+(n-2)\lambda & D & 0 & 0\\ (h_2-h_1^2)D-\lambda h_1 & -h_1D+(n-3)\lambda & D & 0\\ (h_3-h_1h_2)D-2\lambda h_2 & -h_2D & \lambda & D\\ -h_1h_3D-3\lambda h_3 & -h_3D & 0 & 0\end{bmatrix} = \begin{bmatrix}6 & 1 & 0 & 0\\ -25 & -6 & 1 & 0\\ -60 & -11 & 0 & 1\\ -36 & -6 & 0 & 0\end{bmatrix}$$

Then, the gain matrix can be selected as $\boldsymbol{K} = [-85.1800\ \ -2.4400\ \ -4.4600\ \ -3.5000]$ so that $\bar{A}_0 + \boldsymbol{BK}$ is Hurwitz. Performance of $x(t)$ with respect to time is presented in Fig. 9. Fig. 10 shows tracking errors. The observer errors and control inputs are presented in Figs. 11 and 12, respectively.

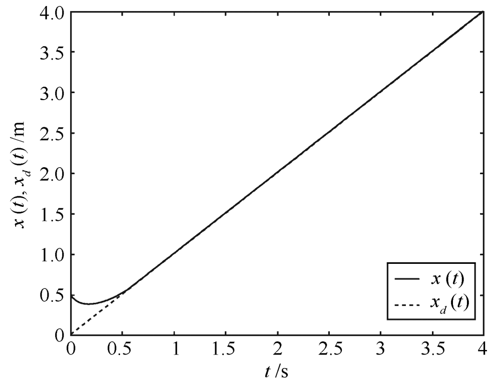

Fig. 9 Plot of $x(t)$ with respect to time (Case C)

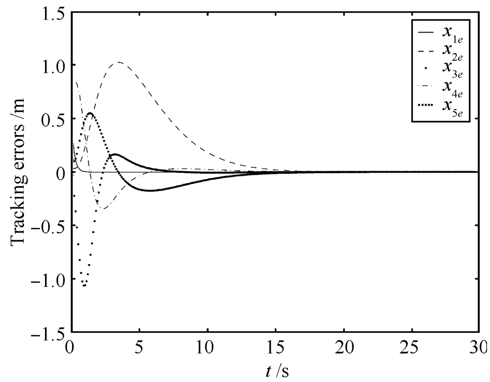

Fig. 10 Tracking errors with respect to time (Case C)

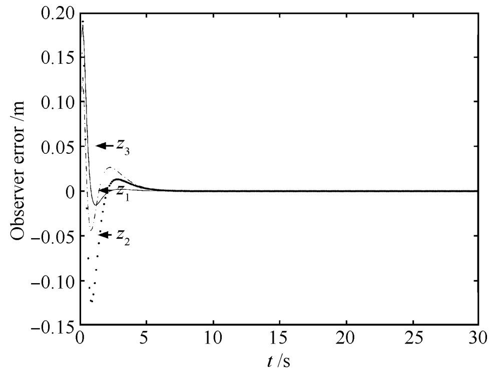

Fig. 11 Observer error with respect to time (Case C)

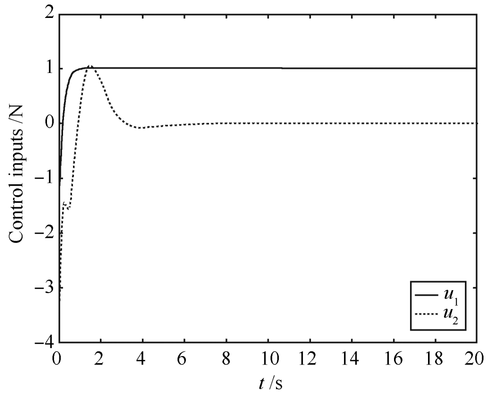

Fig. 12 Control signals with respect to time (Case C)

Case D. u_{1d} converging to zero and $u_{2d} = 0$.

As we can see, if we select $u_{1d} = \mathrm{e}^{-0.5t}(1+\mathrm{e}^{-0.5t})$, $u_{2d} = 0$, and $x_{id}(0) = 0$ $(1 \le i \le 5)$ then the tracking control problem could not be dealt by the controllers presented in [11−15, 28−30]. After simple computation it is easy to get that the reference signal is

$$x_{1d}(t) = -2\mathrm{e}^{-0.5t} - \mathrm{e}^{-t}, \quad x_{id}(t) = 0 \quad (2 \le i \le 5)$$

and Assumption 1 is also satisfied in this case.

Since A_0 in the design of reduced observer is

$$A_0 = \begin{bmatrix} (n-3)\lambda & D & 0 \\ 0 & (n-4)\lambda & D \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0.5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

we can select $\boldsymbol{H} = [-11.5\ -36.25\ -30]^{\mathrm{T}}$ such that $A_0+\boldsymbol{HC}$ is Hurwitzian. Similarly, since

$$\bar{A}_0 = \begin{bmatrix} h_1D+(n-2)\lambda & D & 0 & 0 \\ (h_2-h_1^2)D-\lambda h_1 & -h_1D+(n-3)\lambda & D & 0 \\ (h_3-h_1h_2)D-2\lambda h_2 & -h_2D & \lambda & D \\ -h_1h_3D-3\lambda h_3 & -h_3D & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 13.0000 & 1.0000 & 0 & 0 \\ -101.7500 & -10.5000 & 1.0000 & 0 \\ -423.1250 & -36.2500 & 0.5000 & 1.0000 \\ -390.0000 & -30.0000 & 0 & 0 \end{bmatrix}$$

we can select $\boldsymbol{K} = [884.431\ 20.914\ 13.29\ 4.9]$ such that $\bar{A}_0+\boldsymbol{BK}$ is Hurwitzian. Performance of $x(t)$ with respect to time is presented in Fig. 13 and Fig. 14 shows the tracking errors. The observer errors and control inputs are presented in Fig. 15 and Fig. 16, respectively.

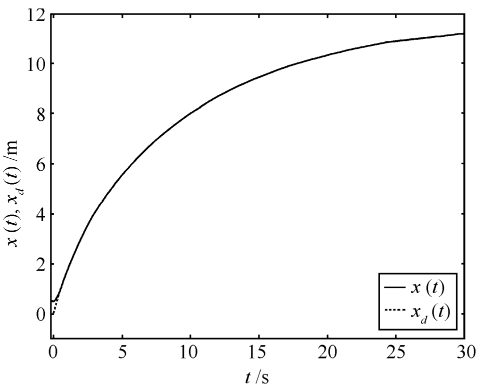

Fig. 13 Plot of $x(t)$ with respect to time (Case D)

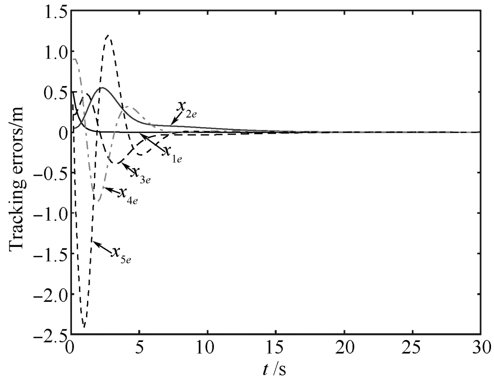

Fig. 14 Tracking errors with respect to time (Case D)

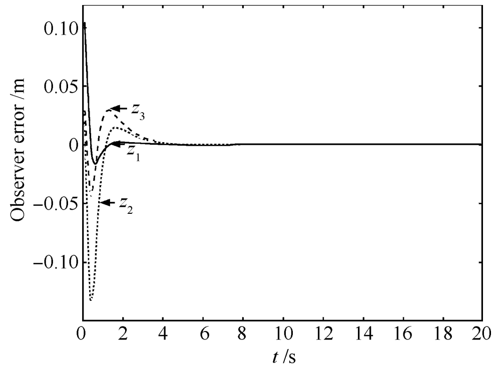

Fig. 15 Observer error with respect to time (Case D)

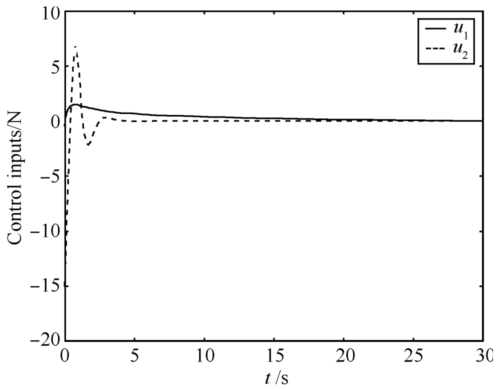

Fig. 16 Control signals with respect to time (Case D)

4 Conclusion

Global $\mathcal{K}$-exponential tracking solutions are obtained for a class of nonholonomic control systems using output feedback. The constructive design procedure has been motivated by our recent tracking approaches proposed in [20−21]. In particular, sufficient conditions on reference trajectories have been given under which the problem of global exponential tracking using output feedback is solvable. Different to earlier research[11−15, 28−30] addressing the tracking problem for nonholonomic chained form systems, our controllers do not require conditions such as persistent excitation or not-converging to zero on reference trajectories. It is under current investigations to explore the feasibility of constructing more general trackers under more relaxed assumptions.

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