Robust Adaptive Fault-tolerant Compensation Control with Actuator Failures and Bounded Disturbances

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Abstract In this paper, direct adaptive state feedback control schemes are developed to solve the robust fault-tolerant compensation control problem for linear time-invariant continuous-time systems with actuator failures and external disturbances. While both eventual faults and upper bound of disturbances are unknown, the adaptive laws are proposed to estimate the unknown controller parameters online. Then, a class of robust adaptive state feedback controllers is constructed for automatically compensating the fault and the disturbance effects based on the information from the adaptive schemes. On the basis of Lyapunov stability theory, it is shown that the resulting adaptive closed-loop system can be guaranteed to be asymptotically stable in the presence of faults on actuators and disturbances. A numerical example of rocket fairing structural-acoustic model and its simulation results are given.

Key words Fault-tolerant control (FTC), robust adaptive control, actuator failures, disturbance rejection, asymptotically stable

In most practical control systems, components′ (including sensors, actuators, and even the plant itself) failures may occur at uncertain time and the size of a fault is also unknown. The faults may lead to performance deterioration or even instability of the system. Therefore, the study of designing fault-tolerant control (FTC) systems, which let the systems operate in safe conditions and with proper performances whenever components are healthy or faulted, has received considerable attention over the past two decades[1−23]. The existing fault-tolerant design approaches can be broadly classified into two groups, namely passive approaches[1−8] and active approaches[9−23]. In the passive approaches, robust control techniques are utilized to design a fixed controller for maintaining the acceptable system stability and performances throughout normal or faulty cases. Recently, several approaches have been developed, such as algebraic Riccati equation based approach[1−3], LMI-based approach[4−7], pole region assignment technique[8], etc. In the passive approach, it is relatively easy to design the controller for the presumed faults because they do not rely on online controller adjustment. However, it has also a limited fault tolerant capability because as the number of possible failures and the degree of system redundancy increase, the controller design becomes more conservative and attainable control performances may not be satisfactory. On the other hand, a fault-tolerant control system based on active approaches can compensate for faults either by selecting a precomputed control law or by synthesizing a new control strategy online. Primarily, there are two typical approaches for fault compensation in active fault-tolerant, such as adaptive approaches[9−18] and fault detection and isolation (FDI) approaches[19−23]. Since the active FTC system provides the flexibility to select different controllers, the most suitable controller can be chosen for the situation and the better performance can be obtained than the passive FTC system.

For the active fault-tolerant design approach based on FDI, the controller reconfiguration or restructure is based on the fault diagnostic information, which is provided by a fault detection and isolation mechanism. However, it should be noted that the FDI mechanism might not always give the exact fault information. Another typical approach for fault compensation is based on adaptive method. In [9], the perfect performance tracking result was obtained by considering the fault model of loss actuator effectiveness. In [13−16], the results of adaptive fault-tolerant control were based on model reference adaptive control, where the outputs of closed-loop systems could track the prescribed referent outputs. However, as we know that external disturbances play an important role in the system, some of above works, such as [9−14], have not considered the disturbances within the system, and the proposed methods may not be suitable for the FTC system when there exist disturbances. Moreover, [15−16] considered the disturbances under some special conditions, such as $\lim_{t\to\infty} \boldsymbol{z}(t) = 0$ ($\boldsymbol{z}(t)$ is disturbance)[15] and constant disturbance[16]. Recently, the disturbance attenuation performances of adaptive FTC system have been addressed in [17], but the system cannot be guaranteed to be asymptotically stable when the disturbance always exists in the system. Therefore, the capability of disturbance rejection for the above FTC systems is very weak. On the other hand, the direct adaptive method proposed in [10] can compensate for the time-varying parameterizable stuck-actuator failures. But for the unparametrizable failures, approximations of the stuck-actuator failures must be employed and the closed-loop system can be guaranteed to be stable rather than asymptotically stable[12]. Furthermore, [14] considered the unparametrizable failures in the system, but the requirement of knowledge of upper bound of failures was needed and asymptotic tracking could not be ensured. In this paper, the newly proposed robust adaptive schemes can solve the problem of FTC with more general actuator failures than the published works[9−18], and make sure the system is asymptotically stable under the influence of actuator unparametrizable time-varying failures and external disturbances.

Here, the robust adaptive compensation design approach can be used for a general actuator fault model, which covers the cases of normal operation, loss of effectiveness, outage, and stuck. Each control effectiveness and the upper bound of disturbances are not necessary to be known. A direct adaptive method is proposed to solve the problem for developing some state feedback controllers. For this purpose, we first propose some adaptive laws to estimate the unknown controller parameters online. Then, the controllers are constructed relying on the updated values of these estimations. Based on the Lyapunov stability theory, the adaptive closed-loop system can be guaranteed to be asymptotically stable in the presence of failures on actuators and disturbances.

The rest of the paper is organized as follows. The FTC problem formulation is described in Section 1. In Section 2,

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the direct adaptive robust state feedback controllers are developed. Section 3 gives a numerical example of rocket fairing structural-acoustic model and its simulation results. Finally, conclusions are given in Section 4.

1 Preliminaries and problem statement

We first introduce our notations. $\mathbf{R}$ stands for the set of real numbers. For a real matrix E, $\{E\}$ represents the induced norm. Given matrices $M_k, k = 1, \cdots, n$, the notation $\text{diag}_{k=1}^{n}\{M_k\}$ denotes the block-diagonal matrix with M_k along the diagonal and denoted as $\text{diag}_k\{M_k\}$ for brevity.

In this paper, we consider a linear time-invariant continuous-time model with the following state-space equation:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B_1\boldsymbol{w}(t) + B_2\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ is the state, $\boldsymbol{u}(t) \in \mathbf{R}^m$ is the control input, and $\boldsymbol{w}(t) \in \mathbf{R}^q$ is a continuous vector function which represents the bounded external disturbances for the system. A, B_1, and B_2 are known real constant matrices with appropriate dimensions.

In this paper, we consider actuator faults including outage, loss of effectiveness, and stuck. Let $u_{ij}^F(t)$ represent the signal from the i-th actuator that has failed in the j-th faulty mode. Then, we denote a general actuator fault model as

$$u_{ij}^F(t) = \rho_i^j(t)u_i(t) + \sigma_i^j u_{si}(t), \quad i = 1, \cdots m, \quad j = 1, \cdots, L \tag{2}$$

where $\rho_i^j(t)$ is the unknown time-varying actuator efficiency factor, the index j denotes the j-th faulty mode, L is the number of total faulty modes, and $\underline{\rho}_i^j$ and $\bar{\rho}_i^j$ represent the known lower and upper bounds of $\rho_i^j(t)$, respectively. $u_{si}(t)$ is the unparametrizable bounded time-varying stuck-actuator fault[14] in the i-th actuator. Note the practical case where we have $0 \le \underline{\rho}_i^j \le \rho_i^j \le \bar{\rho}_i^j$, and σ_i^j is an unknown constant defined as

$$\sigma_i^j = \begin{cases} 0, & \rho_i^j > 0 \\ 0 \text{ or } 1, & \rho_i^j = 0 \end{cases}$$

Then, Table 1 can be given to illustrate the fault model.

Table 1 Fault model

Fault model	$\underline{\rho}_i^j$	$\bar{\rho}_i^j$	σ_i^j
Normal	1	1	0
Outage	0	0	0
Loss of effectiveness	>0	<1	0
Stuck	0	0	1

Denote

$$\boldsymbol{u}_j^F(t) = [u_{1j}^F(t), u_{2j}^F(t), \cdots, u_{mj}^F(t)]^{\mathrm{T}} = \rho^j(t)\boldsymbol{u}(t) + \sigma^j\boldsymbol{u}_s(t)$$

where $\rho^j(t) = \text{diag}_i\{\rho_i^j(t)\}$, $\rho_i^j(t) \in [\underline{\rho}_i^j, \bar{\rho}_i^j]$, $\sigma^j = \text{diag}_i\{\sigma_i^j\}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, L$.

Then, the set of operators with the above structure is denoted by

$$\Delta_{\rho^j} = \{\rho^j(t) : \rho^j(t) = \text{diag}_i\{\rho_i^j(t)\}, \quad \rho_i^j(t) \in [\underline{\rho}_i^j, \bar{\rho}_i^j]\} \tag{3}$$

and we also denote the following set

$$\begin{aligned} N_{\rho^j} = \{&\rho^j(t) : \rho^j(t) = \text{diag}_i\{\rho_i^j(t)\}, \\ &\rho_i^j(t) = \underline{\rho}_i^j \text{ or } \rho_i^j(t) = \bar{\rho}_i^j\} \end{aligned} \tag{4}$$

where $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, L$. Thus, set N_{ρ^j} contains a maximum of 2^m elements.

For the sake of convenient description, for all possible faulty modes L, the following uniform actuator fault model is exploited:

$$\boldsymbol{u}^F(t) = \rho(t)\boldsymbol{u}(t) + \sigma\boldsymbol{u}_s(t) \tag{5}$$

where $\rho(t) = \text{diag}\{\rho_1(t), \cdots, \rho_m(t)\} \in \{\rho^1(t), \cdots, \rho^L(t)\}$.

Hence, the dynamics of system (1) with actuator faults (5) is described by

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B_2\rho(t)\boldsymbol{u}(t) + B_2\sigma\boldsymbol{u}_s(t) + B_1\boldsymbol{w}(t) \tag{6}$$

To ensure the achievement of fault-tolerant objective, the following assumptions in FTC design are also assumed to be valid.

Assumption 1. All the states of system are available at every instant.

Assumption 2. All pairs $\{A, B_2\rho(t)\}$ are uniformly completely controllable for any actuator failure mode $\rho(t) \in \{\rho^1(t), \cdots, \rho^L(t)\}$ under consideration.

Assumption 3. The unparametrizable stuck-actuator fault and external disturbance are piece-wise continuous bounded functions, that is, there exist unknown positive constants $\bar{u}_s$ and $\bar{w}$ such that

$$\|\boldsymbol{u}_s(t)\| \le \bar{u}_s, \quad \|\boldsymbol{w}(t)\| \le \bar{w}$$

respectively.

Assumption 4. For FTC system (6), there exists a matrix function F of appropriate dimensions such that $B_1 = B_2F$.

Assumption 5. $\text{rank}[B_2\rho(t)] = \text{rank}[B_2]$ for any actuator failure mode $\rho(t) \in \{\rho^1(t), \cdots, \rho^L(t)\}$.

Remark 1. It is well known that Assumption 1 is standard for state-feedback system design. Assumption 2 is also standard and denotes the internal stabilizability of each normal and fault isolated system. Assumption 3 is quite natural and is common in the robust fault-tolerant control literature. Assumption 4 defines a matching condition about the disturbances, which physically means that the control signals and disturbances use identical channels, and many systems satisfy this assumption for robust control problem[24]. Assumption 5 introduces a condition of actuator redundancy of the system, and is necessary for completely compensating the stuck-actuator faults and disturbances. Fortunately, many mechanical systems do belong to this class of systems and some designs[10−11] have also been proposed based on the redundant condition. Although it is still under the condition, a novel FTC will be proposed. Furthermore, in terms of (6), if we omit Assumptions 4 and 5, the robust adaptive controller $\boldsymbol{u}(t)$ just can guarantee the closed-loop FTC system signal boundness rather than asymptotically stable.

Then, the main objective of this paper is to construct a robust adaptive state feed-back controller $\boldsymbol{u}(t)$ such that the closed-loop system (6) can be guaranteed to be asymptotically stable even in the cases of actuator failures and disturbance effects all the time.

2 Direct adaptive robust fault-tolerant control system design

In this section, we develop the adaptive laws to update the controller parameters when both the actuator failures and upper bound of disturbances are unknown. Then, a method for designing direct adaptive fault-tolerant controllers to guarantee closed-loop system asymptotically stable via state feedback is presented in Theorem 1.

Consider a linear time-invariant FTC model described by (6) and controller model

$$\boldsymbol{u}(t) = \hat{K}_1(t)\boldsymbol{x}(t) + \boldsymbol{K}_2(t) \tag{7}$$

where $\hat{K}_1(t) = [\hat{\boldsymbol{K}}_{1,1}(t), \hat{\boldsymbol{K}}_{1,2}(t), \cdots, \hat{\boldsymbol{K}}_{1,m}(t)]^{\mathrm{T}} \in \mathbf{R}^{m\times n}$ updated by the following adaptive laws:

$$\frac{\mathrm{d}\hat{\boldsymbol{K}}_{1,i}(t)}{\mathrm{d}t} = -\Gamma_i \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}} P\boldsymbol{b}_{2i}, \quad i = 1, 2, \cdots, m \tag{8}$$

where Γ_i is any positive constant, $\hat{\boldsymbol{K}}_{1,i}(t_0)$ is finite, and $\boldsymbol{b}_{2i}$, $i = 1, 2, \cdots, m$ is the i-th column of B_2; $\boldsymbol{K}_2(t) = [K_{2,1}(t), K_{2,2}(t), \cdots, K_{2,m}(t)]^{\mathrm{T}} \in \mathbf{R}^m$ is given by

$$\boldsymbol{K}_2(t) = \frac{-(\boldsymbol{x}^{\mathrm{T}} PB_2)^{\mathrm{T}} \beta \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel \hat{k}_3(t)}{\parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel^2 \alpha} \tag{9}$$

where α and β are suitable positive constants which satisfy

$$\parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel^2 \alpha \leq \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \sqrt{\underline{\rho}^j} \parallel^2 \beta \tag{10}$$

for any $\underline{\rho}^j = \mathrm{diag}_i\{\underline{\rho}_i^j\} \in \Delta_{\rho^j}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, L$; and $\hat{k}_3(t) \in \mathbf{R}$ is updated by the following adaptive law:

$$\frac{\mathrm{d}\hat{k}_3(t)}{\mathrm{d}t} = \gamma \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel \tag{11}$$

where γ is any positive constant and $\hat{k}_3(t_0)$ is finite. From (11), we can see $\hat{k}_3(t) \geq 0$ if $\hat{k}_3(t_0) \geq 0$.

Therefore, following (6), (7), and Assumption 4, we can write the closed-loop FTC system model as

$$\begin{aligned}\dot{\boldsymbol{x}}(t) = &(A + B_2\rho(t)\hat{K}_1(t))\boldsymbol{x}(t) + \\ &B_2\rho(t)\boldsymbol{K}_2(t) + B_2\sigma\boldsymbol{u}_s(t) + B_2F\boldsymbol{w}(t)\end{aligned} \tag{12}$$

On the other hand, let

$$\tilde{\boldsymbol{K}}_{1,i}(t) = \hat{\boldsymbol{K}}_{1,i}(t) - \boldsymbol{K}_{1,i}, \quad \tilde{k}_3(t) = \hat{k}_3(t) - k_3 \tag{13}$$

Due to $\boldsymbol{K}_{1,i}$ and k_3 are unknown constants, we can write the following error system

$$\frac{\mathrm{d}\tilde{\boldsymbol{K}}_{1,i}(t)}{\mathrm{d}t} = -\Gamma_i \boldsymbol{x}\boldsymbol{x}^{\mathrm{T}} P\boldsymbol{b}_{2i}, \quad \frac{\mathrm{d}\tilde{k}_3(t)}{\mathrm{d}t} = \gamma \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel \tag{14}$$

In the following, by $(\boldsymbol{x}, \tilde{K}_1, \tilde{k}_3)(t)$, we denote a solution of the closed-loop system and the error system. Then, the following theorem can be obtained which shows the global boundedness of the solutions of the adaptive closed-loop system described by (12) and (14).

Theorem 1. Consider the adaptive closed-loop system described by (12) and (14) under Assumptions 1 ∼ 5. The fault-tolerant control system is asymptotically stable for any $\rho(t) \in \Delta_{\rho^j}$ if there exist a positive symmetric matrix P, $\hat{\boldsymbol{K}}_{1,i}(t)$ and $\hat{k}_3(t)$ determined according to the adaptive laws (8) and (11), respectively, and the control gain function $\boldsymbol{K}_2(t)$ given by (9).

Proof. For the adaptive closed-loop system described by (12), we first define a Lyapunov functional candidate as

$$V(\boldsymbol{x}, \tilde{K}_1, \tilde{k}_3) = \boldsymbol{x}^{\mathrm{T}} P\boldsymbol{x} + \sum_{i=1}^{m} \rho_i \tilde{\boldsymbol{K}}_{1,i}^{\mathrm{T}} \Gamma_i^{-1} \tilde{\boldsymbol{K}}_{1,i} + \gamma^{-1}\tilde{k}_3^2 \tag{15}$$

Then, according to (9), the time derivative of V for $t > 0$ associated with a certain failure mode $\rho \in \Delta_{\rho^j}$ is

$$\begin{aligned}\frac{\mathrm{d}V(\boldsymbol{x}, \tilde{K}_1, \tilde{k}_3)}{\mathrm{d}t} = &\boldsymbol{x}^{\mathrm{T}}[(A + B_2\rho\hat{K}_1)^{\mathrm{T}} P + P(A + B_2\rho\hat{K}_1)]\boldsymbol{x} + \\ &2\boldsymbol{x}^{\mathrm{T}} PB_2\rho\boldsymbol{K}_2 + 2\boldsymbol{x}^{\mathrm{T}} PB_2\sigma\boldsymbol{u}_s + \\ &2\boldsymbol{x}^{\mathrm{T}} PB_2F\boldsymbol{w} + \sum_{i=1}^{m} 2\rho_i\tilde{\boldsymbol{K}}_{1,i}^{\mathrm{T}} \Gamma_i^{-1}\dot{\tilde{\boldsymbol{K}}}_{1,i} + \\ &2\gamma^{-1}\tilde{k}_3\dot{\tilde{k}}_3 = \\ &\boldsymbol{x}^{\mathrm{T}}[(A + B_2\rho\hat{K}_1)^{\mathrm{T}} P + P(A + B_2\rho\hat{K}_1)]\boldsymbol{x} - \\ &\frac{2 \parallel \boldsymbol{x}^{\mathrm{T}} PB_2\sqrt{\rho} \parallel^2 \beta \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel \hat{k}_3}{\parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel^2 \alpha} + \\ &2\boldsymbol{x}^{\mathrm{T}} PB_2\sigma\boldsymbol{u}_s + 2\boldsymbol{x}^{\mathrm{T}} PB_2F\boldsymbol{w} + \\ &\sum_{i=1}^{m} 2\rho_i\tilde{\boldsymbol{K}}_{1,i}^{\mathrm{T}} \Gamma_i^{-1}\dot{\tilde{\boldsymbol{K}}}_{1,i} + 2\gamma^{-1}\tilde{k}_3\dot{\tilde{k}}_3\end{aligned} \tag{16}$$

Thus, in the light of inequality (10) and Assumption 3, we can rewrite (16) as

$$\begin{aligned}\frac{\mathrm{d}V(\boldsymbol{x}, \tilde{K}_1, \tilde{k}_3)}{\mathrm{d}t} \leq &\boldsymbol{x}^{\mathrm{T}}[(A + B_2\rho\hat{K}_1)^{\mathrm{T}} P + P(A + B_2\rho\hat{K}_1)]\boldsymbol{x} - \\ &2 \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel \hat{k}_3 + 2 \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel\parallel \sigma \parallel \bar{u}_s + \\ &2 \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel\parallel F \parallel \bar{w} + \\ &\sum_{i=1}^{m} 2\rho_i\tilde{\boldsymbol{K}}_{1,i}^{\mathrm{T}} \Gamma_i^{-1}\dot{\tilde{\boldsymbol{K}}}_{1,i} + 2\gamma^{-1}\tilde{k}_3\dot{\tilde{k}}_3\end{aligned} \tag{17}$$

By Assumption 2, (A, B_2) is stabilizable, there exist constants $K \in \mathbf{R}^{m\times n}$ and $P \in \mathbf{R}^{n\times n}$ such that

$$(A + B_2K)^{\mathrm{T}} P + P(A + B_2K) < 0 \tag{18}$$

The condition $\mathrm{rank}[B_2\rho(t)] = \mathrm{rank}[B_2]$ guarantees that the the linear combinations of columns in B_2 can be reconstructed by those in $B_2\rho(t)$, that is, there exist a K_1 such that

$$B_2\rho K_1 = B_2K \tag{19}$$

for each $\rho \in \Delta_{\rho^j}$. Therefore, for each $\rho \in \Delta_{\rho^j}$, there is a K_1 satisfying

$$(A + B_2\rho K_1)^{\mathrm{T}} P + P(A + B_2\rho K_1) < 0 \tag{20}$$

On the other hand, since $\bar{u}_s$ and $\bar{w}$ are unknown bounded constants, there always exists a constant k_3 such that

$$\parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel k_3 \geq \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel\parallel \sigma \parallel \bar{u}_s + \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel\parallel F \parallel \bar{w} \tag{21}$$

Define

$$Q = -[(A + B_2\rho K_1)^{\mathrm{T}} P + P(A + B_2\rho K_1)] \tag{22}$$

Then, according to the adaptive laws (8) and (11), it follows from (17) that

$$\begin{aligned}\frac{\mathrm{d}V(\boldsymbol{x}, \tilde{K}_1, \tilde{k}_3)}{\mathrm{d}t} \leq &-\boldsymbol{x}^{\mathrm{T}} Q\boldsymbol{x} - 2 \parallel \boldsymbol{x}^{\mathrm{T}} PB_2 \parallel \tilde{k}_3 + \\ &2\boldsymbol{x}^{\mathrm{T}} PB_2\rho\tilde{K}_1\boldsymbol{x} + \sum_{i=1}^{m} 2\rho_i\tilde{\boldsymbol{K}}_{1,i}^{\mathrm{T}} \Gamma_i^{-1}\dot{\tilde{\boldsymbol{K}}}_{1,i} + \\ &2\gamma^{-1}\tilde{k}_3\dot{\tilde{k}}_3 = -\boldsymbol{x}^{\mathrm{T}} Q\boldsymbol{x}\end{aligned} \tag{23}$$

Hence, it is easy to see that $\frac{\mathrm{d}V(\boldsymbol{x}, \tilde{K}_1, \tilde{k}_3)}{\mathrm{d}t} < 0$ for any $\boldsymbol{x} \neq 0$. Thus, the global adaptive fault-tolerant compensation control problem with disturbance rejection is solvable. The solutions of closed-loop FTC system are uniformly bounded, and the state $\boldsymbol{x}(t)$ converges asymptotically to zero. □

Remark 2. Compared with indirect adaptive methods for FTC problem introduced in [9, 17−18], the proposed method can solve more general actuator fault such as time-varying fault effect factor $\rho(t)$, which cannot be solved by the indirect adaptive methods.

Remark 3. Using the fact of spectral norm inequality, the proposed method can also solve the actuator fault such as unparametrizable time-varying bounded stuck faults without the knowledge of upper bound of failures. Obviously, it is a more effective method than existing direct adaptive methods for actuator failure compensation problem introduced in [10−11], where the schemes must be improved for the unparametrizable failures. Furthermore, according to the description of [12], approximations of unparametrizable faults will be employed to achieve approximate compensation of actuator failures. However, the approximation error will appear in the closed-loop system, and the closed-loop asymptotic stability cannot be ensured. Therefore, the proposed method is more suitable to deal with the unparametrizable failures than the approximate compensation method.

3 Numerical example

We consider a rocket fairing structural-acoustic model with external disturbance input added[10]:

$$A = \begin{bmatrix} 0 & 1 & 0.0802 & 1.0415 \\ -0.1980 & -0.115 & -0.0318 & 0.3 \\ -3.0500 & 1.1880 & -0.4650 & 0.9 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1 & 1.55 & 0.75 \\ 0.975 & 0.8 & 0.85 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 1.5 & 1 \\ -2 & -1 \\ -1 & 0.5 \end{bmatrix}$$

Consider the following four possible faulty modes:

Normal mode 1. All actuators are normal, that is, $\rho_1^1 = \rho_2^1 = \rho_3^1 = 1$.

Fault mode 2. The first actuator is outage or stuck, the second and the third actuators may be normal or loss of effectiveness, described by $\rho_1^2 = 0$, $a_2 \le \rho_2^2 \le 1$, $a_3 \le \rho_3^2 \le 1$, $a_2 = 0.3$, and $a_3 = 0.5$. This mode denotes the maximum loss of effectiveness for the second and the third actuators.

Fault mode 3. The second actuator is outage or stuck, the first and third actuators may be normal or loss of effectiveness, that is, $\rho_2^3 = 0$, $b_1 \le \rho_1^3 \le 1$, $b_3 \le \rho_3^3 \le 1$, $b_1 = 0.5$, and $b_3 = 0.3$. This mode denotes the maximum loss of effectiveness for the first and the third actuators.

Fault mode 4. The third actuator is outage or stuck, the first and second actuators may be normal or loss of effectiveness, that is, $\rho_3^4 = 0$, $c_1 \le \rho_1^4 \le 1$, $c_2 \le \rho_2^4 \le 1$, $c_1 = 0.5$, and $c_2 = 0.2$. This mode denotes the maximum loss of effectiveness for the first and the second actuators.

Then, to verify the effectiveness of the proposed adaptive method, simulations are given with the following parameters and initial conditions:

$$\Gamma_i = \mathrm{diag}\{10, 10, 10, 10\}, \quad \gamma = 50, \quad \alpha = 1, \quad \beta = 10$$
$$\boldsymbol{x}(0) = [0, 1, 0.5, -1]^{\mathrm{T}}, \quad \hat{k}_3(0) = 0$$
$$\hat{\boldsymbol{K}}_{1,i}(0) = [0, 0, 0, 0]^{\mathrm{T}}, \quad i = 1, 2, 3$$

The following faulty case is considered in the simulations, that is, before 8 s, the system operates in normal case, and the disturbances $\boldsymbol{w}(t) = [-5\sin(0.1t), 5]^{\mathrm{T}}$ enter into the system at the beginning ($t \ge 0$). At 8 s, some faults in actuators occur: the first actuator has stuck at $u_{s1}(t) = 10 + 3\sin(t) + 2\cos(0.5t)$ and the third actuator loss of effectiveness described by $\rho_3 = 1 - 0.03t$ until loss effectiveness of 50 %.

Fig. 1 is the response curves of the system′s states with robust adaptive state feedback controller in above-mentioned faulty case. Figs. 2 and 3 are the estimated curves of controller parameters $\hat{K}_1$ and $\hat{k}_3$, respectively. It is easy to see the closed-loop FTC system is asymptotically stable in the presence of faults on actuators and external disturbances.

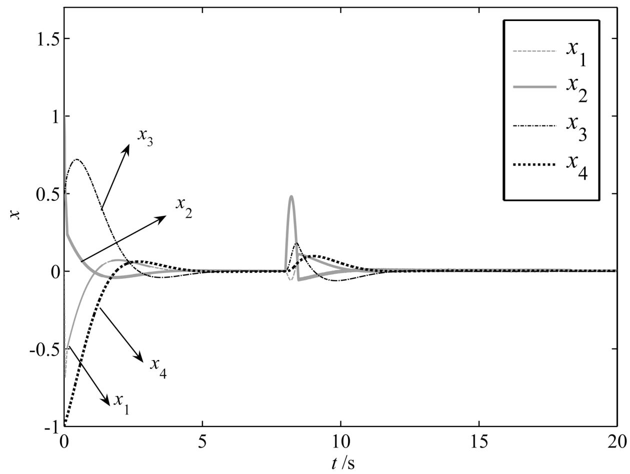

Fig. 1 Response curves of the system state vector $\boldsymbol{x}(t)$

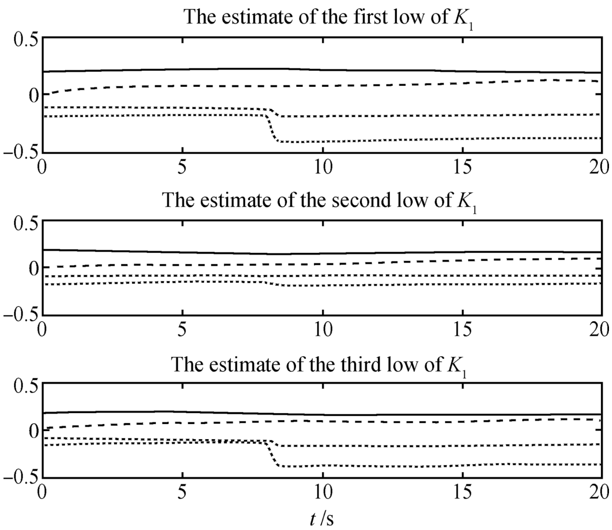

Fig. 2 Response curves of the estimates of controller parameters K_1 (K_{f1} (dash), K_{f2} (solid), K_{f3} (dash-dot), and K_{f4} (dot), $f = 1, 2, 3$)

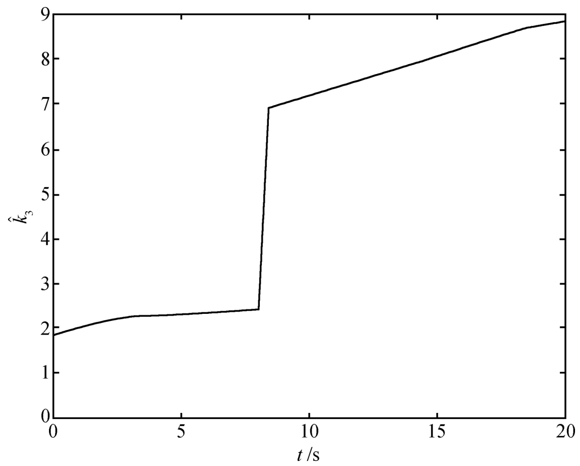

Fig. 3 Response curve of $\hat{k}_3(t)$

4 Conclusion

This paper presents a direct adaptive method for robust fault-tolerant control problem of actuator failure compensation and external disturbance rejection in continuous-time systems. A general actuator failure model is adopted, which covers the cases of normal operation, loss of effectiveness, outage, and stuck. The direct adaptive robust control schemes are based on updating adaptation laws to estimate the controller parameters online. The proposed schemes can construct robust adaptive state feedback controllers for automatically compensating the fault and the disturbance effects for guaranteeing the asymptotically stable of system. A numerical example has shown the effectiveness of the proposed method.

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