LMI Approach to Exponential Stabilization of Distributed Parameter Control Systems with Delay

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Abstract A renovating method for distributed parameter control systems with constants, varying-delays, and multi-varying-delays is put forward. By constructing average Lyapunov functions and employing linear matrix inequality (LMI) and other matrix inequality technologies, several sufficient conditions for exponential stabilization are derived. In this method, the conditions are delay-dependent and at the same time, the upper-bound of exponential convergence rate is obtained. In addition, the distinctive advantage of our method is that the criteria mentioned in the paper are easy to check, so it can be applied to practice easily. Finally, a computation example is given to illustrate the proposed method.

Key words Distributed parameter, stabilization, delay, linear matrix inequality (LMI)

The model of distributed parameter control systems is widely applied in heat processing, migration, and other areas, therefore it is significant to research the control of distributed parameter systems. As we know, variable structure control is the main method applied in the parameters control system at present[1−12]. However, it is especially difficult to avoid the wobble phenomenon[6], and the control that is designed as a tool for operator semigroup theory or a matrix norm theory is based on variable structure control theory[1−12]. Reference [6] has pointed that the controller based on the semigroups operator theory is difficult to use in practice because it is hard to verify compactness, incredulity, and exchangeability of the operator, which are requested. On the other hand, variable structure controller designed on matrix norm theorem is also difficult to apply. Therefore, it is a hot topic to find a practical and effective method for distributed parameter control system. In order to avoid the above-mentioned problems, we recently proposed a way with some useful results[13−14]. However, these results were mainly used by comparative principles. In this method, the stability conditions of the closed-loop system require that all the system parameters should be of absolute value format. This paper is to propose a new method to obtain the stabilization conditions of the distributed parameter control system. By choosing a Lyapunov function, applying distributed control, and using linear matrix inequality (LMI) and the related theory of matrix inequality with the choice of linear state feedback controller, the exponential stabilization of distributed parameter systems with constants, varying delay, and multi-varying-delays is obtained.

1 Description

Consider the following distributed parameter system with multi-varying-delays

$$\begin{aligned}\frac{\partial w_i(\boldsymbol{x},t)}{\partial t} =& D\sum_{k=1}^{m}\frac{\partial^2 w_i(\boldsymbol{x},t)}{\partial x_k^2}+\sum_{j=1}^{n}a_{ij}^0 w_j(\boldsymbol{x},t)+\\ &\sum_{j=1}^{n}a_{ij}w_j(\boldsymbol{x},t-\tau)+\sum_{j=1}^{n}b_{ij}u_j(\boldsymbol{x},t)\\ &i=1,2,\cdots,n\end{aligned} \tag{1}$$

The matrix form of system (1) is

$$\frac{\partial \boldsymbol{W}}{\partial t}=D\triangle\boldsymbol{W}(\boldsymbol{x},t)+A_0\boldsymbol{W}(\boldsymbol{x},t)+A\boldsymbol{W}(\boldsymbol{x},t-\tau)+B\boldsymbol{u}(\boldsymbol{x},t) \tag{2}$$

where $(\boldsymbol{x},t)\in\Omega\times\mathbf{R}_+$, $D>0$, and $\tau>0$ are constants; $A_0=(a_{ij}^0)$, $A=(a_{ij})$, and $B=(b_{ij})$ are constant matrices with corresponding ranks; $\Omega=\{\boldsymbol{x},\parallel\boldsymbol{x}\parallel< l<+\infty\}\subset\mathbf{R}^m$ is the bounded domain with smooth boundary $\partial\Omega$, and $mes\Omega>0$ (mes is short for $measure$). State function $\boldsymbol{W}(\boldsymbol{x},t)=\mathrm{col}(w_1(\boldsymbol{x},t),w_2(\boldsymbol{x},t),\cdots,w_n(\boldsymbol{x},t))\in\mathbf{R}^n$, $\Delta=\sum_{k=1}^m\dfrac{\partial^2}{\partial x_k^2}$ is the Laplace diffusion operator on Ω. And the initial value and boundary value conditions satisfy

$$\boldsymbol{W}(\boldsymbol{x},t)=0,\quad(\boldsymbol{x},t)\in\partial\Omega\times[-\tau,+\infty) \tag{3}$$

$$\frac{\partial\boldsymbol{W}(\boldsymbol{x},t)}{\partial\boldsymbol{n}}=0,\quad(\boldsymbol{x},t)\in\partial\Omega\times[-\tau,+\infty) \tag{4}$$

$$\boldsymbol{W}(\boldsymbol{x},t)=\boldsymbol{\varphi}(\boldsymbol{x},t),\quad(\boldsymbol{x},t)\in\partial\Omega\times[-\tau,0) \tag{5}$$

where $\boldsymbol{n}$ is the unit outward normal vector of $\partial\Omega$ and $\boldsymbol{\varphi}(\boldsymbol{x},t)$ is the suitable smooth function.

2 Main results

In order to get our main results, we first give some lemmas.

Lemma 1[15]. The inequality

$$\begin{pmatrix} Q(\boldsymbol{x}) & S(\boldsymbol{x})\\ S^{\mathrm{T}}(\boldsymbol{x}) & R(\boldsymbol{x})\end{pmatrix}>0 \tag{6}$$

is equal to

$$R(\boldsymbol{x})>0,\quad Q(\boldsymbol{x})-S(\boldsymbol{x})R^{-1}(\boldsymbol{x})S^{\mathrm{T}}(\boldsymbol{x})>0 \tag{7}$$

where $Q(\boldsymbol{x})=Q^{\mathrm{T}}(\boldsymbol{x})$, $R(\boldsymbol{x})=R^{\mathrm{T}}(\boldsymbol{x})$, and $S(\boldsymbol{x})$ is affine on $\boldsymbol{x}$.

Lemma 2[16−17]. Let U_1, U_2, U_3 be real matrices, and $U_3=U_3^{\mathrm{T}}>0$, then for an arbitrary scalar $\beta>0$, the following inequality

$$U_2^{\mathrm{T}}U_1+U_1^{\mathrm{T}}U_2\le\beta^{-1}U_1^{\mathrm{T}}U_3^{-1}U_1+\beta U_2^{\mathrm{T}}U_3U_2 \tag{8}$$

holds. In this paper, we let

$$\boldsymbol{u}(\boldsymbol{x},t)=K\boldsymbol{W}(\boldsymbol{x},t) \tag{9}$$

Theorem 1. For arbitrarily given A_0, A, B, and β, if there exist a matrix K and a positive matrix P, such that the following LMI

Received October 19, 2007; in revised form June 24, 2008
Supported by the Natural Science Foundation of Hunan Province (07JJ6112), the Construct Program of the Key Discipline in Hunan Province (control theory and control engineering), and Scientific Research Fund of Hunan Provincial Education Department (04A012, 07A015)
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DOI: 10.3724/SP.J.1004.2009.00299

$$\begin{pmatrix} A_0 + A_0^{\mathrm{T}} + BK + K^{\mathrm{T}}B^{\mathrm{T}} + 2kI + \beta P & \mathrm{e}^{k\tau}A \\ \mathrm{e}^{k\tau}A^{\mathrm{T}} & -\beta P \end{pmatrix} < 0 \tag{10}$$

holds, then system (2) is exponentially stabilized, and

$$\| \boldsymbol{W}(\boldsymbol{x}, t) \| \leq \sqrt{1 + \beta\lambda_M(P)\frac{1 - \mathrm{e}^{-2k\tau}}{2k}} \| \Phi \| \mathrm{e}^{-kt} \tag{11}$$

where λ_M is the largest matrix eigenvalue of P and $\| \Phi \| = \sup_{-\tau \leq s \leq 0}(\int_\Omega |\boldsymbol{W}(\boldsymbol{x}, s)|^2 \mathrm{d}\boldsymbol{x})^{\frac{1}{2}}$.

Proof. Construct a Lyapunov function:

$$V(t, \boldsymbol{W}(\boldsymbol{x}, t)) = \int_\Omega \mathrm{e}^{2kt}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}x + \beta\int_\Omega\int_{t-\tau}^{t} \mathrm{e}^{2k\theta}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, \theta)P\boldsymbol{W}(\boldsymbol{x}, \theta)\mathrm{d}\theta\mathrm{d}\boldsymbol{x} \tag{12}$$

where P is a positive matrix, $P^{\mathrm{T}} = P$, and $\beta > 0$. By derivation of $V(t, \boldsymbol{W}(\boldsymbol{x}, t))$ along with the solution of (2), we obtain

$$\begin{aligned}
\dot{V} = {} & \int_\Omega \mathrm{e}^{2kt}(\dot{\boldsymbol{W}}^{\mathrm{T}}(\boldsymbol{x}, t)\boldsymbol{W}(\boldsymbol{x}, t) + \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)\dot{\boldsymbol{W}}(\boldsymbol{x}, t))\mathrm{d}\boldsymbol{x} + \\
& \int_\Omega 2k\mathrm{e}^{2kt}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \beta\int_\Omega \mathrm{e}^{2kt}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)P\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} - \\
& \beta\int_\Omega \mathrm{e}^{2k(t-\tau)}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)P\boldsymbol{W}(\boldsymbol{x}, t-\tau)\mathrm{d}\boldsymbol{x} = \\
& 2\int_\Omega \mathrm{e}^{2kt}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)D\Delta\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \int_\Omega \mathrm{e}^{2kt}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)(A_0 + 2kI + A_0^{\mathrm{T}})\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \int_\Omega \mathrm{e}^{2kt}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)A\boldsymbol{W}(\boldsymbol{x}, t-\tau)\mathrm{d}\boldsymbol{x} + \\
& \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)A^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \mathrm{e}^{2kt}\int_\Omega (\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)B\boldsymbol{u}(\boldsymbol{x}, t) + \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{x}, t)B^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}, t))\mathrm{d}\boldsymbol{x} + \\
& \beta\mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)P\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} - \\
& \beta\mathrm{e}^{2k(t-\tau)}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)P\boldsymbol{W}(\boldsymbol{x}, t-\tau)\mathrm{d}\boldsymbol{x} = \\
& 2D\mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)\Delta\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \mathrm{e}^{2kt} \times \\
& \int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)(A_0 + A_0^{\mathrm{T}} + 2kI + \beta P)\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)A\boldsymbol{W}(\boldsymbol{x}, t-\tau)\mathrm{d}\boldsymbol{x} + \\
& \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)A^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)(BK + K^{\mathrm{T}}B^{\mathrm{T}})\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} - \\
& \beta\mathrm{e}^{2k(t-\tau)}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)P\boldsymbol{W}(\boldsymbol{x}, t-\tau)\mathrm{d}\boldsymbol{x}
\end{aligned} \tag{13}$$

According to Lemma 2, it follows that

$$\begin{aligned}
& \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)A\boldsymbol{W}(\boldsymbol{x}, t-\tau) + \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)A^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}, t) \leq \\
& \quad \frac{1}{\beta}\mathrm{e}^{2k\tau}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)AP^{-1}A^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}, t) + \\
& \quad \beta\mathrm{e}^{-2k\tau}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)P\boldsymbol{W}(\boldsymbol{x}, t-\tau)
\end{aligned} \tag{14}$$

Then,

$$\begin{aligned}
& \int_\Omega (\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)A\boldsymbol{W}(\boldsymbol{x}, t-\tau) + \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)A^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}, t))\mathrm{d}\boldsymbol{x} \leq \\
& \quad \int_\Omega \frac{1}{\beta}\mathrm{e}^{2k\tau}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)AP^{-1}A^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \quad \beta\mathrm{e}^{-2k\tau}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t-\tau)P\boldsymbol{W}(\boldsymbol{x}, t-\tau)\mathrm{d}\boldsymbol{x}
\end{aligned} \tag{15}$$

And,

$$\begin{aligned}
& \int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)\Delta\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} = \sum_{i=1}^{n}\int_\Omega w_i(\boldsymbol{x}, t)\Delta w_i(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} = \\
& \quad \sum_{i=1}^{n}\sum_{k=1}^{m}\int_\Omega w_i(\boldsymbol{x}, t)\frac{\partial^2 w_i}{\partial x_k^2}\mathrm{d}\boldsymbol{x} = \\
& \quad \sum_{i=1}^{n}\int_\Omega w_i(\boldsymbol{x}, t)\nabla \cdot \left(\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}\boldsymbol{x} = \\
& \quad \sum_{i=1}^{n}\int_\Omega \nabla \cdot \left(w_i(\boldsymbol{x}, t)\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}\boldsymbol{x} - \\
& \quad \int_\Omega \left(\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\nabla \cdot w_i(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} = \\
& \quad \sum_{i=1}^{n}\int_{\partial\Omega}\left(w_i(\boldsymbol{x}, t)\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}s - \\
& \quad \sum_{i=1}^{n}\sum_{k=1}^{m}\int_\Omega \left(\frac{\partial w_i}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x} = -\sum_{i=1}^{n}\sum_{k=1}^{m}\int_\Omega \left(\frac{\partial w_i}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x}
\end{aligned} \tag{16}$$

where $\nabla = \left(\frac{\partial}{\partial x_1}, \cdots, \frac{\partial}{\partial x_m}\right)$ is the gradient operator, and

$$\left(\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m} = \left(\frac{\partial w_i}{\partial x_1}, \cdots, \frac{\partial w_i}{\partial x_m}\right)$$

Combining (14) ~ (16), we obtain

$$\begin{aligned}
\dot{V} \leq {} & -2D\mathrm{e}^{2kt}\sum_{i=1}^{n}\sum_{k=1}^{m}\int_\Omega \left(\frac{\partial w_i}{\partial x_k}\right)^2\mathrm{d}\boldsymbol{x} + \\
& \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)(A_0 + A_0^{\mathrm{T}} + 2kI + \beta P)\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)(BK + K^{\mathrm{T}}B^{\mathrm{T}})\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} + \\
& \frac{1}{\beta}\mathrm{e}^{2k(t+\tau)}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)AP^{-1}A^{\mathrm{T}}\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x} \leq \\
& \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x}, t)\Big(A_0 + A_0^{\mathrm{T}} + BK + K^{\mathrm{T}}B^{\mathrm{T}} + 2kI + \\
& \frac{1}{\beta}\mathrm{e}^{2k\tau}AP^{-1}A^{\mathrm{T}} + \beta P\Big)\boldsymbol{W}(\boldsymbol{x}, t)\mathrm{d}\boldsymbol{x}
\end{aligned} \tag{17}$$

Therefore, from Schur complement, $\dot{V}(t, \boldsymbol{W}) < 0$ while $R < 0$. Thus, we have

$$V(\boldsymbol{W}(\boldsymbol{x}, t)) \leq V(\boldsymbol{W}(\boldsymbol{x}, 0))$$

and

$$V(\boldsymbol{W}(\boldsymbol{x},0)) = \int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},0)\boldsymbol{W}(\boldsymbol{x},0)\mathrm{d}\boldsymbol{x} + \beta\int_\Omega\int_{-\tau}^{0} \mathrm{e}^{2k\theta}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},\theta)P\boldsymbol{W}(\boldsymbol{x},\theta)\mathrm{d}\theta\mathrm{d}\boldsymbol{x} \leq \|\Phi\|^2 + \beta\lambda_M(P)\|\Phi\|^2\int_{-\tau}^{0}\mathrm{e}^{2k\theta}\mathrm{d}\theta \leq \left\{1+\beta\lambda_M(P)\frac{1-\mathrm{e}^{-2k\tau}}{2k}\right\}\|\Phi\|^2 \tag{18}$$

where $|\boldsymbol{W}|$ denotes Euclid mode of vector $\boldsymbol{W}$. For $V(t,\boldsymbol{W}) \geq \mathrm{e}^{2kt}\|\boldsymbol{W}(\boldsymbol{x},t)\|^2$, we have

$$\|\boldsymbol{W}(\boldsymbol{x},t)\| \leq \sqrt{1+\beta\lambda_M(P)\frac{1-\mathrm{e}^{-2k\tau}}{2k}}\|\Phi\|\mathrm{e}^{-kt} \tag{19}$$

□

Corollary 1. Let $P = I$ in Theorem 1. If

$$R_1 = \begin{pmatrix} A_0 + A_0^{\mathrm{T}} + B + B^{\mathrm{T}} + 2kI + \beta I & \mathrm{e}^{k\tau}A \\ \mathrm{e}^{k\tau}A^{\mathrm{T}} & -\beta I\end{pmatrix} < 0$$

then system (2) is exponentially stabilized, and

$$\|\boldsymbol{W}(\boldsymbol{x},t)\| \leq \sqrt{1+\beta\frac{1-\mathrm{e}^{-2k\tau}}{2k}}\|\Phi\|\mathrm{e}^{-kt} \tag{20}$$

where P is positive, $P^{\mathrm{T}} = P$, and $\beta > 0$.

Theorem 2. For given A_0, A, B, and K, if there exist a matrix K and a positive matrix P such that the following LMI holds:

$$R_2 = \begin{pmatrix} A_0^{\mathrm{T}}P + PA_0 + 2kP + \beta P + P & K^{\mathrm{T}}B^{\mathrm{T}} & \mathrm{e}^{k\tau}PA \\ BK & -P^{-1} & 0 \\ \mathrm{e}^{k\tau}A^{\mathrm{T}}P & 0 & -\beta P\end{pmatrix} < 0 \tag{21}$$

then, system (2) is exponentially stabilized, and

$$\|\boldsymbol{W}(\boldsymbol{x},t)\| \leq \sqrt{\frac{\lambda_M + \dfrac{\lambda_M(1-\mathrm{e}^{-2k\tau})}{2k}}{\lambda_m}}\|\Phi\|\mathrm{e}^{-kt} \tag{22}$$

where λ_M and λ_m are the largest and the smallest eigenvalues of P, respectively.

Proof. Choose a Lyapunov function as

$$V(t,\boldsymbol{W}(\boldsymbol{x},t)) = \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P\boldsymbol{W}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} + \beta\int_\Omega\int_{t-\tau}^{t}\mathrm{e}^{2k\theta}\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},\theta)P\boldsymbol{W}(\boldsymbol{x},\theta)\mathrm{d}\theta\mathrm{d}\boldsymbol{x} \tag{23}$$

where P is positive, $P^{\mathrm{T}} = P$, and $\beta > 0$. Clearly, $V(t,\boldsymbol{W})$ is positive, and

$$\begin{aligned}\dot{V} =\ & \mathrm{e}^{2kt}\int_\Omega(\dot{\boldsymbol{W}}^{\mathrm{T}}(\boldsymbol{x},t)P\boldsymbol{W}(\boldsymbol{x},t) + \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P\dot{\boldsymbol{W}}(\boldsymbol{x},t))\mathrm{d}\boldsymbol{x} + 2k\mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P\boldsymbol{W}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} + \\ & \beta\mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P\boldsymbol{W}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} - \beta\mathrm{e}^{2k(t-\tau)}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t-\tau)P\boldsymbol{W}(\boldsymbol{x},t-\tau)\mathrm{d}\boldsymbol{x} = \\ & \mathrm{e}^{2kt}\int_\Omega(D\Delta\boldsymbol{W}(\boldsymbol{x},t) + A_0\boldsymbol{W}(\boldsymbol{x},t) + A\boldsymbol{W}(\boldsymbol{x},t-\tau) + B\boldsymbol{u}(\boldsymbol{x},t))^{\mathrm{T}}P\boldsymbol{W}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} + \\ & 2k\mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}P\boldsymbol{W}\mathrm{d}\boldsymbol{x} + \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P(D\Delta\boldsymbol{W}(\boldsymbol{x},t) + A_0\boldsymbol{W}(\boldsymbol{x},t) + A\boldsymbol{W}(\boldsymbol{x},t-\tau) + B\boldsymbol{u}(\boldsymbol{x},t))\mathrm{d}\boldsymbol{x} + \\ & \beta\mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P\boldsymbol{W}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} - \beta\mathrm{e}^{2k(t-\tau)}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t-\tau)P\boldsymbol{W}(\boldsymbol{x},t-\tau)\mathrm{d}\boldsymbol{x} = \\ & 2\mathrm{e}^{2kt}D\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P(\Delta\boldsymbol{W}(\boldsymbol{x},t))\mathrm{d}\boldsymbol{x} + \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t-\tau)A^{\mathrm{T}}P\boldsymbol{W}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} + \\ & \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)PA\boldsymbol{W}(\boldsymbol{x},t-\tau)\mathrm{d}\boldsymbol{x} - \beta\mathrm{e}^{2k(t-\tau)}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t-\tau)P\boldsymbol{W}(\boldsymbol{x},t-\tau)\mathrm{d}\boldsymbol{x} + \\ & \mathrm{e}^{2kt}\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)(A_0^{\mathrm{T}}P + PA_0^{\mathrm{T}} + 2kP + \beta P)\boldsymbol{W}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} + \\ & \mathrm{e}^{2kt}\int_\Omega(\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)PB\boldsymbol{u}(\boldsymbol{x},t) + \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{x},t)B^{\mathrm{T}}P\boldsymbol{W}(\boldsymbol{x},t))\mathrm{d}\boldsymbol{x}\end{aligned} \tag{24}$$

and

$$\boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)PB\boldsymbol{u}(\boldsymbol{x},t) + \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{x},t)B^{\mathrm{T}}P\boldsymbol{W}(\boldsymbol{x},t) \leq -[P^{\frac{1}{2}}B\boldsymbol{u}(\boldsymbol{x},t) - P^{\frac{1}{2}}\boldsymbol{W}(\boldsymbol{x},t)]^{\mathrm{T}} \times [P^{\frac{1}{2}}B\boldsymbol{u}(\boldsymbol{x},t) - P^{\frac{1}{2}}\boldsymbol{W}(\boldsymbol{x},t)] + \boldsymbol{u}^{\mathrm{T}}(\boldsymbol{x},t)B^{\mathrm{T}}PB\boldsymbol{u}(\boldsymbol{x},t) + \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P\boldsymbol{W}(\boldsymbol{x},t) \tag{25}$$

$$\begin{aligned}&\int_\Omega \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{x},t)P\Delta\boldsymbol{W}(\boldsymbol{x},t)\mathrm{d}\boldsymbol{x} = \\ &\sum_{i=1}^{n}\sum_{j=1}^{n}\int_\Omega w_j p_{ij}\nabla\left(\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}\boldsymbol{x} = \\ &\sum_{i=1}^{n}\sum_{j=1}^{n}\int_\Omega w_j p_{ij}\cdot\nabla\left(\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}\boldsymbol{x} = \\ &\sum_{i=1}^{n}\sum_{j=1}^{n}\int_\Omega \nabla\left(w_j p_{ij}\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}\boldsymbol{x} - \sum_{i=1}^{n}\sum_{j=1}^{n}\int_\Omega\left(\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\cdot\nabla(w_j p_{ij})\mathrm{d}\boldsymbol{x} = \\ &\sum_{i=1}^{n}\sum_{j=1}^{n}\int_{\partial\Omega}\left(w_j p_{ij}\frac{\partial w_i}{\partial x_k}\right)_{k=1}^{m}\mathrm{d}s - \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{m}\int_\Omega\frac{\partial w_i}{\partial x_k}p_{ij}\frac{\partial w_j}{\partial x_k}\mathrm{d}\boldsymbol{x} = \\ &-\sum_{k=1}^{m}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\int_\Omega\frac{\partial w_i}{\partial x_k}p_{ij}\frac{\partial w_j}{\partial x_k}\mathrm{d}\boldsymbol{x}\right) =\end{aligned}$$

$$-\sum_{k=1}^{m}\int_\Omega\left(\frac{\partial \boldsymbol{W}}{\partial x_k}\right)^{\rm T}P\left(\frac{\partial \boldsymbol{W}}{\partial x_k}\right){\rm d}\boldsymbol{x} \tag{26}$$

where $\frac{\partial \boldsymbol{W}}{\partial x_k}={\rm col}\left(\frac{\partial w_1}{\partial x_k},\cdots,\frac{\partial w_n}{\partial x_k}\right)$. Because P is positive, we have $\left(\frac{\partial \boldsymbol{W}}{\partial x_k}\right)^{\rm T}P\left(\frac{\partial \boldsymbol{W}}{\partial x_k}\right)>0$. By Lemma 2, we have

$$\int_\Omega[\boldsymbol{W}^{\rm T}(\boldsymbol{x},t-\tau)A^{\rm T}P\boldsymbol{W}(\boldsymbol{x},t)+\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)PA\boldsymbol{W}(\boldsymbol{x},t-\tau)]{\rm d}\boldsymbol{x}\le \int_\Omega\Big[\frac{1}{\beta}{\rm e}^{2k\tau}\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)PAP^{-1}A^{\rm T}P\boldsymbol{W}(\boldsymbol{x},t)+ {\rm e}^{-2k\tau}\beta\boldsymbol{W}^{\rm T}(\boldsymbol{x},t-\tau)P\boldsymbol{W}(\boldsymbol{x},t-\tau)\Big]{\rm d}\boldsymbol{x} \tag{27}$$

Combining (24) ~ (27) and condition (21), we get

$$\begin{aligned}\dot V =& -2D{\rm e}^{2kt}\sum_{k=1}^{m}\int_\Omega\left(\frac{\partial \boldsymbol{W}}{\partial x_k}\right)^{\rm T}P\left(\frac{\partial \boldsymbol{W}}{\partial x_k}\right){\rm d}\boldsymbol{x}+\\ &{\rm e}^{2kt}\int_\Omega\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)\Big(A_0^{\rm T}P+PA_0^{\rm T}+2kP+\\ &\beta P+\frac{1}{\beta}{\rm e}^{2k\tau}PAP^{-1}A^{\rm T}P\Big)\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}+\\ &{\rm e}^{2kt}\int_\Omega\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)(K^{\rm T}B^{\rm T}PBK+P)\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}\le\\ &{\rm e}^{2kt}\int_\Omega\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)\Big(A_0^{\rm T}P+PA_0^{\rm T}+2kP+\beta P+\\ &\frac{1}{\beta}{\rm e}^{2k\tau}PAP^{-1}A^{\rm T}P+K^{\rm T}B^{\rm T}PBK+P\Big)\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}\end{aligned} \tag{28}$$

From Schur supplement, $\dot V<0$ is obvious. By a derivation similar to that of Theorem 1, we can also know system (1) is exponentially stabilized. □

Corollary 2. For a given A_0, A, and B, if there exits a matrix K such that the following LMI

$$R_3=\begin{pmatrix}A_0+A_0^{\rm T}+(2k+\beta+1)I & K^{\rm T}B^{\rm T} & {\rm e}^{k\tau}A\\ BK & -I & 0\\ {\rm e}^{k\tau}A^{\rm T} & 0 & -\beta I\end{pmatrix}<0 \tag{29}$$

holds, then system (2) is exponentially stabilized, and

$$\parallel \boldsymbol{W}(\boldsymbol{x},t)\parallel\le\sqrt{1+\frac{1-{\rm e}^{-2k\tau}}{2k}}\parallel\Phi\parallel{\rm e}^{-kt} \tag{30}$$

Moreover, we consider the stabilization of distributed control system with multi-time delays. Now, we study the following system

$$\frac{\partial \boldsymbol{W}}{\partial t}=D\triangle\boldsymbol{W}(\boldsymbol{x},t)+A_0\boldsymbol{W}(\boldsymbol{x},t)+\sum_{q=1}^{z}A_q\boldsymbol{W}(\boldsymbol{x},t-\tau_q)+B\boldsymbol{u}(\boldsymbol{x},t) \tag{31}$$

Theorem 3. If there exist a matrix K and a positive matrix P_q $(q=1,\cdots,z)$, such that the following LMI holds:

$$R_4=\begin{pmatrix}A_0+A_0^{\rm T}+BK+K^{\rm T}B^{\rm T}+\sum_{q=1}^{z}\beta_qP_q & {\rm e}^{k\tau_1}A_1 & \cdots & {\rm e}^{k\tau_z}A_z\\ {\rm e}^{k\tau_1}A_1^{\rm T} & -\beta_1P_1 & 0 & 0\\ \vdots & 0 & \ddots & 0\\ {\rm e}^{k\tau_z}A_z^{\rm T} & 0 & 0 & -\beta_zP_z\end{pmatrix}<0 \tag{32}$$

then, system (31) is exponentially stabilized and

$$\parallel \boldsymbol{W}(\boldsymbol{x},t)\parallel\le\sqrt{1+\sum_{q=1}^{z}\beta_q\lambda_M(P_q)\frac{1-{\rm e}^{-2k\tau_q}}{2k}}\parallel\Phi\parallel{\rm e}^{-kt} \tag{33}$$

where $\lambda_M(P_q)\,(q=1,\cdots,z)$ is the largest matrix eigenvalue of P_q.

Proof. Choose a positive function

$$V(t,\boldsymbol{W}(\boldsymbol{x},t))=\int_\Omega\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}+\sum_{q=1}^{z}\beta_q\int_\Omega\int_{t-\tau_q}^{t}\boldsymbol{W}^{\rm T}(\boldsymbol{x},\theta)P_q\boldsymbol{W}(\boldsymbol{x},\theta){\rm d}\theta{\rm d}\boldsymbol{x} \tag{34}$$

where $\beta_q>0\,(q=1,2,\cdots,n)$, matrix $P_q\,(q=1,\cdots,z)$ is positive, and $P_q^{\rm T}=P_q$. Similar to Lemma 2, we can prove it. □

Next, we consider the stabilization of distributed control system with a varying delay:

$$\frac{\partial \boldsymbol{W}}{\partial t}=D\triangle\boldsymbol{W}(\boldsymbol{x},t)+A_0\boldsymbol{W}(\boldsymbol{x},t)+A\boldsymbol{W}(\boldsymbol{x},t-\tau(t))+B\boldsymbol{u}(\boldsymbol{x},t) \tag{35}$$

where $\tau(t)$ is a non-negative, bounded differential function, and $0\le\tau(t)\le\tau$.

Theorem 4. If $\tau(t)$ satisfies $\dot\tau(t)\le\eta<1$, and there exist a matrix K and a positive matrix P, such that the following LMI holds:

$$R_5=\begin{pmatrix}A_0+A_0^{\rm T}+2kI+\beta P+BK+K^{\rm T}B^{\rm T} & {\rm e}^{k\tau}A\\ {\rm e}^{k\tau}A^{\rm T} & -\beta(1-\dot\tau(t))P\end{pmatrix}<0 \tag{36}$$

then system (35) is stabilized, and

$$\parallel \boldsymbol{W}(\boldsymbol{x},t)\parallel\le\sqrt{1+\beta\lambda_M(P)\frac{1-{\rm e}^{-2k\tau}}{2k}}\parallel\Phi\parallel{\rm e}^{-kt} \tag{37}$$

where λ_M is the largest matrix eigenvalue of P.

Proof. Choose a positive function as

$$V(t,\boldsymbol{W}(\boldsymbol{x},t))={\rm e}^{2kt}\int_\Omega\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}+\beta\int_\Omega\int_{t-\tau(t)}^{t}{\rm e}^{2k\theta}\boldsymbol{W}^{\rm T}(\boldsymbol{x},\theta)P\boldsymbol{W}(\boldsymbol{x},\theta){\rm d}\theta{\rm d}\boldsymbol{x} \tag{38}$$

where P is positive, $P^{\rm T}=P$, and $\beta>0$. We have

$$\begin{aligned}\dot V =& {\rm e}^{2kt}\int_\Omega(\dot{\boldsymbol{W}}^{\rm T}(\boldsymbol{x},t)\boldsymbol{W}(\boldsymbol{x},t)+\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)\dot{\boldsymbol{W}}(\boldsymbol{x},t)){\rm d}\boldsymbol{x}+\\ &\beta{\rm e}^{2kt}\int_\Omega\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)P\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}-\\ &\beta(1-\dot\tau(t)){\rm e}^{2k(t-\tau(t))}\times\end{aligned}$$

$$\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t-\tau(t))P\boldsymbol{W}(\boldsymbol{x},t-\tau(t)){\rm d}\boldsymbol{x} =$$
$$2{\rm e}^{2kt}\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t)D\Delta\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}+$$
$${\rm e}^{2kt}\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t)(A_0+2kI+A_0^{\rm T})\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}+$$
$${\rm e}^{2kt}\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t)A\boldsymbol{W}(\boldsymbol{x},t-\tau(t)){\rm d}\boldsymbol{x}+$$
$${\rm e}^{2kt}\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t-\tau(t))A^{\rm T}\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}+$$
$${\rm e}^{2kt}\int_\Omega (\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)B\boldsymbol{u}(\boldsymbol{x},t)+\boldsymbol{u}^{\rm T}(\boldsymbol{x},t)B^{\rm T}\boldsymbol{W}(\boldsymbol{x},t)){\rm d}\boldsymbol{x}+$$
$$\beta{\rm e}^{2kt}\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t)P\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}-$$
$$\beta(1-\dot\tau(t)){\rm e}^{2k(t-\tau(t))}\times$$
$$\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t-\tau)P\boldsymbol{W}(\boldsymbol{x},t-\tau(t)){\rm d}\boldsymbol{x} \tag{39}$$

also

$$\int_\Omega (\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)A\boldsymbol{W}(\boldsymbol{x},t-\tau)+\boldsymbol{W}^{\rm T}(\boldsymbol{x},t-\tau)A^{\rm T}\boldsymbol{W}(\boldsymbol{x},t)){\rm d}\boldsymbol{x}\le$$
$$\int_\Omega \frac{1}{\beta}(1-\dot\tau(t))^{-1}{\rm e}^{2k\tau(t)}\boldsymbol{W}^{\rm T}(\boldsymbol{x},t)AP^{-1}A^{\rm T}\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}+$$
$$\beta(1-\dot\tau(t)){\rm e}^{-2k\tau(t)}\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t-\tau(t))P\boldsymbol{W}(\boldsymbol{x},t-\tau(t)){\rm d}\boldsymbol{x} \tag{40}$$

By (39), (40), and condition (36), we have

$$\dot V\le\int_\Omega \boldsymbol{W}^{\rm T}(\boldsymbol{x},t)\Big(A_0+2kI+A_0^{\rm T}+BK+K^{\rm T}B^{\rm T}+\beta P+$$
$$\frac{1}{\beta}{\rm e}^{2k\tau(t)}(1-\dot\tau(t))^{-1}AP^{-1}A^{\rm T}\Big)\boldsymbol{W}(\boldsymbol{x},t){\rm d}\boldsymbol{x}<0 \tag{41}$$

The rest is similar to the proof of Theorem 3, so is omitted. □

Next, we consider stabilization of distributed parameter control system with multi-varying delays, and the following system is mainly studied.

$$\frac{\partial \boldsymbol{W}}{\partial t}=D\triangle\boldsymbol{W}(\boldsymbol{x},t)+A_0\boldsymbol{W}(\boldsymbol{x},t)+$$
$$\sum_{q=1}^{z}A_q\boldsymbol{W}(\boldsymbol{x},t-\tau_q(t))+B\boldsymbol{u}(\boldsymbol{x},t) \tag{42}$$

where $\tau_q(t)$ is a non-negative bounded differential function and $0\le\tau_q(t)\le\tau_q$, $q=1,\cdots,z$.

Theorem 5. If $\tau_q(t)$ satisfies $\dot\tau_q(t)\le\eta_q<1$ ($q=1,\cdots,z$) and there are a matrix K and some positive matrices $P_q\,(q=1,\cdots,z)$ such that the following LMI (43) holds, system (42) is exponentially stabilized, and in the inequality below

$$\parallel \boldsymbol{W}(\boldsymbol{x},t)\parallel\le\sqrt{1+\sum_{q=1}^{z}\beta_q\lambda_M(P_q)\frac{1-{\rm e}^{-2k\tau_q}}{2k}}\parallel\Phi\parallel{\rm e}^{-kt} \tag{44}$$

where λ_M, λ_m are the largest and the smallest eigenvalues of P, respectively.

The proof of Theorem 5 is similar to that of Theorem 4 and thus is omitted for brevity.

$$R_6=\begin{pmatrix} A_0+A_0^{\rm T}+BK+K^{\rm T}B^{\rm T}+2kI+\sum_{q=1}^{z}\beta_qP_q & {\rm e}^{k\tau_1}A_1 & \cdots & {\rm e}^{k\tau_z}A_z\\ {\rm e}^{k\tau_1}A_1^{\rm T} & -\beta_1(1-\dot\tau_1(t))P_1 & 0 & 0\\ \vdots & \vdots & \ddots & \vdots\\ {\rm e}^{k\tau_z}A_z^{\rm T} & 0 & 0 & -\beta_z(1-\dot\tau_z(t))P_z\end{pmatrix}<0 \tag{43}$$

3 Calculation of parameter $\boldsymbol{\beta}$ and exponential convergence rate $\boldsymbol{k}$

In the above theorem, we set a scalar β and k at first, then obtain a feedback matrix K by Matlab. What is the range of β that can keep our given system exponentially stabilized for the obtained K? We just offer the way to ensure the range of β for Theorem 1. The calculations of the range of β in other theorems are similar to it, so we omit them.

Based on Theorem 1, k is given in constraint condition (10) when we solve the optimization problem to ensure the system is stable. The supremum of β can be obtained by solving the following optimization problem.

$$\max\beta$$
$$\text{s.t. } P>0 \text{ and } (10)$$

This is a quasi-convex optimization problem, and it can be solved by the tool of LMI in Matlab. Suppose the optimal solution is $\overline{\beta}$. Then, system (2) is exponentially stabilized when $\beta\le\overline{\beta}$ in (10).

Moreover, when $\overline{\beta}$ is ensured, we can obtain supremum of the exponential convergence rate k by solving the following optimization problem

$$\max k$$
$$\text{s.t. } P>0 \text{ and } (10)$$

Note that in the first optimization problem, k is given in constraint condition (10), and at the same time, β is given in constraint condition (10) when we solve it.

4 Example

In order to state the problem, we give a simple example. In system (1), we set $m=n=2$, where $A_0=\begin{pmatrix}-2 & 0.7\\ 0.5 & -0.5\end{pmatrix}$, $A=\begin{pmatrix}-1 & -0.1\\ 3 & -0.3\end{pmatrix}$, and $B=\begin{pmatrix}0.5 & 0\\ 0 & 3\end{pmatrix}$. By Corollary 2, we can get a feedback matrix $K=\begin{pmatrix}-0.1 & 0\\ 0 & -1.7\end{pmatrix}$ and know system (1) is exponentially stabilized for L_2-norm. The trajectories of the example based on L_2-norm is given in Fig. 1.

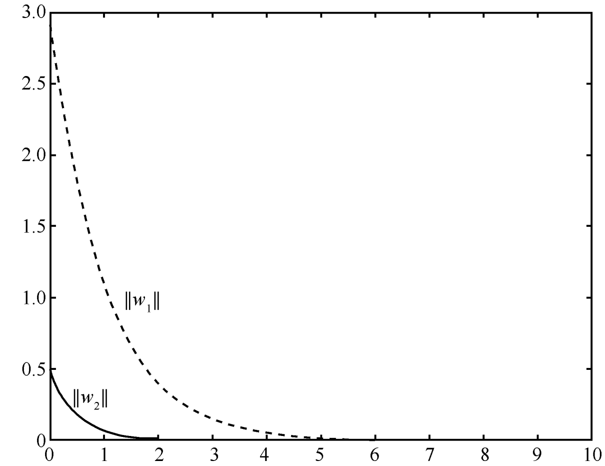

Fig. 1 The trajectories of the example based on L_2-norm

5 Conclusion

Based on Lyapunov stability theory, the exponential stabilization for some classes of distributed parameter control systems with time delays is investigated by using inequality analysis as the main mathematic tool. Sufficient conditions are found for exponential stabilization for the distributed parameter control systems with delays and some theorems of the exponential stabilization for the distributed parameter control systems are established. The biggest advantage of this method is that by choosing Lyapunov function, we come to the conclusion that the stabilization conditions of the closed-loop system consist in an LMI. In addition, the difficulty in analysis dealing with those distributed parameter systems can be reduced by taking off the partial part through technical processing in the deviation of Lyapunov function. Applying this new method, we get the above conclusion, by which the obtained linear matrix inequality can be tested easily. This provides a solid theoretical basis for the designers of distributed parameter control systems.

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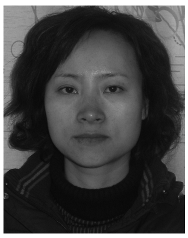

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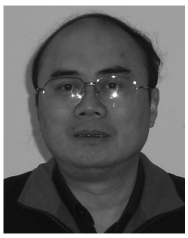

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