

不确定奇异时滞系统的鲁棒 H_∞ 故障诊断滤波器设计

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摘要 研究一类受参数不确定性和干扰影响的奇异时滞系统鲁棒故障诊断滤波器设计问题. 把基于观测器的故障诊断滤波器作为残差产生器, 将故障诊断滤波器设计归结为 H_∞ 滤波问题, 使产生的残差信号即为故障的 H_∞ 估计, 给出了鲁棒 H_∞ 故障诊断滤波器存在的充分条件, 并利用锥面互补线性化迭代算法得到了故障诊断滤波器设计的线性矩阵不等式求解方法. 算例验证了算法的有效性.

关键词 奇异时滞系统, 故障诊断, 滤波器, 线性矩阵不等式
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Designing Robust H_∞ Fault Detection Filter for Singular Time-delay Systems with Uncertainty

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Abstract The robust fault detection filter design problem for a class of singular time-delay systems with parameter uncertainty and disturbance is studied. Using an observer-based fault detection filter as the residual generator, the fault detection filter design is converted to an H_∞ filtering problem such that the generated residual is the H_∞ estimation of the fault. Sufficient conditions are given to guarantee the H_∞ fault detection filter exists. By using the cone complementarity linearization iterative algorithm, the linear matrix inequality method to design the fault detection filter is given. A numerical example is given to illustrate the effectiveness of the proposed algorithm.

Key words Singular time-delay systems, fault detection, filter, linear matrix inequality (LMI)

基于观测器的故障检测与分离 (Fault detection and isolation, FDI) 技术经过三十多年的发展已比较成熟^[1-3]. 综观取得的研究成果, 基本可以分为两类: 1) 将 FDI 问题转化为一个最小化问题, 使残差对干扰的鲁棒性指标与残差对故障的灵敏度指标的比率最小化; 2) 将 FDI 问题转化为 H_∞ 滤波, 即“最小化”残差与故障 (或加权故障) 之间的误差.

奇异系统又称广义系统, 是比正常状态空间系统更一般的系统. 自上世纪 70 年代以来, 奇异系统理论取得了明显进展, 但对于奇异系统 FDI 问题的研究成果尚少^[4-11]. 文献 [4-10] 研究了奇异系统的故障检测问题, 但未考虑时滞. 文献 [11] 在可实现残差与干扰全解耦的情况下, 研究了模型不确定

性奇异时滞系统干扰可全解耦的鲁棒故障诊断问题. 但是, 对于一般的模型不确定性系统, 残差与干扰的全解耦难以实现. 而对奇异系统基于 H_∞ 滤波 FDI 问题的研究尚未开展.

本文研究一类模型不确定性奇异时滞系统的鲁棒故障诊断问题, 将故障诊断滤波器 (Fault detection filter, FDF) 设计归结为 H_∞ 滤波, 给出了奇异时滞系统鲁棒 H_∞ -FDF 存在的充分条件, 并应用锥面互补 (Cone complementarity) 线性化迭代算法及线性矩阵不等式 (Linear matrix inequality, LMI) 技术, 求解 FDF 问题的解. 本文给出的 H_∞ 滤波器的求解条件依赖于时滞, 此结果在关于奇异时滞系统滤波器设计的文献中是首次给出.

1 问题描述

考虑不确定奇异时滞系统

$$\begin{cases} E\dot{\mathbf{x}}(t) = (A + \Delta A)\mathbf{x}(t) + (A_\tau + \Delta A_\tau)\mathbf{x}(t - \tau) + \\ \quad (B + \Delta B)\mathbf{u}(t) + B_f\mathbf{f}(t) + B_d\mathbf{d}(t) \\ \mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t) + D_f\mathbf{f}(t) + D_d\mathbf{d}(t) \\ \mathbf{x}(\theta) = \phi(\theta), \quad \theta \in [-\tau, 0] \end{cases} \quad (1)$$

其中 $\mathbf{x} \in \mathbf{R}^n$, $\mathbf{u} \in \mathbf{R}^r$, $\mathbf{y} \in \mathbf{R}^m$, $\mathbf{f} \in \mathbf{R}^l$ 和 $\mathbf{d} \in \mathbf{R}^g$ 分别为状态、控制输入、测量输出、故障和未知输入向量. 假设 \mathbf{u} , \mathbf{f} , \mathbf{d} 均为 L_2 范数有界信号. $\text{rank } E =$

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$p, 0 < p < n$. τ 是未知常时滞, $0 < \tau \leq \tau_m, \tau_m$ 是已知常数. $\phi(\theta)$ 为 $[-\tau, 0]$ 上的实值连续初始函数向量. $E, A, A_\tau, B, C, D, B_f, B_d, D_f$ 和 D_d 为具有适当维数的已知实常数矩阵. $\Delta A, \Delta A_\tau$ 和 ΔB 为参数不确定性矩阵, 且

$$\begin{bmatrix} \Delta A & \Delta A_\tau & \Delta B \end{bmatrix} = MF(\sigma) \begin{bmatrix} N_A & N_\tau & N_B \end{bmatrix} \quad (2)$$

$$F(\sigma)F^T(\sigma) \leq I$$

其中 M, N_A, N_τ 和 N_B 是具有适当维数的已知实常数矩阵. 不失一般性, 本文假设 $E = \text{diag}\{I, 0\}$.

本文的主要目的: 给定标量 $\gamma > 0$, 设计残差产生器, 使产生的残差 r 满足

$$\|r - W_f(s)f\|_2 \leq \gamma \|w\|_2 \quad (3)$$

即残差是 $W_f(s)$ 描述的频率范围内故障的 H_∞ 估计, 其中 $w = \begin{bmatrix} u^T & f^T & d^T \end{bmatrix}^T$, $W_f(s)$ 是给定的稳定加权矩阵.

注 1. $W_f(s) = I$ 时, 残差即为故障的 H_∞ 估计, 表示可能发生的故障为全频率范围. 为了提高故障估计的性能指标, 通常选取适当的加权函数 $W_f(s)$, 则式 (3) 表示求得的残差为 $W_f(s)$ 描述频段范围的 H_∞ 估计故障.

不失一般性, 设 $W_f(s)$ 的一个最小实现为

$$\begin{cases} \dot{x}_f(t) = A_W x_f(t) + B_W f(t), & x_f(0) = 0 \\ r_f(t) = C_W x_f(t) \end{cases} \quad (4)$$

其中 $x_f \in \mathbf{R}^{n_f}, r_f \in \mathbf{R}^l, A_W, B_W, C_W$ 是已知常数矩阵. 由式 (1) 和 (4) 可得

$$\begin{cases} E_s \dot{x}_s(t) = (A_s + \Delta A_s)x_s(t) + (A_{\tau s} + \Delta A_{\tau s}) \times \\ \quad x_s(t - \tau) + (B_s + \Delta B_s)w(t) \\ y(t) = C_s x_s(t) + D_s w(t) \\ r_f(t) = C_{sf} x_s(t) \\ x_s(\theta) = \phi_s(\theta), \theta \in [-\tau, 0] \end{cases} \quad (5)$$

其中

$$E_s = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, A_s = \begin{bmatrix} A & 0 \\ 0 & A_W \end{bmatrix}$$

$$A_{\tau s} = \begin{bmatrix} A_\tau & 0 \\ 0 & 0 \end{bmatrix}, B_s = \begin{bmatrix} B & B_f & B_d \\ 0 & B_W & 0 \end{bmatrix}$$

$$x_s = \begin{bmatrix} x \\ x_f \end{bmatrix}, \phi_s(\theta) = \begin{bmatrix} \phi(\theta) \\ 0 \end{bmatrix}, \theta \in [-\tau, 0]$$

$$C_s = \begin{bmatrix} C & 0 \end{bmatrix}, D_s = \begin{bmatrix} D & D_f & D_d \end{bmatrix}$$

$$C_{sf} = \begin{bmatrix} 0 & C_W \end{bmatrix}, \Delta A_s = \bar{M}F(\sigma)\bar{N}_A$$

$$\Delta A_{\tau s} = \bar{M}F(\sigma)\bar{N}_\tau, \Delta B_s = \bar{M}F(\sigma)\bar{N}_B$$

$$\bar{M} = \begin{bmatrix} M^T & 0 \end{bmatrix}^T, \bar{N}_A = \begin{bmatrix} N_A & 0 \end{bmatrix}$$

$$\bar{N}_\tau = \begin{bmatrix} N_\tau & 0 \end{bmatrix}, \bar{N}_B = \begin{bmatrix} N_B & 0 \end{bmatrix}$$

选取如下形式的残差产生器

$$\begin{cases} E_s \dot{\hat{x}}_s(t) = H_1 \hat{x}_s(t) + H_2 y(t) + H_3 u(t) \\ \hat{x}_s(0) = 0 \\ r(t) = H_4 \hat{x}_s(t) \end{cases} \quad (6)$$

其中, $\hat{x}_s \in \mathbf{R}^{\hat{n}} (\hat{n} = n + n_f), r \in \mathbf{R}^l$ 分别为滤波器的状态和残差, H_1, H_2, H_3 和 H_4 是要确定的矩阵.

令 $e(t) = \begin{bmatrix} x_s^T(t) & \hat{x}_s^T(t) \end{bmatrix}^T, r_e(t) = r(t) - r_f(t)$, 则有

$$\begin{cases} E_c \dot{e}(t) = (A_c + \Delta A_c)e(t) + (A_{\tau c} + \Delta A_{\tau c}) \times \\ \quad e(t - \tau) + (B_c + \Delta B_c)w(t) \\ r_e(t) = C_c e(t) \\ e(\theta) = \phi_e(\theta), \theta \in [-\tau, 0] \end{cases} \quad (7)$$

其中

$$E_c = \begin{bmatrix} E_s & 0 \\ 0 & E_s \end{bmatrix}, A_c = \begin{bmatrix} A_s & 0 \\ H_2 C_s & H_1 \end{bmatrix}$$

$$A_{\tau c} = \begin{bmatrix} A_{\tau s} & 0 \\ 0 & 0 \end{bmatrix}, C_c = \begin{bmatrix} -C_{sf} & H_4 \end{bmatrix}$$

$$B_c = \begin{bmatrix} B_s \\ H_2 D_s + H_3 \begin{bmatrix} I & 0 & 0 \end{bmatrix} \end{bmatrix}$$

$$\Delta A_c = \begin{bmatrix} \bar{M} \\ 0 \end{bmatrix} F(\sigma) \begin{bmatrix} \bar{N}_A & 0 \end{bmatrix}$$

$$\Delta A_{\tau c} = \begin{bmatrix} \bar{M} \\ 0 \end{bmatrix} F(\sigma) \begin{bmatrix} \bar{N}_\tau & 0 \end{bmatrix}$$

$$\Delta B_c = \begin{bmatrix} \bar{M} \\ 0 \end{bmatrix} F(\sigma) \bar{N}_B$$

$$\phi_e(\theta) = \begin{bmatrix} \phi_s(\theta) \\ 0 \end{bmatrix}, \theta \in [-\tau, 0]$$

此时, 目标 (3) 即为

$$\|r_e\|_2 \leq \gamma \|w\|_2 \quad (8)$$

从而可将本文的主要问题归结为: 设计系数矩阵 H_1, H_2, H_3 和 H_4 , 使系统 (6) 为系统 (1) 的鲁棒 H_∞ -FDF, 即系统 (7) 对所有满足式 (2) 的 $\Delta A,$

$$\begin{aligned} \Gamma_{29} &= P_2^T B_s + W_2^T D_s + W_3 \begin{bmatrix} I & 0 & 0 \end{bmatrix} \\ \Gamma_{33} &= -Q_1 - Q_2 - Q_2^T - Q_3, \Gamma_{34} = -Q_1 - Q_2^T \\ \Gamma_{55} &= -\tau_m Z_1 - \tau_m Z_2 - \tau_m Z_2^T - \tau_m Z_3 \\ \Gamma_{56} &= -\tau_m Z_1 - \tau_m Z_2^T, \Gamma_{78} = -Q_1 - Q_2^T \\ \Gamma_{77} &= -Q_1 - Q_2 - Q_2^T - Q_3 \end{aligned}$$

则系统 (6) 是系统 (1) 的鲁棒 H_∞ -FDF. 此时, 系统 (6) 的参数矩阵可选择为

$$\begin{aligned} H_1 &= (P_1^T - P_2^T)^{-1} W_1^T, \quad H_2 = (P_1^T - P_2^T)^{-1} W_2^T \\ H_3 &= (P_1^T - P_2^T)^{-1} W_3, \quad H_4 = W_4^T \end{aligned} \tag{14}$$

证明. 见附录 A. \square

注意到式 (10) 和 (11) 含有非线性项, 可应用文献 [12] 提出的锥面互补线性化迭代算法将其转化为一个凸优化问题. 由式 (11) 知

$$\begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix} \begin{bmatrix} \hat{M}U_1\hat{M}^T & \hat{M}U_2\hat{M}^T \\ * & \hat{M}U_3\hat{M}^T \end{bmatrix} = I \tag{15}$$

又 $\hat{M}^T E_s \hat{M} = \text{diag}(I, I, 0, 0)$, $\hat{M}^{-1} = \hat{M}^T$, 式 (10) 左乘 $\text{diag}(\hat{M}^T, \hat{M}^T, \hat{M}^T, \hat{M}^T)$, 右乘其转置, 并令

$$\hat{M}^T P_2^T \hat{M} = \begin{bmatrix} P_{11} - P_{33} & P_{12} - P_{34} \\ P_{21} - P_{43} & P_{22} - P_{44} \end{bmatrix}$$

$$\hat{M}^T (P_1^T - P_2^T) \hat{M} = \begin{bmatrix} P_{33} & P_{34} \\ P_{43} & P_{44} \end{bmatrix}$$

$$\hat{M}^T P_1^T \hat{M} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}, U_1 = \begin{bmatrix} U_{11} & U_{12} \\ * & U_{22} \end{bmatrix}$$

$$U_2 = \begin{bmatrix} U_{13} & U_{14} \\ U_{23} & U_{24} \end{bmatrix}, U_3 = \begin{bmatrix} U_{33} & U_{34} \\ * & U_{44} \end{bmatrix}$$

$$\begin{bmatrix} (1,1) & \hat{M}^T X_1 \hat{M} + \hat{M}^T X_2^T \hat{M} \\ * & \hat{M}^T X_1 \hat{M} \end{bmatrix} = \begin{bmatrix} X_{ij} \end{bmatrix}_{4 \times 4} \tag{16}$$

$$\begin{bmatrix} (1,1)' & \hat{M}^T Y_1 \hat{M} + \hat{M}^T Y_3 \hat{M} \\ (2,1)' & \hat{M}^T Y_1 \hat{M} \end{bmatrix} = \begin{bmatrix} Y_{ij} \end{bmatrix}_{4 \times 4} \tag{17}$$

其中, $P_{11}, P_{33} \in \mathbf{R}^{(p+n_f) \times (p+n_f)}$, $(1,1) = \hat{M}^T X_1 \hat{M} + \hat{M}^T X_2 \hat{M} + \hat{M}^T X_2^T \hat{M} + \hat{M}^T X_3 \hat{M}$, $(1,1)' = \hat{M}^T Y_1 \hat{M} + \hat{M}^T Y_2 \hat{M} + \hat{M}^T Y_3 \hat{M} + \hat{M}^T Y_4 \hat{M}$, $(2,1)' = \hat{M}^T Y_1 \hat{M} + \hat{M}^T Y_2 \hat{M}$, $X_{ij} = X_{ji}^T$, $1 \leq i, j \leq 4$. 利用式 (9) 及矩阵论知识^[13] 知

$$\begin{cases} P_{11} > 0, P_{21} = 0, P_{33} < 0, P_{43} = 0 \\ Y_{12} = 0, Y_{22} = 0, Y_{32} = 0, Y_{42} = 0 \\ Y_{14} = 0, Y_{24} = 0, Y_{34} = 0, Y_{44} = 0 \end{cases} \tag{18}$$

又注意到式 (15), 则式 (10) 即为

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & Y_{11} & Y_{13} \\ * & X_{22} & X_{23} & X_{24} & Y_{21} & Y_{23} \\ * & * & X_{33} & X_{34} & Y_{31} & Y_{33} \\ * & * & * & X_{44} & Y_{41} & Y_{43} \\ * & * & * & * & P_{11}U_{11}P_{11} & \Omega_1 \\ * & * & * & * & * & \Omega_2 \end{bmatrix} \geq 0 \tag{19}$$

其中

$$\begin{aligned} \Omega_1 &= P_{11}U_{11}(P_{11} - P_{33}) + P_{11}U_{13}P_{33}, \\ \Omega_2 &= (P_{11} - P_{33})U_{11}(P_{11} - P_{33}) + P_{33}U_{13}^T \times \\ &\quad (P_{11} - P_{33}) + (P_{11} - P_{33})U_{13}P_{33} + \\ &\quad P_{33}U_{33}P_{33}. \end{aligned}$$

注意到 $\begin{bmatrix} U_{11} & U_{13} \\ * & U_{33} \end{bmatrix} > 0$, 引入矩阵 $\begin{bmatrix} L_{11} & L_{12} \\ * & L_{22} \end{bmatrix} > 0$, $\begin{bmatrix} K_{11} & K_{12} \\ * & K_{22} \end{bmatrix} > 0$ 和 $\begin{bmatrix} J_{11} & 0 \\ J_{11} - J_{22} & J_{22} \end{bmatrix}$, 其中 $J_{11} > 0, J_{22} < 0$, 由 Schur 补知若下述各式成立, 则式 (19) 成立.

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & Y_{11} & Y_{13} \\ * & X_{22} & X_{23} & X_{24} & Y_{21} & Y_{23} \\ * & * & X_{33} & X_{34} & Y_{31} & Y_{33} \\ * & * & * & X_{44} & Y_{41} & Y_{43} \\ * & * & * & * & L_{11} & L_{12} \\ * & * & * & * & * & L_{22} \end{bmatrix} \geq 0 \tag{20}$$

$$\begin{bmatrix} U_{11} & U_{13} & J_{11} & 0 \\ * & U_{33} & J_{11} - J_{22} & J_{22} \\ * & * & K_{11} & K_{12} \\ * & * & * & K_{22} \end{bmatrix} \geq 0 \tag{21}$$

$$P_{11}J_{11} = I, \quad P_{33}J_{22} = I \tag{22}$$

$$\begin{bmatrix} L_{11} & L_{12} \\ * & L_{22} \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ * & K_{22} \end{bmatrix} = I \tag{23}$$

由以上讨论知, 若存在

$$\begin{bmatrix} U_1 & U_2 \\ * & U_3 \end{bmatrix} > 0, \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix} > 0 \tag{24}$$

$$\begin{bmatrix} L_{11} & L_{12} \\ * & L_{22} \end{bmatrix} > 0, \begin{bmatrix} K_{11} & K_{12} \\ * & K_{22} \end{bmatrix} > 0 \tag{25}$$

$$\begin{bmatrix} X_{ij} \end{bmatrix}_{4 \times 4} \geq 0, \begin{bmatrix} Y_{ij} \end{bmatrix}_{4 \times 4}, P_{11} > 0 \tag{26}$$

$$P_{33} < 0, J_{11} > 0, J_{22} < 0 \quad (27) \quad 4, 1 \leq s \leq t \leq 2:$$

满足式 (15), (18), (20) ~ (23), 则存在 $P_1, P_2, X_i, Z_i, U_i, Y_j, 1 \leq i \leq 3, 1 \leq j \leq 4$, 满足式 (10) 和 (11), 其中 P_1 非奇异, $P_1 - P_2$ 非奇异, $X_1 \geq 0, X_3 \geq 0$. 此时, $P_1 = \hat{M} \begin{bmatrix} P_{11} & 0 \\ P_{12}^T & P_{22}^T \end{bmatrix} \hat{M}^T, P_2 = \hat{M} \begin{bmatrix} P_{11} - P_{33} & 0 \\ P_{12}^T - P_{34}^T & P_{22}^T - P_{44}^T \end{bmatrix} \hat{M}^T, P_{22}$ 和 P_{44} 为任意非奇异矩阵, P_{12} 和 P_{34} 为任意矩阵. $X_i, Y_j, 1 \leq i \leq 3, 1 \leq j \leq 4$, 可由式 (16) 和 (17) 确定. 则鲁棒 H_∞ -FDF 设计问题即转化为一个锥面互补问题, 其约束条件均为 LMI, 可归纳为如下算法.

算法 1.

Step 1. 对给定的 $\tau_m > 0$, 找一组可行解 $P_1, P_2, X_i, Q_i, Z_i, U_i, Y_j, W_j, \varepsilon, P_{11}, P_{33}, \begin{bmatrix} X_{ij} \end{bmatrix}_{4 \times 4}, \begin{bmatrix} Y_{ij} \end{bmatrix}_{4 \times 4}, L_{st}, K_{st}, J_{11}, J_{22}, 1 \leq i \leq 3, 1 \leq j \leq 4, 1 \leq s \leq t \leq 2$, 满足式 (12), (13), (18), (20), (21), (24) ~ (27) 及

$$\varepsilon > 0, Q_1 > 0, Q_3 > 0, X_1 \geq 0, X_3 \geq 0 \quad (28)$$

$$\begin{bmatrix} P_{11} & I \\ I & J_{11} \end{bmatrix} \geq 0, \begin{bmatrix} -P_{33} & I \\ I & -J_{22} \end{bmatrix} \geq 0 \quad (29)$$

$$\begin{bmatrix} \hat{M}U_1\hat{M}^T & \hat{M}U_2\hat{M}^T & I & 0 \\ * & \hat{M}U_3\hat{M}^T & 0 & I \\ * & * & Z_1 & Z_2 \\ * & * & * & Z_3 \end{bmatrix} \geq 0 \quad (30)$$

$$\begin{bmatrix} L_{11} & L_{12} & I & 0 \\ * & L_{22} & 0 & I \\ * & * & K_{11} & K_{12} \\ * & * & * & K_{22} \end{bmatrix} \geq 0 \quad (31)$$

若不存在, 停止; 否则, 令 $U_i^{(0)} = U_i, Z_i^{(0)} = Z_i, P_{11}^{(0)} = P_{11}, J_{11}^{(0)} = J_{11}, P_{33}^{(0)} = P_{33}, J_{22}^{(0)} = J_{22}, L_{st}^{(0)} = L_{st}, K_{st}^{(0)} = K_{st}, 1 \leq i \leq 3, 1 \leq s \leq t \leq 2$, 并验证式 (10). 若式 (10) 成立, 则系统 (6) 是系统 (1) 的鲁棒 H_∞ -FDF, 其参数矩阵设计为式 (14) 的形式; 否则, 令 $k = 0$, 转至 Step 2.

Step 2. 求出下述凸优化问题的解 $P_1, P_2, X_i, Q_i, Z_i, U_i, Y_j, W_j, \varepsilon, P_{11}, P_{33}, \begin{bmatrix} X_{ij} \end{bmatrix}_{4 \times 4}, \begin{bmatrix} Y_{ij} \end{bmatrix}_{4 \times 4}, L_{st}, K_{st}, J_{11}, J_{22}, 1 \leq i \leq 3, 1 \leq j \leq$

$$\min\{\text{tr}\left(\begin{bmatrix} \hat{M}U_1^{(k)}\hat{M}^T & \hat{M}U_2^{(k)}\hat{M}^T \\ * & \hat{M}U_3^{(k)}\hat{M}^T \end{bmatrix} \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix}\right) + \text{tr}\left(\begin{bmatrix} \hat{M}U_1\hat{M}^T & \hat{M}U_2\hat{M}^T \\ * & \hat{M}U_3\hat{M}^T \end{bmatrix} \begin{bmatrix} Z_1^{(k)} & Z_2^{(k)} \\ * & Z_3^{(k)} \end{bmatrix}\right) + \text{tr}(P_{11}^{(k)}J_{11} + P_{11}J_{11}^{(k)} + P_{33}^{(k)}J_{22} + P_{33}J_{22}^{(k)}) + \text{tr}\left(\begin{bmatrix} L_{11}^{(k)} & L_{12}^{(k)} \\ * & L_{22}^{(k)} \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ * & K_{22} \end{bmatrix}\right) + \left. \begin{bmatrix} L_{11} & L_{12} \\ * & L_{22} \end{bmatrix} \begin{bmatrix} K_{11}^{(k)} & K_{12}^{(k)} \\ * & K_{22}^{(k)} \end{bmatrix}\right)\}$$

约束条件: (12), (13), (18), (20), (21), (24) ~ (31).

令 $U_i^{(k+1)} = U_i, Z_i^{(k+1)} = Z_i, P_{11}^{(k+1)} = P_{11}, J_{11}^{(k+1)} = J_{11}, P_{33}^{(k+1)} = P_{33}, J_{22}^{(k+1)} = J_{22}, L_{st}^{(k+1)} = L_{st}, K_{st}^{(k+1)} = K_{st}, 1 \leq i \leq 3, 1 \leq s \leq t \leq 2$.

Step 3. 验证式 (10). 若式 (10) 成立, 则系统 (6) 是系统 (1) 的鲁棒 H_∞ -FDF, 其参数矩阵设计为式 (14) 的形式. 若在设定的迭代次数内式 (10) 均不满足, 停止; 否则, 令 k 取 $k + 1$, 转至 Step 2.

3 算例

考虑不确定奇异时滞系统 (1), 其中

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -3 & 0 \\ 1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}$$

$$A_\tau = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B_f = \begin{bmatrix} 0.9 \\ 0.4 \end{bmatrix}, B_d = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$D_f = \begin{bmatrix} 0.7 \\ 0.5 \end{bmatrix}, D_d = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, M = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$$

$$N_A = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, N_\tau = \begin{bmatrix} 0.05 & 0.05 \end{bmatrix}$$

$$N_B = 0.1, \tau_m = 3.$$

取 $\gamma = 0.5, W_f(s) = 1/(s + 1)$, 即 $A_W = -1, B_W = 1, C_W = 1$. 应用算法 1 解得

$$H_1 = \begin{bmatrix} -2.5643 & -3.1249 & 0.1152 & 0.0049 \\ 1.2165 & -3.5360 & 0.0400 & 0.0006 \\ -3.3844 & 3.0003 & -1.0238 & 0.0475 \\ -0.0051 & 0.0092 & -0.0003 & -1.3775 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} -0.8819 & 3.0672 \\ -0.2775 & 1.2928 \\ 3.2723 & -2.6911 \\ 0.0059 & -0.0085 \end{bmatrix}$$

$$H_3 = \begin{bmatrix} 0.5515 \\ 0.2256 \\ -0.0187 \\ -0.0002 \end{bmatrix}, H_4 = \begin{bmatrix} 0.0245 \\ 0.0001 \\ 0.9909 \\ 0.6266 \end{bmatrix}^T$$

在零初始条件下, 取 $F(\sigma) = 1, u = 0$, 干扰 d 为图 1 所示能量为 0.5 的白噪声, 故障 f 及时滞 $\tau = 1.5$ 时的残差信号如图 2 所示.

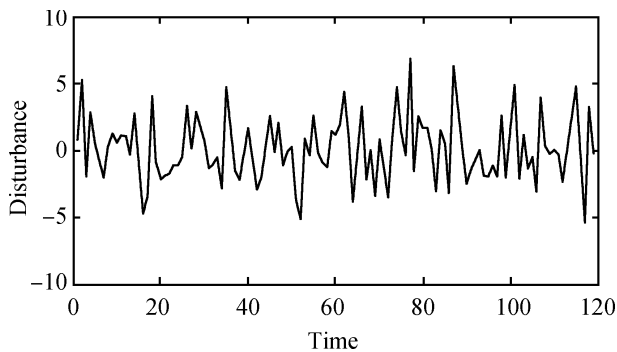


图 1 干扰信号
Fig. 1 Disturbance signal

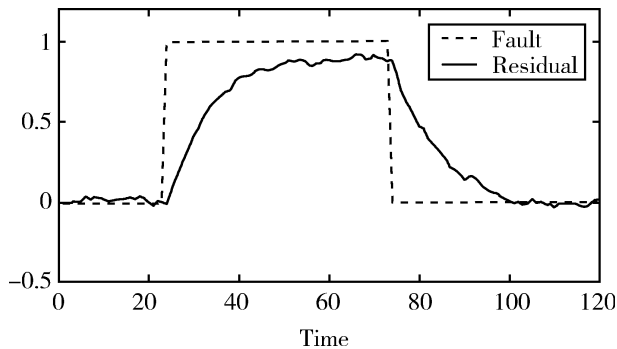


图 2 故障信号及残差信号
Fig. 2 Fault signal and residual signal

4 结语

本文研究了不确定奇异时滞系统有限频率范围内的鲁棒 H_∞ -FDF 问题. 对于给定描述故障频率范围的加权传递函数矩阵, 应用一般结构形式的基于观测器的故障诊断滤波器作为残差产生器, 将不确定奇异时滞系统的残差产生器设计归结为鲁棒 H_∞ 滤波问题, 推导并证明了鲁棒 H_∞ -FDF 存在的充分条件. 应用一种锥面互补线性化迭代算法, 将鲁棒 H_∞ -FDF 参数矩阵的求解转化为受 LMI 约束的最小化问题, 并给出了具体的求解算法. 值得提出的是, 本文得到的依赖于时滞的 H_∞ 滤波器设计结果对于求解奇异时滞系统 H_∞ 滤波问题, 目前在文献中尚未发现. 算例进一步验证了本文提出算法的有效性.

附录 A 定理 1 的证明

证明. 设式 (9)~(13) 成立, 且滤波器 (6) 的系数矩阵取为式 (14). 首先整理式 (9). 令 $P_1 = [\tilde{P}_{ij}]_{4 \times 4}$, 则由式 (9) 及矩阵论知识^[13] 知, $\tilde{P}_{12} = 0, \tilde{P}_{14} = 0, \tilde{P}_{32} = 0, \tilde{P}_{34} = 0$, 且 $\tilde{P}_{31} = \tilde{P}_{13}^T$, 则有

$$P_1^T E_s = E_s P_1 \geq 0, P_2^T E_s = E_s P_2 \geq 0 \quad (A1)$$

易证 P_1 非奇异, 不妨设 $P_1 - P_2$ 非奇异^[14]. 令 $\Pi_1 = \begin{bmatrix} P_1^{-1} & I \\ P_1^{-1} & 0 \end{bmatrix}, \Pi_2 = \begin{bmatrix} I & P_2 \\ 0 & P_1 - P_2 \end{bmatrix}, \bar{P} = \Pi_2 \Pi_1^{-1}$, 由文献 [14] 定理 1 知, $\bar{P} = \begin{bmatrix} P_2 & P_1 - P_2 \\ * & -(P_1 - P_2) \end{bmatrix}$ 非奇异且满足 $E_c \bar{P} = \bar{P}^T E_c \geq 0$. 令 $\bar{P} = P^T$, 则有

$$P E_c = E_c P^T \geq 0 \quad (A2)$$

下面整理式 (13). 令

$$Q = \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix}, X = \begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix}$$

$$Y = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix}, \bar{Z} = \begin{bmatrix} Z_1 & Z_2 \\ * & Z_3 \end{bmatrix}$$

式 (13) 左乘 $\text{diag}((P_1^T)^{-1}, I, (P_1^T)^{-1}, I, (P_1^T)^{-1}, I, (P_1^T)^{-1}, I, I, I, I, I)$, 右乘其转置, 并令 $Z^{-1} = P^{-1} \bar{Z} (P_1^T)^{-1}$, 易知存在 $\varepsilon > 0$ 使得式 (13) 成立, 当且仅当式 (A3) (见本页下方) 对所有满足式 (2) 的 $\Delta A, \Delta A_\tau$ 和 ΔB 成立, 其中 $\tilde{\Gamma}_{11} = (A_c + \Delta A_c)^T P^T + P(A_c + \Delta A_c) + Q + \tau_m X + Y + Y^T + C_c^T C_c$.

$$\begin{bmatrix} \tilde{\Gamma}_{11} & P(A_{\tau c} + \Delta A_{\tau c}) - Y & \tau_m(A_c + \Delta A_c)^T Z & P(A_{\tau c} + \Delta A_{\tau c}) & P(B_c + \Delta B_c) \\ * & -Q & \tau_m(A_{\tau c} + \Delta A_{\tau c})^T Z & 0 & 0 \\ * & * & -\tau_m Z & 0 & 0 \\ * & * & * & -Q & 0 \\ * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (A3)$$

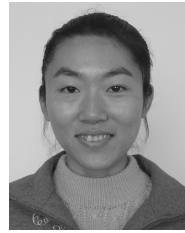
将式 (15) 代入式 (10), 类似可知, 式 (10) 和 (12) 可分别整理为

$$\begin{bmatrix} X & Y \\ * & E_c Z E_c \end{bmatrix} \geq 0, \tau_m X + Y + Y^T \geq 0 \quad (\text{A4})$$

由上述讨论知, 存在矩阵 $Q > 0$ 、 $X \geq 0$ 、 $Z > 0$ 和矩阵 P 、 Y , 使得式 (A2)~(A4) 对所有满足式 (2) 的 ΔA 、 ΔA_τ 和 ΔB 都成立. 由文献 [15] 引理 2 知系统 (6) 是系统 (1) 的鲁棒 H_∞ -FDF. \square

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