Control Decoherence by Quantum Generalized Measurement

ZHANG Ming¹ OU Bao-Quan² DAI Hong-Yi² HU De-Wen¹

Abstract This paper explores the potential of combating the decoherence by associating quantum generalized measurement with decoherence-free subspaces (DFS). It is demonstrated that associating quantum generalized subspace projector measurement (QGSPM) and quantum generalized case-projector measurement (QGCPM) with the operator conditions for DFS will enhance the capability of suppressing the decoherence for both Markovian and non-Markovian open quantum systems. It is emphasized that quantum measurement can be regarded as a means to manipulate quantum states. This method is advantageous because the coherent control Hamilton can be constructively designed.

Key words Quantum measurement, decoherence-free subspace, decoherence control

Quantum information and quantum computation^[1] have become extremely active research areas. For the use of quantum information to progress beyond mere theoretical constructs into the realm of testable and useful implementations and experiments, it is essential to develop active and passive techniques for preserving quantum coherence and overcoming decoherence [2-4]. The theory of decoherencefree subspaces and subsystems^[5-9] is an important passive approach of controlling decoherence. Recently, it has been suggested^[10] that quantum generalized measurement^[11-14] should be used to overcome the decoherence. It is gradually realized that the ability to control decoherence will be enhanced if more resources can be put into use. However, the potential of combating the decoherence by associating quantum generalized measurement with DFS has not been fully explored yet.

Recently, the concept of the quantum generalized subspace projector measurement $(QGSPM)^{[15]}$ has been proposed. The distinguished properties of QGSPM has been revealed: no matter what the state of the system is before the measurement and what the measured result occurs, the state after the measurement can be collapsed onto the specified subspace. This paper furthermore suggests that QGSPM and its special case, quantum generalized caseprojector measurement (QGCPM)^[15] should be used to control the decoherence. It is emphasized in this paper that quantum measurements can be regarded as a means to manipulate quantum states not just as a means to get the information encoded in quantum states.

This paper is organized as follows. In Section 1, we review the basic concept of DFS and give the operator conditions for the existence of DFS. The concepts of QGSPM and QGCPM are reviewed, and their distinguished properties are discussed in Section 2. In Section 3, the main results

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of the paper are presented for both Markovian and non-Markovian open quantum systems. Some extensions and discussions are given for time-local nonMarkovian master equations in Section 4. The paper concludes with Section 5.

1 Decoherence-free subspaces

In this section, we will discuss the concept of decoherence-free subspace (DFS) in the narrow sense of a quantum dynamical semigroup master equation and give the operator conditions for the existence of decoherencefree subspaces.

For a quantum open dynamic system, a subspace of the system Hilbert space is called the decoherence-free subspace^[5-6] if the evolution confined on this subspace is unitary. Here, we just present the concept of decoherence-free subspace in the narrow sense of a quantum dynamical semigroup master equation^[16–17].

In many practical situations, a quantum dynamical semigroup master equation is an appropriate way to describe the evolution of the quantum open system. By assuming that 1) the evolution of system density matrix is an oneparameter semigroup; 2) the system density matrix retains the properties of a density matrix including complete positivity; 3) the system and bath density matrices are initially decoupled, Lindblad^[16] has shown that the most general evolution of the system density matrix can be governed by the master equation.

Let us consider a quantum open system subject to Markovian decoherence and controlled via a control Hamiltonian H. The system dynamics is governed by a master equation of the following form

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho] + L(\rho) \tag{1}$$

where Lindbladian is

$$L(\rho) = \frac{1}{2} \sum_{i} \gamma_i \left(\left[G_i, \rho G_i^+ \right] + \left[G_i \rho, G_i^+ \right] \right)$$
(2)

and $\gamma_i \geq 0$.

Now, the concept of decoherence-free subspaces can be given in the sense of a quantum dynamical semigroup mas-

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College of Mechatronics Engineering and Automation, National University of Defense Technology, Changsha 410073, P.R. China
 College of Science, National University of Defense Technology, Changsha 410073, P.R. China

ter equation.

Definition 1. For the quantum open system described by (1) and (2), a subspace S of the system Hilbert space H_s is called a decoherence-free subspace, and if for any density operator ρ confined on subspace S, (2) satisfies $L(\rho) = 0$.

Remark 1. Select a collection $\{|f\rangle\}_{f=1,2,\cdots,m_0}$ of orthonormal basis in S and extend it to a collection $\{|f\rangle\}_{f=1,2,\cdots,n}$ of orthonormal basis in Hilbert Space H_s . The density operator ρ confined on subspace S can be written as $\rho = \sum_{j,k=1}^{m_0} \rho_{jk} |j\rangle \langle k|$. It is also easy to conclude that S is a decoherence-free subspace if all G_i in (2) can be expressed as

$$G_i = c_i \sum_{j=1}^{m_0} |j\rangle\langle j| + \sum_{j,k=m_0+1}^n g_{jk}^i |j\rangle\langle k|$$
(3)

We call (3) the operator condition for the existence of decoherence-free subspaces.

2 Quantum generalized measurement

2.1 The postulate of quantum measurement

Let us consider the finite-dimensional quantum system Q with its Hilbert space H_s . Quantum measurement is described by a collection $\{M_m\}_{m \in \{1,2,\dots,l\}}$ of measurement operators. These are operators acting on the state space of the quantum system Q. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is ρ prior to the measurement, then the probability that result m occurs is given by

$$p(m) = \operatorname{tr}(M_m^+ M_m \rho) = \operatorname{tr}(M_m \rho M_m^+)$$
(4)

and the state of system after measurement is

$$\frac{M_m \rho M_m^+}{\operatorname{tr}(M_m \rho M_m^+)} \tag{5}$$

The measurement operators satisfy the completeness equation

$$\sum_{m=1}^{l} M_m^+ M_m = I$$
 (6)

where I is the identity operator.

2.2 The concepts of QGSPM and QGCPM

Definition 2 (QGSPM). Suppose that $\{|\varphi_i\rangle\}_{i\in J}$ is an orthonormal basis for H_s with $J = \{1, 2, \dots, n\}$, the index set for H_s , and $\{|\psi_j\rangle\}_{j\in\{1,2,\dots,n_0\}}$, a set of pure states in a subspace B of H_s . Then, QGSPM is defined as follows: Let $\{m\}_{m\in J_{out}=\{1,2,\dots,l\}}$ refer to the outcomes of measurement. When the measurement result m occurs, the corresponding generalized operator can be formed as

$$\overline{\Gamma}(m) = \sum_{i=1}^{n} \sum_{j=1}^{n_0} \Gamma(m, i, j) |\psi_j\rangle \langle \varphi_i|$$
(7)

where $\Gamma(m, i, j) \in \mathbf{C}$, the set of complex number. If the quantum generalized operators given by (7) satisfy the the

completeness equation, i.e., the following equations hold

$$\sum_{m=1}^{l} \sum_{j_1=1}^{n_0} \sum_{j_2=1}^{n_0} \Gamma^*(m, i_1, j_1) \Gamma(m, i_2, j_2) \langle \psi_{j_1} | \psi_{j_2} \rangle = \delta_{i_1 i_2} \quad (8)$$

where $i_1, i_2 \in \{1, 2, \dots, n\}$, the corresponding measurement is called the quantum generalized subspace-projector measurement (QGSPM).

Definition 3 (QGCPM). Let $\{m\}_{m \in J_{out}}$ refer to the outcomes of measurement that may occur in the experiment, where $J_{out} = \{1, 2, \dots, l\}$ is the index set for measurement outcomes and $J_{out} = \biguplus_{i=1}^{n_0} J_{out}^i$ with $J_{out}^i \cap J_{out}^j = \emptyset$ if $i \neq j$. When the measurement result m occurs, where $m \in J_{out}^i$, the corresponding generalized operator can be expressed as

$$\overline{\Gamma}(m) = \sum_{j=1}^{n} \Gamma(m, j) |\psi_i\rangle \langle \varphi_j|$$
(9)

where $\Gamma(m, j) \in \mathbf{C}, m \in J_{out}^{i}, i = 1, 2, \dots, n_{0}, j \in \{1, 2, \dots, n\}$. If the quantum generalized operators given by (8) satisfy the the completeness equation, the following equations hold

$$\sum_{m=1}^{l} \Gamma^*(m, j_1) \Gamma(m, j_2) = \delta_{j_1 j_2}$$
(10)

with $j_1, j_2 \in \{1, 2, \dots, n\}$, and the corresponding measurement is called quantum generalized case-projector measurement (QGCPM).

Obviously, QGCPM is a special case of QGSPM and both of them are special quantum generalized measurements. As for the physical realization of the quantum generalized measurement, the detailed discussions can be found in many references, for example [1, 18]. It is quite wellknown that the quantum generalized measurement is realizable in principle if ancillary systems can be introduced^[1].

2.3 Properties of QGSPM and QGCPM

Lemma 1 (Property of QGSPM). Consider a finitedimensional quantum system S with its Hilbert space H_s . Suppose that the quantum generalized measurement operators for QGSPM is given by (7) with (8). Let ρ_0 be an arbitrary initial density operator. If the state of quantum system S is ρ_0 before the measurement and no matter what measurement result m occurs, the state of the quantum system S after the above mentioned measurement must be the density operator ρ_s confined on the subspace B.

Lemma 2 (Property of QGCPM)^[15]. For the finitedimensional quantum system S with its Hilbert space H_s , the quantum generalized measurement operators for QGCPM are given by (9) with (10). Suppose ρ_0 is an arbitrary initial density operator. If the state of quantum system S is ρ_0 before the measurement, and the measurement result m occurs where $m \in J_{out}^i$, then the state after the measurement must be the pure state $\rho_s = |\psi_i\rangle\langle\psi_i|$, i.e., $|\psi_i\rangle$ in H_s . That is, no matter what measurement result occurs, the state of system can be reduced to one of the pure states $\{|\psi_i\rangle\}_{i=1,2,\cdots,n_0}$.

3 Control decoherence by QGM

In this section, we can demonstrate that the ability to perform QGCPM or QGSPM on a quantum open system, combined with the ability of coherence control and operator conditions for DFS, permits one to suppress quantum decoherence.

3.1 Control decoherence by QGSPM

Denote a set of orthonormal basis in S as $\{|f\rangle\}_{f=1,2,\cdots,m_0}$. We can extend it to a set of orthonormal basis in Hilbert space H_s , $\{|f\rangle\}_{f=1,2,\cdots,n}$.

Theorem 1. Consider an *n*-dimension quantum controlled open system with its *n*-dimensional Hilbert space H_s ; the system dynamics is governed by the following master equation

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H + H_c, \rho] + \frac{1}{2} \sum_i \gamma_i ([G_i, \rho G_i^+] + [G_i \rho, G_i^+]) \quad (11)$$

where G_i satisfies condition (3). Then, the ability to perform QGSPM on quantum open system, combined with the ability of coherence control H_c , permits one to produce a density operator ρ_s confined on the subspace S and store it in the open system stably.

Proof. The constructive demonstration will be given as follows.

For quantum controlled open system given by (11), we will introduce the coherent control as follows

$$H_c = \sum_{f_1, f_2 = m_0 + 1}^n h_{f_1 f_2} |f_1\rangle \langle f_2| - H$$
(12)

with $h_{f_1f_2} = h_{f_2f_1}^*$.

Now, the dynamical equation of quantum controlled open system can be written as

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} \left[\sum_{f_1, f_2=m_0+1}^n h_{f_1f_2} |f_1\rangle \langle f_2|, \rho \right] + \frac{1}{2} \sum_i \gamma_i \left(\left[G_i, \rho G_i^+ \right] + \left[G_i \rho, G_i^+ \right] \right)$$
(13)

Set the subspace *B* as the decoherence-free subspace *S* and choose QGSPM operators as (7) with (8). By Lemma 1, the state of system after the generalized measurement must be a density operator ρ_s confined on the subspace *S*. In the following, we will demonstrate that ρ_s is the stationary solution for controlled open quantum system (13).

Without losing generality, we suppose that $\rho_s = \sum_{j,k=1}^{m_0} \rho_{jk}^s |j\rangle \langle k|$. Now, for $j, k = 1, 2, \cdots, m_0$, we have $[\sum_{f_1, f_2=m_0+1}^n h_{f_1f_2} |f_1\rangle \langle f_2|, |j\rangle \langle k|] = 0$, and

$$\left[\sum_{f_1, f_2=m_0+1}^n h_{f_1f_2} |f_1\rangle \langle f_2|, \sum_{j,k=1}^{m_0} \rho_{jk}^s |j\rangle \langle k|\right] = 0 \qquad (14)$$

By (2), we can conclude that for $j, k = 1, 2, \cdots, m_0$

$$L(|j\rangle\langle k|) = \frac{1}{2}\sum_{i}\gamma_i([G_i,|j\rangle\langle k|G_i^+] + [G_i|j\rangle\langle k|,G_i^+]) = 0$$
(15)

and thus

$$L(\sum_{j,k=1}^{m_0} \rho_{jk}^s |j\rangle \langle k|) = 0$$
 (16)

Therefore, $\rho_s = \sum_{j,k=1}^{m_0} \rho_{jk} |j\rangle \langle k|$ can make both sides of (13) equal to zero simultaneously and is the stationary solution of (13).

Remark 2. For the quantum open system given by (1) with (2) and (3), the ability to perform QGSPM on the quantum open system, associated with the ability of coherence control H_c , allows one to generate a density operator ρ_s confined on the subspace S and store it in the open system stably. Moreover, the method proposed in Theorem 1 is advantageous to the open-loop coherent control Hamilton such that it can be concretely constructed.

3.2 Control decoherence by QGCPM

Theorem 2. Consider an *n*-dimension quantum controlled open system with its *n*-dimensional Hilbert space H_s , whose dynamics is governed by the master (11), where G_i satisfies condition (3); then for a set of pure states $\{|\psi_i\rangle \in S\}_{i=1,2,\dots,n_0}$, the ability to perform QGCPM on quantum open system, combined with the ability of coherence control H_c , permits one to produce one of pure states $\{|\psi_i\rangle \in S\}_{i=1,2,\dots,n_0}$ and store it in the open system stably.

Remark 3. When QGCPM is reduced to a special type of quantum generalized measurement recently constructed in [10] and $h_{f_1f_2} = \delta_{f_1f_2}h_{f_1}$ with $f_1, f_2 = n_0 + 1, \dots, n$, we can get Theorem 4 in [10]. Therefore, Theorem 2 extends the related result about decoherent control in [10].

Furthermore, we can extend the above theorems to a more general case and have the following theorem:

Theorem 3. Consider controlled open quantum system

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H_{int} + H_c, \rho] + \frac{1}{2} \sum_i \gamma_i(t) ([G_i, \rho G_i^+] + [G_i \rho, G_i^+])$$
(17)

where H_{int} is the internal Hamiltonian, H_c is the coherent control Hamiltonian, and $\gamma_i(t)$ is complex function.

1) If G_i satisfies condition (3), then the ability to perform QGSPM on quantum open system, combined with the ability of coherence control H_c , permits one to produce a density operator ρ_s confined on the subspace S and store it in the open system stably.

2) If G_i satisfies condition (3), then for a set of pure states $\{|\psi_i\rangle \in S\}_{i=1,2,\cdots,n_0}$, the ability to perform QGCPM on quantum open system, combined with the ability of coherence control H_c , permits one to produce one of pure states $\{|\psi_i\rangle \in S\}_{i=1,2,\cdots,n_0}$ and store it in the open system stably.

Remark 4. Now, if $\gamma_i(t)$ in (17) is negative, (17) can be used to describe the controlled open nonMarkovian quantum dynamical systems.

4 Extensions and discussions

In fact, the operator conditions (3) for DFS can be also written as the following matrix form

$$G_{i} = \begin{pmatrix} c_{i}(t)I_{n_{0}} & 0\\ 0 & G_{i}^{22}(t) \end{pmatrix}$$
(18)

By means of this kind of representation, we can get the DFS conditions for time-local non-Markovian master equations^[19-20] and have the following result.

Result. Let us consider the following controlled master equations for the density matrix $\rho(t)$ of an open system which have the following general form:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H + H_c, \rho] + \sum_{\alpha} [C_{\alpha}(t)\rho(t)D_{\alpha}^+(t) + D_{\alpha}(t)\rho(t)C_{\alpha}^+(t)] + \frac{1}{2} \sum_{\alpha} \{D_{\alpha}^+(t)C_{\alpha}(t) + C_{\alpha}^+(t)D_{\alpha}(t), \rho(t)\}$$
(19)

where the Hamiltonian H, $C_{\alpha}(t)$, and $D_{\alpha}(t)$ are given, and H_c is the coherent control Hamiltonian to be designed. Suppose that $C_{\alpha}(t)$ and $D_{\alpha}(t)$ are given by the following equations

$$C_{\alpha}(t) = \begin{pmatrix} c_{\alpha}(t)I_{n_0} & 0\\ 0 & C_{\alpha}^{22}(t) \end{pmatrix}$$
(20)

and

$$D_{\alpha}(t) = \begin{pmatrix} d_{\alpha}(t)I_{n_0} & 0\\ 0 & D_{\alpha}^{22}(t) \end{pmatrix}$$
(21)

where $c_{\alpha}(t)$ and $d_{\alpha}(t)$ are complex functions of time t. Then, we have the following results:

1) The ability to perform QGSPM on quantum open system, combined with the ability of coherence control H_c , permits one to produce a density operator ρ_s confined on the subspace S and store it in the open system stably.

2) For a set of pure states $\{|\psi_i\rangle \in S\}_{i=1,2,\cdots,n_0}$, the ability to perform QGCPM on quantum open system, combined with the ability of coherence control H_c , permits one to produce one of pure states $\{|\psi_i\rangle \in S\}_{i=1,2,\cdots,n_0}$ and store it in the open system stably.

Remark 5. 1) (20) and (21) can be considered as the operator conditions for DFS of time-local nonMarkovian master (19). 2) In the above case, we just need to construct such a coherent control Hamiltonian H_c that H_0+H_c satisfies the following equation

$$H_0 + H_c = \begin{pmatrix} H_{11}(t) & 0\\ 0 & H_{22}(t) \end{pmatrix}$$
(22)

with $H_{11}(t)$, an n_0 -dimensional Hermitian operator, and $H_{22}(t)$, an $(n - n_0)$ -dimensional operator.

5 Conclusion

In conclusion, the ability to perform QGSPM or QGCPM on open quantum systems, combined with the

ability of coherent control and the operator conditions for the existence of decoherence-free subspaces, allows one to suppress decoherence and store quantum states stably. This research further establishes the link between quantum generalized measurements and decoherence control. The method proposed in this paper is advantageous in that the open-loop coherent control Hamilton can be constructively designed. It is emphasized in this paper that quantum measurement can be regarded as a means to manipulate quantum states and suppress decoherence. In fact, we have developed this idea in a recent research $^{[21]}$. It should be mentioned here that we have explained why decoherencefree subspaces/subsystems can be used for quantum computation from the view point of control theory [22-23]. In our opinions, making full use of structure features of quantum systems is also important for manipulating quantum states and combating decoherence.

References

- Nielsen M A, Chuang I L. Quantum Computation and Quantum Information. Cambridge: Cambridge University Press, 2000. 1–83
- 2 Zurek W H. Decoherence and the transition from quantum to classical. *Physics Today*, 1991, **44**(10): 36–44
- 3 Unruh W G. Maintaining coherence in quantum computers. Physical Review Online Archive, 1995, 51(2): 992–997
- 4 Landauer R. The physical nature of information. Physical Letters A, 1996, 217: 188–193
- 5 Zanardi P, Rasetti M. Noiseless quantum codes. Physical Review Letters, 1997, 79(17): 3306-3309
- 6 Lidar D A, Chuang I L, Whaley K B. Decoherence-free subspaces for quantum computation. *Physical Review Letters*, 1998, **81**(12): 2594–2597
- 7 Duan L M, Guo G C. Reducing decoherence in quantumcomputer memory with all quantum bits coupling to the same environment. *Physical Review A*, 1998, **57**(2): 737-741
- 8 Zanardi P. Dissipation and decoherence in a quantum register. Physical Review A, 1998, 57(5): 3276-3284
- 9 Lidar D A, Bacon D, Whaley K B. Concatenating decoherence-free subspaces with quantum error correcting codes. *Physical Review Letters*, 1999, 82(22): 4556-4559
- 10 Zhang M, Dai H Y, Zhu X C, Li X W, Hu D W. Control of the quantum open system via quantum generalized measurement. *Physical Review A*, 2006, **73**(3): 2101–2105
- Kraus K. State, Effects and Operations: Fundamental Notions of Quantum Theory. Berlin: Springer-Verlag, 1983. 1-87
- 12 Braginsky V B, Khahili F Y. Quantum Measurement. Cambridge: Cambridge University Press, 1992. 32–35
- 13 Busch P, Grabowski M, Lahti P J. Operational Quantum Physics. Berlin: Springer-Verlag, 1995. 34–36

- 14 Gardiner C W. Quantum Noise (Second Edition). Berlin: Springer-Verlag, 2002. 1
- 15 Zhang Ming, Dong Guo-Hua, Dai Hong-Yi, Hu De-Wen. Quantum generalized subspace projector measurement and measurement induced entanglement. *Chinese Physics Let*ters, 2006, **23**(5): 1072–1075
- 16 Lindblad G. On the generators of quantum dynamical semigroups. Communications in Mathematical Physics, 1976, 48(2): 119–130
- 17 Alicki R, Lendi K. Quantum dynamical semigroups and applications. Lecture Notes in Physics, 1987, 286: 1–130
- 18 Peres A. Quantum Theory: Concepts and Methods. Dordrecht: Kluwer Academic Publishers, 1993. 62-63
- 19 Breuer H P, Burgarth D, Petruccione F. Non-Markovian dynamics in a spin star system: exact solution and approximation. *Physical Review B*, 2004, **70**(4): 5323-5332
- 20 Breuer H P, Petruccione F. The Theory of Open Quantum Systems. Oxford: Oxford University Press, 2002
- 21 Zhang M, Dai H Y, Xi Z, Xie H W, Hu D W. Combating dephasing decoherence by periodically performing tracking control and projective measurement. *Physical Review A*, 2007, **76**(4): 2335-2341
- 22 Zhang M, Dai H Y, Dong G H, Xie H W, Hu D W. Controllable subsystems of quantum dynamical systems. *Quantum Information and Computation*, 2007, 7(5-6): 469–478
- 23 Zhang M, Dai H Y, Gong E L, Xie H W, Hu D W. Controllable subspaces of open quantum dynamical systems. Communication Theory Physics, 2008, 49(1): 61-64



ZHANG Ming Received his Ph. D. degree from National University of Defence Technology in 2007. He is currently an associate professor at National University of Defence Technology. His research interest covers quantum control, quantum information processing, and robust control. Corresponding author of this paper. E-mail: zhangming@nudt.edu.cn



OU Bao-Quan Ph. D. candidate in College of Science, National University of Defence Technology. His research interest covers quantum information and quantum computation.

E-mail: BQOU@nudt.edu.cn



DAI Hong-Yi Received his Ph. D. degree from National University of Defence Technology in 2003. He is currently an associate professor at National University of Defence Technology. His research interest covers quantum information and quantum optics. E-mail: daihongyi1@163.com



HU De-Wen Received his Ph. D. degree from National University of Defence Technology in 1999. He is currently a professor at National University of Defence Technology. His research interest covers image processing, system identification and control, neural networks, and cognitive science. E-mail: dwhu@nudt.edu.cn