Optimal Robust Estimation for Linear Uncertain Systems with Single Delayed Measurement

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Abstract This paper deals with the optimal robust estimation problem for linear uncertain systems with single delayed measurement. The optimal robust estimator is derived based on the reorganized innovation analysis approach. The calculation of the optimal robust estimator involves solving two Riccati difference equations of the same dimensions as that of the original systems and one Lyapunov equation. A numerical example is given to show the effectiveness of the proposed approach.

Key words Innovation, projection, optimal robust estimation, Riccati difference equation

It is well known that the Kalman filtering formulation requires perfect system model and complete statistical knowledge[1−2]. However, many models which we may obtain are uncertain or lack statistical properties. In this case, the performance of conventional Kalman approach becomes poor. To get a good performance in the case of systems with uncertainty, many researchers have focused on robust estimation where the estimator is concerned with the design of estimation that is robust with respect to model uncertainty and lack of statistical properties on the exogenous signals[3−8]. Among these previous works, [3−4, 7−8] discuss the systems with deterministic uncertainty whereas [5−6] discuss the systems with random uncertainty.

On the other hand, estimation of time-delay systems has received much attention[9−11]. In the case of discrete time systems, the estimator for time-delay systems can be obtained by using state augmentation and standard Kalman filtering formulation. However, the state augmentation may usually lead to much expensive calculations when the time-delay or system dimension is large. Recently, [12−13] have proposed a new approach of reorganized innovation analysis. By using the new approach, the estimator for discrete time systems with delayed measurement is designed by solving two standard Riccati difference equations of the same dimensions as the original systems. As the application of the new approach, some complicated problems such as H_∞ filtering and H_∞ fixed-lag smoothing for time-delay systems have been also solved[14−15].

In this paper, we will study the optimal robust estimation for linear discrete time-delay systems with stochastic uncertainties in the system matrices. By applying the reorganized innovation analysis approach developed in our previous works[12−13], the delayed measurement is transformed into delay-free measurements and the associated innovation sequence is thus obtained. The stochastic uncertainties in the systems are dealt with by introducing the fictitious noise where the covariance matrix can be calculated recursively by one Lyapunov equation. The optimal robust estimator is finally derived by the projection formula. Compared with the traditional approach of state augmentation, a significant advantage of the proposed approach is that it can provide a simple solution.

The paper is organized as follows. In Section 1, the mathematical framework and problem statements are illustrated. We will present optimal robust estimation based on reorganized innovation approach and projection theory in Hilbert space in Section 2. The comparison of the computational cost between the presented approach and the conventional state augmented method is given in Section 3. A numerical example is presented to demonstrate the validity of the proposed method in Section 4. The paper is concluded in Section 5.

1 Problem statement

Consider the following linear stochastic uncertain system

$$\boldsymbol{x}(k+1) = \left[A+\sum_{i=1}^{m} A_i(k)\,\xi_i(k)\right]\boldsymbol{x}(k)+B\boldsymbol{u}(k) \quad (1)$$

$$\boldsymbol{y}(k) = \left[C+\sum_{i=1}^{m} C_i(k)\,\xi_i(k)\right]\boldsymbol{x}(k)+\boldsymbol{v}_{(1)}(k) \quad (2)$$

$$\boldsymbol{z}(k) = \left[H+\sum_{i=1}^{m} H_i(k_d)\,\xi_i(k_d)\right]\boldsymbol{x}(k_d)+\boldsymbol{v}_{(2)}(k) \quad (3)$$

$$k_d = k-d$$

where $\boldsymbol{x}(k)\in\mathbf{R}^n$, $\boldsymbol{y}(k)\in\mathbf{R}^p$, and $\boldsymbol{z}(k)\in\mathbf{R}^q$ represent the state, current measurement output, and delayed measurement output. $\boldsymbol{u}(k)\in\mathbf{R}^r$ is the input noise, $\boldsymbol{v}_{(1)}(k)\in\mathbf{R}^p$, and $\boldsymbol{v}_{(2)}(k)\in\mathbf{R}^q$ are the current measurement noise and delayed measurement noise, respectively.

Assumption 1. The initial state $\boldsymbol{x}(0)$, $\boldsymbol{u}(k)$, $\boldsymbol{v}_{(1)}(k)$, $\boldsymbol{v}_{(2)}(k)$, and the stochastic perturbs $\xi_i(k)$, $i=1,\cdots,m$, are zero, which means mutually uncorrelated white noises with $\varepsilon\left[\boldsymbol{x}(0)\boldsymbol{x}^{\mathrm{T}}(0)\right]=P_0$, $\varepsilon\left[\boldsymbol{u}(k)\boldsymbol{u}^{\mathrm{T}}(k)\right]=Q_u$, $\varepsilon\left[\boldsymbol{v}_{(1)}(k)\boldsymbol{v}_{(1)}^{\mathrm{T}}(k)\right]=Q_{v_{(1)}}$, $\varepsilon\left[\boldsymbol{v}_{(2)}(k)\boldsymbol{v}_{(2)}^{\mathrm{T}}(k)\right]=Q_{v_{(2)}}$, and $\varepsilon\left[\xi_i(k)\,\xi_j(l)\right]=\delta(i-j)\,\delta(k-l)$, respectively, where $\delta(k)$ is the Dirac Delta function, T stands for the transpose, and ε denotes the mathematical expectation.

In (3), $\boldsymbol{z}(k)$ is a delayed measurement. It is clear that systems (1)∼(3) are not standard forms to which the Kalman filtering is applicable. Let $\bar{\boldsymbol{y}}(k)$ denote the observation of the systems (1)∼(3) at time k; then we have

$$\bar{\boldsymbol{y}}(k)=\begin{cases}\boldsymbol{y}(k), & 0\le k<d\\ \begin{bmatrix}\boldsymbol{y}(k)\\ \boldsymbol{z}(k)\end{bmatrix}, & k\ge d\end{cases}$$

Thus, (2)∼(3) can be rewritten as

$$\bar{\boldsymbol{y}}(k)=\begin{cases}[C+\sum\limits_{i=1}^{m} C_i(k)\,\xi_i(k)]\boldsymbol{x}(k)+\boldsymbol{v}_s(k), & 0\le k<d\\ \mathcal{A}\begin{bmatrix}\boldsymbol{x}(k)\\ \boldsymbol{x}(k_d)\end{bmatrix}+\boldsymbol{v}_s(k), & k\ge d\end{cases}$$

where

$$\mathcal{A}=\begin{bmatrix}C+\sum\limits_{i=1}^{m} C_i(k)\,\xi_i(k) & 0\\ 0 & H+\sum\limits_{i=1}^{m} H_i(k_d)\,\xi_i(k_d)\end{bmatrix}$$

$$\boldsymbol{v}_s(k)=\begin{cases}\boldsymbol{v}_{(1)}(k), & 0\le k<d\\ \begin{bmatrix}\boldsymbol{v}_{(1)}(k)\\ \boldsymbol{v}_{(2)}(k)\end{bmatrix}, & k\ge d\end{cases}$$

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is white noise with zero mean and covariance matrix

$$Q_{v_s} = \begin{cases} Q_{v_{(1)}}, & 0 \le k < d \\ \begin{bmatrix} Q_{v_{(1)}} & 0 \\ 0 & Q_{v_{(2)}} \end{bmatrix}, & k \ge d \end{cases}$$

The optimal robust estimation problem is described as follows. Given an integer l ($l < d$) and observation $\left\{\{\bar{\boldsymbol{y}}(k)\}_{k=0}^{N}\right\}$, find an optimal robust estimator $\hat{\boldsymbol{x}}(k-l|k)$ of $\boldsymbol{x}(k)$ to minimize the following mean square estimation error

$$\varepsilon\left\{[\boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k-l|k)]^{\mathrm{T}}[\boldsymbol{x}(k) - \hat{\boldsymbol{x}}(k-l|k)]\right\}$$

where ε is the mathematical expectation over $\boldsymbol{u}(k)$, $\boldsymbol{v}_{(1)}(k)$, $\boldsymbol{v}_{(2)}(k)$, and $\xi_i(k)$, $i = 1, \cdots, m$. Depending on l, the optimal robust estimator $\hat{\boldsymbol{x}}(k-l|k)$ will be an optimal robust filter ($l = 0$), or an optimal robust fixed-lag smoother ($0 < l < d$).

Remark 1. The above problems have important applications in the real world, such as signal detection for the IS-136 communication system[16−17], the design of equalizers for the communication channel, and speech processing[5−6]. Different from the previous works, the problem considered in this paper is a complicated problem which is involved in the delayed measurements .

2 Optimal robust estimation

In this section, we shall present the optimal robust estimation based on the reorganized innovation analysis approach and projection theory in Hilbert space.

2.1 Fictitious noise

First, systems (1)~(3) can be rewritten as

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + \bar{\boldsymbol{u}}(k) &(4)\\ \boldsymbol{y}(k) &= C\boldsymbol{x}(k) + \bar{\boldsymbol{v}}_{(1)}(k) &(5)\\ \boldsymbol{z}(k) &= H\boldsymbol{x}(k_d) + \bar{\boldsymbol{v}}_{(2)}(k) &(6) \end{aligned}$$

where

$$\bar{\boldsymbol{u}}(k) = \sum_{i=1}^{m} A_i(k)\xi_i(k)\boldsymbol{x}(k) + B\boldsymbol{u}(k) \quad (7)$$

$$\bar{\boldsymbol{v}}_{(1)}(k) = \sum_{i=1}^{m} C_i(k)\xi_i(k)\boldsymbol{x}(k) + \boldsymbol{v}_{(1)}(k) \quad (8)$$

$$\bar{\boldsymbol{v}}_{(2)}(k) = \sum_{i=1}^{m} H_i(k_d)\xi_i(k_d)\boldsymbol{x}(k_d) + \boldsymbol{v}_{(2)}(k) \quad (9)$$

In (4)~(6), $\bar{\boldsymbol{u}}(k)$ is viewed as the fictitious system noise and $\bar{\boldsymbol{v}}_1(k)$ and $\bar{\boldsymbol{v}}_2(k)$ are viewed as the fictitious observation noises. Now, we calculate expectations and covariance matrices of $\bar{\boldsymbol{u}}(k)$, $\bar{\boldsymbol{v}}_{(1)}(k)$, and $\bar{\boldsymbol{v}}_{(2)}(k)$, respectively. From (7)~(9) and Assumption 1 on the noises $\boldsymbol{u}(k)$, $\boldsymbol{v}_{(1)}(k)$, $\boldsymbol{v}_{(2)}(k)$, and $\xi_i(k)$, $i = 1, \cdots, m$, it is clear that $\bar{\boldsymbol{u}}(k)$, $\bar{\boldsymbol{v}}_{(1)}(k)$, and $\bar{\boldsymbol{v}}_{(2)}(k)$ are random processes with zero means. To calculate the covariance matrices of $\bar{\boldsymbol{u}}(k)$, $\bar{\boldsymbol{v}}_{(1)}(k)$, and $\bar{\boldsymbol{v}}_{(2)}(k)$, we give the following definition

$$D(k) = \varepsilon\left[\boldsymbol{x}(k)\boldsymbol{x}^{\mathrm{T}}(k)\right] \quad (10)$$

By applying (4), it is not difficult to know that $D(k)$ satisfies the following Lyapunov equation

$$D(k+1) = AD(k)A^{\mathrm{T}} + \sum_{i=1}^{m} A_i(k)D(k)A_i^{\mathrm{T}}(k) + BQ_uB^{\mathrm{T}} \quad (11)$$

with the initial value $D(0) = \varepsilon\left[\boldsymbol{x}(0)\boldsymbol{x}^{\mathrm{T}}(0)\right]$. The covariance matrices of $\bar{\boldsymbol{u}}(k)$, $\bar{\boldsymbol{v}}_{(1)}(k)$, and $\bar{\boldsymbol{v}}_{(2)}(k)$ are given in the following lemma.

Lemma 1. The covariance matrices of fictitious noises $\bar{\boldsymbol{u}}(k)$, $\bar{\boldsymbol{v}}_{(1)}(k)$, and $\bar{\boldsymbol{v}}_{(2)}(k)$ can be calculated by

$$\varepsilon\left(\begin{bmatrix} \bar{\boldsymbol{u}}(k) \\ \bar{\boldsymbol{v}}_{(1)}(k) \\ \bar{\boldsymbol{v}}_{(2)}(k) \end{bmatrix}\begin{bmatrix} \bar{\boldsymbol{u}}(k) \\ \bar{\boldsymbol{v}}_{(1)}(k) \\ \bar{\boldsymbol{v}}_{(2)}(k) \end{bmatrix}^{\mathrm{T}}\right) = \begin{bmatrix} \hat{A}^{(1)}(k) & \hat{A}^{(2)}(k) & 0 \\ \hat{A}^{(2)T}(k) & \hat{C}(k) & 0 \\ 0 & 0 & \hat{H}(k) \end{bmatrix} \quad (12)$$

where

$$\begin{aligned} \hat{A}^{(1)}(k) &= \sum_{i=1}^{m} A_i(k)D(k)A_i^{\mathrm{T}}(k) + BQ_uB^{\mathrm{T}} \\ \hat{C}(k) &= \sum_{i=1}^{m} C_i(k)D(k)C_i^{\mathrm{T}}(k) + Q_{v_{(1)}} \\ \hat{H}(k) &= \sum_{i=1}^{m} H_i(k_d)D(k_d)H_i^{\mathrm{T}}(k_d) + Q_{v_{(2)}} \\ \hat{A}^{(2)}(k) &= \sum_{i=1}^{m} A_i(k)D(k)C_i^{\mathrm{T}}(k) \end{aligned}$$

and $D(k)$ is obtained by (11).

Proof. Since $\boldsymbol{u}(k)$, $\boldsymbol{v}_{(1)}(k)$, $\boldsymbol{v}_{(2)}(k)$, and $\xi_i(k)$ are mutually uncorrelated white noises, the covariance matrices of $\bar{\boldsymbol{u}}(k)$, $\bar{\boldsymbol{v}}_{(1)}(k)$, and $\bar{\boldsymbol{v}}_{(2)}(k)$ can be directly computed as (12) by making use of (7)~(9). □

2.2 Reorganized innovation

Having calculated the covariance matrices of the fictitious noises $\bar{\boldsymbol{u}}(k)$, $\bar{\boldsymbol{v}}_{(1)}(k)$, and $\bar{\boldsymbol{v}}_{(2)}(k)$, we now present the optimal robust estimation for systems (1)~(3) by using the Kalman filtering formulation. However, note that $\boldsymbol{z}(k)$ is an additional measurement of state $\boldsymbol{x}(k_d)$, which is received at time instant k, so the measurement $\bar{\boldsymbol{y}}(k)$ for $k \ge d$ contains time-delay d. The linear space spanned by the measurement $\left\{\{\bar{\boldsymbol{y}}(s)\}_{s=0}^{k}\right\}$ is denoted as $\mathcal{L}\left\{\{\bar{\boldsymbol{y}}(s)\}_{s=0}^{k}\right\}$. It is easy to know that the linear space $\mathcal{L}\left\{\{\bar{\boldsymbol{y}}(s)\}_{s=0}^{k}\right\}$ can be equivalently rewritten as

$$\mathcal{L}\left\{\{\boldsymbol{y}_2(i)\}_{i=0}^{k_d}, \boldsymbol{y}_1(k_d+1), \cdots, \boldsymbol{y}_1(k)\right\}$$

where the new observations $\boldsymbol{y}_2(s)$ and $\boldsymbol{y}_1(s)$ are given as follows.

$$\begin{aligned} \boldsymbol{y}_2(s) &= \begin{bmatrix} \boldsymbol{y}(s) \\ \boldsymbol{z}(s+d) \end{bmatrix}, & 0 \le s \le k_d \\ \boldsymbol{y}_1(s) &= \boldsymbol{y}(s), & k_d < s \le k \end{aligned}$$

It is clear that $\boldsymbol{y}_1(s)$ and $\boldsymbol{y}_2(s)$ satisfy

$$\begin{aligned} \boldsymbol{y}_1(s) &= H_1\boldsymbol{x}(s) + \boldsymbol{v}_1(s) &(13)\\ \boldsymbol{y}_2(s) &= H_2\boldsymbol{x}(s) + \boldsymbol{v}_2(s) &(14) \end{aligned}$$

where

$$H_1 = C, \qquad H_2 = \begin{bmatrix} C \\ H \end{bmatrix}$$

and

$$\boldsymbol{v}_1(s) = \bar{\boldsymbol{v}}_{(1)}(s), \qquad \boldsymbol{v}_2(s) = \begin{bmatrix} \bar{\boldsymbol{v}}_{(1)}(s) \\ \bar{\boldsymbol{v}}_{(2)}(s+d) \end{bmatrix}$$

Obviously, the new measurements $\boldsymbol{y}_1(s)$ and $\boldsymbol{y}_2(s)$ are delay-free, and the associated measurement noises $\boldsymbol{v}_1(s)$ and $\boldsymbol{v}_2(s)$ are white noises with zero means and covariance matrices, namely, $Q_{v_1}(s) = \hat{C}(s)$ and $Q_{v_2}(s) = \begin{bmatrix} \hat{C}(s) & 0 \\ 0 & \hat{H}(s+d) \end{bmatrix}$, respectively. The optimal robust estimator $\hat{\boldsymbol{x}}(k-l \mid k)$ becomes the projection of $\boldsymbol{x}(k-l)$ onto the linear space of $\mathcal{L}\left\{\{\boldsymbol{y}_2(i)\}_{i=0}^{k_d}, \boldsymbol{y}_1(k_d+1), \cdots, \boldsymbol{y}_1(k)\right\}$. To calculate the projection, we shall introduce the following stochastic sequence associated with the new observation $\left\{\{\boldsymbol{y}_2(i)\}_{i=0}^{k_d}, \boldsymbol{y}_1(k_d+1), \cdots, \boldsymbol{y}_1(k)\right\}$.

$$\begin{aligned} \boldsymbol{w}(s,1) &= \boldsymbol{y}_1(s) - \hat{\boldsymbol{y}}_1(s,1), \qquad s > k_d & (15)\\ \boldsymbol{w}(s,2) &= \boldsymbol{y}_2(s) - \hat{\boldsymbol{y}}_2(s,2), \qquad 0 \le s \le k_d & (16)\end{aligned}$$

where $\hat{\boldsymbol{y}}_1(s,1)$ is the projection of $\boldsymbol{y}_1(s)$ onto the linear space of $\mathcal{L}\left\{\{\boldsymbol{y}_2(i)\}_{i=0}^{k_d}, \boldsymbol{y}_1(k_d+1), \cdots, \boldsymbol{y}_1(s-1)\right\}$ and $\hat{\boldsymbol{y}}_2(s,2)$ is the projection of $\boldsymbol{y}_2(s)$ onto the linear space of $\mathcal{L}\{\{\boldsymbol{y}_2(i)\}_{i=0}^{s-1}\}$. Based on the discussion in [13], it is known that $\{\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k_d,2), \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(k,1)\}$ is uncorrelated white noise and spans the same linear space as

$$\mathcal{L}\left\{\{\boldsymbol{y}_2(i)\}_{i=0}^{k_d}, \boldsymbol{y}_1(k_d+1), \cdots, \boldsymbol{y}_1(k)\right\}$$

$\{\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k_d,2), \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(k,1)\}$ is called the reorganized innovation sequence. Next, we shall calculate the reorganized innovation $\boldsymbol{w}(s,1)$ and $\boldsymbol{w}(s,2)$. In view of (13)~(14) and (15)~(16), the following equations can be obtained

$$\begin{aligned} \boldsymbol{w}(s,1) &= H_1\tilde{\boldsymbol{x}}(s,1) + \boldsymbol{v}_1(s) & (17)\\ \boldsymbol{w}(s,2) &= H_2\tilde{\boldsymbol{x}}(s,2) + \boldsymbol{v}_2(s) & (18)\end{aligned}$$

with

$$\begin{aligned} \tilde{\boldsymbol{x}}(s,1) &= \boldsymbol{x}(s) - \hat{\boldsymbol{x}}(s,1)\\ \tilde{\boldsymbol{x}}(s,2) &= \boldsymbol{x}(s) - \hat{\boldsymbol{x}}(s,2)\end{aligned}$$

where $\hat{\boldsymbol{x}}(s,1)$ and $\hat{\boldsymbol{x}}(s,2)$ are defined as in $\hat{\boldsymbol{y}}_1(s,1)$ and $\hat{\boldsymbol{y}}_2(s,2)$.

We define the covariance matrices of one-step ahead state estimation error as follows

$$P_1(s) = \varepsilon[\tilde{\boldsymbol{x}}(s,1)\tilde{\boldsymbol{x}}^{\mathrm{T}}(s,1)] \quad (19)$$

$$P_2(s) = \varepsilon[\tilde{\boldsymbol{x}}(s,2)\tilde{\boldsymbol{x}}^{\mathrm{T}}(s,2)] \quad (20)$$

From (17)~(18), the reorganized innovation covariance matrices are given by

$$\begin{aligned} Q_w(s,1) &= \varepsilon[\boldsymbol{w}(s,1)\boldsymbol{w}^{\mathrm{T}}(s,1)] = \\ &\quad H_1P_1(s)H_1^{\mathrm{T}} + \hat{C}(s) & (21)\\ Q_w(s,2) &= \varepsilon[\boldsymbol{w}(s,2)\boldsymbol{w}^{\mathrm{T}}(s,2)] = \\ &\quad H_2P_2(s)H_2^{\mathrm{T}} + \begin{bmatrix} \hat{C}(s) & 0 \\ 0 & \hat{H}(s+d) \end{bmatrix} & (22)\end{aligned}$$

We shall give the following lemma to calculate the covariance matrices of one-step ahead state estimation error.

Lemma 2. The covariance matrices $P_2(s+1)$ and $P_1(s+1)$ obey the following standard Riccati difference equation, respectively

$$\begin{aligned} P_2(s+1) = {} & AP_2(s)A^{\mathrm{T}} + \hat{A}^{(1)}(s) - \\ & (AP_2(s)H_2^{\mathrm{T}} + [\ \hat{A}^{(2)}(s) \quad 0\ ])Q_w^{-1}(s,2) \times \\ & (AP_2(s)H_2^{\mathrm{T}} + [\ \hat{A}^{(2)}(s) \quad 0\ ])^{\mathrm{T}} \qquad (23)\end{aligned}$$

and

$$\begin{aligned} P_1(s+1) = {} & AP_1(s)A^{\mathrm{T}} + \hat{A}^{(1)}(s) - \\ & (AP_1(s)H_1^{\mathrm{T}} + \hat{A}^{(2)}(s))Q_w^{-1}(s,1) \times \\ & (AP_1(s)H_1^{\mathrm{T}} + \hat{A}^{(2)}(s))^{\mathrm{T}} \qquad (24)\end{aligned}$$

where $Q_w(s,2)$ and $Q_w(s,1)$ are calculated by (22) and (21), respectively.

Proof. Note that $\hat{\boldsymbol{x}}(s+1,1)$ is the projection of $\boldsymbol{x}(s+1)$ onto the linear space $\mathcal{L}\{\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k_d,2), \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(s,1)\}$. Using the projection formula, we get

$$\begin{aligned} \hat{\boldsymbol{x}}(s+1,1) = {} & \mathrm{Proj}\{\boldsymbol{x}(s+1)|\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k_d,2), \\ & \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(s,1)\} = \\ & A\hat{\boldsymbol{x}}(s,1) + \varepsilon\left[\boldsymbol{x}(s+1)\boldsymbol{w}^{\mathrm{T}}(s,1)\right] \times \\ & Q_w^{-1}(s,1)\boldsymbol{w}(s,1) = \\ & A\hat{\boldsymbol{x}}(s,1) + \\ & \varepsilon\left[(A\boldsymbol{x}(s)+\bar{\boldsymbol{u}}(s))(H_1\tilde{\boldsymbol{x}}(s,1)+\boldsymbol{v}_1(s))^{\mathrm{T}}\right] \times \\ & Q_w^{-1}(s,1)\boldsymbol{w}(s,1) = \\ & A\hat{\boldsymbol{x}}(s,1) + (AP_1(s)H_1^{\mathrm{T}} + \hat{A}^{(2)}(s)) \times \\ & Q_w^{-1}(s,1)\boldsymbol{w}(s,1) \qquad (25)\end{aligned}$$

It follows from (19) and (25) that

$$\begin{aligned} \tilde{\boldsymbol{x}}(s+1,1) = {} & A\tilde{\boldsymbol{x}}(s,1) + \bar{\boldsymbol{u}}(s) - (AP_1(s)H_1^{\mathrm{T}} + \\ & \hat{A}^{(2)}(s))Q_w^{-1}(s,1)\boldsymbol{w}(s,1) \qquad (26)\end{aligned}$$

Since $\tilde{\boldsymbol{x}}(s+1,1)$ is uncorrelated with $\boldsymbol{w}(s,1)$, then (24) can be directly obtained by using (26) and (19). Similarly, we can prove (23). □

Remark 2. It is clear that (23) is a standard Riccati equation with initial value $P_2(0) = P_0$. It is worth pointing out that $P_2(k_d)$ is the initial value of the Riccati equation (24).

2.3 Main result

We are in the position to present the optimal robust filter based on the reorganized innovation approach and Riccati equation in the following theorem.

Theorem 1. Consider systems (1)~(3) with single delayed measurement. The optimal robust filter $\hat{\boldsymbol{x}}(k|k)$ can be calculated by

$$\begin{aligned} \hat{\boldsymbol{x}}(k|k) = {} & \hat{\boldsymbol{x}}(k,1) + P_1(k)H_1^{\mathrm{T}}Q_w^{-1}(k,1) \times \\ & [\boldsymbol{y}_1(k) - H_1\hat{\boldsymbol{x}}(k,1)] \qquad (27)\end{aligned}$$

where $\hat{\boldsymbol{x}}(k,1)$ is given by the following recursion.

$$\begin{aligned} \hat{\boldsymbol{x}}(s+1,1) = {} & A\hat{\boldsymbol{x}}(s,1) + (AP_1(s)H_1^{\mathrm{T}} + \hat{A}^{(2)}(s)) \times \\ & Q_w^{-1}(s,1)[\boldsymbol{y}_1(s) - H_1\hat{\boldsymbol{x}}(s,1)] \qquad (28)\end{aligned}$$

with initial value $\hat{\boldsymbol{x}}(k_d+1,1) = \hat{\boldsymbol{x}}(k_d+1,2)$ can be computed by

$$\begin{aligned}\hat{\boldsymbol{x}}(s+1,2) \;=\;& A\hat{\boldsymbol{x}}(s,2) + (AP_2(s)H_2^{\mathrm{T}} + \\ & [\ \hat{A}^{(2)}(s) \quad 0\ ]) \times \\ & Q_w^{-1}(s,2)[\boldsymbol{y}_2(s) - H_2\hat{\boldsymbol{x}}(s,2)] \end{aligned} \tag{29}$$

with initial value $\hat{\boldsymbol{x}}(0,2) = \boldsymbol{x}(0)$.

Proof. Since $\hat{\boldsymbol{x}}(k|k)$ is the projection of $\boldsymbol{x}(k)$ onto the linear space $\mathcal{L}\{\{\boldsymbol{w}(i,2)\}_{i=0}^{k_d}, \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(k,1)\}$, by applying projection formula, we can obtain

$$\begin{aligned}\hat{\boldsymbol{x}}(k|k) \;=\;& \mathrm{Proj}\{\boldsymbol{x}(k)|\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k_d,2), \\ & \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(k,1)\} = \\ & \hat{\boldsymbol{x}}(k,1) + \varepsilon\left[\boldsymbol{x}(k)\boldsymbol{w}^{\mathrm{T}}(k,1)\right] \times \\ & Q_w^{-1}(k,1)\boldsymbol{w}(k,1) = \\ & \hat{\boldsymbol{x}}(k,1) + P_1(k)H_1^{\mathrm{T}}Q_w^{-1}(k,1) \times \\ & [\boldsymbol{y}_1(k) - H_1\hat{\boldsymbol{x}}(k,1)]\end{aligned}$$

which is (27).

For $s > k_d$, since $\hat{\boldsymbol{x}}(s+1,1)$ is the projection of $\boldsymbol{x}(s)$ onto the linear space of $\mathcal{L}\{\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k_d,2), \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(s,1)\}$, $\hat{\boldsymbol{x}}(s+1,1)$ can be expressed by

$$\begin{aligned}\hat{\boldsymbol{x}}(s+1,1) \;=\;& \mathrm{Proj}\{\boldsymbol{x}(s+1)|\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k_d,2), \\ & \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(s,1)\} = \\ & A\hat{\boldsymbol{x}}(s,1) + \varepsilon\left[\boldsymbol{x}(s+1)\boldsymbol{w}^{\mathrm{T}}(s,1)\right] \times \\ & Q_w^{-1}(s,1)\boldsymbol{w}(s,1) = \\ & A\hat{\boldsymbol{x}}(s,1) + (AP_1(s)H_1^{\mathrm{T}} + \hat{A}^{(2)}(s)) \times \\ & Q_w^{-1}(s,1)[\boldsymbol{y}_1(s) - H_1\hat{\boldsymbol{x}}(s,1)]\end{aligned}$$

which is (28). Further, note that $\hat{\boldsymbol{x}}(s+1,2)$ is the projection of $\boldsymbol{x}(s)$ onto the linear space $\mathcal{L}\{\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(s,2)\}$. By making use of the similar approach, (29) is obtained. □

Similarly, we address the optimal robust fixed-lag smoother, which is given in the following theorem.

Theorem 2. Consider systems (1)∼(3) and give an integer l satisfying $0 < l < d$. The optimal robust fixed-lag smoother $\hat{\boldsymbol{x}}(k-l|k)$ is formulated as

$$\begin{aligned}\hat{\boldsymbol{x}}(k-l|k) \;=\;& \hat{\boldsymbol{x}}(k-l,1) + \sum_{i=0}^{l}\mathcal{S}(k-i) \times \\ & [\boldsymbol{y}_1(k-i) - H_1\hat{\boldsymbol{x}}(k-i,1)]\end{aligned} \tag{30}$$

where

$$\begin{aligned}\mathcal{S}(k-i) =\;& P_1(k-l)M^{\mathrm{T}}(k-l)\cdots M^{\mathrm{T}}(k-i-1) \times \\ & H_1^{\mathrm{T}}Q_w^{-1}(k-i,1) \\ M(s) =\;& A - \left[AP_1(s)H_1^{\mathrm{T}} + \hat{A}^{(2)}(s)\right] \times \\ & Q_w^{-1}(s,1)H_1, \quad s = k_d+1, \cdots, k\end{aligned}$$

and $\hat{\boldsymbol{x}}(k-i,1)$, $i = 0, \cdots, l$, is computed by (28).

Proof. Note that $\hat{\boldsymbol{x}}(k-l|k)$ is the projection of $\boldsymbol{x}(k-l)$ onto the linear space $\mathcal{L}\{\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k,1)\}$. Using projection formula, we obtain that

$$\begin{aligned}\hat{\boldsymbol{x}}(k-l|k) =\;& \mathrm{Proj}\{\boldsymbol{x}(k-l)|\boldsymbol{w}(0,2), \cdots, \boldsymbol{w}(k_d,2), \\ & \boldsymbol{w}(k_d+1,1), \cdots, \boldsymbol{w}(k,1)\} = \\ & \hat{\boldsymbol{x}}(k-l,1) + \sum_{i=0}^{l}\varepsilon\left[\boldsymbol{x}(k-l)\boldsymbol{w}^{\mathrm{T}}(k-i,1)\right] \times \\ & Q_w^{-1}(k-i,1)\boldsymbol{w}(k-i,1)\end{aligned} \tag{31}$$

From (31), it is easily known that the key is to calculate $\varepsilon\left[\boldsymbol{x}(k-l)\boldsymbol{w}^{\mathrm{T}}(k-i,1)\right]$. In view of (17), we have

$$\boldsymbol{w}(k-i,1) = H_1\tilde{\boldsymbol{x}}(k-i,1) + \boldsymbol{v}_1(k-i) \tag{32}$$

(26) can be expressed as

$$\begin{aligned}\tilde{\boldsymbol{x}}(s+1,1) \;=\;& M(s)\tilde{\boldsymbol{x}}(s,1) + \bar{\boldsymbol{u}}(s) - \\ & \left[AP_1(s)H_1^{\mathrm{T}} + \hat{A}^{(2)}(s)\right] \times \\ & Q_w^{-1}(s,1)\boldsymbol{v}_1(s)\end{aligned} \tag{33}$$

where $M(s) = A - \left[AP_1(s)H_1^{\mathrm{T}} + \hat{A}^{(2)}(s)\right]Q_w^{-1}(s,1)H_1$, and $s = k_d+1, \cdots, k$. Similar to (33), $\tilde{\boldsymbol{x}}(k-i)$ in (32) is easily written as

$$\begin{aligned}\tilde{\boldsymbol{x}}(k-i,1) =\;& M(k-i-1)M(k-i-2)\cdots M(k-l) \times \\ & \tilde{\boldsymbol{x}}(k-l,1) + f(\cdot)\end{aligned} \tag{34}$$

where $f(\cdot)$ is a linear function of $\bar{\boldsymbol{u}}(k-i-1), \cdots, \bar{\boldsymbol{u}}(k-l)$, $\boldsymbol{v}_1(k-i), \cdots, \boldsymbol{v}_1(k-l)$, which are uncorrelated with $\boldsymbol{x}(k-l)$.

Substituting (34) into (32) yields

$$\begin{aligned}\boldsymbol{w}(k-i,1) \;=\;& H_1M(k-i-1) \times \\ & M(k-i-2)\cdots M(k-l) \times \\ & \tilde{\boldsymbol{x}}(k-l,1) + f(\cdot)\end{aligned} \tag{35}$$

It follows from (35) that

$$\begin{aligned}\varepsilon\left[\boldsymbol{x}(k-l)\boldsymbol{w}^{\mathrm{T}}(k-i,1)\right] \;=\;& P_1(k-l)M^{\mathrm{T}}(k-l) \times \cdots \times \\ & M^{\mathrm{T}}(k-i-1)H_1^{\mathrm{T}}\end{aligned} \tag{36}$$

Moreover, with the support of (31) and (36), we can easily obtain (30). □

Remark 3. We have studied the optimal robust fixed-lag smoothing problem for the case of $0 < l < d$ in Theorem 2. It should be pointed out that the optimal robust fixed-lag smoothing problem for the case of $l \geq d$ can also be solved via a similar approach.

3 Comparison of computational costs

The purpose of this section is to compare the computational costs of the presented approach and the state augmented method for the optimal robust filter.

Now, we introduce an augmented state

$$\boldsymbol{x}_a^{\mathrm{T}}(k) = \left[\boldsymbol{x}^{\mathrm{T}}(k) \quad \boldsymbol{x}^{\mathrm{T}}(k-1) \quad \cdots \quad \boldsymbol{x}^{\mathrm{T}}(k_d)\right]$$

Then the original systems (1)∼(3) can be written as an augmented state space model

$$\boldsymbol{x}_a(k+1) = \bar{A}_a(k)\boldsymbol{x}_a(k) + B_a\boldsymbol{u}(k) \tag{37}$$

$$\boldsymbol{y}_a(k) = \bar{H}_a(k)\boldsymbol{x}_a(k) + \boldsymbol{v}(k) \tag{38}$$

where

$$\begin{aligned} \boldsymbol{y}_a^{\mathrm{T}}(k) &= \begin{bmatrix} \boldsymbol{y}^{\mathrm{T}}(k) & \boldsymbol{z}^{\mathrm{T}}(k) \end{bmatrix} \\ \boldsymbol{v}^{\mathrm{T}}(k) &= \begin{bmatrix} \boldsymbol{v}_{(1)}^{\mathrm{T}}(k) & \boldsymbol{v}_{(2)}^{\mathrm{T}}(k) \end{bmatrix} \\ B_a^T &= \begin{bmatrix} B^{\mathrm{T}} & 0 & \cdots & 0 \end{bmatrix} \end{aligned}$$

and

$$\bar{H}_a(k) = \begin{bmatrix} C + \sum_{i=1}^{m} C_i(k)\xi_i(k) & \cdots & 0 \\ 0 & \cdots & H + \sum_{i=1}^{m} H_i(k_d)\xi_i(k_d) \end{bmatrix}$$

$$\bar{A}_a(k) = \begin{bmatrix} A + \sum_{i=1}^{m} A_i(k)\xi_i(k) & & & & \\ I_n & 0 & & & \\ & I_n & \ddots & & \\ & & \ddots & \ddots & \\ & & & I_n & 0 \end{bmatrix}$$

Note that (37) and (38) are easily rewritten as follows

$$\boldsymbol{x}_a(k+1) = A_a\boldsymbol{x}_a(k) + \boldsymbol{u}_a(k) \tag{39}$$

$$\boldsymbol{y}_a(k) = H_a\boldsymbol{x}_a(k) + \bar{\boldsymbol{v}}(k) \tag{40}$$

where

$$\begin{aligned} \boldsymbol{u}_a^{\mathrm{T}}(k) &= \begin{bmatrix} \bar{\boldsymbol{u}}^{\mathrm{T}}(k) & 0 & \cdots & 0 \end{bmatrix} \\ \bar{\boldsymbol{v}}^{\mathrm{T}}(k) &= \begin{bmatrix} \bar{\boldsymbol{v}}_{(1)}^{\mathrm{T}}(k) & \bar{\boldsymbol{v}}_{(2)}^{\mathrm{T}}(k) \end{bmatrix} \end{aligned}$$

$$H_a = \begin{bmatrix} C & \cdots & 0 \\ 0 & \cdots & H \end{bmatrix}, A_a = \begin{bmatrix} A & & & & \\ I_n & 0 & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ & & & I_n & 0 \end{bmatrix}.$$

Obviously, the optimal robust filter $\hat{\boldsymbol{x}}(k|k)$ is obtained by

$$\hat{\boldsymbol{x}}(k|k) = \begin{bmatrix} I_n & 0 & \cdots & 0 \end{bmatrix} \hat{\boldsymbol{x}}_a(k|k)$$

where $\hat{\boldsymbol{x}}_a(k|k)$ is the optimal robust filter of the above augmented systems (39) and (40). In view of [2], the optimal robust filter $\hat{\boldsymbol{x}}_a(k|k)$ is directly given by

$$\begin{aligned} \hat{\boldsymbol{x}}_a(k+1|k+1) = {} & [A_a + P_a(k+1)H_a^{\mathrm{T}}Q_w^{-1}(k+1)H_aA_a] \times \\ & \hat{\boldsymbol{x}}_a(k|k) + P_a(k+1)H_a^{\mathrm{T}} \times \\ & Q_w^{-1}(k+1)\boldsymbol{y}_a(k+1) \end{aligned}$$

where the matrix $P_a(k)$ satisfies the following Riccati difference equation

$$\begin{aligned} P_a(k+1) = {} & A_aP_a(k)A_a^{\mathrm{T}} + Q_{u_a}(k) - \\ & (A_aP_a(k)H_a^{\mathrm{T}} + \begin{bmatrix} \hat{A}^{(2)}(k) & 0 \end{bmatrix})Q_w^{-1}(k) \times \\ & (A_aP_a(k)H_a^{\mathrm{T}} + \begin{bmatrix} \hat{A}^2(k) & 0 \end{bmatrix})^{\mathrm{T}} \end{aligned}$$

with $Q_w(k) = H_aP_a(k)H_a^{\mathrm{T}} + \begin{bmatrix} \hat{C}(k) & 0 \\ 0 & \hat{H}(k) \end{bmatrix}$ and $Q_{u_a}(k) = \begin{bmatrix} \hat{A}^{(1)}(k) & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$.

Traditionally, since additions are much faster than multiplications and divisions, it is the number of multiplications and divisions that is used as the operational count. Let $MD_{filter,aug}$ and $MD_{filter,new}$ denote the numbers of multiplications and divisions for the augmented method, and the new approach in one step, respectively. We suppose that $p = q$ and the multiplications and divisions of the algorithm are from left to right. Then $MD_{filter,aug}$ and $MD_{filter,new}$ are directly computed as

$$\begin{aligned} MD_{filter,aug} = {} & 4(n+nd)^3 + 8p(n+nd)^2 + (n+nd)^2 + \\ & 8p^2(n+nd) + 2p(n+nd) + \\ & 8p^3 + n(n+nd) + n^3 + 3n^3m + \\ & nr^2 + n^2r + 2np^2m + 3n^2pm \end{aligned} \tag{41}$$

$$\begin{aligned} MD_{filter,new} = {} & 3n^3 + 9n^2p + 7np^2 + 3n^3m + nr^2 + \\ & n^2r + 3n^2pm + 2np^2m + p^3 + \\ & [4n^3 + 8n^2p + 4np^2 + 8p^3]d \end{aligned} \tag{42}$$

According to (41) and (42), we have that the order of delay d in $MD_{filter,aug}$ is 3 whereas the order of d in $MD_{filter,new}$ is 1. Thus, when the delay d is large, it is easy to know that $MD_{filter,new} \ll MD_{filter,aug}$, which implies that the presented approach is more effective. To show this point, a numerical example is given.

Example 1. Consider systems (1)$\sim$(3) with $n = 1, m = 9, r = 3$, and $p = q = 1$. The MD numbers of the proposed approach and the state augmented approach are compared in Table 1 for various values of d.

Table 1 Computational cost for various time-delay d

d	1	3	5	8
$MD_{filter,new}$	128	176	224	296
$MD_{filter,aug}$	183	537	1 347	3 837

Remark 4. It is worth emphasizing that via a way similar to the above, the comparison of the operational counts costed by the optimal robust smoothers, which are provided by the traditional augmentation method and the presented approach, can easily be given.

4 Numerical example

In this section, a numerical example will be given to verify the efficiency of the proposed approach. Consider systems (1)$\sim$(3) with $d = 30$, $m = 3$,

$$\begin{aligned} A &= \begin{bmatrix} 0.96 & 0 \\ 0.9 & 0.2 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0.38 \\ 0.6 \end{bmatrix} \\ \boldsymbol{C} &= \begin{bmatrix} 0.34 & 0 \end{bmatrix}, \quad \boldsymbol{H} = \begin{bmatrix} 1 & 0.36 \end{bmatrix} \end{aligned}$$

and

$$\begin{aligned} A_i(k) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{C}_i(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \\ \boldsymbol{H}_i(k_d) &= \begin{bmatrix} 1 & 0 \end{bmatrix}, \qquad i = 1, 2, 3. \end{aligned}$$

Here, $\boldsymbol{u}(k)$, $\boldsymbol{v}_{(1)}(k)$, $\boldsymbol{v}_{(2)}(k)$, and $\xi_i(k)$, $i = 1, 2, 3$, are mutually uncorrelated white noises with zero means and covariances 0.5. By applying Theorem 1 and Theorem 2, the optimal robust filter and the optimal robust fixed-lag smoother ($l = 5$) are designed, respectively. The simulation results are shown in Figs. 1$\sim$4, respectively.

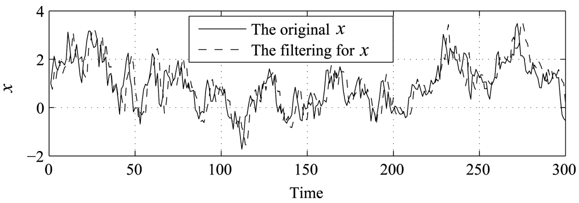

Fig. 1 Tracking performance of the filter $\hat{\boldsymbol{x}}_1(k|k)$

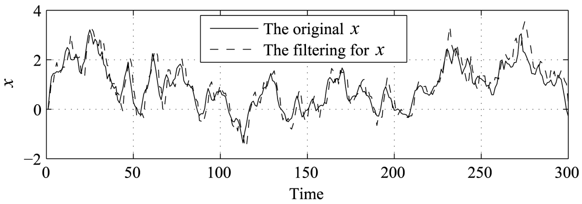

Fig. 2 Tracking performance of the filter $\hat{\boldsymbol{x}}_2(k|k)$

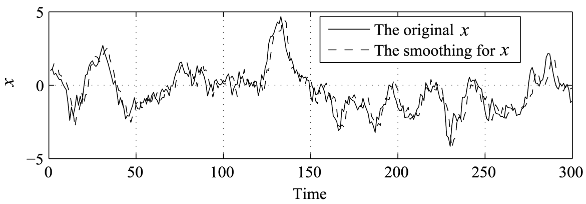

Fig. 3 Tracking performance of the smoother $\hat{\boldsymbol{x}}_1(k-l|k)$

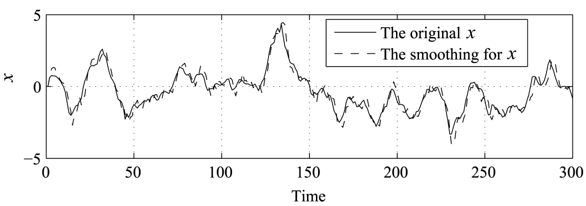

Fig. 4 Tracking performance of the smoother $\hat{\boldsymbol{x}}_2(k-l|k)$

It can be observed from Figs. 1∼4 that the optimal robust estimator can track the original state very well.

5 Conclusion

In this paper, we have studied the optimal robust estimation problem for the linear discrete uncertain systems with single delayed measurement. By applying the reorganized innovation analysis theory, a new approach, which is new to our knowledge, has been presented. The optimal robust estimators are designed by calculating two Riccati difference equations of the same dimensions as that of the original systems and one Lyapunov equation. Compared with the state augmentation method, the presented approach is much simpler for derivation and calculation, especially when the time-delay is large. It is worth pointing out that the presented approach is easily extended to systems with multiple delayed measurements.

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