

基于全驱系统方法的高阶严反馈系统时变输出约束控制

蔡光斌¹ 肖永强¹ 胡昌华¹ 杨小冈¹ 凡永华²

摘要 针对输出受不对称时变约束的不确定高阶严反馈系统, 提出一种基于全驱系统方法的高阶自适应动态面输出约束控制方法. 所研究的高阶严反馈系统, 每个子系统都是高阶形式, 通过非线性转换函数将原输出约束系统转换为新的无约束系统, 从而将原系统输出约束问题转化为新系统输出有界的问题. 进一步结合全驱系统方法和自适应动态面控制, 直接将每个高阶子系统作为一个整体进行控制器设计, 而不需要将其转化为一阶系统形式, 有效简化了设计步骤; 同时通过引入一系列低通滤波器来获得虚拟控制律的高阶导数, 以代替复杂的微分运算. 基于 Lyapunov 稳定性理论证明闭环系统所有信号是一致最终有界的, 系统输出在满足约束的条件下能有效跟踪期望的参考信号, 且可通过调整参数使得系统跟踪误差收敛到零附近的足够小的邻域内. 最后, 通过对柔性关节机械臂系统进行仿真, 验证了所提出控制方法的有效性.

关键词 全驱系统方法, 高阶严反馈系统, 时变输出约束, 非线性转换函数, 自适应动态面控制

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Time-varying Output Constraint Control of High-order Strict-feedback Systems Based on Fully Actuated System Approach

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Abstract A high-order adaptive dynamic surface output constraint control method based on fully actuated system approach is proposed for uncertain high-order strict-feedback systems with asymmetric time-varying output constraints. The high-order strict-feedback system studied in this paper, each subsystem is a high-order form. The original output constraint system is transformed into a new unconstrained system by nonlinear transformation function, so that the original system output constraint problem is transformed into the new system output bounded problem. Furthermore, combined with the fully actuated system approach and the adaptive dynamic surface control method, the controller is designed directly for each high-order subsystem as a whole without converting it into a first-order system form, which effectively reduces the design steps. At the same time, a series of low-pass filters are introduced to obtain the high-order derivative of the virtual control law to replace the complex differential operation. Based on Lyapunov theory, it is proved that all signals of the closed-loop system are uniformly ultimately bounded. The system output can effectively track the desired reference signal without violating the constraints, and the system tracking error can be converged to any small neighborhood of the origin by adjusting the parameters. Finally, the effectiveness of the control method is verified by simulating the flexible joint manipulator system.

Key words Fully actuated system approach, high-order strict-feedback systems, time-varying output constraints, nonlinear transformation function, adaptive dynamic surface control

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实际系统由于安全和性能方面的要求, 以及一些物理器件的限制, 会不可避免地受到各种约束条件的限制, 如果约束条件得不到满足, 系统性能将可能急剧下降甚至出现不稳定. 为满足实际应用需求, 有必要研究系统在不同约束条件下的控制问题. 常见的约束条件包括: 输出约束、状态约束和输入约束等, 其中输出约束是指系统输出需满足特定的约束条件. 现有的输出约束控制方法主要有模型预测控制^[1]、参考轨线调节^[2] 以及基于不变集的方法^[3]

等,但这些方法均基于数值计算,计算量较大.与这些方法不同,基于障碍 Lyapunov 函数 (Barrier Lyapunov function, BLF) 的设计是处理输出约束问题的一种常用的方法,其基本思想是当自变量的值趋于某些区域边界时, BLF 的值趋于无穷大,通过保证 BLF 的有界性,进而使系统输出满足约束条件^[4]. 经过近些年的发展,所研究的问题由对称常值输出约束问题^[4]发展到非对称时变输出约束问题^[5],应用方面也包括在直升机系统^[6]、航天器交会对接^[7] 等领域的实际应用. 然而,大多数基于 BLF 的输出约束控制方法,需要将输出约束问题转换为系统跟踪误差约束问题,会造成系统输出初始值较为保守的问题.

针对上述问题,有学者提出了基于非线性转换函数的输出约束控制方法,将输出约束问题转换为非线性转换函数的有界性问题,即无需将其转换为跟踪误差约束问题,设计过程较为简单,且系统输出初值可以在整个约束区域内选取,放宽了对系统输出初值选取的要求^[8-9]. 基于这一方法,学者们研究了严反馈系统、切换系统、随机系统、纯反馈系统、MIMO (Multiple-input and multiple-output) 系统等定常输出约束以及时变输出约束控制问题^[8-14],同时在应用方面也研究了四旋翼飞行器^[15]、机器人^[16] 等实际系统的输出约束控制问题,在这一领域取得了较多的研究成果.

然而,上述方法是应用一阶系统方法(状态空间方法)进行研究的. 由于牛顿定律、拉格朗日定律、角动量定律等物理定律的应用,大多数实际系统最初被建模为二阶或高阶形式. 文献^[17-20]指出,当采用一阶系统方法设计控制器时,需要对二阶或高阶系统降阶,将其转化为一阶系统,这样会破坏系统的全驱特性、增大系统阶数、增加控制器的设计步骤. 针对这一问题,有学者提出了全驱系统方法,该方法直接对高阶系统设计控制器而不需要降阶,通过设计伪线性状态反馈控制律,将闭环系统转化为具有特定结构的线性定常系统,并基于参数化设计方法配置闭环系统的特征多项式系数矩阵实现期望的系统特性,从而满足系统控制需求^[17-20].

目前,基于全驱系统方法,在自适应控制、最优控制、预测控制、故障诊断、亚全驱系统控制^[21-27] 等领域取得了较多的研究成果,且都具有良好的控制效果. 同时全驱系统方法也出色应用到了航天器制导、控制,以及飞行器制导控制一体化^[28-30] 等领域. 尤其针对高阶严反馈系统,文献^[31-33] 基于全驱系统方法提出高阶反步法,不需要将高阶系统转化为多个一阶子系统,所需推导的步骤少,过程简洁高效. 同时,在其基础上,文献^[21-22, 34] 进一步

研究了高阶严反馈系统动态面、指令滤波控制等问题. 上述工作取得了较多的研究成果,但尚未考虑系统输出约束控制问题. 文献^[35] 研究了高阶严反馈系统的输出约束问题,但其研究的是基于 BLF 的对称常值输出约束控制问题.

由于许多实际系统的状态参数和控制参数是时变的,因此研究时变输出约束更具有一般性. 受上述研究工作的启发,本文研究高阶严反馈系统的非对称时变输出约束控制问题,旨在实现直接对高阶系统设计控制器,而不需要将其转化为一阶系统. 本文提出的高阶自适应动态面输出约束控制方法具有以下特点:

1) 不同于状态空间框架下一阶严反馈系统的输出约束问题,本文针对高阶严反馈系统,研究其输出约束问题. 基于全驱系统方法,直接研究高阶系统,不需要将其转化为一阶系统,减少了控制器设计的步骤,从而减轻计算负担.

2) 与基于 BLF 的输出在对称常值约束下的控制问题相比,本文在其基础上进一步研究输出在非对称时变约束下的控制问题. 非对称时变约束相比于对称常值约束更加复杂,更具一般性. 且与基于 BLF 的方法相比,本文基于非线性转换函数的方法能够将系统输出初值可行区间扩大为整个约束区间,避免了系统输出初始值较为保守的问题.

本文结构安排如下:第 1 节是问题描述,介绍不确定高阶严反馈系统,并给出一些必要的假设和引理;第 2 节和第 3 节分别给出高阶自适应动态面输出约束控制器的设计和稳定性证明,从理论上证明系统输出约束得到满足,且闭环系统所有信号是一致最终有界的;第 4 节基于柔性关节机械臂系统进行仿真,验证本文所提出控制方法的有效性;最后,在第 5 节给出本文的结论.

为了简化表达,定义一些符号如下: $\lambda_{\min}(P_i)$ 和 $\lambda_{\max}(P_i)$ 分别表示矩阵 P_i 的最小和最大特征值. 矩阵 $P_i > 0$ 表示矩阵 P_i 是一个正定对称矩阵. 以及

$$x^{(0\sim n)} = \begin{bmatrix} x \\ \dot{x} \\ \vdots \\ x^{(n)} \end{bmatrix}$$

$$x_{i\sim j}^{(0\sim n)} = \begin{bmatrix} x_i^{(0\sim n)} \\ x_{i+1}^{(0\sim n)} \\ \vdots \\ x_j^{(0\sim n)} \end{bmatrix}, \quad j \geq i$$

$$A^{0\sim n-1} = [A_0 \quad A_1 \quad \cdots \quad A_{n-1}]$$

$$\Phi(A^{0\sim n-1}) = \begin{bmatrix} 0 & \mathbf{I} & & \\ & & \ddots & \\ & & & \mathbf{I} \\ -A_0 & -A_1 & \cdots & -A_{n-1} \end{bmatrix}$$

1 问题描述

考虑含有模型不确定性的高阶严反馈系统^[33]

$$\left\{ \begin{aligned} x_1^{(m_1)} &= f_{10}(x_1^{(0\sim m_1-1)}) + \\ & f_1(x_1^{(0\sim m_1-1)})\theta_1 + \\ & g_1(x_1^{(0\sim m_1-1)})x_2 \\ x_2^{(m_2)} &= f_{20}(x_i^{(0\sim m_i-1)}|_{i=1\sim 2}) + \\ & f_2(x_i^{(0\sim m_i-1)}|_{i=1\sim 2})\theta_2 + \\ & g_2(x_i^{(0\sim m_i-1)}|_{i=1\sim 2})x_3 \\ & \vdots \\ x_n^{(m_n)} &= f_{n0}(x_i^{(0\sim m_i-1)}|_{i=1\sim n}) + \\ & f_n(x_i^{(0\sim m_i-1)}|_{i=1\sim n})\theta_n + \\ & g_n(x_i^{(0\sim m_i-1)}|_{i=1\sim n})u \end{aligned} \right. \quad (1)$$

其中, $x_i, i = 1, 2, \dots, n$ 为系统状态量, $m_i, i = 1, 2, \dots, n$ 为正整数, u 为系统输入, $f_{j0}(x_i^{(0\sim m_i-1)}|_{i=1\sim j})$ 和 $g_j(x_i^{(0\sim m_i-1)}|_{i=1\sim j}), j = 1, 2, \dots, n$ 为两组已知的光滑函数, $\theta_i, i = 1, 2, \dots, n$ 为未知的常数向量, $f_j(x_i^{(0\sim m_i-1)}|_{i=1\sim j})$ 为一组已知的函数向量. 系统输出 $y = x_1$ 需在时变且不对称开区间内运行, 即

$$x_1 \in D_1 = \{(x_1, t) \in \mathbf{R} \times \mathbf{R}_+ : -F_1(t) < x_1 < F_2(t)\} \quad (2)$$

其中, $F_1(t)$ 和 $F_2(t)$ 为已知时变约束边界函数.

注 1. 在工程实际中, 根据物理定律建模得到的系统模型一般是高阶的, 因此本文研究的高阶严反馈系统 (1) 相比于传统一阶形式的严反馈系统更贴近于物理实际. 同时, 全驱系统方法是基于系统的全驱特性直接对原始的高阶系统模型设计控制器, 而不需要将其转化为一阶形式. 针对高阶系统 (1), 本文无需将其转化为一阶形式, 直接对每个高阶子系统设计控制器, 尤其对反步法而言, 更少的设计步骤更有利于降低算法的复杂度.

本文控制目标是: 设计高阶自适应动态面控制器使得系统 (1) 输出在满足约束条件 (2) 的情况下有效跟踪期望的参考信号, 闭环系统所有信号都是一致最终有界的, 且跟踪误差能够收敛到零附近的足够小的邻域内.

为实现本文控制目标, 给出必要的假设以及引

理. 首先, 系统 (1) 需要满足如下的全驱系统假设.

假设 1^[33]. $g_j(x_i^{(0\sim m_i-1)}|_{i=1\sim j}) \neq 0, j = 1, 2, \dots, n$.

假设 2^[9, 13, 21]. 期望的参考信号 $y_d(t)$ 是光滑且可以得到, 并满足

$$y_d, \dot{y}_d, \dots, y_d^{(m_1+1)} \in \Omega_0 = \left\{ \left[\begin{array}{cccc} y_d & \dot{y}_d & \cdots & y_d^{(m_1+1)} \end{array} \right]^T \mid y_d^2 + \dot{y}_d^2 + \cdots + (y_d^{(m_1+1)})^2 \leq r_0^2 \right\}$$

其中, r_0 为一常数.

假设 3^[9, 13]. 约束边界函数 $F_1(t)$ 和 $F_2(t)$ 都是正的函数, 且存在常数 $\varepsilon_F > 0$ 使得 $F_2(t) + F_1(t) \geq \varepsilon_F > 0$. 且存在满足不等式 $-\underline{\kappa}_d(t) < \bar{\kappa}_d(t)$ 的实变函数 $\underline{\kappa}_d(t), \bar{\kappa}_d(t)$; 以及满足不等式 $F_2(t) - \bar{\kappa}_d(t) \geq \bar{\varepsilon}_d, F_1(t) - \underline{\kappa}_d(t) \geq \underline{\varepsilon}_d$ 的常数 $\bar{\varepsilon}_d$ 和 $\underline{\varepsilon}_d$. 同时, 定义集合 $y_d(t) \in \Omega_d = \{y_d \in \mathbf{R} \mid -\underline{\kappa}_d(t) \leq y_d \leq \bar{\kappa}_d(t)\}$.

假设 4^[9, 13]. 约束边界函数 $F_1(t)$ 和 $F_2(t)$ 以及其导数 $F_1^{(i)}$ 和 $F_2^{(i)} (1 \leq i \leq n)$ 是连续且有界的.

注 2. 假设 1 为全驱系统假设, 相比于传统一阶严反馈系统要求控制增益非零, 高阶严反馈系统也要求控制增益非零. 假设 2 要求期望的跟踪信号及其各阶导数是连续有界的, 在状态空间框架下的动态面控制方法同样要求跟踪信号及其各阶导数连续可导, 这个假设也是容易实现的. 假设 3 是为了保证约束上下边界不交叉, 且期望的跟踪信号在约束边界内, 这是符合工程实际的. 假设 4 要求约束边界函数及其各阶导数连续有界, 也是合理的.

本文中, 通过选取 $A^{0\sim n-1}$ 使得矩阵 $\Phi(A^{0\sim n-1})$ 是 Hurwitz 的. 为了方便后续控制器设计, 给出如下引理.

引理 1^[33]. 对任意 $\mu > 0$, 存在 $A^{0\sim n-1}$ 使得

$$\lambda_{\min} [\Phi(A^{0\sim n-1})] < -\frac{\mu}{2} \quad (3)$$

进一步可得存在正定对称矩阵 $P(A^{0\sim n-1})$, 使得

$$\Phi^T(A^{0\sim n-1})P(A^{0\sim n-1}) + P(A^{0\sim n-1})\Phi(A^{0\sim n-1}) < -\mu P(A^{0\sim n-1}) \quad (4)$$

根据文献 [9] 和文献 [13], 引入如下非线性转换函数

$$\zeta_1 = \frac{x_1}{(F_1(t) + x_1)(F_2(t) - x_1)} \quad (5)$$

在系统输出初值在区间 D_1 内确保 $\zeta_1(0)$ 有界的前提下, 只需设计控制器使得 ζ_1 在 $[0, \infty)$ 内有界, 输出约束条件 (2) 就能始终得到满足.

对转换函数式 (5) 求导可得

$$\dot{\zeta}_1 = \sigma_1 \dot{x}_1 + \sigma_2 \quad (6)$$

其中

$$\sigma_1 = \frac{F_1(t)F_2(t) + x_1^2}{(F_1(t) + x_1)^2(F_2(t) - x_1)^2} \quad (7)$$

$$\sigma_2 = -\frac{[\dot{F}_1 F_2 + F_1 \dot{F}_2 + (\dot{F}_2 - \dot{F}_1)x_1]x_1}{(F_1(t) + x_1)^2(F_2(t) - x_1)^2} \quad (8)$$

定义

$$x_0 = \dot{x}_1 \quad (9)$$

将式 (9) 与式 (6) 结合可得

$$\dot{\zeta}_1 = \sigma_1 x_0 + \sigma_2 \quad (10)$$

进而将含有输出约束的原系统 (1) 转化为如下系统

$$\left\{ \begin{array}{l} \dot{\zeta}_1 = \sigma_1 x_0 + \sigma_2 \\ x_0^{(m_1-1)} = f_{10}(x_1^{(0 \sim m_1-1)}) + \\ \quad f_1(x_1^{(0 \sim m_1-1)})\theta_1 + \\ \quad g_1(x_1^{(0 \sim m_1-1)})x_2 \\ x_2^{(m_2)} = f_{20}(x_i^{(0 \sim m_i-1)}|_{i=1 \sim 2}) + \\ \quad f_2(x_i^{(0 \sim m_i-1)}|_{i=1 \sim 2})\theta_2 + \\ \quad g_2(x_i^{(0 \sim m_i-1)}|_{i=1 \sim 2})x_3 \\ \quad \vdots \\ x_n^{(m_n)} = f_{n0}(x_i^{(0 \sim m_i-1)}|_{i=1 \sim n}) + \\ \quad f_n(x_i^{(0 \sim m_i-1)}|_{i=1 \sim n})\theta_n + \\ \quad g_n(x_i^{(0 \sim m_i-1)}|_{i=1 \sim n})u \end{array} \right. \quad (11)$$

当系统输出初始值在 D_1 内, 只需设计控制器使得系统 (11) 的第 1 个状态 ζ_1 有界, 则原系统 (1) 的输出约束就能得到满足^[9, 13].

2 控制器设计

针对输出受时变约束的不确定高阶严反馈系统 (1), 基于全驱系统方法^[21, 33], 设计高阶自适应动态面控制器, 主要步骤如下.

步骤 1. 定义如下坐标变换

$$z_0 = \zeta_1 - x_{0c} \quad (12)$$

其中

$$x_{0c} = \frac{y_d}{(F_1(t) + y_d)(F_2(t) - y_d)} \quad (13)$$

在区间 Ω_d 内是一个连续的有界函数.

定义

$$z_1 = x_0 - x_{1c} \quad (14)$$

虚拟控制律 x_{1d} 通过一系列低通滤波器^[21] 得到 x_{1c}

$$\alpha_{1i} \dot{\omega}_{1i} + \omega_{1i} = \omega_{1(i-1)}, i = 1, 2, \dots, m_1 - 1 \quad (15)$$

其中, $\omega_{10} = x_{1d}$, $\omega_{1(m_1-1)} = x_{1c}$, 设计的滤波参数 $\alpha_{1i} > 0$, $i = 1, 2, \dots, m_1 - 1$ 为常值, 初值 $\omega_{10}(0) = \omega_{11}(0) = \dots = \omega_{1(m_1-1)}(0)$.

定义滤波误差为

$$s_{1i} = \omega_{1i} - \omega_{1(i-1)}, i = 1, 2, \dots, m_1 - 1 \quad (16)$$

可以得到

$$x_0 = x_{1d} + z_1 + \sum_{i=1}^{m_1-1} s_{1i} \quad (17)$$

以及

$$\left\{ \begin{array}{l} \dot{s}_{11} = -\frac{1}{\alpha_{11}} s_{11} - \dot{x}_{1d} \\ \dot{s}_{1i} = -\frac{1}{\alpha_{1i}} s_{1i} + \frac{1}{\alpha_{1(i-1)}} s_{1(i-1)}, \\ \quad i = 2, 3, \dots, m_1 - 1 \end{array} \right. \quad (18)$$

选取 Lyapunov 函数为

$$V_0 = \frac{1}{2} z_0^2 \quad (19)$$

对式 (19) 求导, 可得

$$\dot{V}_0 = z_0 \dot{z}_0 = z_0 (\sigma_1 x_0 + \sigma_2 - \sigma_{d1} \dot{y}_d - \sigma_{d2}) \quad (20)$$

其中

$$\sigma_{d1} = \frac{F_1 F_2 + y_d^2}{(F_1 + y_d)^2 (F_2 - y_d)^2} \quad (21)$$

$$\sigma_{d2} = -\frac{[\dot{F}_1 F_2 + F_1 \dot{F}_2 + (\dot{F}_2 - \dot{F}_1)y_d]y_d}{(F_1 + y_d)^2 (F_2 - y_d)^2} \quad (22)$$

取虚拟控制律为

$$x_{1d} = \frac{-\sigma_2 + \sigma_{d1} \dot{y}_d + \sigma_{d2} - k_0 z_0}{\sigma_1} \quad (23)$$

其中, $k_0 > 0$ 为一常数, 则可得

$$\dot{V}_0 = -k_0 z_0^2 + \sigma_1 z_0 \left(z_1 + \sum_{i=1}^{m_1-1} s_{1i} \right) \quad (24)$$

由式 (14) 可得

$$z_1^{(0 \sim m_1-2)} = x_0^{(0 \sim m_1-2)} - x_{1c}^{(0 \sim m_1-2)} \quad (25)$$

定义

$$z_2 = x_2 - x_{2c} \quad (26)$$

虚拟控制律 x_{2d} 通过一系列低通滤波器得到 x_{2c}

$$\alpha_{2i} \dot{\omega}_{2i} + \omega_{2i} = \omega_{2(i-1)}, i = 1, 2, \dots, m_2 \quad (27)$$

其中, $\omega_{20} = x_{2d}$, $\omega_{2m_2} = x_{2c}$, 设计的滤波参数 $\alpha_{2i} > 0$, $i = 1, 2, \dots, m_2$ 为常值, 初值 $\omega_{20}(0) = \omega_{21}(0) =$

$\dots = \omega_{2m_2}(0)$.
定义滤波误差

$$s_{2i} = \omega_{2i} - \omega_{2(i-1)}, i = 1, 2, \dots, m_2 \quad (28)$$

可得

$$x_2 = x_{2d} + z_2 + \sum_{i=1}^{m_2} s_{2i} \quad (29)$$

以及

$$\begin{cases} \dot{s}_{21} = -\frac{1}{\alpha_{21}}s_{21} - \dot{x}_{2d} \\ \dot{s}_{2i} = -\frac{1}{\alpha_{2i}}s_{2i} + \frac{1}{\alpha_{2(i-1)}}s_{2(i-1)}, i = 2, 3, \dots, m_2 \end{cases} \quad (30)$$

设 $\hat{\theta}_1$ 为 θ_1 的估计值, 定义估计误差为

$$\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1 \quad (31)$$

进一步可得

$$\begin{aligned} z_1^{(m_1-1)} &= x_0^{(m_1-1)} - x_{1c}^{(m_1-1)} = \\ & f_{10} + f_1\hat{\theta}_1 + g_1 \left(x_{2d} + z_2 + \sum_{i=1}^{m_2} s_{2i} \right) - \\ & f_1\tilde{\theta}_1 - x_{1c}^{(m_1-1)} \end{aligned} \quad (32)$$

取虚拟控制律为

$$x_{2d} = \frac{1}{g_1} \left(-f_{10} - f_1\hat{\theta}_1 - A_1^{0\sim m_1-2} z_1^{(0\sim m_1-2)} + x_{1c}^{(m_1-1)} \right) \quad (33)$$

其中, $A_1^{0\sim m_1-2}$ 为待设计参数. 结合式 (32) 和式 (33), 可得

$$\begin{aligned} z_1^{(m_1-1)} + A_1^{0\sim m_1-2} z_1^{(0\sim m_1-2)} &= \\ g_1 \left(z_2 + \sum_{i=1}^{m_2} s_{2i} \right) - f_1\tilde{\theta}_1 \end{aligned} \quad (34)$$

定义

$$V_{z1} = \left(z_1^{(0\sim m_1-2)} \right)^T P_1 z_1^{(0\sim m_1-2)} \quad (35)$$

其中, 矩阵 P_1 为引理 1 中描述的正定矩阵.

对式 (35) 求导, 结合式 (34), 并根据引理 1 可得

$$\begin{aligned} \dot{V}_{z1} &= \left(\dot{z}_1^{(0\sim m_1-2)} \right)^T P_1 z_1^{(0\sim m_1-2)} + \\ & \left(z_1^{(0\sim m_1-2)} \right)^T P_1 \dot{z}_1^{(0\sim m_1-2)} = \\ & \left(z_1^{(0\sim m_1-2)} \right)^T \left(\Phi_1^T P_1 + P_1 \Phi_1 \right) z_1^{(0\sim m_1-2)} - \end{aligned}$$

$$\begin{aligned} & 2 \left(z_1^{(0\sim m_1-2)} \right)^T P_{1L} f_1 \tilde{\theta}_1 + \\ & 2 \left(z_1^{(0\sim m_1-2)} \right)^T P_{1L} g_1 \left(z_2 + \sum_{i=1}^{m_2} s_{2i} \right) \leq \\ & -\mu_1 \left(z_1^{(0\sim m_1-2)} \right)^T P_1 z_1^{(0\sim m_1-2)} - \\ & 2 \left(z_1^{(0\sim m_1-2)} \right)^T P_{1L} f_1 \tilde{\theta}_1 + \\ & 2 \left(z_1^{(0\sim m_1-2)} \right)^T P_1 \begin{bmatrix} 0 \\ g_1 \end{bmatrix} \left(z_2 + \sum_{i=1}^{m_2} s_{2i} \right) \end{aligned} \quad (36)$$

式中, 下标 L 表示相应矩阵的最后一列.

对式 (31) 求导, 可得

$$\dot{\tilde{\theta}}_1 = \dot{\hat{\theta}}_1 \quad (37)$$

定义

$$V_{\theta_1} = \tilde{\theta}_1^T \tilde{\theta}_1 \quad (38)$$

以及

$$V_1 = V_{z1} + V_{\theta_1} \quad (39)$$

对式 (39) 求导, 可得

$$\begin{aligned} \dot{V}_1 &\leq -\mu_1 \left(z_1^{(0\sim m_1-2)} \right)^T P_1 z_1^{(0\sim m_1-2)} + \\ & 2\tilde{\theta}_1^T \left[\dot{\hat{\theta}}_1 - \left(z_1^{(0\sim m_1-2)} \right)^T P_{1L} f_1 \right] + \\ & 2 \left(z_1^{(0\sim m_1-2)} \right)^T P_1 \begin{bmatrix} 0 \\ g_1 \end{bmatrix} \left(z_2 + \sum_{i=1}^{m_2} s_{2i} \right) \end{aligned} \quad (40)$$

取自适应律为

$$\dot{\hat{\theta}}_1 = \left(z_1^{(0\sim m_1-2)} \right)^T P_{1L} f_1 - \gamma_1 \hat{\theta}_1 \quad (41)$$

其中, 常值 $\gamma_1 > 0$ 为设计参数, 进一步可得

$$\begin{aligned} \dot{V}_1 &\leq -\mu_1 \left(z_1^{(0\sim m_1-2)} \right)^T P_1 z_1^{(0\sim m_1-2)} - 2\gamma_1 \tilde{\theta}_1^T \hat{\theta}_1 + \\ & 2 \left(z_1^{(0\sim m_1-2)} \right)^T P_1 \begin{bmatrix} 0 \\ g_1 \end{bmatrix} \left(z_2 + \sum_{i=1}^{m_2} s_{2i} \right) \end{aligned} \quad (42)$$

步骤 i ($i = 2, 3, \dots, n-1$).

定义

$$z_{i+1} = x_{i+1} - x_{(i+1)c} \quad (43)$$

其中, 虚拟控制律 $x_{(i+1)d}$ 通过一系列低通滤波器得到 $x_{(i+1)c}$

$$\begin{aligned} \alpha_{(i+1)j} \dot{\omega}_{(i+1)j} + \omega_{(i+1)j} &= \omega_{(i+1)(j-1)}, \\ j &= 1, 2, \dots, m_{i+1} \end{aligned} \quad (44)$$

其中, $\omega_{(i+1)0} = x_{(i+1)d}$, $\omega_{(i+1)m_{i+1}} = x_{(i+1)c}$, 设计的滤波参数 $\alpha_{(i+1)j} > 0$, $j = 1, 2, \dots, m_{i+1}$ 为常值,

初值 $\omega_{(i+1)0}(0) = \omega_{(i+1)1}(0) = \dots = \omega_{(i+1)m_{i+1}}(0)$.

定义滤波误差为

$$s_{(i+1)j} = \omega_{(i+1)j} - \omega_{(i+1)(j-1)}, j = 1, 2, \dots, m_{i+1} \quad (45)$$

进一步可得

$$x_{i+1} = x_{(i+1)d} + z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \quad (46)$$

以及

$$\begin{cases} \dot{s}_{(i+1)1} = -\frac{1}{\alpha_{(i+1)1}} s_{(i+1)1} - \dot{x}_{(i+1)d} \\ \dot{s}_{(i+1)j} = -\frac{1}{\alpha_{(i+1)j}} s_{(i+1)j} + \frac{1}{\alpha_{(i+1)(j-1)}} s_{(i+1)(j-1)}, \\ j = 2, 3, \dots, m_{i+1} \end{cases} \quad (47)$$

设 $\hat{\theta}_i$ 为对 θ_i 的估计, 定义估计误差为

$$\tilde{\theta}_i = \hat{\theta}_i - \theta_i \quad (48)$$

进一步可得

$$\begin{aligned} z_i^{(m_i)} &= x_i^{(m_i)} - x_{ic}^{(m_i)} = \\ & f_{i0} + f_i \hat{\theta}_i + g_i \left(x_{(i+1)d} + z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \right) - \\ & f_i \tilde{\theta}_i - x_{ic}^{(m_i)} \end{aligned} \quad (49)$$

取虚拟控制律为

$$x_{(i+1)d} = \frac{1}{g_i} \left(-f_{i0} - f_i \hat{\theta}_i - A_i^{0 \sim m_i - 1} z_i^{(0 \sim m_i - 1)} + x_{ic}^{(m_i)} \right) \quad (50)$$

其中, $A_i^{0 \sim m_i - 1}$ 为设计参数. 进一步可得

$$\begin{aligned} z_i^{(m_i)} + A_i^{0 \sim m_i - 1} z_i^{(0 \sim m_i - 1)} &= \\ g_i \left(z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \right) - f_i \tilde{\theta}_i \end{aligned} \quad (51)$$

定义

$$V_{zi} = \left(z_i^{(0 \sim m_i - 1)} \right)^T P_i z_i^{(0 \sim m_i - 1)} \quad (52)$$

对式 (52) 求导, 可得

$$\begin{aligned} \dot{V}_{zi} &\leq -\mu_i \left(z_i^{(0 \sim m_i - 1)} \right)^T P_i z_i^{(0 \sim m_i - 1)} - \\ & 2 \left(z_i^{(0 \sim m_i - 1)} \right)^T P_i L f_i \tilde{\theta}_i + \\ & 2 \left(z_i^{(0 \sim m_i - 1)} \right)^T P_i \begin{bmatrix} 0 \\ g_i \end{bmatrix} \left(z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \right) \end{aligned} \quad (53)$$

对式 (48) 求导, 可得

$$\dot{\tilde{\theta}}_i = \dot{\hat{\theta}}_i \quad (54)$$

定义

$$V_{\theta_i} = \tilde{\theta}_i^T \tilde{\theta}_i \quad (55)$$

$$V_i = V_{zi} + V_{\theta_i} \quad (56)$$

取自适应律为

$$\dot{\hat{\theta}}_i = \left(z_i^{(0 \sim m_i - 1)} \right)^T P_i L f_i - \gamma_i \hat{\theta}_i \quad (57)$$

其中, $\gamma_i > 0$ 为设计参数. 进一步可得

$$\begin{aligned} \dot{V}_i &\leq -\mu_i \left(z_i^{(0 \sim m_i - 1)} \right)^T P_i z_i^{(0 \sim m_i - 1)} - 2\gamma_i \tilde{\theta}_i^T \hat{\theta}_i + \\ & 2 \left(z_i^{(0 \sim m_i - 1)} \right)^T P_i \begin{bmatrix} 0 \\ g_i \end{bmatrix} \left(z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \right) \end{aligned} \quad (58)$$

步骤 n.

设 $\hat{\theta}_n$ 为对 θ_n 的估计, 定义估计误差为

$$\tilde{\theta}_n = \hat{\theta}_n - \theta_n \quad (59)$$

进一步可得

$$\begin{aligned} z_n^{(m_n)} &= x_n^{(m_n)} - x_{nc}^{(m_n)} = \\ & f_{n0} + f_n \hat{\theta}_n + g_n u - f_n \tilde{\theta}_n - x_{nc}^{(m_n)} \end{aligned} \quad (60)$$

取控制律为

$$\begin{aligned} u &= \frac{1}{g_n} \left(-f_{n0} - f_n \hat{\theta}_n - \right. \\ & \left. A_n^{0 \sim m_n - 1} z_n^{(0 \sim m_n - 1)} + x_{nc}^{(m_n)} \right) \end{aligned} \quad (61)$$

其中, $A_n^{0 \sim m_n - 1}$ 为设计参数, 可得

$$z_n^{(m_n)} + A_n^{0 \sim m_n - 1} z_n^{(0 \sim m_n - 1)} = -f_n \tilde{\theta}_n \quad (62)$$

定义

$$V_{zn} = \left(z_n^{(0 \sim m_n - 1)} \right)^T P_n z_n^{(0 \sim m_n - 1)} \quad (63)$$

对式 (63) 求导, 可得

$$\begin{aligned} \dot{V}_{zn} &\leq -\mu_n \left(z_n^{(0 \sim m_n - 1)} \right)^T P_n z_n^{(0 \sim m_n - 1)} - \\ & 2 \left(z_n^{(0 \sim m_n - 1)} \right)^T P_n L f_n \tilde{\theta}_n \end{aligned} \quad (64)$$

对式 (59) 求导, 可得

$$\dot{\tilde{\theta}}_n = \dot{\hat{\theta}}_n \quad (65)$$

定义

$$V_{\theta_n} = \tilde{\theta}_n^T \tilde{\theta}_n \quad (66)$$

以及

$$V_n = V_{zn} + V_{\theta_n} \quad (67)$$

取自适应律为

$$\dot{\hat{\theta}}_n = \left(z_n^{(0 \sim m_n - 1)} \right)^T P_n L f_n - \gamma_n \hat{\theta}_n \quad (68)$$

其中, $\gamma_n > 0$ 为设计参数, 可得

$$\dot{V}_n \leq -\mu_n \left(z_n^{(0 \sim m_n - 1)} \right)^T P_n z_n^{(0 \sim m_n - 1)} - 2\gamma_n \tilde{\theta}_n^T \hat{\theta}_n \quad (69)$$

3 稳定性证明

结合第 2 节中的控制器设计过程, 给出定理 1, 并根据 Lyapunov 理论给出其证明.

定理 1. 考虑满足假设 1 ~ 4 下的具有时变输出约束的高阶严反馈系统 (1). 由虚拟控制律 (23), (33), (50), 控制律 (61), 自适应律 (41), (57), (68), 组成的高阶自适应动态面输出约束控制, 能够使闭环系统所有信号是一致最终有界的, 系统输出满足时变约束条件, 并能有效跟踪期望的参考信号, 且跟踪误差可通过调整参数收敛到零附近足够小的邻域内.

证明.

定义

$$\bar{V}_n = \sum_{i=0}^n V_i \quad (70)$$

则可以得到

$$\begin{aligned} \dot{\bar{V}}_n \leq & -k_0 z_0^2 + \sigma_1 z_0 \left(z_1 + \sum_{i=1}^{m_1-1} s_{1i} \right) - \\ & \mu_1 \left(z_1^{(0 \sim m_1-2)} \right)^T P_1 z_1^{(0 \sim m_1-2)} - \sum_{i=1}^n 2\gamma_i \tilde{\theta}_i^T \hat{\theta}_i - \\ & \sum_{i=2}^n \left(\mu_i \left(z_i^{(0 \sim m_i-1)} \right)^T P_i z_i^{(0 \sim m_i-1)} \right) + \\ & \sum_{i=2}^{n-1} 2 \left(z_i^{(0 \sim m_i-1)} \right)^T P_i \begin{bmatrix} 0 \\ g_i \end{bmatrix} \times \\ & \left(z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \right) + \\ & 2 \left(z_1^{(0 \sim m_1-2)} \right)^T P_1 \begin{bmatrix} 0 \\ g_1 \end{bmatrix} \left(z_2 + \sum_{i=1}^{m_2} s_{2i} \right) \end{aligned} \quad (71)$$

由假设 3 以及式 (7) 可知, $\sigma_1 > 0$, 根据 Young's 不等式可得

$$\sigma_1 z_0 \left(z_1 + \sum_{i=1}^{m_1-1} s_{1i} \right) \leq \frac{\sigma_1}{2} \left(z_1^2 + m_1 z_0^2 + \sum_{i=1}^{m_1-1} s_{1i}^2 \right) \quad (72)$$

结合式 (71) 和式 (72), 可得

$$\begin{aligned} \dot{\bar{V}}_n \leq & - \left(k_0 - \frac{m_1 \sigma_1}{2} \right) z_0^2 + \\ & \frac{\sigma_1}{2 \lambda_{\min}(P_1)} \left(z_1^{(0 \sim m_1-2)} \right)^T P_1 z_1^{(0 \sim m_1-2)} + \\ & \frac{\sigma_1}{2} \sum_{i=1}^{m_1-1} s_{1i}^2 - \mu_1 \left(z_1^{(0 \sim m_1-2)} \right)^T P_1 z_1^{(0 \sim m_1-2)} - \\ & \sum_{i=1}^n 2\gamma_i \tilde{\theta}_i^T \hat{\theta}_i - \\ & \sum_{i=2}^n \left(\mu_i \left(z_i^{(0 \sim m_i-1)} \right)^T P_i z_i^{(0 \sim m_i-1)} \right) + \\ & \sum_{i=2}^{n-1} 2 \left(z_i^{(0 \sim m_i-1)} \right)^T P_i \begin{bmatrix} 0 \\ g_i \end{bmatrix} \times \\ & \left(z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \right) + \\ & 2 \left(z_1^{(0 \sim m_1-2)} \right)^T P_1 \begin{bmatrix} 0 \\ g_1 \end{bmatrix} \left(z_2 + \sum_{i=1}^{m_2} s_{2i} \right) \end{aligned} \quad (73)$$

且有如下不等式成立

$$\begin{aligned} & \left(z_i^{(0 \sim m_i-1)} \right)^T P_i \begin{bmatrix} 0 \\ g_i \end{bmatrix} \left(z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \right) = \\ & \left(z_i^{(0 \sim m_i-1)} \right)^T P_i^{\frac{1}{2}} P_i^{\frac{1}{2}} \begin{bmatrix} 0 \\ g_i \end{bmatrix} \left(z_{i+1} + \sum_{j=1}^{m_{i+1}} s_{(i+1)j} \right) \leq \\ & \frac{1}{2} (m_{i+1} + 1) \left\| P_i^{\frac{1}{2}} z_i^{(0 \sim m_i-1)} \right\|^2 + \\ & \frac{1}{2} g_i^2 \left\| P_i^{\frac{1}{2}} \right\|^2 \left(z_{i+1}^2 + \sum_{j=1}^{m_{i+1}} s_{(i+1)j}^2 \right) \leq \\ & \frac{1}{2} (m_{i+1} + 1) \left(z_i^{(0 \sim m_i-1)} \right)^T P_i z_i^{(0 \sim m_i-1)} + \\ & \frac{1}{2} \lambda_{\max}(P_i) g_i^2 \left(\left\| z_{i+1}^{(0 \sim m_{i+1}-1)} \right\|^2 + \sum_{j=1}^{m_{i+1}} s_{(i+1)j}^2 \right) \end{aligned} \quad (74)$$

定义

$$V_{s1} = \frac{1}{2} \sum_{i=1}^{m_1-1} s_{1i}^2 \quad (75)$$

以及

$$V_{si} = \frac{1}{2} \sum_{j=1}^{m_i} s_{ij}^2, \quad i = 2, 3, \dots, n \quad (76)$$

进一步可得

$$\left\{ \begin{aligned} \dot{V}_{s1} &\leq -\frac{1}{2\alpha_{11}}s_{11}^2 - \sum_{i=2}^{m_1-2} \left[\left(\frac{1}{2\alpha_{1i}} - \frac{1}{2\alpha_{1(i-1)}} \right) s_{1i}^2 \right] - \\ &\quad \left(\frac{1}{\alpha_{1(m_1-1)}} - \frac{1}{2\alpha_{1(m_1-2)}} \right) s_{1(m_1-1)}^2 - s_{11}\dot{x}_{1d} \\ \dot{V}_{si} &\leq -\frac{1}{2\alpha_{i1}}s_{i1}^2 - \sum_{j=2}^{m_i-1} \left[\left(\frac{1}{2\alpha_{ij}} - \frac{1}{2\alpha_{i(j-1)}} \right) s_{ij}^2 \right] - \\ &\quad \left(\frac{1}{\alpha_{im_i}} - \frac{1}{2\alpha_{i(m_i-1)}} \right) s_{im_i}^2 - s_{i1}\dot{x}_{id}, \end{aligned} \right. \quad i = 2, 3, \dots, n \quad (77)$$

定义

$$V = \bar{V}_n + \sum_{i=1}^n V_{si} \quad (78)$$

且结合如下不等式

$$-2\gamma_i\tilde{\theta}_i^T\theta_i < \gamma_i\tilde{\theta}_i^T\tilde{\theta}_i + \gamma_i\theta_i^T\theta_i, \quad i = 1, 2, \dots, n \quad (79)$$

可得

$$\begin{aligned} \dot{V} &\leq -\left(k_0 - \frac{m_1\sigma_1}{2}\right)z_0^2 + \\ &\quad \frac{\sigma_1}{2\lambda_{\min}(P_1)}\left(z_1^{(0\sim m_1-2)}\right)^T P_1 z_1^{(0\sim m_1-2)} + \\ &\quad \frac{\sigma_1}{2} \sum_{i=1}^{m_1-1} s_{1i}^2 - \mu_1 \left(z_1^{(0\sim m_1-2)}\right)^T P_1 z_1^{(0\sim m_1-2)} - \\ &\quad \sum_{i=1}^n \gamma_i \tilde{\theta}_i^T \tilde{\theta}_i + \sum_{i=1}^n \gamma_i \theta_i^T \theta_i - \sum_{i=2}^n s_{i1} \dot{x}_{id} - \\ &\quad \sum_{i=2}^n \left(\mu_i \left(z_i^{(0\sim m_i-1)}\right)^T P_i z_i^{(0\sim m_i-1)} \right) - \\ &\quad \sum_{i=2}^n \frac{1}{2\alpha_{i1}} s_{i1}^2 + \\ &\quad (m_2 + 1) \left(z_1^{(0\sim m_1-2)}\right)^T P_1 z_1^{(0\sim m_1-2)} - s_{11} \dot{x}_{1d} + \\ &\quad \sum_{i=2}^{n-1} (m_{i+1} + 1) \left(z_i^{(0\sim m_i-1)}\right)^T P_i z_i^{(0\sim m_i-1)} + \\ &\quad \lambda_{\max}(P_1) g_1^2 \left(\left\| z_2^{(0\sim m_2-1)} \right\|^2 + \sum_{j=1}^{m_2} s_{2j}^2 \right) + \\ &\quad \sum_{i=2}^{n-1} \lambda_{\max}(P_i) g_i^2 \times \\ &\quad \left(\left\| z_{i+1}^{(0\sim m_{i+1}-1)} \right\|^2 + \sum_{j=1}^{m_{i+1}} s_{(i+1)j}^2 \right) - \end{aligned}$$

$$\begin{aligned} &\frac{1}{2\alpha_{11}}s_{11}^2 - \sum_{i=2}^{m_1-2} \left[\left(\frac{1}{2\alpha_{1i}} - \frac{1}{2\alpha_{1(i-1)}} \right) s_{1i}^2 \right] - \\ &\quad \left(\frac{1}{\alpha_{1(m_1-1)}} - \frac{1}{2\alpha_{1(m_1-2)}} \right) s_{1(m_1-1)}^2 - \\ &\quad \sum_{i=2}^n \sum_{j=2}^{m_i-1} \left[\left(\frac{1}{2\alpha_{ij}} - \frac{1}{2\alpha_{i(j-1)}} \right) s_{ij}^2 \right] - \\ &\quad \sum_{i=2}^n \left(\frac{1}{\alpha_{im_i}} - \frac{1}{2\alpha_{i(m_i-1)}} \right) s_{im_i}^2 \end{aligned} \quad (80)$$

经计算可知, $\dot{x}_{id}, \forall i \in \{1, 2, 3, \dots, n\}$ 是变量 $F_1, F_2, \dot{F}_1, \dot{F}_2, z_0, z_j^{(0\sim m_j-1)}, j = 1, 2, \dots, i-1, s_{11}, \dots, s_{1(m_1-1)}, s_{21}, \dots, s_{2m_2}, \dots, s_{i1}, \dots, s_{im_i}, y_d^{(0\sim m_i+1)}, \tilde{\theta}_j, j = 1, 2, \dots, i-1$ 的函数, 且依赖于设计参数 $\alpha_{11}, \dots, \alpha_{1(m_1-1)}, \alpha_{21}, \dots, \alpha_{2m_2}, \dots, \alpha_{i1}, \dots, \alpha_{im_i}, A_j^{(0\sim m_j-1)}, P_j, j = 1, 2, \dots, i-1$, 以及 k_0 . 因此, 存在非负连续函数 η_i , 它是上述变量的函数, 且依赖于上述设计参数, 使得 $|\dot{x}_{id}| \leq \eta_i$.

对任意 $V(0) > 0$, 集合

$$\Omega = \left\{ z_0, z_1^{(0\sim m_1-2)}, z_i^{(0\sim m_i-1)} \Big|_{i=2\sim n}, s_{11}, \dots, s_{1(m_1-1)}, s_{21}, \dots, s_{2m_2}, \dots, s_{i1}, \dots, s_{im_i}, \tilde{\theta}_i, i = 1, \dots, n \mid V \leq V(0) \right\} \quad (81)$$

是一个紧集, 由于 $[y_d \quad \dot{y}_d \quad \dots \quad y_d^{(m_1+1)}]^T \in \Omega_0$, 所以, $\eta_i, \forall i \in \{1, 2, \dots, n\}$ 在 $\Omega \times \Omega_0$ 上有最大值, 将其定义为 $\bar{\eta}_i$. 因此在 Ω 上, 可得

$$\begin{aligned} \dot{V} &\leq -\left(k_0 - \frac{m_1\sigma_1}{2}\right)z_0^2 - \\ &\quad \left(\mu_1 - \frac{\sigma_1}{2\lambda_{\min}(P_1)} - m_2 - 1\right) \times \\ &\quad \left(z_1^{(0\sim m_1-2)}\right)^T P_1 z_1^{(0\sim m_1-2)} - \sum_{i=2}^{n-1} \left(\mu_i - m_{i+1} - \right. \\ &\quad \left. 1 - \frac{\lambda_{\max}(P_{i-1})}{\lambda_{\min}(P_i)} g_{i-1}^2\right) \left(z_i^{(0\sim m_i-1)}\right)^T P_i z_i^{(0\sim m_i-1)} - \\ &\quad \left(\mu_n - \frac{\lambda_{\max}(P_{n-1})}{\lambda_{\min}(P_n)} g_{n-1}^2\right) \left(z_n^{(0\sim m_n-1)}\right)^T \times \\ &\quad P_n z_n^{(0\sim m_n-1)} - \\ &\quad \left(\frac{1}{2\alpha_{11}} - \frac{\sigma_1}{2} - \frac{\bar{\eta}_1^2}{4\bar{\epsilon}}\right) s_{11}^2 - \sum_{i=2}^n \left(\frac{1}{2\alpha_{i1}} - \frac{\bar{\eta}_i^2}{4\bar{\epsilon}}\right) s_{i1}^2 - \\ &\quad \sum_{i=2}^{m_1-2} \left(\frac{1}{2\alpha_{1i}} - \frac{1}{2\alpha_{1(i-1)}} - \frac{\sigma_1}{2}\right) s_{1i}^2 - \\ &\quad \left(\frac{1}{\alpha_{1(m_1-1)}} - \frac{1}{2\alpha_{1(m_1-2)}} - \frac{\sigma_1}{2}\right) s_{1(m_1-1)}^2 - \end{aligned}$$

$$\sum_{i=2}^n \left(\frac{1}{\alpha_{im_i}} - \frac{1}{2\alpha_{i(m_i-1)}} - \lambda_{\max}(P_{i-1}) g_{i-1}^2 \right) s_{im_i}^2 - \sum_{i=2}^n \sum_{j=2}^{m_i-1} \left[\left(\frac{1}{2\alpha_{ij}} - \frac{1}{2\alpha_{i(j-1)}} - \lambda_{\max}(P_{i-1}) g_{i-1}^2 \right) s_{ij}^2 \right] - \sum_{i=1}^n \gamma_i \tilde{\theta}_i^T \tilde{\theta}_i + \sum_{i=1}^n (\gamma_i \theta_i^T \theta_i + \bar{\varepsilon}) \quad (82)$$

其中, 常数 $\bar{\varepsilon} > 0$, 且有如下不等式在推导过程中被用到

$$\sum_{i=1}^n |s_{i1}| \bar{\eta}_i \leq \sum_{i=1}^n \frac{\bar{\eta}_i^2}{4\bar{\varepsilon}} s_{i1}^2 + n\bar{\varepsilon} \quad (83)$$

以及

$$\begin{aligned} & \sum_{i=1}^{n-1} \lambda_{\max}(P_i) g_i^2 \left\| z_{i+1}^{(0 \sim m_{i+1}-1)} \right\|^2 \leq \\ & \sum_{i=1}^{n-1} \frac{\lambda_{\max}(P_i)}{\lambda_{\min}(P_{i+1})} g_i^2 \left(z_{i+1}^{(0 \sim m_{i+1}-1)} \right)^T \times \\ & P_{i+1} z_{i+1}^{(0 \sim m_{i+1}-1)} = \\ & \sum_{i=2}^n \frac{\lambda_{\max}(P_{i-1})}{\lambda_{\min}(P_i)} g_{i-1}^2 \left(z_i^{(0 \sim m_i-1)} \right)^T P_i z_i^{(0 \sim m_i-1)} \end{aligned} \quad (84)$$

因此, 只需设计参数满足

$$\left\{ \begin{aligned} & 2k_0 - m_1 \sigma_1 \geq \kappa; \quad \mu_1 - \frac{\sigma_1}{2\lambda_{\min}(P_1)} - m_2 - 1 \geq \kappa; \\ & \mu_i - m_{i+1} - 1 - \frac{\lambda_{\max}(P_{i-1})}{\lambda_{\min}(P_i)} g_{i-1}^2 \geq \kappa, \\ & \quad \quad \quad i = 2, 3, \dots, n-1; \\ & \mu_n - \frac{\lambda_{\max}(P_{n-1})}{\lambda_{\min}(P_n)} g_{n-1}^2 \geq \kappa; \\ & \gamma_i \geq \kappa, \quad i = 1, 2, \dots, n; \\ & \frac{1}{2\alpha_{11}} - \frac{\sigma_1}{2} - \frac{\bar{\eta}_1^2}{4\bar{\varepsilon}} \geq \kappa; \\ & \frac{1}{2\alpha_{1i}} - \frac{1}{2\alpha_{1(i-1)}} - \frac{\sigma_1}{2} \geq \kappa, \quad i = 2, 3, \dots, m_1 - 2; \\ & \frac{1}{\alpha_{1(m_1-1)}} - \frac{1}{2\alpha_{1(m_1-2)}} - \frac{\sigma_1}{2} \geq \kappa; \\ & \frac{1}{2\alpha_{i1}} - \frac{\bar{\eta}_i^2}{4\bar{\varepsilon}} \geq \kappa, \quad i = 2, 3, \dots, n; \\ & \frac{1}{\alpha_{im_i}} - \frac{1}{2\alpha_{i(m_i-1)}} - \lambda_{\max}(P_{i-1}) g_{i-1}^2 \geq \kappa, \\ & \quad \quad \quad i = 2, 3, \dots, n; \\ & \left(\frac{1}{2\alpha_{ij}} - \frac{1}{2\alpha_{i(j-1)}} - \lambda_{\max}(P_{i-1}) g_{i-1}^2 \right) y_{ij}^2 \geq \kappa, \\ & \quad \quad \quad i = 2, 3, \dots, n, j = 2, 3, \dots, m_i - 1 \end{aligned} \right. \quad (85)$$

其中, $\kappa > 0$ 为一常数. 进一步可得

$$\dot{V} \leq -\kappa V + \varepsilon \quad (86)$$

其中

$$\varepsilon = \sum_{i=1}^n (\gamma_i \theta_i^T \theta_i + \bar{\varepsilon}) \quad (87)$$

对式 (86), 若选取参数满足 $\kappa \geq \varepsilon/V(0)$, 则 $\dot{V} \leq 0$. 所以对任意 $t \geq 0$, 有 $V(t) \leq V(0)$ 成立. 因此可知 Ω 是一个不变集. 根据比较原理^[36] 可得

$$0 \leq V(t) \leq \frac{\varepsilon}{\kappa} + \left(V(0) - \frac{\varepsilon}{\kappa} \right) e^{-\kappa t}, \quad \forall t \geq 0 \quad (88)$$

因此, 可得 $V(t)$ 是一致最终有界的. 进一步可知, 坐标转换误差 z_i , $i = 0, 1, \dots, n$, 滤波误差 $s_{11}, \dots, s_{1(m_1-1)}, s_{21}, \dots, s_{2m_2}, \dots, s_{i1}, \dots, s_{im_i}$ 以及参数估计误差 $\tilde{\theta}_i$, $i = 1, \dots, n$ 是一致最终有界的, 即闭环系统所有信号都是一致最终有界的. 且由于 $\lim_{t \rightarrow \infty} V(t) \rightarrow \varepsilon/\kappa$, 而 ε 不依赖于 κ , 所以可通过调整参数使得 ε/κ 足够小. 由于 $z_0 = \zeta_1 - x_{0c}$, 且 x_{0c} 在 Ω_d 内有界, 所以可知 ζ_1 有界, 从而 $x_1 \in D_1$, 即输出约束始终得到满足. 定义跟踪误差为

$$e = x_1 - y_d \quad (89)$$

由式 (5), (12) 和 (13), 可得

$$e = \delta_0 z_0 \quad (90)$$

其中

$$\delta_0 = \frac{(F_1 + x_1)(F_2 - x_1)(F_1 + y_d)(F_2 - y_d)}{F_1 F_2 + x_1 y_d} \quad (91)$$

根据上述分析可知, 存在两个正常数 $\bar{\delta}_0$ 和 $\underline{\delta}_0$, 使得 $0 < \underline{\delta}_0 \leq \delta_0 \leq \bar{\delta}_0 < +\infty$. 进一步由 z_0 一致最终有界可知, e 是一致最终有界的. \square

注 3. 由上述分析可以看出, 本文不需要将高阶反馈系统转化为一阶形式, 直接对其设计控制器, 会简化一些步骤. 且可通过调整设计参数使得 ε/κ 足够小以提高跟踪精度, 然而这样会增大控制输入, 因此在实际应用中, 应考虑跟踪精度和控制输入之间的权衡.

4 仿真分析

为验证所提算法的有效性, 选取柔性关节机械臂系统对本文算法进行仿真验证, 机械臂系统模型为^[37-38]

$$\begin{cases} I_Q \ddot{q} + MgL \sin q + Kq + \Delta_1 = K\theta \\ J \ddot{\theta} + K(\theta - q) + \Delta_2 = \tau \end{cases} \quad (92)$$

其中, $q \in \mathbf{R}^n$ 表示连杆位置, 其一阶导数和二阶导数分别代表连杆的速度和加速度, $I_Q \in \mathbf{R}^{n \times n}$ 是连

杆转动惯量; $M \in \mathbf{R}^n$ 和 $L \in \mathbf{R}^n$ 分别为连杆质量和长度; g 为重力加速度; $J \in \mathbf{R}^{n \times n}$ 是电机转动惯量; $\theta, \dot{\theta}, \ddot{\theta} \in \mathbf{R}^n$ 表示转子的角位移、角速度和角加速度; $K \in \mathbf{R}^{n \times n}$ 表示等效弹簧弹性系数; $\tau \in \mathbf{R}^n$ 为电机控制力矩; Δ_1 和 Δ_2 表示系统模型不确定性, 其表达式为 $\Delta_1 = 0.1(MgL \sin q + Kq)$ 和 $\Delta_2 = 0.1K(\theta - q)$. 机械臂模型参数^[30]为 $I_Q = 1$ (kg·m²), $J = 1$ (kg·m²), $M = 0.5$ kg, $L = 1$ m, $K = 40$ (N·m·rad⁻¹), $g = 10$ (m·s⁻²).

定义 $x_1 = q$, $x_2 = \theta$, $u = \tau$, 可得

$$\begin{cases} \ddot{x}_1 = f_{10}(x_1, \dot{x}_1) + f_1(x_1, \dot{x}_1)\theta_1 + g_1(x_1, \dot{x}_1)x_2 \\ \ddot{x}_2 = f_{20}(x_{1\sim 2}^{(0\sim 1)}) + f_2(x_{1\sim 2}^{(0\sim 1)})\theta_2 + g_2(x_{1\sim 2}^{(0\sim 1)})u \end{cases} \quad (93)$$

其中, $g_1 = 40$, $g_2 = 1$, $f_1 = f_{10} = -5 \sin x_1 - 40x_1$, $f_2 = f_{20} = 40x_1 - 40x_2$. θ_1 和 θ_2 为系统未知的模型不确定参数, 其真实值为 0.1.

为验证本文方法的有效性, 首先将本文方法与不考虑输出约束的自适应动态面控制方法^[39]进行对比仿真. 令 $x_1 = q$, $x_2 = \dot{q}$, $x_3 = \theta$, $x_4 = \dot{\theta}$, 将机械臂系统 (92) 转化为如下一阶系统的形式, 如式 (94) 所示, 再对其设计自适应动态面控制器.

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{MgL}{I_Q} \sin x_1 - \frac{K}{I_Q} x_1 - \frac{\Delta_1}{I_Q} + \frac{K}{I_Q} x_3 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -\frac{K}{J} (x_3 - x_1) - \frac{\Delta_2}{J} + \frac{1}{J} u \end{cases} \quad (94)$$

情况 1. 期望的跟踪信号为 $y_d = 0.5 \sin(0.5t)$, 约束边界为 $F_1 = 0.65 + 0.2 \sin(t)$ 、 $F_2 = 0.7 + 0.2 \sin(t)$, 其他相关参数选为 $\alpha_{11} = 0.04$, $\alpha_{21} = 0.03$, $\alpha_{22} = 0.01$, $a_1 = 10$, $P_1 = 1$, $a_2^{0\sim 1} = [40, 10]$, $\gamma_1 = 10$, $\gamma_2 = 10$, $P_2 = \begin{bmatrix} 2.1750 & 0.0125 \\ 0.0125 & 0.0512 \end{bmatrix}$. 系统初值选为 $x_1(0) = 0.6$ 、 $x_2(0) = x_3(0) = x_4(0) = 0$, $\hat{\theta}_1(0) = 0$, $\hat{\theta}_2(0) = 0$. 仿真时的相关参数和初值, 二者保持一致, 将本文方法得到的系统输出记为 x_1 、将不考虑输出约束的自适应动态面控制方法得到的系统输出记为 x_1 wu, 得到仿真结果如图 1 所示.

情况 2. 增加更为复杂的约束边界进行仿真对比, 期望的跟踪信号为 $y_d = \cos(2t) + \sin(0.5t)$, 约束边界为 $F_1 = 1.01 \times 2^{-2t} + 1 - \cos(2t) - \sin(0.5t)$, $F_2 = 0.56 \times 2^{-1.2t} + 0.84 + \cos(2t) + \sin(0.5t)$, 系统初值设置为 $x_1(0) = 2$, $x_2(0) = 1$, $x_3(0) = x_4(0) = 0.5$. 其他相关参数及记法与情况 1 一致, 得到系统输出仿真结果如图 2 所示.

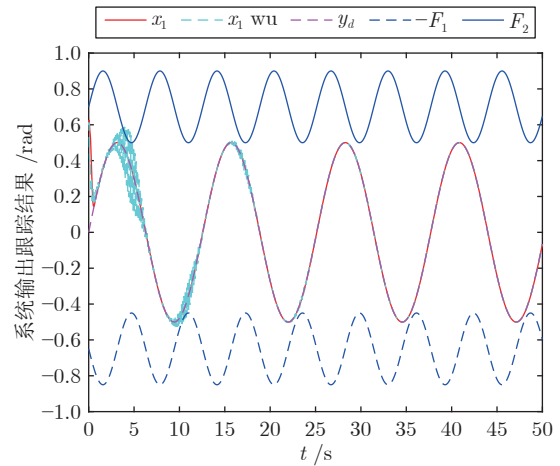


图 1 情况 1 下的系统输出跟踪结果

Fig.1 System output tracking results in Case 1

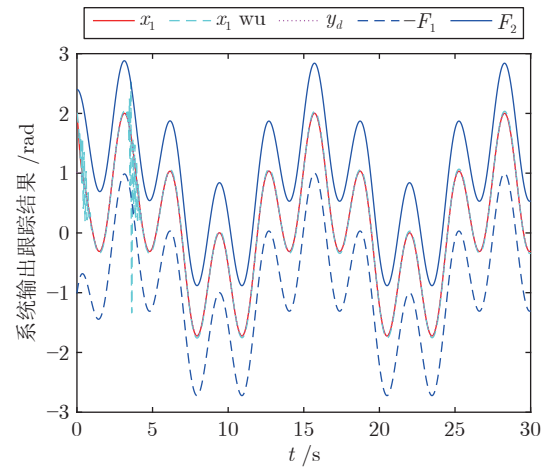


图 2 情况 2 下的系统输出跟踪结果

Fig.2 System output tracking results in Case 2

由图 1 和图 2 可知, 系统本身存在输出约束, 但是自适应动态面控制器不考虑输出约束问题, 就无法始终保持系统输出在约束范围内. 从仿真结果来看其中存在系统输出超出约束边界的情形, 这对于实际工程比如无人车系统、隧道无人机系统等, 超出约束边界就意味着无人车冲出跑道、无人机撞到隧道边界, 出现事故. 而本文算法能够有效使系统输出始终保持在约束范围内, 不违反约束条件.

为说明本文算法的先进性, 将本文算法与基于非对称障碍 Lyapunov 函数 (BLF), 即参考文献 [5] 中的方法以及传统基于非线性映射函数 (Nonlinear mapping, NM), 即参考文献 [8-9] 中的方法, 在情况 1 和情况 2 两种情况下进行对比仿真, 相关参数和初值保持不变. 将对比的两种方法简称为一阶 BLF 方法和一阶 NM 方法. 将本文方法仿真结果记为 $x_{1,h}$, e_h , $\hat{\theta}_1(h)$, $\hat{\theta}_2(h)$, u_h ; 将一阶 BLF 方法仿

真结果记为 $x_{1,f1}, e_{f1}, \hat{\theta}_1(f1), \hat{\theta}_2(f1), u_{f1}$; 将一阶 NM 方法仿真结果记为 $x_{1,f2}, e_{f2}, \hat{\theta}_1(f2), \hat{\theta}_2(f2), u_{f2}$. 情况 1 的仿真结果如图 3 ~ 7 所示; 情况 2 的仿真结果如图 8 ~ 12 所示.

上述仿真结果中, 图 3、图 8 和图 4、图 9 分别为系统输出跟踪结果和跟踪误差, 可知三种控制方法下的系统输出均能有效地跟踪期望信号, 且系统输出不违反约束条件, 始终保持在约束范围内; 相比于一阶 BLF 方法和一阶 NM 方法, 在相同的控制参数及相同的初值条件下, 本文方法收敛速度和精度都略优. 图 5、图 6 和图 10、图 11 为系统参数估计结果, 可知三种控制方法下的参数估计值均能有效收敛到参数真实值附近, 且本文方法收敛精度和收敛速度更好. 图 7 和图 12 为系统控制输入, 可知三种控制方法的输入均在合理范围内, 相比于一阶 BLF 方法和一阶 NM 方法, 本文方法控制输入初值更小.

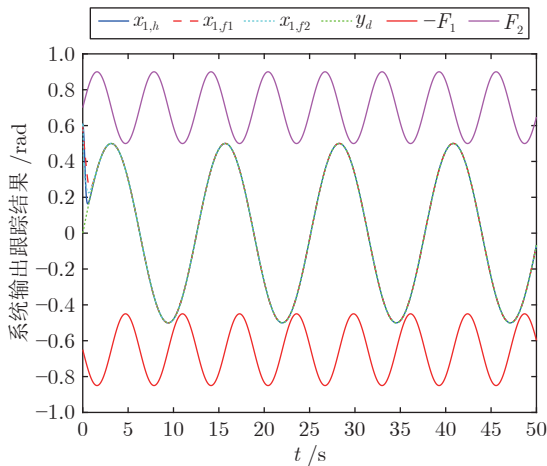


图 3 系统输出跟踪结果 (情况 1)

Fig.3 System output tracking results (Case 1)

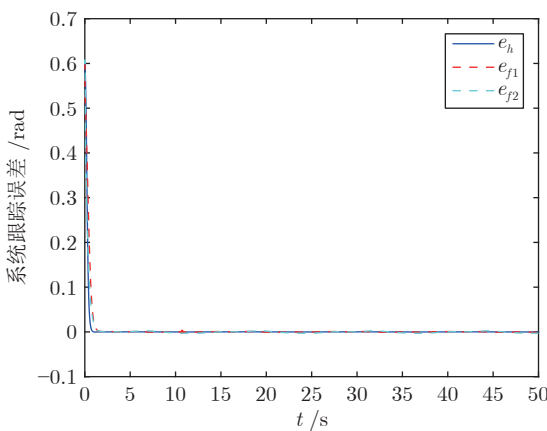


图 4 系统跟踪误差 (情况 1)

Fig.4 System tracking errors (Case 1)

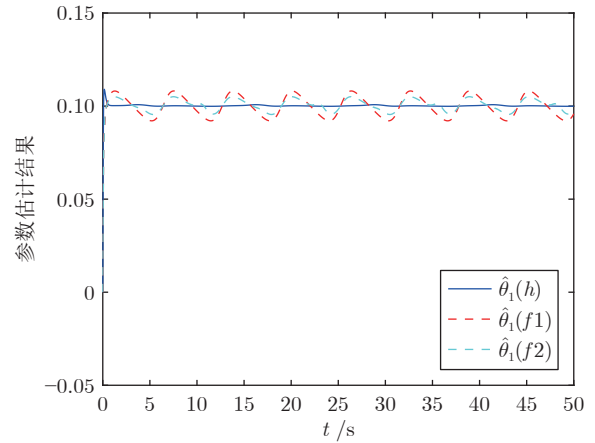


图 5 参数估计 $\hat{\theta}_1(h), \hat{\theta}_1(f1), \hat{\theta}_1(f2)$ (情况 1)

Fig.5 Parameter estimation $\hat{\theta}_1(h), \hat{\theta}_1(f1),$ and $\hat{\theta}_1(f2)$ (Case 1)

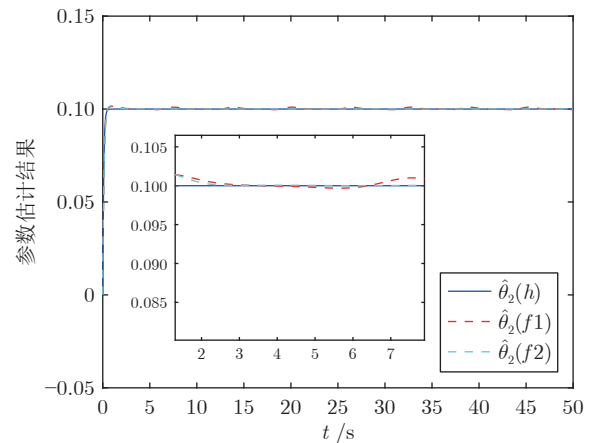


图 6 参数估计 $\hat{\theta}_2(h), \hat{\theta}_2(f1), \hat{\theta}_2(f2)$ (情况 1)

Fig.6 Parameter estimation $\hat{\theta}_2(h), \hat{\theta}_2(f1),$ and $\hat{\theta}_2(f2)$ (Case 1)

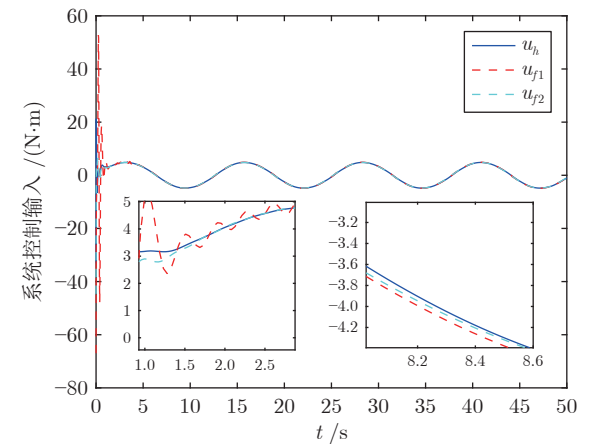


图 7 系统控制输入 (情况 1)

Fig.7 System control inputs (Case 1)

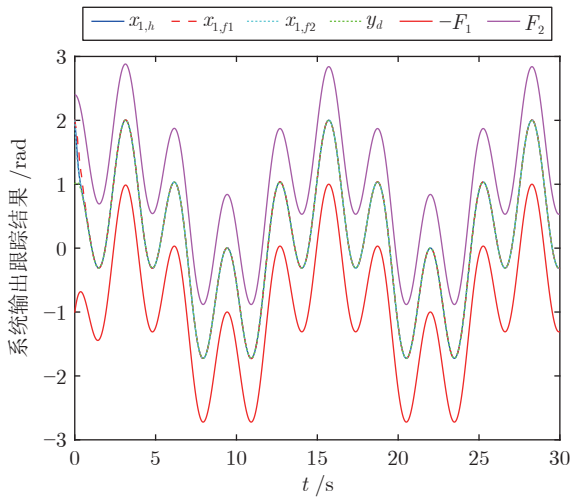


图 8 系统输出跟踪结果 (情况 2)

Fig.8 System output tracking results (Case 2)

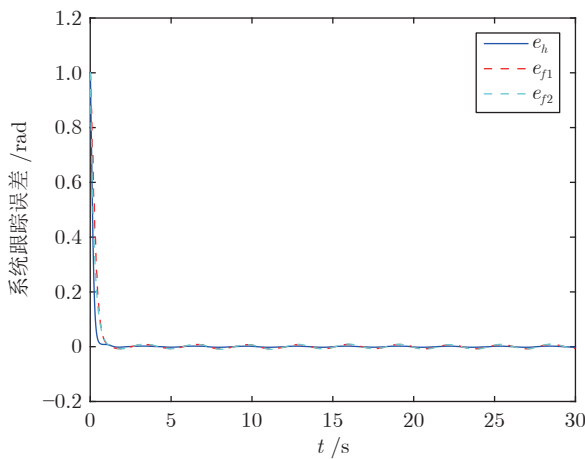


图 9 系统跟踪误差 (情况 2)

Fig.9 System tracking errors (Case 2)

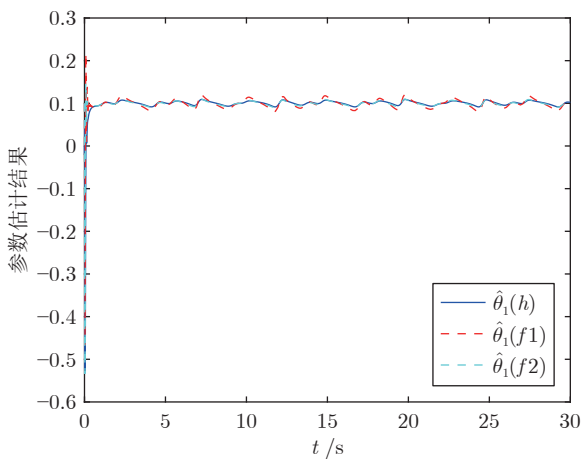


图 10 参数估计 $\hat{\theta}_1(h)$, $\hat{\theta}_1(f1)$, $\hat{\theta}_1(f2)$ (情况 2)

Fig.10 Parameter estimation $\hat{\theta}_1(h)$, $\hat{\theta}_1(f1)$, and $\hat{\theta}_1(f2)$ (Case 2)

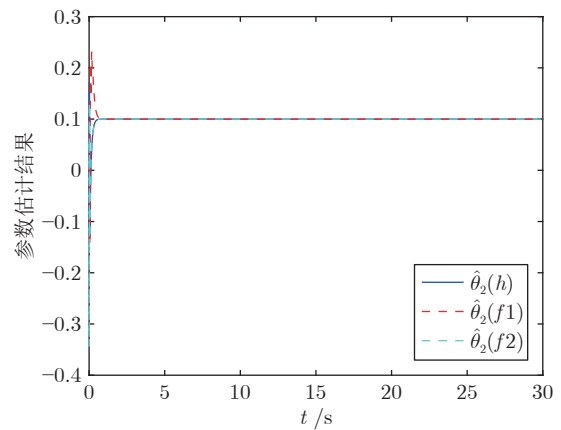


图 11 参数估计 $\hat{\theta}_2(h)$, $\hat{\theta}_2(f1)$, $\hat{\theta}_2(f2)$ (情况 2)

Fig.11 Parameter estimation $\hat{\theta}_2(h)$, $\hat{\theta}_2(f1)$, and $\hat{\theta}_2(f2)$ (Case 2)

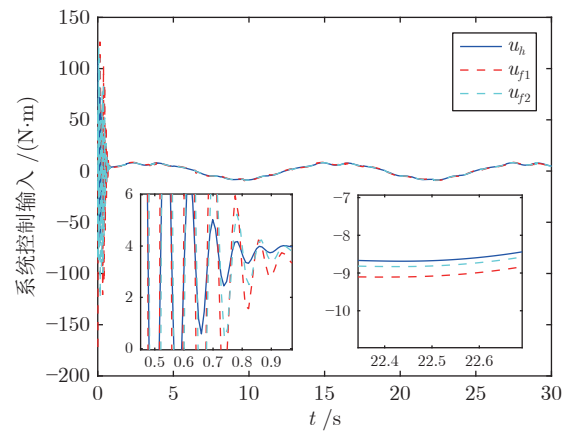


图 12 系统控制输入 (情况 2)

Fig.12 System control inputs (Case 2)

注 4. 对于反步法/动态面控制方法而言, 随着系统阶数的增多, 控制器设计步骤也会增多, 不仅会增加控制器设计的复杂度、引起微分爆炸问题, 还会增加中间步骤的误差. 而本文方法直接对高阶系统设计控制器, 不仅省去了将高阶系统转换为一阶系统这一步骤, 还能减少控制器设计步骤, 从而有效减弱上述问题.

为进一步说明本文方法的优势, 将本文方法与上述一阶 BLF 方法以及一阶 NM 方法在相同的仿真参数下, 并在同一台计算机同一个仿真软件下运行 50 次, 比较三者情况 1 和情况 2 两种情况下的运行时间, 结果如表 1 所示. 可知相比于于一阶 NM 方法和一阶 BLF 方法, 由于本文方法设计步骤更少, 所需仿真时间更少, 具有更高的效率.

由上述仿真结果可知, 本文所提出的算法不需要将原始系统转化为 4 个一阶子系统, 设计步骤更少, 能使系统始终满足输出约束要求. 相比于于一阶

表 1 三种算法运算时间对比 (s)
Table 1 Comparison of operation time of three algorithms (s)

	本文方法	一阶 BLF 方法	一阶 NM 方法
情况 1	44.380383	58.879631	49.213382
情况 2	35.173545	47.965324	41.693068

系统方法, 本文方法在收敛精度、收敛速度方面略优, 控制输入初值更小、仿真时间更少。

注 5. 一阶 BLF 方法、一阶 NM 方法以及本文方法均能有效处理输出约束问题, 对应的文献从理论方面说明了三种方法能确保闭环系统所有信号一致最终有界, 输出约束能始终满足, 而且本文的仿真也证实了三种方法都能有效处理输出约束问题。从仿真结果来看, 本文方法的收敛精度、收敛速度略优于一阶 BLF 方法和一阶 NM 方法, 性能指标方面并没有很突出的优势。这是因为本文的优势主要在于能减少设计步骤、降低算法设计复杂度, 从而减弱反步法/动态面控制方法的微分爆炸问题, 这对于反步法/动态面控制是很有意义的。从仿真结果中也能证实本文方法控制输入初值更小、算法运算时间更少, 具有更高的效率。

5 结束语

针对不确定高阶严反馈系统, 基于全驱系统方法, 本文提出一种高阶自适应动态面时变输出约束控制方法。首先通过非线性转换函数将原输出约束系统转换为无约束的系统, 进而可通过确保新系统的有界性使得原系统输出约束得到满足。然后基于全驱系统方法, 在高阶动态面控制方法的框架下直接对每个高阶子系统设计控制器, 而不需要将其转化为一阶系统形式, 减少了一些步骤; 通过引入一系列低通滤波器来获得虚拟控制律的高阶导数, 进而将复杂的高阶求导运算转化为简单的代数运算, 更加简洁高效。通过 Lyapunov 稳定性理论证明闭环系统所有信号都是一致最终有界的, 系统输出约束条件始终得到满足, 且可通过调整参数使得系统跟踪误差收敛到原点的任意小邻域内。最后通过对实际的机械臂系统进行仿真, 验证了本文所提控制方法的有效性。

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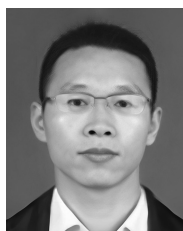


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