具有未建模动态的互联大系统事件触发自适应模糊控制

赵光同 [1,2] 曹 亮 [1,2] 周 琪 [1,2] 李鸿一 [1,2]

摘 要 针对一类具有未建模动态及执行器故障的非严格反馈非线性互联大系统, 提出一种基于事件触发机制的模糊分散自适应输出反馈控制算法. 首先, 通过设计模糊状态观测器估计系统中不可测的状态, 并引入李雅普诺夫函数约束未建模动态. 然后, 提出一种基于事件触发机制的自适应容错控制器补偿多个执行器故障产生的影响. 最后, 利用障碍李雅普诺夫函数实现对系统输出的约束, 并证明闭环系统中所有信号均是半全局一致最终有界的, 且设计的事件触发机制可以避免 Zeno 行为. 数值仿真结果验证所提出设计方案的可行性及有效性.

关键词 非线性互联大系统, 未建模动态, 执行器故障, 事件触发

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Event-triggered Adaptive Fuzzy Control for Interconnected Large-scale Systems With Unmodeled Dynamics

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Abstract This paper develops an event-triggered fuzzy decentralized adaptive output feedback control method for a class of nonstrict-feedback interconnected large-scale nonlinear systems with unmodeled dynamics and actuator faults. Firstly, a fuzzy state observer is designed to estimate the unmeasurable states, and unmodeled dynamics will be addressed by using the Lyapunov function method. Furthermore, an event-triggered-based adaptive fault-tolerant controller is proposed to compensate the effect of multiple actuator faults. Finally, by using the barrier Lyapunov function, the contravention of the output constraint will be excluded. And all signals of the closed-loop system will be ensured to be semiglobally uniformly ultimately bounded and the Zeno behavior will be avoided. The numerical simulation results illustrate the effectiveness and availability of the proposed design method.

Key words Interconnected large-scale nonlinear systems, unmodeled dynamics, actuator faults, event-triggered

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非线性互联大系统具有结构复杂、控制分散、子系统关联性强、模型维数高等特点, 广泛应用于机器人、化工过程、智能电网和航空航天等领域. 因此, 针对此类系统的控制问题, 吸引众多学者的关注. 由于神经网络/模糊逻辑系统能够拟合非线性系统中复杂未知的连续函数, 为解决非线性互联大系统的控制问题起到巨大作用. 文献 [1−4] 针对非线性互联大系统, 基于反步法技术, 利用神经网络/模糊逻辑系统逼近复杂系统中的非线性项, 使设计的自适应控制器达到良好的控制效果. 然而, 上述成果仅考虑了系统状态可测的情况. 在实际工程中, 系统状态往往不可测量或测量成本过高, 导致已有的大部分成果将失效. 因此, 学者们在文献 [5−9] 中通过构造观测器来估计系统中的未知状态, 提出了有效的输出反馈控制方案. 针对互联大系统, 文献 [8−9] 设计了模糊观测器准确观测系统未知状态, 并构造自适应容错控制器补偿执行器故障带来的影响. 尽管互联大系统的研究成果已较为丰富, 但仍存在一些问题尚待解决, 如系统存在未建模动态或执行器故障时, 如何设计有效的控制器等问题.

在实际工程应用中, 为了保证控制性能和安全等指标, 往往需要系统输出受限于一定范围. 对此,

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文献 [10−12] 基于障碍李雅普诺夫函数来设计控制器, 保证了系统输出和跟踪误差受限. 但是以上设计方法只能处理静态受限的情况, 难以实现快速的收敛. 此外, 由于实际非线性系统存在复杂性、不确定性和时变性等特点, 难以获得准确数学模型, 不可避免地存在未建模动态问题[13]. 对此, 文献 [14−18] 给出相应的解决方法, 其中文献 [14−15] 基于小增益定理证明控制方案对未建模动态处理的有效性. 文献 [16−17] 通过构建辅助信号模型, 抵消了未建模动态的影响. 但目前针对非线性互联大系统的未建模动态研究尚处于起步阶段, 需要进一步探讨.

受复杂工作环境的影响, 实际系统中普遍存在执行器故障问题. 执行器故障不但会影响控制器的性能, 甚至会导致系统完全失控, 造成安全隐患[19−21]. 因此, 如何设计分散自适应容错控制器, 保证系统在发生故障情况下能正常运行, 引起许多学者的关注. 为了补偿故障对系统的影响, 文献 [8−9, 21] 提出了有效的容错控制方案. 随着网络化控制的普及, 如何减少控制器与执行器之间的通信带宽, 节约通信资源, 变得尤其重要. 而基于事件触发机制设计的控制器, 不但能实现此要求, 而且可以保证控制效果不会降低, 所以得到了广泛研究[21−24]. 因此, 文献 [21] 设计了一种基于事件触发机制的分散自适应容错控制器, 补偿了执行器故障对互联大系统的影响, 并保证了系统持续工作的稳定性. 虽然已有成果分别研究了非线性互联大系统的未建模动态, 执行器故障和输出受限问题. 但当它们共存于非线性互联大系统时, 如何处理耦合项、未建模动态和输出受限, 并补偿执行器故障的影响, 是一项具有挑战性的工作.

因此, 针对一类具有未建模动态及执行器故障的非严格反馈非线性互联大系统, 如何基于事件触发机制, 设计有效的分散自适应输出反馈容错控制方案是一个值得深入研究的问题. 对此本文主要完成以下工作: 1) 构建模糊观测器, 估计系统不可测的状态. 通过引入一类李雅普诺夫函数, 约束系统未建模动态, 从而保证控制器的设计不受未建模动态的影响. 2) 基于事件触发机制设计分散自适应容错控制器, 在减少通信带宽的同时, 补偿执行器故障对系统的影响. 3) 基于一类时变新型的障碍李雅普诺夫函数, 使得系统跟踪误差能快速收敛至原点附近较小区域内, 从而满足性能要求, 保证系统输出受限的约束.

本文的组织结构如下: 第 1 节介绍互联大系统的模型和相关的假设及定理; 第 2 节提出观测器和控制器的设计方案, 并进行稳定性分析; 第 3 节通过数值仿真验证该方法的有效性; 第 4 节对全文的工作进行总结.

1 问题描述及相关介绍

1.1 问题描述

本文考虑的非线性互联大系统模型由 N 个子系统所构成, 其中, 第 i 个子系统的模型为:

$$\begin{cases}\dot{\boldsymbol{z}}_i = q_i(\boldsymbol{z}_i, y_i)\\ \dot{x}_{i,j} = x_{i,j+1} + f_{i,j}(\boldsymbol{X}_i) + \Delta_{i,j}(y_i, \boldsymbol{z}_i) + H_{i,j}(\boldsymbol{y})\\ \dot{x}_{i,n} = \displaystyle\sum_{q=1}^{m} b_{i,q}u_{i,q} + f_{i,n}(\boldsymbol{X}_i) + \Delta_{i,n}(y_i, \boldsymbol{z}_i) + H_{i,n}(\boldsymbol{y})\\ y_i = x_{i,1}\end{cases} \tag{1}$$

其中, $i = 1, \cdots, N$. $j = 1, \cdots, n-1$. $\boldsymbol{X}_i = [x_{i,1}, \cdots, x_{i,n}]^{\mathrm{T}} \in \mathbf{R}^n$ 为子系统状态, $\boldsymbol{z}_i \in \mathbf{R}^m$ 为未建模动态. $f_{i,j}(\boldsymbol{X}_i)$, $q_i(\boldsymbol{z}_i, y_i)$ 是未知的光滑函数, $\Delta_{i,j}(y_i, \boldsymbol{z}_i)$ 代表未知的动态. $\boldsymbol{y} = [y_1, \cdots, y_N]^{\mathrm{T}} \in \mathbf{R}^N$ 为各个子系统的输出, $H_{i,j}(\boldsymbol{y})$ 是子系统的互联项, 是未知的光滑函数. $b_{i,q}$ 是已知的控制增益. $u_{i,q}$ 为第 q 个执行器的输出, 即执行器对系统的控制输入. 本文假设仅系统的输出 y_i 是可测的. 基于文献 [21], 执行器故障模型可表示为

$$u_{i,q} = k_{i,q}v_{i,q} + \bar{u}_{i,q},\ t \in [t_{i,q}^s, t_{i,q}^e) \tag{2}$$

其中, $q = 1, \cdots, m$. $k_{i,q} \in [0, 1]$, $\bar{u}_{i,q}$ 为执行器故障参数. $v_{i,q}$ 为将要设计的控制器. $t_{i,q}^s, t_{i,q}^e$ 分别表示执行器发生故障的起始时间和终止时间. 故障模型有如下 4 种情况: 1) 当 $\bar{u}_{i,q} = 0, 0 < k_{i,q} < 1$, 执行器存在部分失效. 2) 当 $\bar{u}_{i,q} = 0, k_{i,q} = 1$, 执行器正常运行. 3) 当 $\bar{u}_{i,q} \neq 0, k_{i,q} = 0$, 执行器完全失效. 4) 当 $\bar{u}_{i,q} \neq 0, k_{i,q} = 1$, 执行器发生偏差故障.

本文的控制目标是: 1) 设计模糊观测器对系统状态进行观测, 并通过设计二阶滤波器避免反步递推方法本身存在的 "复杂度爆炸" 问题. 2) 系统输出 y_i 能跟踪参考信号 $y_{i,d}$, 且跟踪误差满足 $|y_i - y_{i,d}| \le k_i(t)$. 3) 在事件触发机制下, 设计容错控制器能够补偿执行器故障对系统的影响.

注 1. 若能保证系统跟踪误差受限, 可进一步得到输出满足: $-k_i(t) + y_{i,d} \le y_i \le k_i(t) + y_{i,d}$, 保证系统输出受限.

假设 1[8]**.** 参考信号 $y_{i,d}$ 及其导数 $\dot{y}_{i,d}$ 是有界的.

假设 2[21]**.** 每个子系统最多容许 $m-1$ 个执行器同时发生故障.

假设 3[8]**.** 函数 $f_{i,j}(\cdot)$ 满足局部 Lipschitz 条件, 即存在常数 $\varpi_{i,j}, i = 1, \cdots, N, j = 1, \cdots, n$ 使得:

$$|f_{i,j}(\boldsymbol{X}) - f_{i,j}(\boldsymbol{Y})| \le \varpi_{i,j}\|\boldsymbol{X}-\boldsymbol{Y}\| \tag{3}$$

其中, $\|\cdot\|$是向量的 2-范数.

假设 4[11]**.** 非线性互联项满足:

$$|H_{i,j}(\boldsymbol{y})| \le \sum_{k=1}^{p}\sum_{l=1}^{N}\lambda_{l,i,j}^{k}|y_l|^k \tag{4}$$

其中, $\lambda_{l,i,j}^{k}$是未知常数, p 是已知常数.

假设 5[25]**.** 对于系统未建模动态 $\Delta_{i,j}(y_i,\boldsymbol{z}_i)$, 存在两个未知的非负的光滑函数 $\varphi_{ij1}(y_i),\varphi_{ij2}(y_i)$, 且 $\varphi_{ij1}(0)=\varphi_{ij2}(0)=0$, 满足:

$$|\Delta_{i,j}(y_i,\boldsymbol{z}_i)| \le \varphi_{ij1}(y_i)+\varphi_{ij2}(y_i)\|\boldsymbol{z}_i\| \tag{5}$$

假设 6[18]**.** 未建模动态 $\dot{\boldsymbol{z}}_i=q_i(\boldsymbol{z}_i,0,t)-q_i(0,0,t)$ 是全局指数稳定的. 即, 当 $\boldsymbol{z}_i=0$ 时, 存在一个李雅普诺夫函数 $W_i(\boldsymbol{z}_i,t)$满足:

$$\begin{aligned}&r_{i,1}\|\boldsymbol{z}_i\|^4 \le W_i(\boldsymbol{z}_i,t) \le r_{i,2}\|\boldsymbol{z}_i\|^4\\&\frac{\partial W_i}{\partial t}+\frac{\partial W_i}{\partial \boldsymbol{z}_i}(q_i(\boldsymbol{z}_i,0,t)-q_i(0,0,t)) \le -r_{i,3}\|\boldsymbol{z}_i\|^4\\&\left\|\frac{\partial W_i}{\partial \boldsymbol{z}_i}\right\| \le r_{i,4}\|\boldsymbol{z}_i\|^3\end{aligned} \tag{6}$$

其中, $\|q_i(0,0,t)\| \le r_{i,5}, \forall t \ge 0$. $r_{i,1}$, $r_{i,2}$, $r_{i,3}$, $r_{i,4}$, $r_{i,5}$ 为正常数.

假设 7[25]**.** 存在未知的正定函数 $h_i(\cdot)$ 且 $h_i(0)=0$, 使得: $\|q_i(\boldsymbol{z}_i,y_i,t)-q_i(\boldsymbol{z}_i,0,t)\| \le h_i(y_i)$.

引理 1[26]**.** 对于任意的 $\epsilon>0$ 和 $\eta\in\mathbf{R}$, 以下的不等式成立:

$$0 \le |\eta|-\eta\tanh\left(\frac{\eta}{\epsilon}\right) \le \kappa\epsilon,\ \kappa\approx 0.2785$$

1.2 指数相关的障碍李雅普诺夫函数

为了更好地保证系统跟踪误差受限, 一种指数相关的障碍李雅普诺夫函数被提出[27], 形式如下:

$$V=\ln\frac{k^2(t)}{k^2(t)-s^2}$$

其中, $k(t)=k_a\mathrm{e}^{-rt}+k_b$ 是时变函数, $k_a>k_b>0$, $r>0$. 与文献 [10−12] 中设计的障碍李雅普诺夫函数不同, 本文选取的障碍李雅普诺夫函数结合了指数型性能函数, 该性能函数可以保证变量的收敛速度, 进一步可以提高系统的暂态性能.

引理 2[27]**.** 对于任意的 $k(t)$, 当且仅当 $|s|<k(t)$, 以下不等式成立:

$$\ln\frac{k^2(t)}{k^2(t)-s^2}<\frac{s^2}{k^2(t)-s^2}$$

引理 3[28]**.** 对于任何定义在紧集 U 上的连续函数 $f(\boldsymbol{x})$, 存在模糊逻辑系统 $\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{x})$ 及任意大于 0 的常数 ϵ, 使得下列不等式成立:

$$\sup_{x\in U}\left|f(\boldsymbol{x})-\boldsymbol{\theta}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{x})\right| \le \epsilon$$

其中, $\boldsymbol{\phi}(\boldsymbol{x})=[\phi_1(\boldsymbol{x}),\cdots,\phi_L(\boldsymbol{x})]^{\mathrm{T}}$ 为模糊基函数, $\boldsymbol{\theta}\in\mathbf{R}^L$ 是理想权重向量. $\phi_i(\boldsymbol{x})$ 可以选取高斯函数:

$$\phi_i(\boldsymbol{x})=\exp\left[\frac{-(\boldsymbol{x}-\boldsymbol{\delta}_i)^{\mathrm{T}}(\boldsymbol{x}-\boldsymbol{\delta}_i)}{\varsigma_i^2}\right]$$

其中, $\boldsymbol{\delta}_i$, ς_i 分别代表高斯函数的中心和宽度.

2 观测器及控制器的设计

2.1 模糊观测器的设计

采用模糊逻辑系统来逼近未知非线性函数:

$$\hat{f}_{i,j}(\hat{\boldsymbol{X}}_i|\boldsymbol{\theta}_{i,j})=\boldsymbol{\theta}_{i,j}^{\mathrm{T}}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i) \tag{7}$$

其中, $\hat{\boldsymbol{X}}_i$ 为稍后设计的观测器对状态 $\boldsymbol{X}_i$ 的估计. 定义理想权重向量 $\boldsymbol{\theta}_{i,j}^{*}$ 为:

$$\boldsymbol{\theta}_{i,j}^{*}=\arg\min_{\boldsymbol{\theta}_{i,j}\in\boldsymbol{\Omega}_{i,j}}\left[\sup_{\hat{\boldsymbol{X}}_i\in\boldsymbol{U}_i}|\hat{f}_{i,j}(\hat{\boldsymbol{X}}_i|\boldsymbol{\theta}_{i,j})-f_{i,j}(\hat{\boldsymbol{X}}_i)|\right] \tag{8}$$

其中, $\boldsymbol{\Omega}_{i,j}, \boldsymbol{U}_i$ 为紧集, 定义模糊逼近误差 $\epsilon_{i,j}$ 为:

$$\epsilon_{i,j}=\left|\hat{f}_{i,j}(\hat{\boldsymbol{X}}_i|\boldsymbol{\theta}_{i,j}^{*})-f_{i,j}(\hat{\boldsymbol{X}}_i)\right| \tag{9}$$

其中, $\epsilon_{i,j}$ 满足: $|\epsilon_{i,j}| \le \epsilon_{i,j}^{*}$, $\epsilon_{i,j}^{*}$ 为未知常数.

由此, 系统 (1) 可表示为如下形式:

$$\begin{aligned}\dot{\boldsymbol{X}}_i=&\boldsymbol{A}\boldsymbol{X}_i+\sum_{j=1}^{n}\boldsymbol{B}_{i,j}\boldsymbol{\theta}_{i,j}^{*\mathrm{T}}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)+\boldsymbol{\epsilon}_i+\Delta\boldsymbol{F}_i+\\&\boldsymbol{B}_{i,n}\sum_{q=1}^{m}b_{i,q}u_{i,q}+\boldsymbol{H}_i(\boldsymbol{y})+\boldsymbol{\Delta}_i(y_i,\boldsymbol{z}_i)\end{aligned} \tag{10}$$

其中, $\boldsymbol{A}=\begin{bmatrix}\boldsymbol{0}_{(n-1)\times 1} & \boldsymbol{I}_{n-1}\\ 0 & \boldsymbol{0}_{1\times(n-1)}\end{bmatrix}\in\mathbf{R}^{n\times n}$, $\boldsymbol{B}_{i,j}=[\overbrace{0,\cdots,0}^{j-1},1,0,\cdots,0]^{\mathrm{T}}$ (共 n 项: $\underbrace{\quad}_{n}$), $\Delta\boldsymbol{F}_i=[\Delta f_{i,1},\cdots,\Delta f_{i,n}]^{\mathrm{T}}$, $\Delta f_{i,j}=f_{i,j}(\boldsymbol{X}_i)-f_{i,j}(\hat{\boldsymbol{X}}_i)$, $\boldsymbol{\epsilon}_i=[\epsilon_{i,1},\cdots,\epsilon_{i,n}]^{\mathrm{T}}$, $\boldsymbol{\Delta}_i(y_i,\boldsymbol{z}_i)=[\Delta_{i,1}(y_i,\boldsymbol{z}_i),\cdots,\Delta_{i,n}(y_i,\boldsymbol{z}_i)]^{\mathrm{T}}$, $\boldsymbol{H}_i(\boldsymbol{y})=[H_{i,1}(\boldsymbol{y}),\cdots,H_{i,n}(\boldsymbol{y})]^{\mathrm{T}}$.

为了估计系统中不可测的状态, 设计如下的模糊观测器:

$$\begin{aligned}\dot{\hat{\boldsymbol{X}}}_i=&\boldsymbol{A}_{0,i}\hat{\boldsymbol{X}}_i+\boldsymbol{L}_iy_i+\sum_{j=1}^{n}\boldsymbol{B}_{i,j}\boldsymbol{\theta}_{i,j}^{\mathrm{T}}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)+\\&\boldsymbol{B}_{i,n}\sum_{q=1}^{m}b_{i,q}u_{i,q}\end{aligned} \tag{11}$$

其中, $\boldsymbol{A}_{0,i}=\boldsymbol{A}-\left[\boldsymbol{L}_i,\boldsymbol{0}_{n\times(n-1)}\right]$, $\boldsymbol{L}_i=[l_{i,1},\cdots,l_{i,n}]^{\mathrm{T}}$,

$\boldsymbol{\theta}_{i,j}$ 为对 $\boldsymbol{\theta}_{i,j}^*$ 的估计. 通过设计适当的 $\boldsymbol{L}_i$ 使得正定对称矩阵 $\boldsymbol{P}_i$ 满足如下等式:

$$\boldsymbol{A}_{0,i}^{\mathrm{T}}\boldsymbol{P}_i+\boldsymbol{P}_i\boldsymbol{A}_{0,i}=-2\boldsymbol{Q}_i \tag{12}$$

其中, $\boldsymbol{Q}_i$ 为正定矩阵.

定义观测误差为 $\boldsymbol{e}_i=\boldsymbol{X}_i-\hat{\boldsymbol{X}}_i$, 由式 (10) 和 (11) 可得到如下观测误差系统:

$$\dot{\boldsymbol{e}}_i=\boldsymbol{A}_{0,i}\boldsymbol{e}_i+\sum_{j=1}^n\boldsymbol{B}_{i,j}\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)+\boldsymbol{\epsilon}_i+\Delta\boldsymbol{F}_i+\boldsymbol{H}_i(\boldsymbol{y})+\boldsymbol{\Delta}_i(y_i,\boldsymbol{z}_i) \tag{13}$$

其中, $\tilde{\boldsymbol{\theta}}_{i,j}=\boldsymbol{\theta}_{i,j}^*-\boldsymbol{\theta}_{i,j}$. 为了保证误差系统的稳定性, 可选取如下李雅普诺夫函数:

$$V_0=\sum_{i=1}^N\frac{1}{2}\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{e}_i \tag{14}$$

其导数为:

$$\dot{V}_0=\sum_{i=1}^N\Bigg[-\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{Q}_i\boldsymbol{e}_i+\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{P}_i\Bigg(\sum_{j=1}^n\boldsymbol{B}_{i,j}\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)+\boldsymbol{\epsilon}_i+\Delta\boldsymbol{F}_i+\boldsymbol{H}_i(\boldsymbol{y})+\boldsymbol{\Delta}_i(y_i,\boldsymbol{z}_i)\Bigg)\Bigg] \tag{15}$$

根据 Young's 不等式[4−21] 及模糊基函数的性质 $0\le\boldsymbol{\phi}_{i,j}^{\mathrm{T}}(\hat{\boldsymbol{X}}_i)\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)\le 1$, 得:

$$\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{P}_i\sum_{j=1}^n\boldsymbol{B}_{i,j}\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)\le\sum_{j=1}^n\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,j}+\frac{\|\boldsymbol{P}_i\|^2\|\boldsymbol{e}_i\|^2}{4} \tag{16}$$

根据假设 3 ~ 5, 可得到如下不等式:

$$\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{P}_i(\boldsymbol{\epsilon}_i+\Delta\boldsymbol{F}_i)\le\|\boldsymbol{e}_i\|^2+\frac{\|\boldsymbol{P}_i\|^2\sum\limits_{j=1}^n\varpi_{i,j}^2}{2}\|\boldsymbol{e}_i\|^2+\frac{\|\boldsymbol{P}_i\|^2\|\boldsymbol{\epsilon}_i^*\|^2}{2} \tag{17}$$

$$\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{\Delta}_i(y_i,\boldsymbol{z}_i)\le\frac{\|\boldsymbol{P}_i\|^2\|\boldsymbol{e}_i\|^2}{4}+\sum_{j=1}^n|\Delta_{i,j}(y_i,\boldsymbol{z}_i)|^2\le\frac{\|\boldsymbol{P}_i\|^2\|\boldsymbol{e}_i\|^2}{4}+\sum_{j=1}^n\bar{\varphi}_{ij}(y_i)+\frac{r_{i,3}}{16\lambda_0}\|\boldsymbol{z}_i\|^4 \tag{18}$$

$$\sum_{i=1}^N\boldsymbol{e}_i^{\mathrm{T}}\boldsymbol{P}_i\boldsymbol{H}_i(\boldsymbol{y})\le\frac{\sum\limits_{i=1}^N\|\boldsymbol{P}_i\|^2\|\boldsymbol{e}_i\|^2}{4}+\sum_{i=1}^N\|\boldsymbol{H}_i(\boldsymbol{y})\|^2\le\frac{\sum\limits_{i=1}^N\|\boldsymbol{P}_i\|^2\|\boldsymbol{e}_i\|^2}{4}+\sum_{i=1}^N\sum_{k=1}^p pN3^{2k}\left(\sum_{l=1}^N\sum_{j=1}^n\lambda_{i,l,j}^{2k}\right)\times\left(|y_{i,d}|^{2k}+|s_{i,1}|^{2k}+|q_{i,1}|^{2k}\right) \tag{19}$$

其中, $\boldsymbol{\epsilon}_i^*=[\epsilon_{i,1}^*,\cdots,\epsilon_{i,n}^*]^{\mathrm{T}}$, $\bar{\varphi}_{ij}(y_i)=2\varphi_{ij1}^2(y_i)+(16n\lambda_0/r_{i,3})\varphi_{ij2}^4(y_i)$, λ_0 为正常数. $q_{i,1},s_{i,1}$ 稍后定义. 将式 (16) ~ (19) 代入式 (15) 可得:

$$V_0\le\sum_{i=1}^N\Bigg[-\eta_{i,0}\|\boldsymbol{e}_i\|^2+\frac{\|\boldsymbol{P}_i\|^2\|\boldsymbol{\epsilon}_i^*\|^2}{2}+\frac{r_{i3}}{16\lambda_0}\|\boldsymbol{z}_i\|^4+\sum_{j=1}^n\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,j}+\sum_{k=1}^p pN3^{2k}\left(\sum_{l=1}^N\sum_{j=1}^n\lambda_{i,l,j}^{2k}\right)\times\left(|y_{i,d}|^{2k}+|s_{i,1}|^{2k}+|q_{i,1}|^{2k}\right)+\sum_{j=1}^n\bar{\varphi}_{ij}(y_i)\Bigg] \tag{20}$$

其中, $\eta_{i,0}=\lambda_{\min}(\boldsymbol{Q}_i)-1-\dfrac{\|\boldsymbol{P}_i\|^2\sum_{j=1}^n\varpi_{i,j}^2}{2}-\dfrac{3\|\boldsymbol{P}_i\|^2}{4}$.

2.2 自适应事件触发控制器设计及稳定性证明

接下来, 根据反步法原理, 构建如下误差变换:

$$\begin{aligned}z_{i,1}&=x_{i,1}-y_{i,d}\\z_{i,j}&=\hat{x}_{i,j}-\nu_{i,j-1}\end{aligned} \tag{21}$$

其中, $z_{i,j}$ $(i=1,\cdots,N,j=2,\cdots,n)$ 为误差变量, $\nu_{i,j-1}$ 为滤波器的输出.

注 2. 在反步法设计中, 对虚拟控制的求导容易出现 "计算爆炸" 问题, 为降低计算量, 利用滤波器估计虚拟控制器导数的思想被相应提出[29−32].

本文采用如下的二阶滤波器:

$$\begin{cases}\dot{\nu}_{i,j}=p_{i,j}\hat{\nu}_{i,j}\\\dot{\hat{\nu}}_{i,j}=-2\omega_{i,j}p_{i,j}\hat{\nu}_{i,j}-p_{i,j}(\nu_{i,j}-\alpha_{i,j})\end{cases} \tag{22}$$

其中, $p_{i,j}$ 和 $\omega_{i,j}\in(0,1]$ 为需要设计的正数, 且 $\nu_{i,j}(0)=\alpha_{i,j}(0)$, $\hat{\nu}_{i,j}(0)=0$, 虚拟控制器 $\alpha_{i,j}$ 为滤波器输入.

由于滤波过程容易产生误差, 影响控制效果[33]. 对此引入补偿信号 $q_{i,j}$ 进行处理, 定义如下新的误差变换:

$$s_{i,j}=z_{i,j}-q_{i,j},\ \ j=1,\cdots,n \tag{23}$$

基于反步递推控制方法, 对每一个子系统设计的虚拟控制器、补偿信号导数及参数自适应律如下:

$$\alpha_{i,1} = -c_{i,1}z_{i,1} - \frac{13z_{i,1}}{4(k_i^2(t)-s_{i,1}^2)} - \bar{k}_i(t)s_{i,1} + \dot{y}_{i,d} - \boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\phi}_{i,1}(\hat{x}_{i,1}) - \frac{s_{i,1}}{k_i^2(t)-s_{i,1}^2}\Theta_{i,1} - (k_i^2(t)-s_{i,1}^2)\beta_i\sum_{k=1}^{p}3^{2k}|s_{i,1}|^{2k-1} \tag{24}$$

$$\dot{q}_{i,1} = -c_{i,1}q_{i,1} - \frac{13q_{i,1}}{4\left(k_i^2(t)-s_{i,1}^2\right)} + q_{i,2} + \nu_{i,1} - \alpha_{i,1} \tag{25}$$

$$\alpha_{i,2} = -c_{i,2}z_{i,2} - \frac{3z_{i,2}}{4} - l_{i,2}e_{i,1} + \dot{\nu}_{i,1} - \boldsymbol{\theta}_{i,2}^{\mathrm{T}}\boldsymbol{\phi}_{i,2}(\hat{\boldsymbol{X}}_{i,2}) - s_{i,2}\Theta_{i,2} \tag{26}$$

$$\dot{q_{i,2}} = -c_{i,2}q_{i,2} - \frac{3q_{i,2}}{4} + q_{i,3} + \nu_{i,2} - \alpha_{i,2} \tag{27}$$

对于 $j=3,\cdots,n-1$, 虚拟控制器 $\alpha_{i,j}$ 和补偿信号导数 $\dot{q}_{i,j}$, 以及 $\alpha_{i,n}$, $\dot{q}_{i,n}$ 被设计为

$$\alpha_{i,j} = -c_{i,j}z_{i,j} - \frac{z_{i,j}}{4} - z_{i,j-1} - l_{i,j}e_{i,1} + \dot{\nu}_{i,j-1} - \boldsymbol{\theta}_{i,j}^{\mathrm{T}}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_{i,j}) - s_{i,j}\Theta_{i,j} \tag{28}$$

$$\dot{q}_{i,j} = -c_{i,j}q_{i,j} - \frac{q_{i,j}}{4} - q_{i,j-1} + q_{i,j+1} + \nu_{i,j} - \alpha_{i,j} \tag{29}$$

$$\alpha_{i,n} = c_{i,n}z_{i,n} + z_{i,n-1} + l_{i,n}e_{i,1} - \dot{\nu}_{i,n-1} + s_{i,n}\Theta_{i,n} \tag{30}$$

$$\dot{q}_{i,n} = -c_{i,n}q_{i,n} - q_{i,n-1} \tag{31}$$

设计如下的参数自适应律:

$$\begin{aligned}\dot{\boldsymbol{\theta}}_{i,1} &= \boldsymbol{\Gamma}_{i,1}\left(\frac{s_{i,1}}{k_i^2(t)-s_{i,1}^2}\boldsymbol{\phi}_{i,1}(\hat{x}_{i,1}) - \rho_{i,1,1}\boldsymbol{\theta}_{i,1}\right)\\ \dot{\Theta}_{i,1} &= \gamma_{i,1,2}\left(\frac{s_{i,1}^2}{(k_i^2(t)-s_{i,1}^2)^2} - \rho_{i,1,2}\Theta_{i,1}\right)\\ \dot{\beta}_i &= \gamma_i\left(\sum_{k=1}^{p}3^{2k}|s_{i,1}|^{2k} - \rho_i\beta_i\right)\end{aligned} \tag{32}$$

对于 $j=2,\cdots,n-1$, $\dot{\boldsymbol{\theta}}_{i,j}$ 和 $\dot{\Theta}_{i,j}$ 被设计为

$$\begin{aligned}\dot{\boldsymbol{\theta}}_{i,j} &= \boldsymbol{\Gamma}_{i,j}(s_{i,j}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_{i,j}) - \rho_{i,j,1}\boldsymbol{\theta}_{i,j})\\ \dot{\Theta}_{i,j} &= \gamma_{i,j,2}(s_{i,j}^2 - \rho_{i,j,2}\Theta_{i,j})\end{aligned} \tag{33}$$

以及

$$\begin{aligned}\dot{\boldsymbol{\theta}}_{i,n} &= \boldsymbol{\Gamma}_{i,n}(-s_{i,n}\boldsymbol{\phi}_{i,n}(\hat{\boldsymbol{X}}_{i}) - \rho_{i,n,1}\boldsymbol{\theta}_{i,n})\\ \dot{\Theta}_{i,n} &= \gamma_{i,n,2}(s_{i,n}^2 - \rho_{i,n,2}\Theta_{i,n})\\ \dot{\varsigma}_i &= \gamma_{i,n,3}(|s_{i,n}|\,\|\boldsymbol{E}\| - \rho_{i,n,3}\varsigma_i)\\ \dot{\xi}_i &= \gamma_{i,n,4}(s_{i,n}\alpha_{i,n} - \rho_{i,n,4}\xi_i)\end{aligned} \tag{34}$$

其中, 对于 $j=1,\cdots,n$, β_i, $\Theta_{i,j}$, ς_i, ξ_i 分别为对 β_i^*, $\Theta_{i,j}^*$, ς_i^*, ξ_i^* 的估计. β_i^*, $\Theta_{i,j}^*$, ς_i^*, ξ_i^* 的定义后面给出. $\hat{\boldsymbol{X}}_{i,j}=[\hat{x}_{i,1},\cdots,\hat{x}_{i,j}]^{\mathrm{T}}$. $\boldsymbol{\Gamma}_{i,j}$ 为设计的正定矩阵, $\boldsymbol{E}=[1,\cdots,1]_{m\times 1}^{\mathrm{T}}$. $c_{i,j}$, γ_i, $\gamma_{i,j,2}$, $\gamma_{i,n,3}$, $\gamma_{i,n,4}$, ρ_i, $\rho_{i,j,1}$, $\rho_{i,j,2}$, $\rho_{i,n,3}$, $\rho_{i,n,4}$ 为设计的正常数. $\alpha_{i,1}$ 中: $\bar{k}_i(t)=\sqrt{\left(\frac{\dot{k}_i(t)}{k_i(t)}\right)^2+b_i}$, $b_i>0$ 为设计常数.

系统稳定性的证明过程如下.

第 1 步. 定义李雅普诺夫函数为

$$V_1 = V_0 + \sum_{i=1}^{N}\left[\frac{1}{2}\ln\frac{k_i^2(t)}{k_i^2(t)-s_{i,1}^2} + \frac{\tilde{\Theta}_{i,1}^2}{2\gamma_{i,1,2}} + \frac{\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\Gamma}_{i,1}^{-1}\tilde{\boldsymbol{\theta}}_{i,1}}{2} + \frac{\tilde{\beta}_i^2}{2\gamma_i} + \frac{W_i(\boldsymbol{z}_i,t)}{\lambda_0}\right] \tag{35}$$

对 V_1 求导得:

$$\dot{V}_1 = \dot{V}_0 + \sum_{i=1}^{N}\left[\frac{s_{i,1}}{k_i^2(t)-s_{i,1}^2}\left(\dot{s}_{i,1} - \frac{\dot{k}_i(t)s_{i,1}}{k_i(t)}\right) + \frac{\dot{W}_i}{\lambda_0} - \frac{\tilde{\Theta}_{i,1}\dot{\Theta}_{i,1}}{\gamma_{i,1,2}} - \tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\Gamma}_{i,1}^{-1}\dot{\boldsymbol{\theta}}_{i,1} - \frac{\tilde{\beta}_i\dot{\beta}_i}{\gamma_i}\right] \tag{36}$$

其中, 对于 $j=1,\cdots,n$, 令 $\tilde{\beta}_i=\beta_i^*-\beta_i$, $\tilde{\Theta}_{i,j}=\Theta_{i,j}^*-\Theta_{i,j}$, $\tilde{\varsigma}_i=\varsigma_i^*-\varsigma_i$, $\tilde{\xi}_i=\xi_i^*-\xi_i$.

$$\begin{aligned}\dot{s}_{i,1} =\ & s_{i,2} + q_{i,2} + \nu_{i,1} + \alpha_{i,1} - \alpha_{i,1} + e_{i,2} + \Delta f_{i,1} + \boldsymbol{\theta}_{i,1}^{*\mathrm{T}}\boldsymbol{\phi}_{i,1}(\hat{\boldsymbol{X}}_i) - \boldsymbol{\theta}_{i,1}^{*\mathrm{T}}\boldsymbol{\phi}_{i,1}(\hat{x}_{i,1}) +\\ & \boldsymbol{\theta}_{i,1}^{\mathrm{T}}\boldsymbol{\phi}_{i,1}(\hat{x}_{i,1}) + \tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\boldsymbol{\phi}_{i,1}(\hat{x}_{i,1}) + \epsilon_{i,1} + H_{i,1}(\boldsymbol{y}) + \Delta_{i,1}(y_i,\boldsymbol{z}_i) - \dot{y}_{i,d} - \dot{q}_{i,1}\end{aligned} \tag{37}$$

由 Young′s 不等式和模糊基函数的性质, 再结合假设 3 ~ 5 得:

$$\frac{s_{i,1}}{k_i^2(t)-s_{i,1}^2}(\boldsymbol{\theta}_{i,1}^{*\mathrm{T}}\boldsymbol{\phi}_{i,1}(\hat{\boldsymbol{X}}_i) - \boldsymbol{\theta}_{i,1}^{*\mathrm{T}}\boldsymbol{\phi}_{i,1}(\hat{x}_{i,1})) \le \frac{s_{i,1}^2}{(k_i^2(t)-s_{i,1}^2)^2}\Theta_{i,1}^* + 1 \tag{38}$$

$$\frac{s_{i,1}(e_{i,2}+\Delta f_{i,1}+\epsilon_{i,1})}{k_i^2(t)-s_{i,1}^2} \le \frac{2s_{i,1}^2}{(k_i^2(t)-s_{i,1}^2)^2} + \frac{\|\boldsymbol{e}_i\|^2+|\epsilon_{i,1}^*|^2}{2} + \frac{\varpi_{i,1}^2\|\boldsymbol{e}_i\|^2}{4} \tag{39}$$

$$\sum_{i=1}^{N}\frac{s_{i,1}H_{i,1}(\boldsymbol{y})}{k_i^2(t)-s_{i,1}^2} \le \sum_{i=1}^{N}\left(\frac{s_{i,1}^2}{4(k_i^2(t)-s_{i,1}^2)^2} + \sum_{k=1}^{p}pN3^{2k}\times\left(\sum_{l=1}^{N}\lambda_{i,l,1}^{2k}\right)(|y_{i,d}|^{2k}+|s_{i,1}|^{2k}+|q_{i,1}|^{2k})\right) \tag{40}$$

$$\frac{s_{i,1}}{k_i^2(t)-s_{i,1}^2}\Delta_{i,1}(y_i,\boldsymbol{z}_i)\le \frac{s_{i,1}^2}{2(k_i^2(t)-s_{i,1}^2)^2}+\bar{\varphi}_{i11}(y_i)+\frac{r_{i,3}}{16\lambda_0}\|\boldsymbol{z}_i\|^4 \quad (41)$$

$$\frac{s_{i,1}s_{i,2}}{k_i^2(t)-s_{i,1}^2}\le\frac{s_{i,1}^2}{2(k_i^2(t)-s_{i,1}^2)^2}+\frac{s_{i,2}^2}{2} \quad (42)$$

其中, $\Theta_{i,1}^*=\|\boldsymbol{\theta}_{i,1}^*\|^2$, $\bar{\varphi}_{i11}(y_i)=\varphi_{i11}^2(y_i)+\dfrac{4\lambda_0}{r_{i,3}}\varphi_{i12}^4(y_i)$.

把式 (20), (38) ~ (42) 代入式 (36) 可得:

$$\begin{aligned}\dot{V}_1\le&\sum_{i=1}^N\Bigg[-\eta_{i,0}\|\boldsymbol{e}_i\|^2+\frac{\|\boldsymbol{e}_i\|^2}{2}+\frac{\varpi_{i,1}^2\|\boldsymbol{e}_i\|^2}{4}+\\&\sum_{j=1}^n\tilde{\boldsymbol{\theta}}_{i,j}^{\rm T}\tilde{\boldsymbol{\theta}}_{i,j}+\frac{r_{i,3}}{8\lambda_0}\|\boldsymbol{z}_i\|^4+\frac{\dot{W}_i}{\lambda_0}+\frac{s_{i,1}}{k_i^2(t)-s_{i,1}^2}\\&\Big(q_{i,2}+\nu_{i,1}+\alpha_{i,1}-\alpha_{i,1}-\dot{q}_{i,1}+\boldsymbol{\theta}_{i,1}^{\rm T}\boldsymbol{\phi}_{i,1}(\hat{x}_{i,1})-\\&\dot{y}_{i,d}-\frac{\dot{k}_i(t)s_{i,1}}{k_i(t)}\Big)+\frac{s_{i,2}^2}{2}+\frac{13s_{i,1}^2}{4(k_i^2(t)-s_{i,1}^2)^2}+\\&\frac{s_{i,1}^2}{(k_i^2(t)-s_{i,1}^2)^2}\Theta_{i,1}^*+\sum_{j=1}^n\bar{\varphi}_{ij}(y_i)+\bar{\varphi}_{i11}(y_i)+\\&\beta_i^*\sum_{k=1}^p3^{2k}\left(|y_{i,d}|^{2k}+|s_{i,1}|^{2k}+|q_{i,1}|^{2k}\right)+1+\\&\tilde{\boldsymbol{\theta}}_{i,1}^{\rm T}\boldsymbol{\Gamma}_{i,1}^{-1}\left(\frac{\boldsymbol{\Gamma}_{i,1}s_{i,1}}{k_i^2(t)-s_{i,1}^2}\boldsymbol{\phi}_{i,1}(\hat{x}_{i,1})-\dot{\boldsymbol{\theta}}_{i,1}\right)-\frac{\tilde{\beta}_i\dot{\beta}_i}{\gamma_i}-\\&\frac{\tilde{\Theta}_{i,1}\dot{\Theta}_{i,1}}{\gamma_{i,1,2}}+\frac{\|\varepsilon_{i,1}^*\|^2}{2}+\frac{\|\boldsymbol{P}_i\|^2\|\boldsymbol{\varepsilon}_i^*\|^2}{2}\Bigg]\end{aligned} \quad (43)$$

其中, $\beta_i^*=\max_{1\le k\le p}\Big\{pN\sum_{l=1}^N\sum_{j=1}^n\lambda_{i,l,j}^{2k}+pN\sum_{l=1}^N\lambda_{i,l,1}^{2k}\Big\}$. 根据假设 6, 7 和文献 [18], $\dfrac{W_i(\boldsymbol{z}_i,t)}{\lambda_0}$ 的导数为:

$$\begin{aligned}\frac{\dot{W}_i(\boldsymbol{z}_i,t)}{\lambda_0}=&\\&\frac{1}{\lambda_0}\left[\frac{\partial W_i}{\partial\boldsymbol{z}_i}(q_i(\boldsymbol{z}_i,0,t)-q_i(0,0,t))+\frac{\partial W_i}{\partial t}+\right.\\&\left.\frac{\partial W_i}{\partial\boldsymbol{z}_i}(q_i(\boldsymbol{z}_i,y_i,t)-q_i(\boldsymbol{z}_i,0,t))+\frac{\partial W_i}{\partial\boldsymbol{z}_i}q_i(0,0,t)\right]\le\\&-\frac{5r_{i,3}}{8\lambda_0}\|\boldsymbol{z}_i\|^4+\frac{16r_{i,4}^4}{\lambda_0r_{i,3}^3}h_i^4(y_i)+\frac{16r_{i,4}^4r_{i,5}^4}{\lambda_0r_{i,3}^3}\end{aligned} \quad (44)$$

注 3. $\alpha_{i,1}$中的 $\bar{k}_i(t)$是为了保证 $\bar{k}_i(t)+\dot{k}_i(t)/k_i(t)\ge 0$. 所以, $\left(\bar{k}_i(t)+\dfrac{\dot{k}_i(t)}{k_i(t)}\right)s_{i,1}^2\ge0$.

将虚拟控制器 (24), 补偿信号 (25), 自适应律 (32) 和 (44) 代入式 (43) 得到:

$$\begin{aligned}\dot{V}_1\le&\sum_{i=1}^N\Bigg[-\eta_{i,1}\|\boldsymbol{e}_i\|^2-\frac{c_{i,1}s_{i,1}^2}{k_i^2(t)-s_{i,1}^2}-\frac{r_{i,3}}{2\lambda_0}\|\boldsymbol{z}_i\|^4+\\&\rho_{i,1,1}\tilde{\boldsymbol{\theta}}_{i,1}^{\rm T}\boldsymbol{\theta}_{i,1}+\rho_{i,1,2}\tilde{\Theta}_{i,1}\Theta_{i,1}+\rho_i\tilde{\beta}_i\beta_i+\\&\sum_{j=1}^n\tilde{\boldsymbol{\theta}}_{i,j}^{\rm T}\tilde{\boldsymbol{\theta}}_{i,j}+\frac{s_{i,2}^2}{2}+D_{i,1}+M_i\Bigg]\end{aligned} \quad (45)$$

其中, $\eta_{i,1}=\eta_{i,0}-\dfrac{1}{2}-\dfrac{\varpi_{i,1}^2}{4}$, $M_i=\sum_{j=1}^n\bar{\varphi}_{ij}(y_i)+\bar{\varphi}_{i11}(y_i)+\dfrac{16r_{i,4}^4}{\lambda_0r_{i,3}^3}h_i^4(y_i)+\beta_i^*\sum_{k=1}^p3^{2k}(|y_{i,d}|^{2k}+|q_{i,1}|^{2k})$, $D_{i,1}=\dfrac{\|\boldsymbol{P}_i\|^2\|\boldsymbol{\varepsilon}_i^*\|^2}{2}+\dfrac{\|\varepsilon_{i,1}^*\|^2}{2}+1+\dfrac{16r_{i,4}^4r_{i,5}^4}{\lambda_0r_{i,3}^3}$.

注 4. 因为, M_i 是非负函数, $y_{i,d}$ 是有界的且输出 y_i 是受限的, $q_{i,1}$ 作为补偿信号也是收敛有界的[31], 所以存在正数 $\bar{M}_i$ 满足 $\bar{M}_i\ge M_i$.

第 j 步 ($j=2,\cdots,n-1$). 定义李雅普诺夫函数

$$V_j=V_{j-1}+\sum_{i=1}^N\left[\frac{1}{2}s_{i,j}^2+\frac{\tilde{\boldsymbol{\theta}}_{i,j}^{\rm T}\boldsymbol{\Gamma}_{i,j}^{-1}\tilde{\boldsymbol{\theta}}_{i,j}}{2}+\frac{\tilde{\Theta}_{i,j}^2}{2\gamma_{i,j,2}}\right]$$

求导得:

$$\begin{aligned}\dot{V}_j=&\dot{V}_{j-1}+\sum_{i=1}^N\Bigg[s_{i,j}\Big(s_{i,j+1}+q_{i,j+1}+\nu_{i,j}+\\&\alpha_{i,j}-\alpha_{i,j}+\boldsymbol{\theta}_{i,j}^{*\rm T}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)-\tilde{\boldsymbol{\theta}}_{i,j}^{\rm T}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)-\\&\boldsymbol{\theta}_{i,j}^{*\rm T}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_{i,j})+\boldsymbol{\theta}_{i,j}^{\rm T}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_{i,j})+\\&\tilde{\boldsymbol{\theta}}_{i,j}^{\rm T}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_{i,j})+l_{i,j}e_1-\dot{\nu}_{i,j-1}-\dot{q}_{i,j}\Big)-\\&\tilde{\boldsymbol{\theta}}_{i,j}^{\rm T}\boldsymbol{\Gamma}_{i,j}^{-1}\dot{\boldsymbol{\theta}}_{i,j}-\frac{\tilde{\Theta}_{i,j}\dot{\Theta}_{i,j}}{\gamma_{i,j,2}}\Bigg]\end{aligned} \quad (46)$$

与第 1 步相似, 可得如下不等式:

$$\begin{aligned}&s_{i,j}\left(\boldsymbol{\theta}_{i,j}^{*\rm T}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)-\boldsymbol{\theta}_{i,j}^{*\rm T}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_{i,j})-\tilde{\boldsymbol{\theta}}_{i,j}^{\rm T}\boldsymbol{\phi}_{i,j}(\hat{\boldsymbol{X}}_i)\right)\le\\&s_{i,j}^2\Theta_{i,j}^*+1+\frac{s_{i,j}^2}{4}+\tilde{\boldsymbol{\theta}}_{i,j}^{\rm T}\tilde{\boldsymbol{\theta}}_{i,j}\end{aligned} \quad (47)$$

其中, $\Theta_{i,j}^*=\|\boldsymbol{\theta}_{i,j}^*\|^2$. 若 $j=2$, 将式 (26)~(27), (47) 代入式 (46) 得到:

$$\begin{aligned}\dot{V}_2\le&\dot{V}_1+\sum_{i=1}^N\Bigg[-c_{i,2}s_{i,2}^2-\frac{s_{i,2}^2}{2}+s_{i,2}s_{i,3}+\\&\tilde{\boldsymbol{\theta}}_{i,2}^{\rm T}\tilde{\boldsymbol{\theta}}_{i,2}+\frac{\tilde{\Theta}_{i,2}}{\gamma_{i,2,2}}(\gamma_{i,2,2}s_{i,2}^2-\dot{\Theta}_{i,2})+1+\\&\tilde{\boldsymbol{\theta}}_{i,2}^{\rm T}\boldsymbol{\Gamma}_{i,2}^{-1}(\boldsymbol{\Gamma}_{i,2}s_{i,2}\boldsymbol{\phi}_{i,2}(\hat{\boldsymbol{X}}_{i,2})-\dot{\boldsymbol{\theta}}_{i,2})\Bigg]\end{aligned} \quad (48)$$

若 $j=k\ge3$, 将式 (28) ~ (29), (47) 代入式 (46) 得到:

$$\dot{V}_k \le \dot{V}_{k-1} + \sum_{i=1}^{N}\Big[-c_{i,k}s_{i,k}^2 - s_{i,k-1}s_{i,k} + \tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,k} + \tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\boldsymbol{\Gamma}_{i,k}^{-1}(\boldsymbol{\Gamma}_{i,k}s_{i,k}\boldsymbol{\phi}_{i,k}(\hat{\boldsymbol{X}}_{i,k}) - \dot{\boldsymbol{\theta}}_{i,k}) + \frac{\tilde{\Theta}_{i,k}}{\gamma_{i,k,2}}(\gamma_{i,k,2}s_{i,k}^2 - \dot{\Theta}_{i,k}) + s_{i,k}s_{i,k+1} + 1\Big] \tag{49}$$

将式 (33) 代入式 (48) 及 (49) 可得:

$$\dot{V}_j \le \sum_{i=1}^{N}\Big[-\eta_{i,1}\|\boldsymbol{e}_i\|^2 - \frac{c_{i,1}s_{i,1}^2}{k_i^2(t)-s_{i,1}^2} - \frac{r_{i,3}}{2\lambda_0}\|\boldsymbol{z}_i\|^4 - \sum_{k=2}^{j}c_{i,k}s_{i,k}^2 + s_{i,j}s_{i,j+1} + \sum_{k=1}^{j}\rho_{i,k,2}\tilde{\Theta}_{i,k}\Theta_{i,k} + \sum_{k=2}^{j}\tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,k} + \sum_{k=1}^{n}\tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,k} + \sum_{k=1}^{j}\rho_{i,k,1}\tilde{\boldsymbol{\theta}}_{i,k}^{\mathrm{T}}\boldsymbol{\theta}_{i,k} + \rho_i\tilde{\beta}_i\beta_i + D_{i,j} + \bar{M}_i\Big] \tag{50}$$

其中, $D_{i,j} = D_{i,j-1} + 1$. $j = 2, \cdots, n-1$.

第 n 步. 设计如下事件触发机制和执行器故障的容错控制器:

$$\sum_{q=1}^{m} b_{i,q}u_{iq} = \sum_{q=1}^{m} b_{i,q}\left(k_{i,q}v_{i,q} + \bar{u}_{i,q}\right)$$
$$v_{i,q}(t) = o_{i,q}(t_k^q), t \in [t_k^q, t_{k+1}^q) \tag{51}$$

执行器触发条件为:

$$t_{k+1}^q = \inf\{t > t_k^q \,|\, |e_{i,q}| \ge \mu_{i,q}|v_{i,q}| + \tau_{i,q}\}$$

其中, $e_{i,q} = v_{i,q}(t) - o_{i,q}(t)$, $\mu_{i,q}, \tau_{i,q}$ 为设计参数. 根据触发条件, 进一步可得到:

$$v_{i,q}(t) = \frac{o_{i,q}(t)}{1+\kappa_{i,1}^q\mu_{i,q}} - \frac{\kappa_{i,2}^q\tau_{i,q}}{1+\kappa_{i,1}^q\mu_{i,q}}$$

其中, $\kappa_{i,1}^q, \kappa_{i,2}^q \in [-1, 1]$. 所以, 可得到

$$s_{i,n}\sum_{q=1}^{m} b_{i,q}u_{i,q} = s_{i,n}\sum_{q=1}^{m}\frac{b_{i,q}k_{i,q}o_{i,q}(t)}{1+\kappa_{i,1}^q\mu_{i,q}} + s_{i,n}\boldsymbol{\Lambda}\boldsymbol{E}$$

其中, $\boldsymbol{\Lambda} = \begin{bmatrix} b_{i,1}\bar{u}_{i,1} - \dfrac{b_{i,1}k_{i,1}\kappa_{i,2}^1\tau_{i,1}}{1+\kappa_{i,1}^1\mu_{i,1}} \\ \vdots \\ b_{i,m}\bar{u}_{i,m} - \dfrac{b_{i,m}k_{i,m}\kappa_{i,2}^m\tau_{i,m}}{1+\kappa_{i,1}^m\mu_{i,m}} \end{bmatrix}^{\mathrm{T}}$,

$\boldsymbol{E} = [1, \cdots, 1]_{m\times 1}^{\mathrm{T}}$. 令 $\varsigma_i^* = \xi_i^*\sup\|\boldsymbol{\Lambda}\|$, $\xi_i^* = \dfrac{1}{\chi_i}$, $\chi_i = \inf\sum_{q=1}^{m}|b_{i,q}|k_{i,q}$, 及上述 $o_{i,q}(t)$的表达式为:

$$o_{i,q} = -(1+\mu_{i,q})\,\mathrm{sign}(b_{i,q})B_i \tag{52}$$

其中, $B_i = \varsigma_i\|\boldsymbol{E}\|\tanh\left((s_{i,n}\varsigma_i\|\boldsymbol{E}\|)/\iota_i\right) + \xi_i\alpha_{i,n}\tanh\left((s_{i,n}\xi_i\alpha_{i,n})/\iota_i\right)$, ι_i 为设计的常数. 化简可得:

$$s_{i,n}\sum_{q=1}^{m}b_{i,q}u_{i,q} \le -\chi_i s_{i,n}B_i + \chi_i\varsigma_i^*|s_{i,n}|\|\boldsymbol{E}\| = -\chi_i s_{i,n}\varsigma_i\|\boldsymbol{E}\|\tanh\left(\frac{s_{i,n}\varsigma_i\|\boldsymbol{E}\|}{\iota_i}\right) + \chi_i\varsigma_i|s_{i,n}|\|\boldsymbol{E}\| + \chi_i\tilde{\varsigma}_i|s_{i,n}|\|\boldsymbol{E}\| - \chi_i s_{i,n}\xi_i\alpha_{i,n}\tanh\left(\frac{s_{i,n}\xi_i\alpha_{i,n}}{\iota_i}\right) \tag{53}$$

由引理 1 可得:

$$\chi_i\varsigma_i|s_{i,n}|\|\boldsymbol{E}\| \le \chi_i s_{i,n}\varsigma_i\|\boldsymbol{E}\|\tanh\left(\frac{s_{i,n}\varsigma_i\|\boldsymbol{E}\|}{\iota_i}\right) + 0.2785\iota_i\chi_i \tag{54}$$

$$-\chi_i s_{i,n}\xi_i\alpha_{i,n}\tanh\left(\frac{s_{i,n}\xi_i\alpha_{i,n}}{\iota_i}\right) \le \chi_i s_{i,n}(\xi_i\alpha_{i,n} - \tilde{\xi}_i\alpha_{i,n}) - \chi_i s_{i,n}\xi_i\alpha_{i,n} + 0.2785\iota_i\chi_i = -s_{i,n}\alpha_{i,n} + \chi_i s_{i,n}\tilde{\xi}_i\alpha_{i,n} + 0.2785\iota_i\chi_i \tag{55}$$

根据式 (53) ~ (55) 可得:

$$s_{i,n}\sum_{q=1}^{m}b_{i,q}u_{i,q} \le -s_{i,n}\alpha_{i,n} + \chi_i s_{i,n}\tilde{\xi}_i\alpha_{i,n} + \chi_i\tilde{\varsigma}_i|s_{i,n}|\|\boldsymbol{E}\| + 0.557\iota_i\chi_i \tag{56}$$

选取第 n 步的李雅普诺夫函数为:

$$V_n = V_{n-1} + \sum_{i=1}^{N}\left[\frac{1}{2}s_{i,n}^2 + \frac{\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\boldsymbol{\Gamma}_{i,n}^{-1}\tilde{\boldsymbol{\theta}}_{i,n}}{2} + \frac{\tilde{\Theta}_{i,n}^2}{2\gamma_{i,n,2}} + \frac{\chi_i\tilde{\varsigma}_i^2}{2\gamma_{i,n,3}} + \frac{\chi_i\tilde{\xi}_i^2}{2\gamma_{i,n,4}}\right] \tag{57}$$

进一步可以得到:

$$\dot{V}_n \le \dot{V}_{n-1} + \sum_{i=1}^{N}\Big[s_{i,n}\Big(-\alpha_{i,n} + l_{i,n}e_1 - \dot{q}_{i,n} + \boldsymbol{\theta}_{i,n}^{*\mathrm{T}}\boldsymbol{\phi}_{i,n}(\hat{\boldsymbol{X}}_i) - \tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\boldsymbol{\phi}_{i,n}(\hat{\boldsymbol{X}}_i) - \dot{\nu}_{i,n-1}\Big) + 0.557\iota_i\chi_i + \chi_i\tilde{\varsigma}_i|s_{i,n}|\|\boldsymbol{E}\| + \chi_i s_{i,n}\tilde{\xi}_i a_n - \tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\boldsymbol{\Gamma}_{i,n}^{-1}\dot{\boldsymbol{\theta}}_{i,n} - \frac{\tilde{\Theta}_{i,n}\dot{\Theta}_{i,n}}{\gamma_{i,n,2}} - \frac{\chi_i\tilde{\varsigma}_i\dot{\varsigma}_i}{\gamma_{i,n,3}} - \frac{\chi_i\tilde{\xi}_i\dot{\xi}_i}{\gamma_{i,n,4}}\Big] \tag{58}$$

参考式 (38) 的计算过程可得:

$$s_{i,n}\boldsymbol{\theta}_{i,n}^{*\mathrm{T}}\boldsymbol{\phi}_{i,n}\left(\hat{\boldsymbol{X}}_i\right) \le s_{i,n}^2\Theta_{i,n}^* + \frac{1}{4} \tag{59}$$

其中, $\Theta_{i,n}^* = \|\boldsymbol{\theta}_{i,n}^*\|^2$. 将式 (30) ~ (31), (34) 及 (59)

代入式 (58) 得:

$$\dot{V}_n \le \sum_{i=1}^{N}\Bigg[-\eta_{i,1}\|\boldsymbol{e}_i\|^2-\frac{c_{i,1}s_{i,1}^2}{k_i^2(t)-s_{i,1}^2}-\frac{r_{i,3}}{2\lambda_0}\|\boldsymbol{z}_i\|^4- \sum_{j=2}^{n}c_{i,j}s_{i,j}^2+D_{i,n}+\rho_i\tilde{\beta}_i\beta_i+\sum_{j=1}^{n}\rho_{i,j,1}\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\boldsymbol{\theta}_{i,j}+ \sum_{j=1}^{n}\rho_{i,j,2}\tilde{\Theta}_{i,j}\Theta_{i,j}+\rho_{i,n,3}\chi_i\tilde{\varsigma}_i\varsigma_i+\rho_{i,n,4}\chi_i\tilde{\xi}_i\xi_i+ 2\sum_{j=2}^{n-1}\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,j}+\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,1}+\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,n}+\bar{M}_i\Bigg] \tag{60}$$

其中, $D_{i,n}=D_{i,n-1}+\dfrac{1}{4}+0.557\iota_i\chi_i$. 根据 Young′s 不等式可得:

$$\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\boldsymbol{\theta}_{i,j}\le-\frac{\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,j}}{2}+\frac{\boldsymbol{\theta}_{i,j}^{*\mathrm{T}}\boldsymbol{\theta}_{i,j}^{*}}{2}$$
$$\tilde{\Theta}_{i,j}\Theta_{i,j}\le-\frac{\tilde{\Theta}_{i,j}^2}{2}+\frac{\Theta_{i,j}^{*2}}{2},\ \tilde{\beta}_i\beta_i\le-\frac{\tilde{\beta}_i^2}{2}+\frac{\beta_i^{*2}}{2}$$
$$\tilde{\varsigma}_i\varsigma_i\le-\frac{\tilde{\varsigma}_i^2}{2}+\frac{\varsigma_i^{*2}}{2},\ \tilde{\xi}_i\xi_i\le-\frac{\tilde{\xi}_i^2}{2}+\frac{\xi_i^{*2}}{2} \tag{61}$$

把式 (61) 代入式 (60) 得到:

$$\dot{V}_n \le \sum_{i=1}^{N}\Bigg[-\eta_{i,1}\|\boldsymbol{e}_i\|^2-\frac{c_{i,1}s_{i,1}^2}{k_i^2(t)-s_{i,1}^2}-\frac{r_{i,3}}{2\lambda_0}\|\boldsymbol{z}_i\|^4- \sum_{j=2}^{n}c_{i,j}s_{i,j}^2-\sum_{j=2}^{n-1}\left(\frac{\rho_{i,j,1}}{2}-2\right)\tilde{\boldsymbol{\theta}}_{i,j}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,j}- \left(\frac{\rho_{i,1,1}}{2}-1\right)\tilde{\boldsymbol{\theta}}_{i,1}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,1}-\left(\frac{\rho_{i,n,1}}{2}-1\right)\tilde{\boldsymbol{\theta}}_{i,n}^{\mathrm{T}}\tilde{\boldsymbol{\theta}}_{i,n}- \sum_{j=1}^{n}\frac{\rho_{i,j,2}}{2}\tilde{\Theta}_{i,j}^2-\frac{\rho_i}{2}\tilde{\beta}_i^2-\frac{\rho_{i,n,3}}{2}\chi_i\tilde{\varsigma}_i^2- \frac{\rho_{i,n,4}}{2}\chi_i\tilde{\xi}_i^2+\bar{D}_{i,n}\Bigg] \tag{62}$$

其中, $\bar{D}_{i,n}=D_{i,n}+\sum_{j=1}^{n}\dfrac{\rho_{i,j,1}}{2}\boldsymbol{\theta}_{i,j}^{*\mathrm{T}}\boldsymbol{\theta}_{i,j}^{*}+\sum_{j=1}^{n}\dfrac{\rho_{i,j,2}}{2}\Theta_{i,j}^{*2}+\dfrac{\rho_i}{2}\beta_i^{*2}+\dfrac{\rho_{i,n,3}}{2}\chi_i\varsigma_i^{*2}+\dfrac{\rho_{i,n,4}}{2}\chi_i\xi_i^{*2}+\bar{M}_i$. 根据引理 2 得: $-\dfrac{c_{i,1}s_{i,1}^2}{k_i^2(t)-s_{i,1}^2}\le-c_{i,1}\ln\dfrac{k_i^2(t)}{k_i^2(t)-s_{i,1}^2}$, 在令 $D_n=\sum_{i=1}^{N}\bar{D}_{i,n}$. 式 (62) 进一步化简为:

$$\dot{V}_n\le-CV_n+D_n \tag{63}$$

其中, $C=\min 2\Bigg\{\eta_{i,1}/\lambda_{\min}(\boldsymbol{P}_i), c_{i,j}, \dfrac{r_{i,3}}{4r_{i,2}}, \dfrac{\rho_{i,j,2}\gamma_{i,j,2}}{2}, \dfrac{\rho_i\gamma_i}{2}, \dfrac{\rho_{i,n,3}\gamma_{i,n,3}}{2\chi_i}, \dfrac{\rho_{i,n,4}\gamma_{i,n,4}}{2\chi_i}, \dfrac{\lambda_{\min}(\boldsymbol{\Gamma}_{i,j})(\rho_{i,j,1}-4)}{2}, \dfrac{\lambda_{\min}(\boldsymbol{\Gamma}_{i,1})(\rho_{i,1,1}-2)}{2}, \dfrac{\lambda_{\min}(\boldsymbol{\Gamma}_{i,n})(\rho_{i,n,1}-2)}{2}\Bigg\}$.

对式 (63) 积分可得:

$$V_n\le V_n(0)\,\mathrm{e}^{-Ct}+\frac{D_n}{C}(1-\mathrm{e}^{-Ct}) \tag{64}$$

因此, 闭环系统的所有信号都是半全局一致最终有界的.

注 5. 通过设计合适的参数, 使得 D_n 足够小或 C 足够大, 就可以使得闭环系统中所有信号收敛到原点附近一个很小的紧集内. 因为 $s_{i,1}\to k_i(t)$ 时, V_n 会无穷大, 但 $\lim_{t\to\infty}V_n\to\dfrac{D_n}{C}$, 所以 $s_{i,1}$ 是严格小于 $k_i(t)$ 的, 而补偿信号 $q_{i,1}$ 也会收敛, 所以系统跟踪误差 $z_{i,1}$ 最终会收敛到一个紧集内, 进一步保证系统的输出是受限的.

接下来, 证明上述所设计基于事件触发机制的控制器可以避免 Zeno 行为. 首先, 假设存在一个时间常数 $t_q^*>0$ 满足 $\forall w\in Z^+, \{t_{w+1}^q-t_w^q\ge t_q^*\}$.

根据 $e_{i,q}=v_{i,q}(t)-o_{i,q}(t), t\in\{t_w^q, t_{w+1}^q\}$, 对 $e_{i,q}$ 微分可以得到:

$$\frac{\mathrm{d}}{\mathrm{d}t}|e_{i,q}|=\mathrm{sign}(e_{i,q})\,\dot{e}_{i,q}\le|\dot{o}_{i,q}| \tag{65}$$

根据式 (52) , $o_{i,q}$ 是可微的且 $\dot{o}_{i,q}$ 是闭环系统中有界的信号函数. 所以, 存在一个常数 $a_{i,q}>0$, 满足: $\dot{o}_{i,q}\le a_{i,q}$. 因为 $e_{i,q}(t_w^q)=0$, $\lim_{t\to t_{w+1}^q}e_{i,q}(t)\ge\tau_{i,q}$, 可以得到: $t_q^*>\dfrac{\tau_{i,q}}{a_{i,q}}>0$. 所以, 不会有 Zeno 行为发生. □

根据上述设计方案与分析, 得到以下定理.

定理 1. 对于含有未建模动态, 状态不可测, 执行器故障的非严格反馈非线性互联大系统 (1) , 在满足假设 1~7 的情况下, 通过设计观测器 (11) , 滤波器 (22) 及补偿信号 (25), (27), (29), (31) , 虚拟控制器 (24), (26), (28), 事件触发容错控制器 (51) ~ (52), 以及参数自适应律 (32) ~ (34) , 可以保证闭环系统的所有信号是半全局一致最终有界的, 且避免了 Zeno 行为.

注 6. 根据事件触发条件可以看出, 控制器触发的时间间隔由控制器 $v_{i,q}$ 大小决定. 这种触发机制的优势在于: 当 $v_{i,q}$ 很大时, 能增大控制器触发的时间间隔; 若 $v_{i,q}$ 很小时, 触发的时间间隔就很小, 这就保证控制器输出精准的控制量, 保证系统精准的控制和性能; 同时, 参数 $\tau_{i,q}$ 能避免 Zeno 现象出现.

3　仿真及结果

本文考虑有两个子系统组成的非线性互联系统, 系统模型如下 $(i=1,2)$:

$$\begin{cases}\dot{z}_i = -z_i + x_{i,1}^2 + 0.5\\ \dot{x}_{i,1} = x_{i,2} + f_{i,1}(\boldsymbol{X}_i)\\ \dot{x}_{i,2} = \sum\limits_{q=1}^{2} b_{i,q}u_{i,q} + f_{i,2}(\boldsymbol{X}_i) + H_{i,2}(\boldsymbol{y}) + x_{i,1}z_i\\ y_i = x_{i,1}\end{cases}$$

其中, $f_{1,1}(\boldsymbol{X}_1) = 0.5x_{1,1}\sin(x_{1,1}x_{1,2})$, $f_{1,2}(\boldsymbol{X}_1) = -4x_{1,2}^2x_{1,1}$, $f_{2,1}(\boldsymbol{X}_2) = 0.5x_{2,1}x_{2,2}$, $f_{2,2}(\boldsymbol{X}_2) = -2x_{2,2}^2x_{2,1}$, $H_{1,2}(\boldsymbol{y}) = -x_{1,1}^3 + x_{2,1}$, $H_{2,2}(\boldsymbol{y}) = x_{1,1} - x_{2,1}$.

选取系统初始状态及相关参数如下: $z_1(0) = z_2(0) = 0, x_{1,1}(0) = x_{2,1}(0) = 0.1, x_{1,2}(0) = x_{2,2}(0) = 0$, 观测器参数及观测器系统状态初始值为: $l_{1,1} = 45$, $l_{1,2} = 40, l_{2,1} = 45, l_{2,2} = 40, \hat{x}_{1,1}(0) = 0.1, \hat{x}_{1,2}(0) = 0$, $\hat{x}_{2,1}(0) = 0.1, \hat{x}_{2,2}(0) = 0$. 控制器主要参数为: $c_{1,1} = 5$, $c_{1,2} = 4$, $c_{2,1} = 4$, $c_{2,2} = 3.5$, $\boldsymbol{\Gamma}_{1,1} = \boldsymbol{\Gamma}_{1,2} = \boldsymbol{\Gamma}_{2,1} = \boldsymbol{\Gamma}_{2,2} = \mathrm{diag}\{0.1\ 0.1\}, \gamma_{1,1,2} = \gamma_1 = \gamma_{1,2,2} = \gamma_{1,2,3} = \gamma_{1,2,4} = \gamma_{2,1,2} = \gamma_2 = \gamma_{2,2,2} = \gamma_{2,2,3} = \gamma_{2,2,4} = 0.03$, $\rho_{1,1,1} = \rho_{2,1,1} = 3, \rho_{1,2,1} = \rho_{2,2,1} = 3.2, \rho_{1,1,2} = \rho_{1,2,2} = \rho_{2,1,2} = \rho_{2,2,2} = 1$, $\rho_1 = 1$, $\rho_2 = 1.5$, $\rho_{1,2,3} = \rho_{2,2,3} = 1.2, \rho_{1,2,4} = \rho_{2,2,4} = 1$.

执行器故障模型及事件触发参数设置如下:

$$u_{1,1} = \bar{u}_{1,1},\ u_{2,1} = k_{2,1}v_{2,1},\ t \in [2i, 2i+1)$$
$$u_{1,2} = k_{1,2}v_{1,2},\ u_{2,2} = k_{2,2}v_{2,2},\ t \in [2i+1, 2i+2)$$

其中, $i = (0, 1, 2, \cdots), \bar{u}_{1,1} = 2, k_{2,1} = 0.2, k_{1,2} = 0.2$, $k_{2,2} = 0.3$. $b_{1,1} = b_{1,2} = b_{2,1} = b_{2,2} = 0.6, \mu_{1,1} = \mu_{1,2} = 0.8$, $\mu_{2,1} = \mu_{2,2} = 0.5, \tau_{1,1} = \tau_{1,2} = 0.8, \tau_{2,1} = \tau_{2,2} = 0.6$, $\iota_1 = \iota_2 = 0.5$.

选取跟踪参考信号分别为, $y_{1,d}(t) = \sin(\frac{\pi t}{20})$, $y_{2,d}(t) = \sin(\frac{\pi t}{15})$. $z_{i,1} = y_i - y_{i,d}$ 的受限函数设置为 $k_i(t) = \mathrm{e}^{-t} + 0.2$, $i = 1, 2$. 仿真结果如图所示.

由图 1 可得, 设计的观测器能准确对系统状态进行估计, 提出的控制方案可以保证系统跟踪误差能够收敛于原点较小的领域内, 同时保证系统输出受限满足: $y_{i,d} - k_i(t) \le y_i \le k_i(t) + y_{i,d}$. 从图 2 可以看出, 未建模动态是稳定有界的, 轨迹曲线与输出相似. 从图 3 可以看出, 虚拟控制器含有非常高频率的分量, 曲线剧烈抖动, 显然它的导数是非常复杂的. 而滤波器能消除高频分量, 且有效地估计虚拟控制器. 图 4 表明事件触发机制能避免执行器持续更新, 减少了通信的次数, 且由图可以看出, 执行器发生故障对执行器总输出不会有太大影响, 说明设计的容错控制器能有效补偿故障影响. 仿真结果表明了该控制方案的有效性.

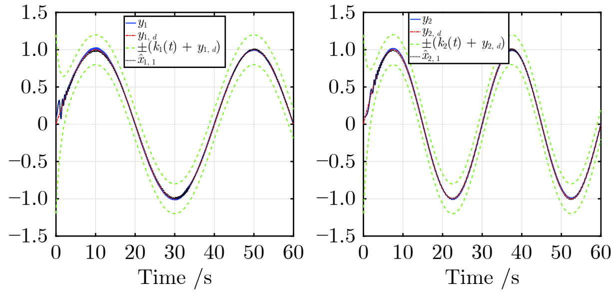

图 1　子系统的输出 y_1, y_2 和观测状态 $\hat{x}_{1,1}, \hat{x}_{2,1}$ 的响应曲线

Fig. 1　Trajectories of output y_1, y_2 and observer $\hat{x}_{1,1}, \hat{x}_{2,1}$

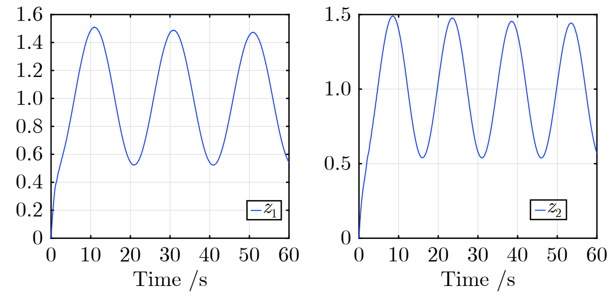

图 2　子系统未建模动态 z_1, z_2 响应曲线

Fig. 2　Trajectories of unmodeled dynamics z_1, z_2

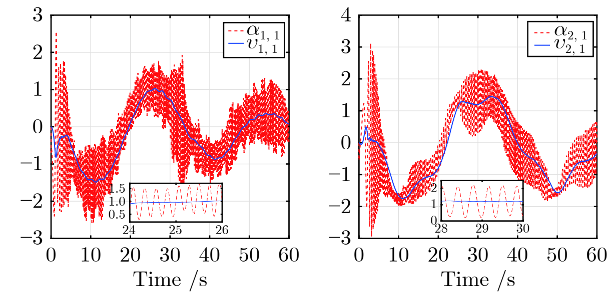

图 3　滤波器输入及输出的响应曲线

Fig. 3　Trajectories of filter′s input and output

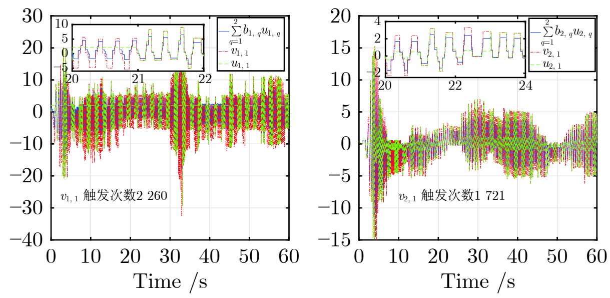

图 4　子系统第一个执行器输出的响应曲线

Fig. 4　Trajectories of the first actuator′s output

4 结论

本文主要解决了存在未建模动态和状态不可测情况下, 非严格反馈非线性互联大系统的输出受限和执行器故障等问题. 通过设计状态观测器对系统状态进行估计, 引入李雅普诺夫函数来约束未建模动态. 基于事件触发机制和时变障碍李雅普诺夫函数设计自适应容错控制器, 不仅保证了系统在发生执行器故障时, 系统输出受限, 还避免了 Zeno 行为, 并且闭环系统所有信号是半全局一致最终有界的. 最后, 通过仿真验证了所提出控制策略的有效性. 在未来工作中, 研究如何将事件触发机制应用于无人机系统的协同编队问题.

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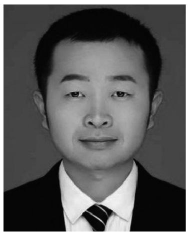

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