

# 具有指定性能和全状态约束的多智能体系统事件触发控制

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**摘要** 针对一类非严格反馈的非线性多智能体系统一致性跟踪问题,在考虑全状态约束和指定性能的基础上提出了一种事件触发自适应控制算法。首先,通过设计性能函数,使跟踪误差在规定时间内收敛于指定范围。然后,在反步法中引入 Barrier Lyapunov 函数使所有状态满足约束条件,结合动态面技术解决传统反步法产生的“计算爆炸”问题,并利用径向基函数神经网络(Radial basis function neural networks, RBF NNs)处理系统中的未知非线性函数。最后基于 Lyapunov 稳定性理论证明系统中所有信号都是半全局一致最终有界的,跟踪误差收敛于原点的有界邻域内且满足指定性能。仿真结果验证了该控制算法的有效性。

**关键词** 指定性能, 全状态约束, 事件触发控制, 多智能体系统

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## Event-Triggered Control for Multi-Agent Systems With Prescribed Performance and Full State Constraints

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**Abstract** In this paper, an event-triggered adaptive control algorithm is proposed to address the consensus tracking problem of non-strict feedback nonlinear multi-agent systems with prescribed performance and full state constraints. With the aid of performance function, tracking error converges to a specified range within a specified time. By introducing the Barrier Lyapunov function into the backstepping approach, the full states of the systems satisfy the constraints. And the dynamic surface technique is employed to dispose the problem of “explosion of complexity”. Moreover, the radial basis function neural networks (RBF NNs) are utilized to approximate unknown nonlinear functions in the systems. According to the Lyapunov stability theory, all signals are semi-globally uniformly ultimately bounded, and the tracking error converges to a bounded neighborhood of the origin with prescribed performance. Finally, the effectiveness of the proposed control algorithm is verified by simulation results.

**Key words** Prescribed performance, full state constraints, event-triggered control, multi-agent systems

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随着工业和军事应用的发展,愈来愈多的工作需要多个智能体协同完成,因此多智能体协同控制

迅速成为学界的热门课题<sup>[1–8]</sup>。由于多智能体协同控制只需接收智能体之间的局部信息,从而大大降低了系统的通讯成本和能耗,且具有更好的灵活性和鲁棒性,所以广泛应用于多机器人协同控制、无人机编队和传感器网络等领域。多智能体协同控制主要包括一致性问题<sup>[1–6]</sup>、集群问题<sup>[7]</sup>、编队问题<sup>[8]</sup>等,其中一致性跟踪问题作为协同控制的一项基本问题,受到了学者们的广泛关注<sup>[1–5]</sup>。

在多智能体系统中,每个智能体的微处理器一般会受到计算能力和通信带宽的限制,因此如何在多智能体的一致性跟踪控制过程中降低通讯成本、减小计算压力成为了亟需解决的问题。针对这一问题,有学者在控制器中加入了事件触发机制,并获得了显著的成果<sup>[9–12]</sup>。但是,事件触发机制可能发生奇诺现象,即触发事件会在有限时间内被无数次触发。为了解决这一问题,文献[13]研究了几种不同

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事件触发机制的最小事件触发时间间隔的基本性质. 文献 [14] 则结合自适应控制解决了事件触发控制中输入状态稳定性的问题. 将事件触发机制加入到控制器设计中显著降低了控制过程的能耗和成本, 因此, 事件触发控制成为了多智能体一致性跟踪控制的一种重要方法.

由于受到物理器件和运行条件的限制, 系统通常需要对其状态进行约束, 否则将会导致系统不稳定、性能急剧下降等现象, 甚至会造成重大的安全事故. 因此, 解决系统的状态约束问题意义重大. 全状态约束作为一种常见的状态约束形式, 一直是控制领域的研究热点与难点. 文献 [3, 15–16] 均使用 Barrier Lyapunov 函数对具有全状态约束的系统进行了研究. 文献 [15] 研究了一类具有参数不确定性的非线性系统的全状态约束问题, 文献 [16] 进一步研究了一类随机非线性系统的全状态约束问题, 文献 [3] 则研究了一类具有全状态约束和未知扰动的多智能体一致性跟踪问题. 此外, 系统在保证稳定运行的同时, 通常还会对收敛速率, 最大超调量, 稳态误差等性能指标提出要求. 因此, 在控制器的设计过程中结合指定性能具有重要意义<sup>[17–21]</sup>.

基于以上讨论, 本文针对一类具有全状态约束的非严格反馈多智能体系统一致性跟踪控制问题进行了研究, 改进了控制算法. 与现有结果相比, 本文的优势在于考虑了指定性能, 对跟踪误差进行了指定性能变换, 从而获得了更小的稳态误差. 此外, 在控制器设计过程中加入了固定阈值的事件触发机制, 设计了一个基于事件触发的自适应控制算法, 降低了智能体之间的通讯成本以及控制成本. 在控制算法的设计过程中, 用径向基函数神经网络 (Radial basis function neural networks, RBF NNs) 对系统中的未知非线性函数进行处理, 从而解决了控制器设计中不满足匹配条件和未知非线性函数的问题. 除此之外, 在反步法中引入动态面技术, 有效地避免了传统反步法中“计算爆炸”的问题.

## 1 预备知识与问题描述

### 1.1 代数图论

本文用图论来描述智能体之间的通信拓扑. 智能体之间的有向图记为  $\zeta = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ , 其中,  $\mathcal{V} = (1, 2, \dots, N)$  是节点数, 表示系统中含有  $N$  个智能体.  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$  为节点的边,  $\mathcal{A} = [a_{i,j}] \in \mathbf{R}^{N \times N}$  为邻接矩阵. 节点  $j$  到  $i$  的边记为  $(\mathcal{V}_j, \mathcal{V}_i) \in \mathcal{E}$ , 表示智能体  $i$  能够接收到智能体  $j$  的信息, 智能体  $i$  的邻居节点的集合定义为  $\mathcal{N}_i = \{\mathcal{V}_j | (\mathcal{V}_j, \mathcal{V}_i) \in \mathcal{E}, i \neq j\}$ . 对于邻接矩阵  $\mathcal{A}$ , 如果节点  $j$  的信息能被节点  $i$  接收到, 那么  $a_{i,j} > 0$ , 否则  $a_{i,j} = 0$ . 定义节点  $i$

的度为  $d_i = \sum_{j \in \mathcal{N}_i} a_{i,j}$ , 对角矩阵  $\mathcal{D} = \text{diag}\{d_1, d_2, \dots, d_N\}$ . 有向图  $\zeta$  的拉普拉斯矩阵为  $\mathcal{L} = \mathcal{D} - \mathcal{A}$ . 定义拓展图  $\bar{\zeta} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ , 其中  $\bar{\mathcal{V}} = (0, 1, 2, \dots, N)$ ,  $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ , 0 表示领导者, 同样的, 当节点  $i$  能够接收领导者 0 的信号时  $a_{i,0} > 0$ , 否则  $a_{i,0} < 0$ .

**引理 1**<sup>[22]</sup>. 如果存在一条路径能够从根节点到达所有其他节点, 那么称有向图  $\zeta$  具有一个生成树. 定义  $B = \text{diag}\{a_{1,0}, \dots, a_{N,0}\}$ , 其中节点 0 称为生成树的根, 则矩阵  $\mathcal{L} + B$  是非奇异的.

### 1.2 系统描述

对于具有  $N$  个同构智能体的多智能体系统, 第  $i$  个智能体可以用以下  $n$  阶非严格反馈的非线性系统描述

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1} + f_{i,m}(x_i), \quad m = 1, 2, \dots, n-1 \\ \dot{x}_{i,n} &= u_i + f_{i,n}(x_i) \\ y_i &= x_{i,1} \end{aligned} \tag{1}$$

其中,  $x_i = [x_{i,1}, x_{i,2}, \dots, x_{i,n}]^T \in \mathbf{R}^n$  表示第  $i$  个智能体的状态向量,  $y_i \in \mathbf{R}$  和  $u_i \in \mathbf{R}$  分别表示第  $i$  个智能体的输出和控制器输入,  $f_{i,m}(x_i)$  和  $f_{i,n}(x_i)$  是未知光滑的非线性函数,  $i = 1, 2, \dots, N$ .

**假设 1**<sup>[1]</sup>. 有向图  $\zeta$  具有一个生成树. 节点 0 的期望轨迹  $y_d$  是一个虚拟的领导者, 且只能被部分智能体直接获取.  $y_d$  已知,  $y_d$  及其  $n$  阶导数都是连续且有界的.

**定义 1**<sup>[23]</sup>. 如果对于任意一个先验紧集  $\Omega \in \mathbf{R}^n$ ,  $x(0) \in \Omega$ , 存在有界的  $\varepsilon > 0$  以及常数  $N(\varepsilon, x(0))$ , 使得

$$\|x(t)\| < \varepsilon, \quad \forall t \geq t_0 + N$$

则系统 (1) 的解是半全局一致最终有界的.

### 1.3 径向基函数神经网络

在本文中, RBF NNs 用于逼近系统中的非线性函数<sup>[24–25]</sup>

$$f(\mathcal{Z}) = W^T S(\mathcal{Z})$$

其中,  $S(\mathcal{Z}) = [S_1(\mathcal{Z}), S_2(\mathcal{Z}), \dots, S_k(\mathcal{Z})]^T$  表示基函数向量,  $k$  为 RBF NNs 的节点数,  $W = [W_1, W_2, \dots, W_k]^T \in \mathbf{R}^k$  为权重向量. 存在一个恒定的理想权重向量  $W^*$ , 使以下方程成立

$$f(\mathcal{Z}) = W^{*T} S(\mathcal{Z}) + \delta(\mathcal{Z}), \quad \forall \mathcal{Z} \in \Omega \subset \mathbf{R}^q$$

$$W^* = \arg \min_{W \in \mathbf{R}^k} \{ \sup_{\mathcal{Z} \in \Omega} |f(\mathcal{Z}) - W^T S(\mathcal{Z})| \}$$

其中,  $W^* = [W_1^*, W_2^*, \dots, W_k^*]^T \in \mathbf{R}^k$ , 逼近误差  $\delta(\mathcal{Z})$  满足  $|\delta(\mathcal{Z})| \leq \varepsilon$ ,  $\varepsilon > 0$ . 本文使用的高斯基函数如下

$$S_i(\mathcal{Z}) = \exp\left(-\frac{(\mathcal{Z} - \iota_i)^T(\mathcal{Z} - \iota_i)}{\omega_i^2}\right)$$

其中,  $\iota_i = [\iota_{i1}, \iota_{i2}, \dots, \iota_{iq}]^T$  和  $\omega_i$  ( $i = 1, 2, \dots, k$ ), 分别表示高斯函数的中心和宽度.

引理 2<sup>[26]</sup>. 令

$$S(\bar{x}_q) = [S_1(\bar{x}_q), S_2(\bar{x}_q), \dots, S_k(\bar{x}_q)]^T$$

为 RBF NNs 的基函数向量, 其中,  $\bar{x}_q = [x_1, x_2, \dots, x_q]^T$ , 对于任意的正整数  $p \leq q$ , 则有

$$\|S(\bar{x}_q)\|^2 \leq \|S(\bar{x}_p)\|^2$$

## 2 事件触发自适应控制算法设计

本文使用反步法和动态面技术设计了一个事件触发自适应控制算法使多智能体系统达到以下控制目标: 1) 所有智能体的输出都能对期望轨迹进行跟踪并满足指定性能; 2) 系统中所有的信号都是半全局一致最终有界的.

定义以下坐标变换

$$\begin{aligned} \tilde{s}_{i,1} &= \sum_{j \in \mathcal{N}_i} a_{i,j}(y_i - y_j) + a_{i,0}(y_i - y_d) \\ s_{i,m} &= x_{i,m} - z_{i,m} \\ \lambda_{i,m} &= z_{i,m} - \alpha_{i,m-1}, \quad m = 2, 3, \dots, n \end{aligned} \quad (2)$$

其中,  $\tilde{s}_{i,1}$  表示跟踪误差,  $s_{i,m}$  表示虚拟误差面,  $z_{i,m}$  表示一阶滤波器的输出信号,  $\lambda_{i,m}$  为滤波误差,  $\alpha_{i,m-1}$  为虚拟控制信号.

指定性能可以用以下不等式进行描述:

$$-\delta_{\min}\mu(t) < \tilde{s}_{i,1}(t) < \delta_{\max}\mu(t), \quad \forall t > 0$$

其中,  $\delta_{\min}$  和  $\delta_{\max}$  是可调节的参数, 性能函数  $\mu(t)$  有界且严格单调递减, 其形式为  $\mu(t) = (\mu_0 - \mu_\infty)e^{-vt} + \mu_\infty$ , 式中,  $v$ ,  $\mu_0$  和  $\mu_\infty$  都是正实数,  $\mu_0 = \mu(0)$ , 选取适当的  $\mu_0$ , 使得  $\mu_0 > \mu_\infty$ ,  $-\delta_{\min}\mu(0) < \tilde{s}_{i,1}(0) < \delta_{\max}\mu(0)$ .

为了满足指定性能, 进行如下等效变换:

$$\tilde{s}_{i,1}(t) = \mu(t)\varpi_i(\varpi_i(t)), \quad \forall t \geq 0$$

其中,  $\varpi_i$  为变换误差,  $\Psi_i(\varpi_i) = \frac{\delta_{\max}e^{\varpi_i} - \delta_{\min}e^{-\varpi_i}}{e^{\varpi_i} + e^{-\varpi_i}}$ , 由于函数  $\Psi_i(\varpi_i)$  是严格单调递增的, 且  $\frac{\partial\Psi_i}{\partial\varpi_i} = \frac{2(\delta_{\max} + \delta_{\min})}{(e^{\varpi_i} + e^{-\varpi_i})^2} > 0$ , 则有

$$\varpi_i(t) = \Psi_i^{-1}\left(\frac{\tilde{s}_{i,1}(t)}{\mu(t)}\right) = \frac{1}{2}\ln\frac{\Psi_i + \delta_{\min}}{\delta_{\max} - \Psi_i}$$

其导数为

$$\dot{\varpi}_i(t) = r_i\left(\dot{\tilde{s}}_{i,1} - \frac{\dot{\mu}\tilde{s}_{i,1}}{\mu}\right)$$

其中,  $r_i = \frac{1}{2\mu}[\frac{1}{\Psi_i + \delta_{\min}} - \frac{1}{\Psi_i - \delta_{\max}}]$ . 定义坐标变换

$$s_{i,1}(t) = \varpi_i(t) - \frac{1}{2}\ln\frac{\delta_{\min}}{\delta_{\max}}$$

其导数为

$$\dot{s}_{i,1}(t) = r_i\left(\dot{\tilde{s}}_{i,1} - \frac{\dot{\mu}\tilde{s}_{i,1}}{\mu}\right) \quad (3)$$

定义自适应参数为

$$\theta_{i,m}^* = \|W_{i,m}^*\|^2, \quad m = 1, \dots, n$$

其中,  $W_{i,m}^*$  为 RNF NNs 理想权重向量,  $\hat{\theta}_{i,m}$  是  $\theta_{i,m}^*$  的估计, 且  $\tilde{\theta}_{i,m} = \theta_{i,m}^* - \hat{\theta}_{i,m}$ .

定义  $k_{bl}$  为误差  $s_{i,l}$  的约束, 即  $|s_{i,l}| < k_{bl}$ ,  $i = 1, 2, \dots, N; l = 1, 2, \dots, n$ .

引理 3<sup>[27]</sup>. 对于任意正常数  $k_{bl}$ , 若满足不等式  $|s_{i,l}| < k_{bl}$ ,  $s_{i,l} \in \mathbf{R}$ , 则有

$$\log\frac{k_{bl}^2}{k_{bl}^2 - s_{i,l}^2} < \frac{s_{i,l}^2}{k_{bl}^2 - s_{i,l}^2}$$

引理 4<sup>[4]</sup>. 定义  $s_1 = [s_{1,1}, s_{2,1}, \dots, s_{N,1}]^T$ ,  $y = [y_1, y_2, \dots, y_N]^T$ ,  $\bar{y}_d = [y_d, y_d, \dots, y_d]^T$ , 其中,  $y_d$  的个数为  $N$ . 则有

$$\|y - \bar{y}_d\| \leq \frac{\|s_1\|}{\underline{\sigma}(\mathcal{L} + B)}$$

其中,  $\underline{\sigma}(\mathcal{L} + B)$  是矩阵  $\mathcal{L} + B$  的最小奇异值.

引理 5 (Young's 不等式)<sup>[26]</sup>. 对于  $\forall(x, y) \in \mathbf{R}^n$ , 有以下不等式成立

$$xy \leq \frac{p^a}{a}|x|^a + \frac{1}{bp^b}|y|^b$$

其中,  $p > 0, a > 1, b > 1, (a-1)(b-1) = 1$ .

事件触发自适应控制算法的具体设计步骤如下:

步骤 1. 反步法第 1 步选取的 Barrier Lyapunov 函数为

$$V_{i,1} = \frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2 - s_{i,1}^2} + \frac{1}{2}\tilde{\theta}_{i,1}^2 \quad (4)$$

由式 (1)~(4), 可得

$$\begin{aligned} \dot{V}_{i,1} &= \frac{s_{i,1}r_i}{k_{b1}^2 - s_{i,1}^2} \left[ (d_i + a_{i,0})(s_{i,2} + \lambda_{i,2} + \alpha_{i,1}) + \right. \\ &\quad \left. \bar{f}_{i,1} - \sum_{j \in \mathcal{N}_i} a_{i,j}x_{j,2} - a_{i,0}\dot{y}_d - \frac{\dot{\mu}\tilde{s}_{i,1}}{\mu} \right] - \tilde{\theta}_{i,1}\dot{\tilde{s}}_{i,1} \end{aligned} \quad (5)$$

其中,  $\bar{f}_{i,1} = (d_i + a_{i,0})f_{i,1}(x_i) - \sum_{j \in \mathcal{N}_i} a_{i,j}f_{j,1}(x_j)$ .

用 RBF NNs 逼近  $\bar{f}_{i,1}$ , 可得

$$\bar{f}_{i,1} = W_{i,1}^{*\top} S_{i,1}(Z_{i,1}) + \delta(Z_{i,1})$$

其中,  $Z_{i,1} = [x_i, x_j]^T, \delta(Z_{i,1})$  为逼近误差且  $|\delta(Z_{i,1})| \leq \bar{\varepsilon}_{i,1}$ .

由引理 2 和引理 5, 可得

$$\begin{aligned} \frac{r_i s_{i,1}}{k_{b1}^2 - s_{i,1}^2} \bar{f}_{i,1} &\leq \frac{r_i^2 s_{i,1}^2 \|W_{i,1}^*\|^2 S_{i,1}^T(Z_{i,1}) S_{i,1}(Z_{i,1})}{2p_{i,1}^2(k_{b1}^2 - s_{i,1}^2)^2} + \\ &\quad \frac{1}{2} p_{i,1}^2 + \frac{r_i^2 s_{i,1}^2}{2(k_{b1}^2 - s_{i,1}^2)^2} + \frac{1}{2} \bar{\varepsilon}_{i,1}^2 \leq \\ &\quad \frac{r_i^2 s_{i,1}^2 \theta_{i,1}^* S_{i,1}^T(X_{i,1}) S_{i,1}(X_{i,1})}{2p_{i,1}^2(k_{b1}^2 - s_{i,1}^2)^2} + \\ &\quad \frac{r_i^2 s_{i,1}^2}{2(k_{b1}^2 - s_{i,1}^2)^2} + \frac{p_{i,1}^2 + \bar{\varepsilon}_{i,1}^2}{2} \end{aligned} \quad (6)$$

其中,  $X_{i,1} = [x_{i,1}, x_{j,1}]^T$ .

$$\begin{aligned} \frac{(d_i + a_{i,0}) r_i s_{i,1}}{k_{b1}^2 - s_{i,1}^2} (s_{i,2} + \lambda_{i,2}) &\leq \\ \frac{(d_i + a_{i,0})^2 r_i^2 s_{i,1}^2}{(k_{b1}^2 - s_{i,1}^2)^2} + \frac{s_{i,2}^2 + \lambda_{i,2}^2}{2} & \end{aligned} \quad (7)$$

设计反步法第 1 步的虚拟控制信号  $\alpha_{i,1}$  和自适应律  $\dot{\hat{\theta}}_{i,1}$  分别为

$$\begin{aligned} \alpha_{i,1} &= \frac{1}{d_i + a_{i,0}} \left[ -\frac{c_{i,1} s_{i,1}}{r_i} - \frac{r_i s_{i,1}}{2(k_{b1}^2 - s_{i,1}^2)} - \right. \\ &\quad \frac{r_i s_{i,1}}{2p_{i,1}^2(k_{b1}^2 - s_{i,1}^2)} \hat{\theta}_{i,1} S_{i,1}^T(X_{i,1}) S_{i,1}(X_{i,1}) - \\ &\quad \left. \frac{(d_i + a_{i,0})^2 r_i s_{i,1}}{k_{b1}^2 - s_{i,1}^2} + a_{i,0} \dot{y}_d + \right. \\ &\quad \left. \sum_{j \in \mathcal{N}_i} a_{i,j} x_{j,2} + \frac{\dot{\mu} \tilde{s}_{i,1}}{\mu} \right] \end{aligned} \quad (8)$$

$$\dot{\hat{\theta}}_{i,1} = \frac{r_i^2 s_{i,1}^2}{2p_{i,1}^2(k_{b1}^2 - s_{i,1}^2)^2} S_{i,1}^T(X_{i,1}) S_{i,1}(X_{i,1}) - \sigma_{i,1} \hat{\theta}_{i,1} \quad (9)$$

其中,  $p_{i,1}, c_{i,1}, \sigma_{i,1}$  都是正的设计参数.

将式 (6)~(9) 代入式 (5), 可得

$$\begin{aligned} \dot{V}_{i,1} &\leq -\frac{c_{i,1} s_{i,1}^2}{k_{b1}^2 - s_{i,1}^2} + \frac{1}{2} s_{i,2}^2 + \frac{1}{2} \lambda_{i,2}^2 + \frac{1}{2} p_{i,1}^2 + \\ &\quad \frac{1}{2} \bar{\varepsilon}_{i,1}^2 + \sigma_{i,1} \hat{\theta}_{i,1} \hat{\theta}_{i,1} \end{aligned} \quad (10)$$

**步骤 2.** 基于动态面技术<sup>[28]</sup>, 定义如下一阶滤波器用于解决反步法的“计算爆炸”问题

$$\tau_{i,2} \dot{z}_{i,2} + z_{i,2} = \alpha_{i,1}, \quad z_{i,2}(0) = \alpha_{i,1}(0)$$

其中,  $\tau_{i,2}$  是正的设计参数, 由  $\lambda_{i,2} = z_{i,2} - \alpha_{i,1}$  可得  $\dot{z}_{i,2} = -\frac{\lambda_{i,2}}{\tau_{i,2}}$ , 则有

$$\dot{\lambda}_{i,2} = -\frac{\lambda_{i,2}}{\tau_{i,2}} + M_{i,2}(\cdot) \quad (11)$$

其中,  $M_{i,2}(\cdot) = -\dot{\alpha}_{i,1}$  为连续函数.

**注 1.** 由步骤 1 推导出  $\alpha_{i,1}$  后, 在下一步的虚拟控制信号设计过程中必须对其求导, 并且在之后的每一步中都要对虚拟控制信号进行反复求导, 从而产生“计算爆炸”问题. 引入动态面技术将虚拟控制信号通过一阶低通滤波器得到其估计值  $z_{i,2}$ , 在下一步设计过程中用估计值代替虚拟控制信号可以避免对其进行求导, 简化了控制器结构.

选取第  $m$  ( $m = 2, 3, \dots, n-1$ ) 步的 Barrier Lyapunov 函数为

$$V_{i,m} = V_{i,m-1} + \frac{1}{2} \log \frac{k_{bm}^2}{k_{bm}^2 - s_{i,m}^2} + \frac{1}{2} \tilde{\theta}_{i,m}^2 + \frac{1}{2} \lambda_{i,m}^2$$

由式 (1), (2), (11), 可得

$$\begin{aligned} \dot{V}_{i,m} &= \dot{V}_{i,m-1} + \frac{s_{i,m}}{k_{bm}^2 - s_{i,m}^2} [s_{i,m+1} + \lambda_{i,m+1} + \\ &\quad \alpha_{i,m} + f_{i,m}(x_i) - \dot{z}_{i,m}] - \tilde{\theta}_{i,m} \dot{\hat{\theta}}_{i,m} + \\ &\quad \lambda_{i,m} \left[ -\frac{\lambda_{i,m}}{\tau_{i,m}} + M_{i,m}(\cdot) \right] \end{aligned} \quad (12)$$

由引理 2 和引理 5, 可得

$$\begin{aligned} \frac{s_{i,m} f_{i,m}(x_i)}{k_{bm}^2 - s_{i,m}^2} &\leq \frac{s_{i,m}^2 \theta_{i,m}^* S_{i,m}^T(\bar{x}_{i,m}) S_{i,m}(\bar{x}_{i,m})}{2p_{i,m}^2(k_{bm}^2 - s_{i,m}^2)^2} + \\ &\quad \frac{s_{i,m}^2}{2(k_{bm}^2 - s_{i,m}^2)^2} + \frac{1}{2} p_{i,m}^2 + \frac{1}{2} \bar{\varepsilon}_{i,m}^2 \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{s_{i,m} (s_{i,m+1} + \lambda_{i,m+1})}{k_{bm}^2 - s_{i,m}^2} &\leq \frac{1}{2} s_{i,m+1}^2 + \frac{1}{2} \lambda_{i,m+1}^2 + \\ &\quad \frac{s_{i,m}^2}{(k_{i,m}^2 - s_{i,m}^2)^2} \end{aligned} \quad (14)$$

设计第  $m$  步的虚拟控制信号  $\alpha_{i,m}$  和自适应律  $\dot{\hat{\theta}}_{i,m}$  分别为

$$\begin{aligned} \alpha_{i,m} &= -c_{i,m} s_{i,m} - \frac{3s_{i,m}}{2(k_{bm}^2 - s_{i,m}^2)} - \\ &\quad \frac{s_{i,m} \hat{\theta}_{i,m} S_{i,m}^T(\bar{x}_{i,m}) S_{i,m}(\bar{x}_{i,m})}{2p_{i,m}^2(k_{bm}^2 - s_{i,m}^2)} - \\ &\quad \frac{k_{bm}^2 - s_{i,m}^2}{2} s_{i,m} + \dot{z}_{i,m} \end{aligned} \quad (15)$$

$$\begin{aligned}\dot{\hat{\theta}}_{i,m} &= \frac{s_{i,m}^2}{2p_{i,m}^2(k_{bm}^2 - s_{i,m}^2)^2} S_{i,m}^T(\bar{x}_{i,m}) S_{i,m}(\bar{x}_{i,m}) - \\ &\quad \sigma_{i,m} \hat{\theta}_{i,m}\end{aligned}\quad (16)$$

其中,  $p_{i,m}$ ,  $c_{i,m}$ ,  $\sigma_{i,m}$  都是正的设计参数. 基于递归的思想, 将式 (10), (13)~(16) 代入式 (12), 可得

$$\begin{aligned}\dot{V}_{i,m} &\leq -\sum_{l=1}^m \frac{c_{i,l}s_{i,l}^2}{k_{bl}^2 - s_{i,l}^2} + \sum_{l=1}^m \sigma_{i,l} \tilde{\theta}_{i,l} \hat{\theta}_{i,l} + \\ &\quad \sum_{l=2}^m \left[ -\frac{\lambda_{i,l}^2}{\tau_{i,l}} + \lambda_{i,l} M_{i,l}(\cdot) \right] + \frac{1}{2} \sum_{l=2}^{m+1} \lambda_{i,l}^2 + \\ &\quad \frac{1}{2} \sum_{l=1}^m (p_{i,l}^2 + \bar{\varepsilon}_{i,l}^2) + \frac{1}{2} s_{i,m+1}^2\end{aligned}\quad (17)$$

**步骤 3.** 与步骤 2 相同, 定义滤波器

$$\tau_{i,m+1} \dot{z}_{i,m+1} + z_{i,m+1} = \alpha_{i,m}, z_{i,m+1}(0) = \alpha_{i,m}(0)$$

其中,  $\tau_{i,m+1}$  是正的设计参数, 由  $\lambda_{i,m+1} = z_{i,m+1} - \alpha_{i,m}$ , 可得  $\dot{z}_{i,m+1} = -\frac{\lambda_{i,m+1}}{\tau_{i,m+1}}$ , 则有

$$\dot{\lambda}_{i,m+1} = -\frac{\lambda_{i,m+1}}{\tau_{i,m+1}} + M_{i,m+1}(\cdot)$$

其中,  $M_{i,m+1}(\cdot) = -\dot{\alpha}_{i,m}$  为连续函数.

选取第  $n$  步的 Barrier Lyapunov 函数为

$$V_{i,n} = V_{i,n-1} + \frac{1}{2} \log \frac{k_{bn}^2}{k_{bn}^2 - s_{i,n}^2} + \frac{1}{2} \tilde{\theta}_{i,n}^2 + \frac{1}{2} \lambda_{i,n}^2$$

则  $V_{i,n}$  的导数为

$$\begin{aligned}\dot{V}_{i,n} &= \dot{V}_{i,n-1} + \frac{s_{i,n}}{k_{bn}^2 - s_{i,n}^2} (u_i + f_{i,n}(x_i) - \dot{z}_{i,n}) + \\ &\quad \left[ -\frac{\lambda_{i,n}^2}{\tau_{i,n}} + \lambda_{i,n} M_{i,n}(\cdot) \right] - \tilde{\theta}_{i,n} \dot{\hat{\theta}}_{i,n}\end{aligned}$$

由引理 5 可得

$$\begin{aligned}\frac{s_{i,n} f_{i,n}(x_i)}{k_{bn}^2 - s_{i,n}^2} &\leq \frac{s_{i,n}^2 \theta_{i,n}^* S_{i,n}^T(x_i) S_{i,n}(x_i)}{2p_{i,n}^2(k_{bn}^2 - s_{i,n}^2)^2} + \\ &\quad \frac{s_{i,n}^2}{2(k_{bn}^2 - s_{i,n}^2)^2} + \frac{1}{2} p_{i,n}^2 + \frac{1}{2} \bar{\varepsilon}_{i,n}^2\end{aligned}$$

自适应控制器设计为如下形式

$$w_i(t) = \alpha_{i,n} - \bar{m}_i \tanh \left[ \frac{s_{i,n} \bar{m}_i}{\epsilon_i (k_{bn}^2 - s_{i,n}^2)} \right] \quad (18)$$

$$\alpha_{i,n} = -c_{i,n} s_{i,n} - \frac{s_{i,n}}{2(k_{bn}^2 - s_{i,n}^2)} -$$

$$\begin{aligned}\frac{s_{i,n} \hat{\theta}_{i,n} S_{i,n}^T(x_i) S_{i,n}(x_i)}{2p_{i,n}^2(k_{bn}^2 - s_{i,n}^2)} - \\ \frac{k_{bn}^2 - s_{i,n}^2}{2} s_{i,n} + \dot{z}_{i,n}\end{aligned}\quad (19)$$

$$\begin{aligned}\dot{\hat{\theta}}_{i,n} &= \frac{s_{i,n}^2}{2p_{i,n}^2(k_{bn}^2 - s_{i,n}^2)^2} S_{i,n}^T(x_i) S_{i,n}(x_i) - \\ &\quad \sigma_{i,n} \hat{\theta}_{i,n}\end{aligned}\quad (20)$$

事件触发机制定义为如下形式

$$u_i(t) = w_i(t_k), \forall t \in [t_k, t_{k+1}) \quad (21)$$

$$t_{k+1} = \inf\{t \in \mathbf{R} \mid |e_i(t)| \geq m_i\}, \quad t_1 = 0 \quad (22)$$

其中,  $e_i(t) = w_i(t) - u_i(t)$  表示测量误差,  $p_{i,n}$ ,  $c_{i,n}$ ,  $\sigma_{i,n}$ ,  $\epsilon_i$ ,  $m_i$  以及  $\bar{m}_i$  都是正的设计参数, 且  $m_i < \bar{m}_i$ . 事件触发时刻定义为  $t_k, k \in \mathbf{Z}^+$ . 即当式 (22) 条件被触发时, 控制信号将更新为  $u_i(t_{k+1})$ , 当  $t \in [t_k, t_{k+1})$  时, 控制信号为  $w_i(t_k)$  保持不变. 于是存在一个连续的时变常数  $\varsigma(t)$ , 满足  $\varsigma(t_k) = 0$ ,  $\varsigma(t_{k+1}) = \pm 1$ ,  $|\varsigma(t)| \leq 1, \forall t \in [t_k, t_{k+1})$ , 使得  $w_i(t) = u_i(t) + \varsigma(t)m_i$ . 参考文献 [14] 可得

$$\begin{aligned}\dot{V}_{i,n} &= \dot{V}_{i,n} - \frac{c_{i,n} s_{i,n}^2}{k_{bn}^2 - s_{i,n}^2} - \frac{s_{i,n}^2}{2} + \sigma_{i,n} \tilde{\theta}_{i,n} \hat{\theta}_{i,n} + \\ &\quad 0.2785 \epsilon_i + \left[ -\frac{\lambda_{i,n}^2}{\tau_{i,n}} + \lambda_{i,n} M_{i,n}(\cdot) \right] + \\ &\quad \frac{1}{2} p_{i,n}^2 + \frac{1}{2} \bar{\varepsilon}_{i,n}^2 \leq \\ &\quad \sum_{l=1}^n \frac{-c_{i,l} s_{i,l}^2}{k_{bl}^2 - s_{i,l}^2} + \sum_{l=2}^n \left[ -\frac{\lambda_{i,l}^2}{\tau_{i,l}} + \lambda_{i,l} M_{i,l}(\cdot) \right] + \\ &\quad \sum_{l=1}^n \sigma_{i,l} \tilde{\theta}_{i,l} \hat{\theta}_{i,l} + \frac{1}{2} \sum_{l=2}^n \lambda_{i,l}^2 + \\ &\quad \frac{1}{2} \sum_{l=1}^n (p_{i,l}^2 + \bar{\varepsilon}_{i,l}^2) + 0.2785 \epsilon_i\end{aligned}\quad (23)$$

由引理 5 可得

$$\sigma_{i,l} \tilde{\theta}_{i,l} \hat{\theta}_{i,l} \leq -\frac{\sigma_{i,l} \tilde{\theta}_{i,l}^2}{2} + \frac{\sigma_{i,l} \theta_{i,l}^{*2}}{2} \quad (24)$$

$$\lambda_{i,l} M_{i,l}(\cdot) \leq \frac{\lambda_{i,l}^2 M_{i,l}^2(\cdot)}{2} + \frac{1}{2} \quad (25)$$

由引理 3 可得

$$-\frac{c_{i,l} s_{i,l}^2}{k_{bl}^2 - s_{i,l}^2} \leq -c_{i,l} \log \frac{k_{bl}^2}{k_{bl}^2 - s_{i,l}^2} \quad (26)$$

存在一个标量  $\bar{M}_{i,l} > 0$  满足  $|M_{i,l}(\cdot)| < \bar{M}_{i,l}$ <sup>[28]</sup>. 则将式(24)~(26)代入式(23), 可得

$$\begin{aligned}\dot{V}_{i,n} \leq & -\sum_{l=1}^n c_{i,l} \log \frac{k_{bl}^2}{k_{bl}^2 - s_{i,l}^2} - \sum_{l=1}^n \frac{\sigma_{i,l} \tilde{\theta}_{i,l}^2}{2} - \\ & \sum_{l=2}^n \left( \frac{1}{\tau_{i,l}} - \frac{\bar{M}_{i,l}^2}{2} - \frac{1}{2} \right) \lambda_{i,l}^2 + \Lambda_i\end{aligned}\quad (27)$$

其中,

$$\begin{aligned}\Lambda_i = & \frac{1}{2} \sum_{l=1}^n (p_{i,l}^2 + \varepsilon_{i,l}^2) + 0.2785\epsilon_i + \\ & \sum_{l=1}^n \frac{\sigma_{i,l} \theta_{i,l}^{*2}}{2} + \frac{n-1}{2}\end{aligned}$$

**定理 1.** 在假设 1 成立的条件下, 考虑虚拟控制信号(8), (15), (19), 自适应律(9), (16), (20)及事件触发自适应控制器(18), (21), (22), 能够保证式(1)所描述的多智能体系统满足以下条件: 1) 系统中所有的信号都是半全局一致最终有界的; 2) 跟踪误差收敛于原点的有界邻域内且满足指定性能. 此外, 事件触发的时间间隔  $\{t_{k+1} - t_k\}$  存在一个下界  $t^*$ ,  $t^* > 0$ . 即该事件触发自适应控制器不会发生奇诺现象.

证明. Lyapunov 函数定义为

$$V = \sum_{i=1}^N V_{i,n}$$

由式(27)可得其导数为

$$\begin{aligned}\dot{V} \leq & \sum_{i=1}^N \left[ -\sum_{l=1}^n c_{i,l} \log \frac{k_{bl}^2}{k_{bl}^2 - s_{i,l}^2} - \sum_{l=1}^n \frac{\sigma_{i,l} \tilde{\theta}_{i,l}^2}{2} - \right. \\ & \left. \sum_{l=2}^n \left( \frac{1}{\tau_{i,l}} - \frac{\bar{M}_{i,l}^2}{2} - \frac{1}{2} \right) \lambda_{i,l}^2 + \Lambda_i \right]\end{aligned}\quad (28)$$

选择设计参数使得  $\frac{1}{\tau_{i,l}} - \frac{\bar{M}_{i,l}^2}{2} - \frac{1}{2} > 0$ , 取

$$q = \min \left\{ 2c_{i,l}, 2 \left[ \frac{1}{\tau_{i,l}} - \frac{\bar{M}_{i,l}^2}{2} - \frac{1}{2} \right], \sigma_{i,l} \right\}$$

$$\Lambda = \sum_{i=1}^N \Lambda_i$$

则式(28)可以写为

$$\dot{V}(t) \leq -qV(t) + \Lambda \quad (29)$$

则所有的信号都是半全局一致最终有界的, 由式(29), 得

$$\frac{1}{2}s_{i,1}^2 \leq V(t) \leq e^{-qt}V(0) + \frac{\Lambda}{q}(1 - e^{-qt})$$

基于引理 4, 可得

$$\lim_{t \rightarrow \infty} \|y - \bar{y}_d\| \leq \frac{\sqrt{\frac{2\Lambda}{q}}}{\underline{\sigma}(\mathcal{L} + B)} \quad (30)$$

由式(30)可知, 通过选择合适的设计参数可以使跟踪误差收敛于以原点为中心的有界邻域内.

由  $e_i(t) = w_i(t) - u_i(t)$ ,  $\forall t \in [t_k, t_{k+1})$ , 可得

$$\frac{d}{dt}|e_i| = \frac{d}{dt}(e_i \times e_i)^{\frac{1}{2}} = \text{sign}(e_i)\dot{e}_i \leq |\dot{w}_i|$$

其中,  $\dot{w}_i$  是  $w_i$  的导数, 因为系统中所有信号都是有界的, 所以必然存在一个正常数  $\kappa$ , 使得  $|\dot{w}_i| \leq \kappa$ . 基于  $e_i(t_k) = 0$ ,  $\lim_{t \rightarrow t_{k+1}} e_i(t) = m_i$ , 可以得出事件触发时间间隔的下界  $t^*$  满足  $t^* \geq m_i/\kappa$ . 由此可以证明本文所提出的事件触发机制不发生奇诺现象.

□

### 3 仿真实例

考虑有向拓扑图下由 4 个跟随者和 1 个虚拟领导者组成的多智能体系统, 如图 1 所示. 其中第  $i$  个子系统的动态模型为

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2} + x_{i,1}^2 \sin x_{i,2} \\ \dot{x}_{i,2} &= u_i + x_{i,2} \sin x_{i,1} \\ y_i &= x_{i,1}, \quad i = 1, 2, 3, 4\end{aligned}$$

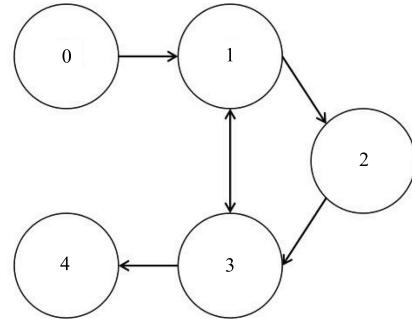
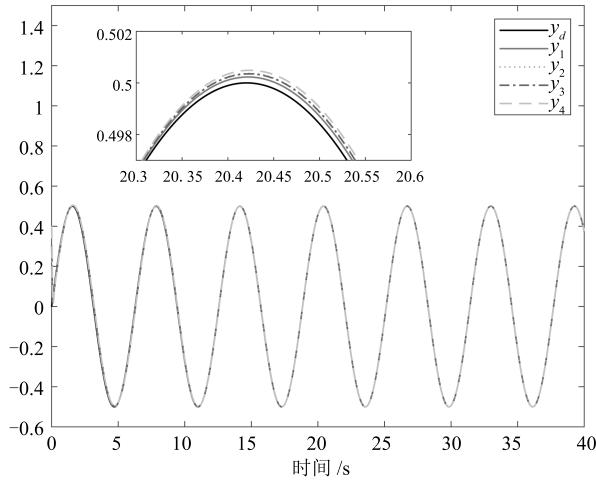
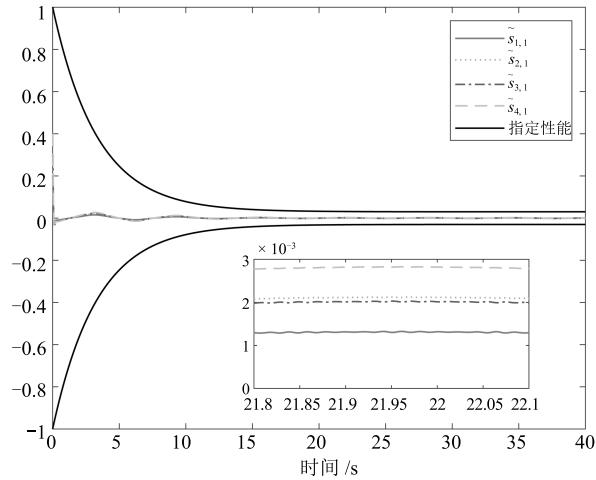
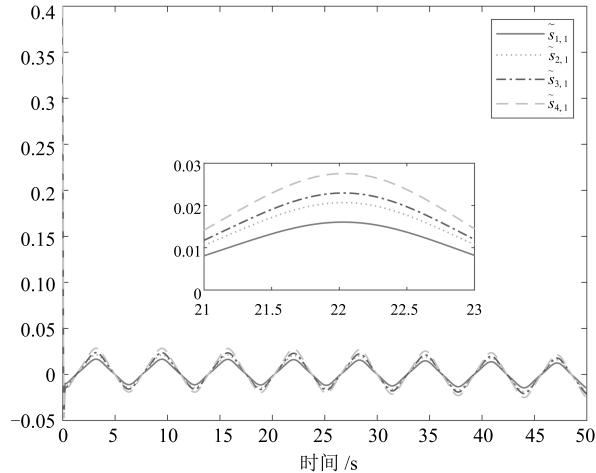
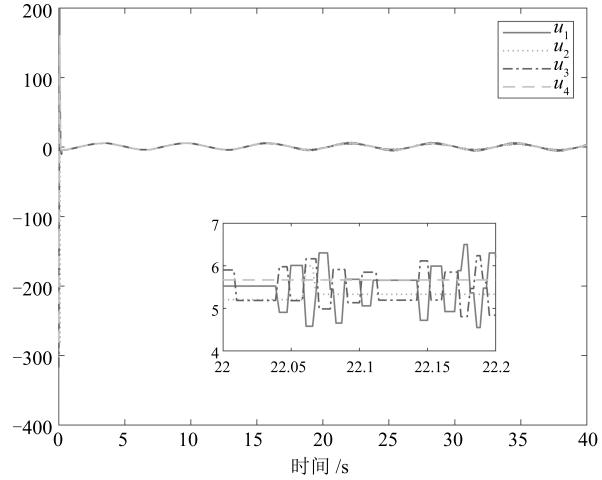
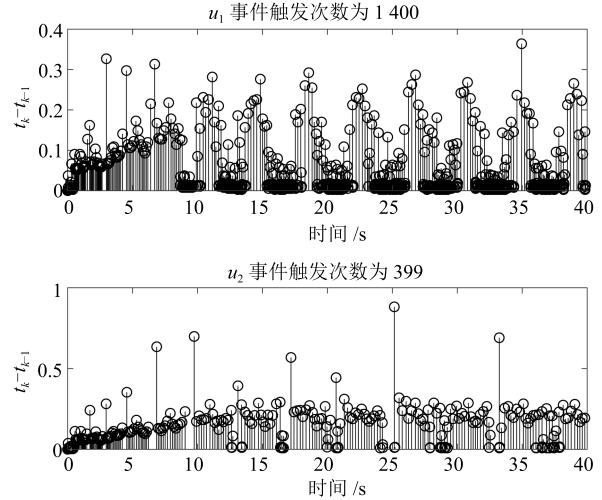
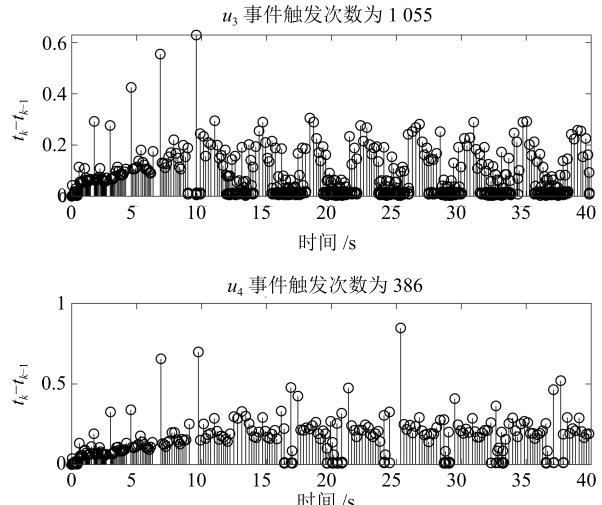


图 1 通信拓扑图

Fig. 1 Communication topology

作为虚拟领导者的期望轨迹为  $y_d = 0.5 \sin t$ . 设计的性能函数为  $\mu(t) = 0.97e^{-0.3t} + 0.03$ . 数值仿真程序中设计参数的值分别设置为  $c_{i,1} = c_{i,2} = 80$ ,  $\sigma_{i,1} = \sigma_{i,2} = 0.001$ ,  $p_{i,1} = p_{i,2} = 10$ ,  $\tau_{i,2} = 0.02$ ,  $\delta_{\min} = 0.999$ ,  $\delta_{\max} = 1$ ,  $k_{b1} = k_{b2} = 2$ ,  $\bar{m}_i = 8$ ,  $m_i = 0.6$ ,  $\epsilon_i = 1.5$ . 初始值设置为:  $x_{i,1}(0) = [0.2, 0.3, 0.3, 0.4]$ ,  $x_{i,2}(0) = [1.0, -1.5, 1.2, -0.6]$ ,  $\hat{\theta}_{i,1}(0) = \hat{\theta}_{i,2}(0) = 0$ ,  $z_{i,2}(0) = 0.1$ .

图 2~7 为仿真结果, 图 2 表示虚拟领导者的期望轨迹  $y_d$  和每个跟随者的输出信号  $y_i$  的响应曲线, 由图可以看出各跟随者在较短的时间内实现了对期

图 2 参考信号  $y_d$  和输出信号  $y_i$ Fig. 2 Reference signal  $y_d$  and output  $y_i$ 图 3 具有指定性能的跟踪误差  $\tilde{s}_{i,1}$ Fig. 3 Tracking errors with prescribed performance  $\tilde{s}_{i,1}$ 图 4 不具有指定性能的跟踪误差  $\tilde{s}_{i,1}$ Fig. 4 Tracking errors without prescribed performance  $\tilde{s}_{i,1}$ 图 5 控制信号  $u_i$ Fig. 5 Control signal  $u_i$ 图 6  $u_1, u_2$  的事件触发时间间隔Fig. 6 Time interval of event-triggered for  $u_1, u_2$ 图 7  $u_3, u_4$  的事件触发时间间隔Fig. 7 Time interval of event-triggered for  $u_3, u_4$

望轨迹的跟踪, 表明本文提出的控制算法快速实现了多智能体的一致性跟踪。图3和图4分别表示具有指定性能的跟踪误差和不具有指定性能的跟踪误差, 虽然跟踪误差均能快速收敛于以原点为中心的有界邻域内, 但通过对比可得, 在不具有指定性能的条件下, 稳态误差接近0.03, 而具有指定性能时稳态误差小于0.003, 因此通过指定性能变换可以显著减小多智能体一致性跟踪控制的稳态误差。控制器输入 $u_i$ 由图5表示, 控制器输入的数值除了在起始阶段的瞬间有较大的波动, 在其余时间均小于7, 表明该控制器性能良好, 能耗较低。图6和图7是 $u_i$ 的事件触发时间间隔和事件触发次数, 横坐标为事件触发时间, 纵坐标为事件触发的时间间隔。控制器 $u_1, u_2, u_3, u_4$ 在40 s内的事件触发次数分别为1400, 399, 1055, 386, 表明事件触发控制器具有减少控制信号更新次数和降低控制成本的优点, 且没有发生奇诺现象。

## 4 结论

本文研究了一类具有全状态约束的非严格反馈多智能体一致性跟踪问题。在控制算法设计过程中考虑了指定性能, 获得了期望的稳态误差。此外, 还设计了一个固定阈值的事件触发机制, 降低了智能体之间的通讯成本和计算压力。本文给出了控制算法的详细推导过程和稳定性证明过程。最后通过仿真结果验证了本文所设计的控制算法的有效性。在未来的研究中, 我们将考虑更为复杂的应用环境, 提高多智能体系统的抗干扰能力。

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