SAR Image Despeckling by Sparse Reconstruction Based on Shearlets

JI Jian[1] LI Xiao[1, 2] XU Shuang-Xing[1, 2] LIU Huan[1] HUANG Jing-Jing[1]

Abstract Synthetic aperture radar (SAR) image is usually polluted by multiplicative speckle noise, which can affect further processing of SAR image. This paper presents a new approach for multiplicative noise removal in SAR images based on sparse coding by shearlets filtering. First, a SAR despeckling model is built by the theory of compressed sensing (CS). Secondly, obtain shearlets coefficient through shearlet transform, each scale coefficient is represented as a unit. For each unit, sparse coefficient is iteratively estimated by using Bayesian estimation based on shearlets domain. The represented units are finally collaboratively aggregated to construct the despeckling image. Our results in SAR image despeckling show the good performance of this algorithm, and prove that the algorithm proposed is robustness to noise, which is not only good for reducing speckle, but also has an advantage in holding information of the edge.

Key words Synthetic aperture radar (SAR), compressed sensing (CS), sparse reconstruction, shearlets

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Speckle noise is always present in SAR images. This noise is produced due to the coherent sum of many elementary scatterers in each resolution cell and gives a grainy appearance to images that make detection and classification tasks more difficult.

The methods of SAR image speckle noise removal basically can be divided into three categories: multi-visual processing, space adaptive filtering and wavelet domain processing. Multi-visual processing can effectively remove speckle noise, but also reduces the image resolution at the same time. Space adaptive filtering techniques including techniques proposed by Lee[1], Kuan et al.[2], and Frost[3], can effectively smooth noise in homogeneous area, but often lead to edge blur, texture loss of faults in the heterogeneous area. And due to the wavelet have two characteristics of the multi-resolution, time-frequency local, which are widely used in SAR image speckle noise removal. But two-dimensional wavelet is constructed by product of the two dimensional orthogonal wavelets. Wavelet transform only have three directions: horizontal, vertical and diagonal. Two dimensional wavelet approximate singularity line by singularity point, cannot sparse representation the contour and edge information of the SAR image. In order to overcome the drawbacks of the wavelet transform, Easley et al. put forward to an effective method of two-dimensional image representation: shearlets transform[4]. Theoretical analysis and experimental results showed shearlets transform denoising effect was better than wavelet denoising.

Sparse representation has been successfully used in the past decade for denoising problem. The work reported here is also built on the same sparsity, but based on a different model such as hidden Markov model[5], Gaussian scale mixture model[6], and so on. These methods can obtain a good denoising result. Image denoising is an inverse problem of image processing. Recent compressed sensing theory[7] provides a powerful tool for dealing with problems including SAR image multiplicative noise removal. Several reconstructed methods have been proposed. Among the existing methods, iterative/thresholding algorithms[8] (IST) is widely applied to compression reconstruction, because the existing transforms (including curvelet[9], contourlet[10], shearlets[4]) can be easily incorporated into IST method, thus can obtain a better signal sparse representation.

In this paper, we propose a denoising method by sparse reconstruction based on shearlets filter. First, we construct denoising model by CS theory and obtain observation value. Secondly, we compute the shearlets transform of the original noise image and obtain shearlets coefficient, each scale coefficient of which is represented as a unit. Instead of dealing with shearlets coefficient, for each unit we iteratively estimate the coefficient by Bayesian estimation based on shearlets until termination condition is satisfied. Finally, the represented units are collaboratively aggregated to construct the denoised image. We show how this denoising model leads to a simple and effective denoising algorithm, with competing performance, equivalent and sometimes surpassing recently published leading alternative denoising methods.

The traditional SAR image despeckling approaches cannot preserve the edge and point target well. Due to these defects, we propose a SAR image denoising algorithm by sparse reconstruction based on CS theory. The algorithm transforms the image denoising to solving an l_1 norm optimization problem. The proposed algorithm reconstructs the image by the iterative threshold method and calculates the threshold function by Bayesian estimation based on shearlet transform. The shearlet transform has the characteristics of shift-invariance and multi-directionality, which can obtain better performance than the wavelet transform in image denoising. The algorithm solves the optimization problem by using the iterative threshold method. It is easy to demonstrate that the algorithm could rapidly converge to the global optimal solution.

The contents of the paper are organized as follows. Section 1 contains the background of compressed sensing and shearlets transform. In Section 2, the SAR image despeckling model is built by the compressed sensing. We show how such denoising model leads to a simple and effective denoising algorithm in Section 3. In Section 4, the process of the algorithm is presented. Section 5 contains the experiments result and analysis. Conclusions are presented in Section 6.

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1 Background

1.1 Compressed sensing[7]

In the compressed sensing, $\forall x$, we obtain its projection in Ψ domain, that is,

$$x = \Psi\alpha \tag{1}$$

where x is input signal, $\alpha = \Psi^{\mathrm{T}}x$ is coefficient of the projection, Ψ is orthogonal basis matrix. Clearly, x and α are equivalent representations of the same signal. When $||\alpha||_0 = K(K \ll N)$, x the signal is K-sparse.

If x is K-sparse in Ψ domain, we may use matrix $\Phi \in \mathbf{R}^{M\times N}$ for the signal x to linear measurement according to CS theory. Obtain the observation value y:

$$y = \Phi x \tag{2}$$

and substitute (1),

$$y = \Phi\Psi\alpha \ = \ A\alpha \tag{3}$$

The measure matrix Φ is incoherent with the sparsifying basis Ψ. A is called senor matrix under the compressed sensing in the absence of noise model.

Since the dimension of the observation vector is much smaller than the dimension of the signal, there exist infinitely many solutions in (3), that is, the question is ill-posed. It is difficult to reconstruct the original signal from the observation vector. To solve this problem, Candes pointed out that we might reconstruct image by solving the following non-convex optimization problem,

$$\min \left\|\Psi^{\mathrm{T}}x\right\|_0 \ \text{s.t.}\ \Phi x = y \tag{4}$$

where $||x||_0$ denotes zero norm, i.e., the number of non-zero elements. But formula (4) is a typical NP-hard problem, which is not easy to solve. Therefore, Candes proposed one-norm instead of zero-norm to solve the problem.

$$\min \left\|\Psi^{\mathrm{T}}x\right\|_1 \ \text{s.t.}\ A\alpha = y \tag{5}$$

Formulas (4) and (5) are equivalent under the premise of sparse image representation. By applying Lagrange multiplier to formula (4), it converts to (6):

$$\min \frac{1}{2}\left\|\Phi x - y\right\|_2^2 + \lambda\left\|\Psi^{\mathrm{T}}x\right\|_1 \tag{6}$$

We consider an approach for solving compressed sensing in the form of (7)

$$f = \min_{x\in\mathbf{R}^{N\times N}} \frac{1}{2}\left\|y - \Phi x\right\|_2^2 + \tau c \tag{7}$$

where $x \in \mathbf{R}^{N\times N}$ is input signal, $\Phi \in \mathbf{R}^{M\times N}$ ($M \ll N$), f is objective function, c is the regularization function, $y = \Phi x$ is observation value.

The recovery of signal x from observation value y is an inverse and ill-posed problem, because$M \ll N$. However, we assume that signal x has a sparse representation expressed by a certain basis function Ψ, then applying a transform Ψ to signal x, the form of the CS-problem with x in transform domain is

$$f = \min_{x\in\mathbf{R}^{N\times N}} \frac{1}{2}\left\|y - \Phi\Psi^{-1}\alpha\right\|_2^2 + \tau c \tag{8}$$

where $x = \Psi^{-1}\alpha$, $\widetilde{\Phi} = \Phi\Psi^{-1}$ stands for sensing matrix, Ψ^{-1} is a sparse inverse transform (e.g., curvelet inverse transform[9], contourlet inverse transform[10], shearlets inverse transform[4] and so on).

Regularization function c has many choices, especially if $c = \|x\|_1$. Problem (9) generalizes the mathematical model of compressed sensing.

$$f = \min_{x\in\mathbf{R}^{N\times N}} \frac{1}{2}\left\|y - \Phi\Psi^{-1}\alpha\right\|_2^2 + \tau\|\alpha\|_1 \tag{9}$$

Here, $\|\cdot\|_1$ stands for l_1 norm.

It has been verified that problem (9) can be used for compressing construction by CS theory[7]. To solve equation (9), we should choose a suitable sparse basis Ψ that signal x can be sparse representation in the Ψ domain. In this paper, we choose shearlets transform as Ψ.

1.2 Shearlets transform

The theory of composite wavelets has been introduced in [4] provides a very effective method for combining geometry and multi-scale analysis by using classical theory of affine system. When dimension $n = 2$, the form of affine system with composite dilations is

$$\begin{aligned}&\chi_{AB}(\psi) = \\ &\quad \{\psi_{j,l,k}(x) = |\det A|^{\frac{j}{2}}\psi(B^l A^j x - k) : j, l \in \mathbf{Z}, k \in \mathbf{Z}^2\}\end{aligned} \tag{10}$$

where $\psi \in L^2(\mathbf{R}^2)$, $A \in \mathbf{R}^{2\times 2}$, $B \in \mathbf{R}^{2\times 2}$ and $|\det B| = 1$. If $\chi_{AB}(\psi)$ satisfies the Parseval framework (also called tight framework), and there exists $f \in L^2(\mathbf{R}^2)$

$$\sum_{j,l,k} |\langle f, \psi_{j,l,k}\rangle|^2 = ||f||^2 \tag{11}$$

then $\chi_{AB}(\psi)$ is called composite wavelets.

Shearlets transform is an example of composite wavelets, where $A = A_0 = \begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}$ and $B = B_0 = \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ stand for anisotropic dilations matrix, and shear matrix, respectively. For any $\xi = (\xi_1, \xi_2) \in \hat{\mathbf{R}}^2$, $\xi_1 \neq 0$, $\psi^{(0)}$ satisfies

$$\hat{\psi}^{(0)}(\xi) = \hat{\psi}^{(0)}(\xi_1, \xi_2) = \hat{\psi}_1(\xi_1)\,\hat{\psi}_2\left(\frac{\xi_2}{\xi_1}\right) \tag{12}$$

where $\hat{\psi}^{(0)}$ stands for Fourier transform of $\psi^{(0)}$, ψ_1 stands for continuous wavelet function, and $\hat{\psi}_1^{(0)} \in C^\infty(\mathbf{R})$, supp $\hat{\psi}_1 \subset [-1/2, -1/16] \cup [1/16, 1/2]$, $\hat{\psi}_2$ stands for bump function and $\hat{\psi}_2 \in C^\infty(\mathbf{R})$, supp $\hat{\psi}_2 \subset [-1, 1]$. Therefore, $\hat{\psi}^{(0)}$ is a continuous and tight support, $\hat{\psi}^{(0)} \in C^\infty(\mathbf{R})$, supp $\hat{\psi}^{(0)} \subset [-1/2, 1/2]^2$.

We suppose

$$\sum_{j\geq 0} |\hat{\psi}_1(2^{-2j}\omega)|^2 = 1, \quad |\omega| \geq \frac{1}{8} \tag{13}$$

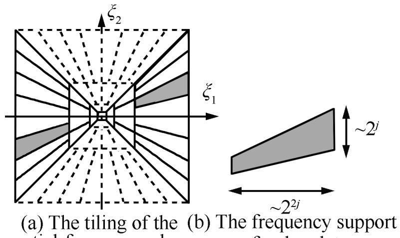
By observing formulas (13) and (14), for any $(\xi_1, \xi_2) \in D_0 = \{(\xi_1, \xi_2) \in \mathbf{R}^2 : |\xi_1| \geq \frac{1}{8}, |\frac{\xi_2}{\xi_1}| \leq 1\}$, there exists

$$\begin{aligned}&\sum_{j\geq 0}\sum_{l=-2^j}^{2^j-1} |\hat{\psi}^{(0)}(\xi A_0^{-j} B_0^{-l})|^2 = \\ &\quad \sum_{j\geq 0}\sum_{l=-2^j}^{2^j-1} |\hat{\psi}_1(2^{-2j}\xi_1)|^2 |\hat{\psi}_2(2^j\frac{\xi_2}{\xi_1} - l)|^2 = 1\end{aligned} \tag{14}$$

The fact that $\hat{\psi}_1$ and $\hat{\psi}_2$ are supported inside $2^{2j} \times 2^j$, (Fig. 1 (b)) oriented along the straight line with slope $l2^{-j}$, implies that the collection of functions defined by

$$\text{supp}\, \hat{\psi}^{(0)}_{j,l,k} \subset \{(\xi_1, \xi_2)\} \tag{15}$$

where $\xi_1 \in [-2^{2j-1}, -2^{2j-4}] \cup [2^{2j-4}, 2^{2j-1}], |\frac{\xi_2}{\xi_1} + l2^{-j}| \leq 2^{-j}$



(a) The tiling of the spatial-frequency plane induced by the shearlets (b) The frequency support of a shearlets

Fig. 1 Spatial-frequency plane and frequency support of shearlets

2 Building SAR image despeckling model by compressed sensing

The approach of sparse reconstruction by separable approximation (SpaRSA) has been introduced in [11]. We solve despeckling problem in (9) by computing a series of iteration value $\{x^t, t = 0, 1, \cdots\}$. The x^t of each iteration can be solved by the equation in (16):

$$x^{t+1} \in \arg\min_z \frac{1}{2}\left\|z^t - u^t\right\|_2^2 + \frac{\tau}{\alpha^t} c(z) \tag{16}$$

where $u^t = x^t - \frac{1}{\alpha^t}\nabla f(x^t)$.

When $c(z) = \|z\|_0 = \sum_i 1_{x_i \neq 0}$ ($\sum_i 1_{x_i \neq 0}$ stands for the sum of $x_i \neq 0$), we compute the first derivative of (16), that is

$$x^{t+1} \in \arg\min_z \frac{1}{2}\left\|z^t - u^t\right\|_2^2 + \sum_{i=1}^{n} \frac{\tau}{\alpha^t} 1_{x_i^t \neq 0} = hard(u^t, \sqrt{\frac{2\tau}{\alpha^t}}) \tag{17}$$

where $hard(\cdot)$ stands for hard-threshold function. Generally, we compute a sequence of $\{x^t, t = 0, 1, \cdots\}$ by

$$x^{t+1} = \Gamma\left(x^t - \frac{1}{\alpha^t}\nabla f(x^t), \delta\right) \tag{18}$$

where $\Gamma(\cdot)$ is threshold function, δ stands for threshold, $\nabla f(x^t)$ stands for the gradient of $\nabla f(x^t)$. In this paper, we choose the form of (19) as $\nabla f(x^t)$

$$\nabla f(x^t) = \nabla f(\Psi^{-1}\alpha^t) = \Phi^{\mathrm{T}}\left(\Phi\Psi^{-1}\alpha^t - y\right) \tag{19}$$

where Ψ stands for shearlets transform. In order to using shearlets threshold function, we choose $\delta = \sigma$ (σ stands for standard deviation of the noise), that is

$$x^{t+1} = \Gamma\left(x^t - \frac{1}{\alpha^t}\Phi^{\mathrm{T}}\left(\Phi\Psi^{-1}\alpha^t - y\right), \sigma\right) \tag{20}$$

The next part will introduce how to compute σ, α^t and how to designing $\Gamma(\cdot)$.

3 SAR image despeckling based on compressed sensing

3.1 Algorithm framework

By the above analysis, our approach to solve SAR image speckle noise removing problems by iteratively computing x using (20), until the stopping criterion is satisfied.

The following pseudo-algorithm shows the framework of the algorithm.

Algorithm 1. SpaRSA

1) Choose factor $\eta > 1$ and constants $\alpha_{\min}$, $\alpha_{\max}$ (with $0 < \alpha_{\min} < \alpha_{\max}$);
2) Choose original SAR image x^0, initialize iteration counter $t \leftarrow 0$, maximum number of iterations $mt \leftarrow 100$;
3) Repeat;
4) Choose $\alpha^t \in [\alpha_{\min}, \alpha_{\max}]$;
5) Repeat;
6) $x^{t+1} \leftarrow$ solution of sub-problem (20);
7) $\alpha_t \leftarrow \eta\alpha_t$;
8) Until x^{t+1} satisfies an acceptance criterion;
9) $t \leftarrow t + 1$;
10) Until stop criterion is satisfied;

The denoising algorithm involves two key steps: setting threshold function $\Gamma(\cdot)$ and estimating standard deviation of the noise σ. In addition, the setting of α^t and acceptance criterion will refer to SpaRSA algorithm[11].

3.2 Bayesian estimation

Consider an image whose pixels are contaminated with independent and identically distributed samples of additive Gaussian noise. Thus, the noisy image can be written as $s = x + n$, where x is the denoising image, s is the input image, n is Gaussian noise. But SAR image exists multiplicative speckle noise, the noise image is written as $s = nx$, we convert multiplicative noise into additive noise through $s = x + x(n-1)$, assuming that the corrupting noise n is independent identically distributed and sampled from a zero mean density of unknown variance σ^2. $s = x+x(n-1)$ and $s = x + n$ have similar forms.

Applying shearlets transform Ψ to x, each coefficient in the shearlets expansion of the noisy image is written as

$$s^j = \omega + Ns \tag{21}$$

where $\omega = \Psi^{-1}\alpha^j$, $Ns = \Psi^{-1}\alpha^j(n-1)$, and j stands for shearlets scale .

A standard estimator for c given the corrupted observation s is the maximum a posteriori (MAP) estimator[12] is:

$$\begin{aligned}\widehat{\omega}(s) &= \arg\max p_{\omega|s}(\omega|s) = \\ &\arg\max_\omega [p_{Ns}(y-\omega)p_\omega(\omega)] = \\ &\arg\max_\omega [\lg(p_{Ns}(y-\omega)) + \lg(p_\omega(\omega))]\end{aligned} \tag{22}$$

According to the assumption $p_{Ns} \sim \mathrm{N}(0, \sigma_n^2)$, there exists

$$\widehat{\omega}(s) = \arg\max_\omega [-\frac{(y-\omega)^2}{2\sigma_n^2} + f(\omega)] \tag{23}$$

where $f(\omega) = \lg(p_\omega(\omega))$. Computing the first derivative of (23), that is

$$\frac{y-\omega}{\sigma_n^2} + f'(\omega) = 0 \tag{24}$$

According to the conclusion of the analysis above, shearlets coefficient follows the generalized Gaussian distribution, that is,

$$p_\omega(\omega) = K(m,l)\exp\left(-\left|\frac{\omega}{m}\right|^l\right) \tag{25}$$

If $l = 1$, then $p_\omega(\omega)$ follows Laplace distribution, that is,

$$\widehat{\omega}(s) = \mathrm{sgn}(s)\left(|y| - \frac{\sqrt{2}\sigma_n^2}{\sigma}\right) \tag{26}$$

Therefore, for each shearlets scale, we can use (27) to estimate the shearlets coefficient.

$$T = \frac{\sqrt{2}\sigma_j^2}{\sigma} \tag{27}$$

where σ_j^2 stands for noise variance of scale j, σ stands for the standard deviation of the shearlets coefficient α^j with scale j.

We will employ hard-threshold method. Next, we will demonstrate the characteristics of shearlets coefficient in histogram by the transforming SAR image (Fig. 3 (a)). We use shearlets transform to obtain shearlets coefficients (two scales, direction = [3 3 3 4 5]), by retaining one layer coefficients and combining all the sub-direction coefficients to a matrix, so we obtain the histogram of the second shearlets coefficient in Fig. 2. As Fig. 2 shows, most of the high-frequence shearlets coefficients of the SAR image distributes near zero. The shape of the histogram similar to the generalized Gaussian distribution with zero mean.

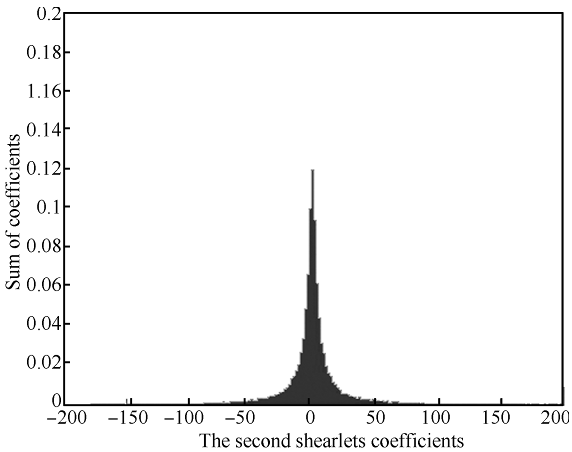


Fig. 2 Histogram of the second shearlets coefficients

3.3 Parameter estimation

3.3.1 The setting of $\boldsymbol{\alpha^t}$

α^t is chosen through Barzilai-Borwein (BB) such that letting $s^t = x^t - x^{t-1}$, $r^t = \nabla f(x^t) - \nabla f(x^{t-1})$, $\alpha^t = (\beta^t)^{-1}$. By solving $s^t = \beta^t r^t$ in the least-squares sense, we can compute α^t by the following equation.

$$\alpha^t = \frac{(r^t)^{\mathrm{T}} r^t}{(r^t)^{\mathrm{T}} s^t} = \frac{\left\|\Phi^{\mathrm{T}}\Phi s^t\right\|^2}{\left\|\Phi s^t\right\|^2} \tag{28}$$

3.3.2 The setting of $\boldsymbol{\sigma^j}$

Here the noise standard deviation is estimated with a heuristic priori

$$\sigma^j = \frac{median\left(|\alpha^j|\right)}{0.6745} \tag{29}$$

where α^j stands for shearlets coefficient with scale j.

3.3.3 Acceptance criterion

The new estimation will be accepted only when its objective function value is smaller than the median value of the objective function value over last t iterations. That is, x^{t+1} will be accepted if

$$f(x^{t+1}) \le \underset{i=1,2,\cdots,t}{median}\, f(x^i) \tag{30}$$

3.3.4 Stopping criterion

The stopping criterion is defined by the relative difference between the objective function values of two subsequent iterations. We terminate at iteration t if:

$$\frac{\left|f(x^{t+1}) - f(x^t)\right|}{f(x^t)} \le tolA \tag{31}$$

where $tolA$ is a constant. Generally, $tolA = 10^{-4}$.

4 The process of algorithm

Base on the above analysis, the procedures of the algorithm are as follows:

Algorithm 2. Shearlets_CS

Task: Denoise a given SAR image x_0;

Algorithm parameters: τ-Lagrange multiplier , c-regularize function, t-iteration counter, mt-maximum iteration number

$$\min_{x\in\mathbf{R}^{N\times N}} \tfrac{1}{2}\left\|y - \Phi x\right\|_2^2 + \tau c = \min_{x\in\mathbf{R}^{N\times N}} \tfrac{1}{2}\left\|y - \Phi\Psi^{-1}\alpha\right\|_2^2 + \tau c$$

1) Initialization: Set $x = x_0, t = 0, mt = 100$, Ψ =shearlets transform;

2) Repeat;

3) Compute objective function value $f(x^t)$;

4) Repeat;

5) Calculate the denoising factor $\Gamma(\cdot)$ and compute α^t and σ^j through Shearlets transform and Bayesian estimation;

6) $x^{t+1} = \Gamma\left(x^t - \frac{1}{\alpha^t}\Phi^{\mathrm{T}}\left(\Phi\Psi^{-1}\alpha^t - y\right), \sigma\right)$

7) Until x^{t+1} satisfies the acceptance criterion;

8) $t \leftarrow t + 1$

9) Until stop criterion is satisfied;

5 Experiments and analysis

In the experiment, we choose two field SAR image of 256 × 256 for testing (Fig. 3 (a) and Fig. 4 (a)). And we select six other denoising methods for comparison, which are Lee filtering[1], Gamma MAP filtering (Gamma_map)[13], wavelet , hard-threshold based on curvelet domain (Curvelet)[9], hard-threshold based on contourlet domain (Contourlet)[10] and hard-threshold based on shearlets domain (shearlets)[4]. The despeckling results of the SAR image obtained by different methods are shown in Fig. 3 (b) ∼ (h) and Fig. 4 (b) ∼ (h).

In order to demonstrate the superiority of the method proposed in this paper (Shearlets_CS), we evaluate the despeckling effect of each algorithm by the following three aspects:

1) Evaluate the ability of keeping the mean of image by computing the mean of the SAR image. The mean value reflects the average brightness of a SAR image.

2) Evaluate the deviation degree of the despeckling image by computing standard deviation of the SAR image. As for the standard deviation (Std), a lower Std gives a cleaner image.

3) The speckle noise removal ability by equivalent number of looks (ENL).

ENL is a parameter of SAR image, which is used to evaluate the preservation of radiation characteristics and the performance of speckle reduction. ENL is widely used to measure smoothing effects of the despeckling methods.

The definition of ENL is:

$$ENL = \frac{E(X)^2}{V(X)} \tag{32}$$

where X stands for original SAR image, $E(X)$ and $V(Y)$ are the mean and standard deviation of the despeckling image, respectively

Table 1 and Table 2 show quantity comparison of seven methods. Fig. 3 and Fig. 4 show original SAR image and despeckled results of seven methods.

Table 1 Quantity comparison of several despeckling methods (labeled area in Fig. 3)

Approach	Mean	Std	Runtime	ENL		
				Reg1	Reg2	Reg3
Original	107.09	53.88		10.67	11.42	6.08
Lee	106.85	46.17	5.40	23.19	35.03	8.63
Gamma_map	105.85	40.19	5.20	39.10	49.46	10.04
Wavelet	107.05	48.72	0.08	41.03	45.92	7.26
Curvelet	102.44	42.10	0.73	50.57	79.58	8.16
Contourlet	101.60	39.35	0.58	58.51	91.87	7.94
Shearlets	102.06	40.66	1.84	65.89	111.48	9.59
Shearlets_CS	107.09	36.08	0.87	110.59	100.78	16.35

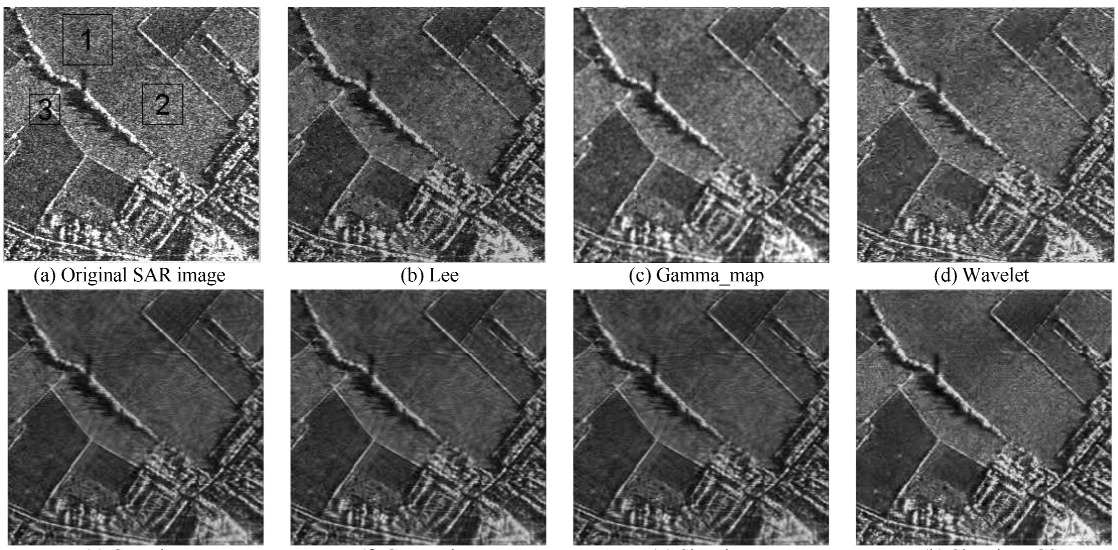
As shown in Table 1, Lee, Gamma_map, wavelet and Shearlets_CS have a good capability in keeping the mean value of images. But Shearlets_CS is the best in mean preserving. As for the standard deviation of image, all methods show an apparent decrease, and Shearlets_CS method performs best. The runtime of wavelet is the shortest, but its performance is poor. Shearlet_CS has the best efficiency through a comprehensive consideration.

In Table 1, we compare the performance of the seven methods. Reg. 1 $\sim$ 3 correspond to the areas indicated by the three rectangles in Fig. 3 (a). The ENL value of the Shearlet_CS method is larger than any other method. The results show the effectiveness of the proposed method. Similarly, Table 2 shows the results of another example.

Table 2 Quantity comparison of several despeckling methods (labeled area in the Fig. 4)

Approach	Mean	Std	Runtime	ENL	
				Reg1	Reg2
Original	144.65	52.86		18.05	16.62
Lee	144.48	43.56	5.30	51.68	43.41
Gamma_map	143.80	39.54	5.16	71.28	48.80
Wavelet	144.65	46.12	0.07	35.79	35.66
Curvelet	140.55	40.40	0.75	108.58	55.78
Contourlet	139.94	38.28	0.58	143.18	56.31
Shearlets	140.33	39.45	1.81	130.08	63.88
Shearlets_CS	144.65	34.85	0.83	151.77	143.99

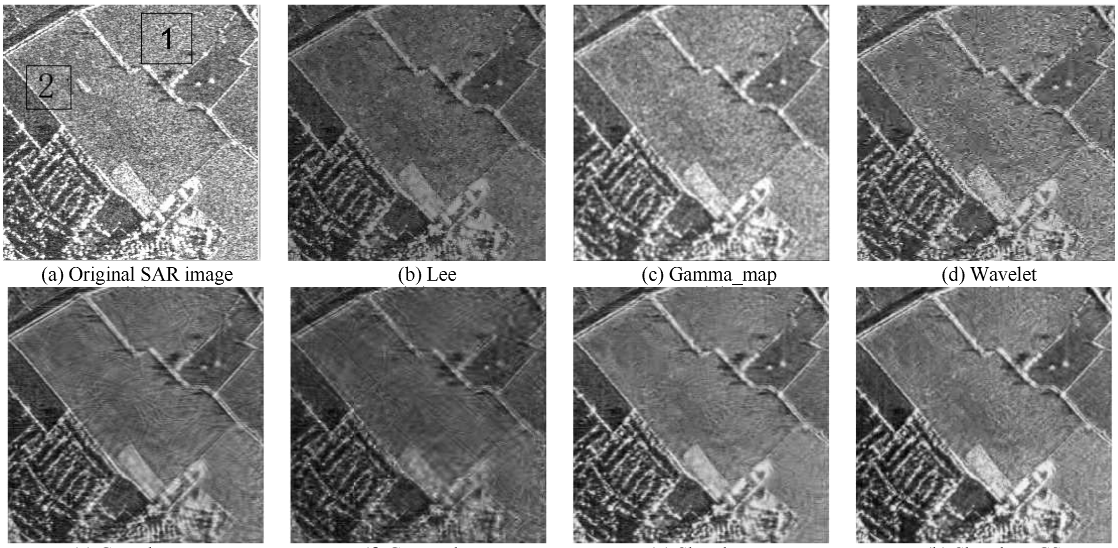
Fig. 3 (b) and Fig. 3 (c) demonstrate that the Lee and the Gamma MAP filter smooth speckle noise to some extent, but edges are blurred and plenty of details are lost. Fig. 3 (d) has a short runtime, but edges and contour do not preserve well. Fig. 3 (e) is the despeckled result of curvelet hard threshold, where the scratch obviously exist in uniform regions, but keep the image′s edges and detailed information well. Fig. 3 (f) is the despeckled image of contourlet hard-threshold, where we still observe scratches in homogenous areas and edge distortion to some degree. Shearlets hard-threshold method performs well in homogenous areas, but it oversmoothes the SAR image and causes fuzzy distortion, which is illustrated in Fig. 3 (g). As illustrated in Fig. 4 (h), the proposed method effectively smoothes the homogenous area, while retaining the detail of edges. Both the image quality and visual effect are superior to the comparative methods.



(a) Original SAR image (b) Lee (c) Gamma_map (d) Wavelet

(e) Curvelet (f) Contourlet (g) Shearlets (h) Shearlets_CS

Fig. 3 Despeckling results of seven methods



(a) Original SAR image (b) Lee (c) Gamma_map (d) Wavelet

(e) Curvelet (f) Contourlet (g) Shearlets (h) Shearlets_CS

Fig. 4 Despeckling results of seven methods

6 Conclusion

This paper introduces a new synthetic aperture radar (SAR) image denoising method using CS model based on Shearlets domain. Seven methods for speckle reduction and enhancement of SAR images have been applied. All the methods significantly reduce the speckle, while the Shearlets_CS methods show the best result in preserving the resolution and the structure of original SAR images. In the end, the visual effect image and detailed measurements show that the Shearlets_CS method based on CS model, is a more effective method, which is not only better in reducing speckle, but also has an advantage in preserving information of target edge

References

1 Lee J S. Digital image enhancement and noise filtering by use of local statistics. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1980, **PAMI-2**(2): 165−168

2 Kuan D T, Sawchuk A A, Strand T C, Chavel P. Adaptive restoration of images with speckle. *IEEE Transactions on Acoustics, Speech and Signal Process*, 1987, **35**(3): 373−383

3 Frost V S, Stiles J A, Shanmugan K S, Holtzman J C. A model for radar images and its application to adaptive digital filtering of multiplicative noise. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1982, **PAMI-4**(2): 157−166

4 Easley G, Labate D, Lim W Q. Sparse directional image representations using the discrete shearlet transform. *Applied and Computational Harmonic Analysis*, 2008, **25**(1): 25−46

5 Jiao X, Wen X B. SAR image segmentation based on Markov random field model and multiscale technology. In: Proceedings of the 6th International Conference on Fuzzy Systems and Knowledge Discovery, 2009. Tianjin, China: IEEE, 2009. 442−446

6 Hou B, Guan H, Jiang J G, Liu K, Jiao L C. SAR image despeckling based on improved directionlet domain Gaussian mixture model. In: Proceedings of the IEEE International Geoscience and Remote Sensing Symposium (IGARSS). Vancouver, BC: IEEE, 2011. 3795−3798

7 Baraniuk R G. Compressive sensing. *IEEE Signal Processing Magazine*, 2007, **24**(4): 118−121

8 Daubechies I, Defrise M, Mol C D. An iterative thresholding algorithm for linear inverse problems with a sparsity constraint. *Communications on Pure and Applied Mathematics*, 2004, **57**(11): 1413−1457

9 Candés E, Demanet L, Donoho D, Ying L X. Fast discrete curvelet transforms. *SIAM Journal on Multiscale Modeling and Simulation*, 2005, **5**(3): 861−899

10 Do M N, Vetterli M. The contourlet transform: an efficient directional multiresolution image representation. *IEEE Transaction on Image Processing*, 2005, **14**(12): 2091−2106

11 Wright S J, Nowak R D, Figueiredo M A T. Sparse reconstruction by separable approximation. *IEEE Transactions on Signal Processing*, 2009, **57**(7): 2479−2493

12 Achim A, Tsakalides P, Bezerianos A. SAR image denoising via Bayesian wavelet shrinkage based on heavy tailed modelling. *IEEE Transactions on Geoscience and Remote Sensing*, 2003, **41**(8): 1773−1784

13 Baraldi A, Parmigiani F. A refined Gamma MAP SAR speckle filter with improved geometrical adaptivity. *IEEE Transactions on Geoscience and Remote Sensing*, 1995, **33**(5): 1245−1257

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