The Design of Reduced-order Observer for Systems with Monotone Nonlinearities

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Abstract Based on the discussion about the existence and design method of full-order observer for systems with monotone nonlinearities, a reduced-order observer design method is developed under the assumption that a linear matrix inequality (LMI) has positive definite matrix solution and the reduced-order observer gain matrix is computed by the solution of LMI. By a linear transformation, a reduced-order observer which does not contain the information of the derivative of the system output is provided. A model is simulated and some conclusions are drawn based on the comparison of the results of reduced-order observer to that of full-order observer. The simulation shows that the design method developed by this paper has good performance.

Key words Linear matrix inequality (LMI), monotone nonlinearities, full-order observer, reduced-order observer

1 Introduction

The design of observer for nonlinear systems has been a very active field and has received more and more attention in the literature during the last four decades. In practice, the state variables of control systems can rarely be measured online directly, so there is a substantial need for a reliable state estimation. For this particular task, a state observer is usually used. Generally speaking, there are several major design methods of state observer for nonlinear systems: extended Kalman filter and extended Luenberger observer[1∼3] as nonlinear observers; nonlinear state transformation method[4∼6] and Lyapunov-like method[7∼12]. Based on state transformation method, the original systems are changed into linear systems or nonlinear canonical forms and then the linear methods are used to complete the observer design procedure. For instance, [13∼14] attempt to find a state transformation to bring the original systems into a canonical form. The Lyapunov-like method introduces the Lyapunov′s stability theory into the observer design and the basic ideal of this approach is to find a Lyapunov function which can guarantee that the error dynamic system has a stable equilibrium of zero. Recently, some intelligent control technics are introduced into nonlinear observers and this leads to some new nonlinear observer design methods[15].

Based on the assumption that a linear matrix inequality (LMI) has positive definite solution, this paper discusses the reduced-order observer design method for a class of nonlinear systems. The present paper is organized as follows. Section 2 summarizes the main result about full-order observer provided by [12]. Under the same assumption, a reduced-order observer design method is developed in Section 3. In Section 4, we apply the reduced-order observer to a system to illustrate its usefulness. Some conclusions are drawn in Section 5.

2 The design of full-order observer

Consider nonlinear system described by

$$\begin{cases} \dot{\boldsymbol{x}} = A\boldsymbol{x} + G\boldsymbol{\gamma}(H\boldsymbol{x}) + \boldsymbol{\rho}(\boldsymbol{y}, \boldsymbol{u}) \\ \boldsymbol{y} = C\boldsymbol{x} \end{cases} \tag{1}$$

where $\boldsymbol{x} \in \mathbf{R}^n$, $\boldsymbol{u} \in \mathbf{R}^m$, and $\boldsymbol{y} \in \mathbf{R}^p$ are state, control input, and output, respectively. $\boldsymbol{\gamma}(\cdot) : \mathbf{R}^r \rightarrow \mathbf{R}^r$ is a nonlinear term. $A \in \mathbf{R}^{n\times n}$, $G \in \mathbf{R}^{n\times r}$, $H \in \mathbf{R}^{r\times n}$, and $C \in \mathbf{R}^{p\times n}$ are all known constant matrices.

Assumption 1. $\boldsymbol{\gamma}(\cdot) : \mathbf{R}^r \rightarrow \mathbf{R}^r$ satisfies

$$\frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{v}} + \left(\frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{v}}\right)^{\mathrm{T}} \geq 0 \quad \forall\boldsymbol{v} \in \mathbf{R}^p \tag{2}$$

that is $\frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{v}} + \left(\frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{v}}\right)^{\mathrm{T}}$ $\forall\boldsymbol{v} \in \mathbf{R}^p$ is a semi-positive definite matrix.

Just as single variable functions, the multivariable function $\boldsymbol{\gamma}$ is said to be monotone if it satisfies condition (2). Thus, system (1) is a system with monotone nonlinearities.

Assumption 2. There exists a constant $\zeta > 0$ and matrices $K \in \mathbf{R}^{r\times p}$ and $L \in \mathbf{R}^{n\times p}$ such that the LMI

$$\begin{pmatrix} (A+LC)^{\mathrm{T}}P + P(A+LC) + \zeta I & PG + (H+KC)^{\mathrm{T}} \\ G^{\mathrm{T}}P + (H+KC) & 0 \end{pmatrix} \leq 0 \tag{3}$$

has positive definite matrix solution P.

Reference [12] has discussed a kind of full-order observer design method and the full-order observer takes the form of

$$\dot{\hat{\boldsymbol{x}}} = A\hat{\boldsymbol{x}} + L(C\hat{\boldsymbol{x}} - \boldsymbol{y}) + G\boldsymbol{\gamma}(H\hat{\boldsymbol{x}} + K(C\hat{\boldsymbol{x}} - \boldsymbol{y})) + \rho(\boldsymbol{y}, \boldsymbol{u}) \tag{4}$$

The main purpose of the full-order observer design method is to find matrices $K \in \mathbf{R}^{r\times p}$ and $L \in \mathbf{R}^{n\times p}$ such that the observer error $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \hat{\boldsymbol{x}}$ approaches to zero as time tends to infinite. By (1) and (4), we know that the error dynamic system of the full-order observer is

$$\dot{\tilde{\boldsymbol{x}}} = (A+LC)\tilde{\boldsymbol{x}} + \mathrm{G}[\boldsymbol{\gamma}(\boldsymbol{v}) - \boldsymbol{\gamma}(\boldsymbol{w})]$$

where $\boldsymbol{v} = H\boldsymbol{x}$ and $\boldsymbol{w} = H\hat{\boldsymbol{x}} + K(C\hat{\boldsymbol{x}} - \boldsymbol{y})$. If $\boldsymbol{\gamma}(\boldsymbol{v}) - \boldsymbol{\gamma}(\boldsymbol{w})$ is regarded as a function of $\boldsymbol{v}$ and $\boldsymbol{\xi} = \boldsymbol{v} - \boldsymbol{w} = (H+KC)\tilde{\boldsymbol{x}}$, and if we denote $\boldsymbol{\phi}(\boldsymbol{v}, \boldsymbol{\xi}) = \boldsymbol{\gamma}(\boldsymbol{v}) - \boldsymbol{\gamma}(\boldsymbol{w})$, $\phi(\boldsymbol{v}, \boldsymbol{\xi})$ will satisfy multivariable sector property

$$\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{v}, \boldsymbol{\xi}) \geq 0 \quad \forall\boldsymbol{\xi} \in \mathbf{R}^r \tag{5}$$

In fact, since

$$\boldsymbol{\phi}(\boldsymbol{v}, \boldsymbol{\xi}) = \boldsymbol{\gamma}(\boldsymbol{v}) - \boldsymbol{\gamma}(\boldsymbol{w}) = \int_{\boldsymbol{w}}^{\boldsymbol{v}} \frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{s}}\mathrm{d}s = \int_0^1 \left[\frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{s}}\right]_{\boldsymbol{s}=\boldsymbol{v}+\lambda(\boldsymbol{w}-\boldsymbol{v})} (\boldsymbol{v}-\boldsymbol{w})\mathrm{d}\lambda = \int_0^1 \left[\frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{s}}\right]_{\boldsymbol{s}=\boldsymbol{v}-\lambda\boldsymbol{\xi}} \boldsymbol{\xi}\mathrm{d}\lambda$$

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based on condition (2) in Assumption 1, we have

$$\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{v},\boldsymbol{\xi}) = \frac{1}{2}\boldsymbol{\xi}^{\mathrm{T}}\int_0^1 \left(\left[\frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{s}}\right] + \left[\frac{\partial\boldsymbol{\gamma}}{\partial\boldsymbol{s}}\right]^{\mathrm{T}}\right)_{s=\boldsymbol{v}-\lambda\boldsymbol{\xi}} \mathrm{d}\lambda \quad \boldsymbol{\xi} \geq 0$$

Now by summarizing the result in [12], we present a theorem about full-order observer as follows:

Theorem 1[12]**.** Under the Assumptions 1 and 2, the system (4) is a full-order observer of system (1).

3 The design of reduced-order observer

Just like linear systems, the observers for nonlinear system can be distinguished as full-order and reduced-order observers. Reduced-order observers only estimate parts of the states which are independent of the outputs of the original system. Hence, they usually have lower dimensions than the full-order ones. This implies that the reduced-order observers can be constructed with fewer integrators, and this will make the whole control system simpler. In this section, we consider the problems of reduced-order observers. Without the loss of generality, we assume that $C = (I_p \quad 0)$. Decomposing A, G, H, and P, the solution of LMI, into block matrices is as follows

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},\ G = \begin{pmatrix} G_1 \\ G_2 \end{pmatrix}$$

$$H = \begin{pmatrix} H_1 & H_2 \end{pmatrix},\ P = \begin{pmatrix} P_1 & P_2 \\ P_2^{\mathrm{T}} & P_3 \end{pmatrix}$$

where $A_{11} \in \mathbf{R}^{p\times p}$, $G_1 \in \mathbf{R}^{p\times r}$, $H_1 \in \mathbf{R}^{r\times p}$, and $P_1 \in \mathbf{R}^{p\times p}$, denote

$$N = P_3^{-1}P_2^{\mathrm{T}} \in \mathbf{R}^{(n-p)\times p} \tag{6}$$

Theorem 2. Under Assumptions 1 and 2, there exists also a reduced-order observer for system (1) with dimensions of $n-p$

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}}_2 = {} & (A_{22} + NA_{12})\hat{\boldsymbol{x}}_2 + (NG_1 + G_2)\boldsymbol{\gamma}(H_1\boldsymbol{y} + H_2\hat{\boldsymbol{x}}_2) + \\ & (NA_{11} + A_{21})\boldsymbol{y} + (N \quad I_{n-p})\rho(\boldsymbol{y},\boldsymbol{u}) - N\dot{\boldsymbol{y}}\end{aligned} \tag{7}$$

where $\hat{\boldsymbol{x}}_2 \in \mathbf{R}^{n-p}$, and N is given by (6) and it serves as the reduced-order observer gain matrix.

Proof. Denote $\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1^{\mathrm{T}} & \boldsymbol{x}_2^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$, where $\boldsymbol{x}_1 \in \mathbf{R}^p$ and $\boldsymbol{x}_2 \in \mathbf{R}^{n-p}$. The original system can be written as two parts:

$$\begin{cases} \dot{\boldsymbol{x}}_1 = (I_p\ 0)[A\begin{pmatrix} \boldsymbol{y} \\ \boldsymbol{x}_2 \end{pmatrix} + G\boldsymbol{\gamma}(H_1\boldsymbol{y} + H_2\boldsymbol{x}_2) + \boldsymbol{\rho}(\boldsymbol{y},\boldsymbol{u})] \\ \dot{\boldsymbol{x}}_2 = (0\ I_{n-p})[A\begin{pmatrix} \boldsymbol{y} \\ \boldsymbol{x}_2 \end{pmatrix} + G\boldsymbol{\gamma}(H_1\boldsymbol{y} + H_2\boldsymbol{x}_2) + \boldsymbol{\rho}(\boldsymbol{y},\boldsymbol{u})] \end{cases}$$

The second equation of above equations is equivalent to

$$\begin{aligned}\dot{\boldsymbol{x}}_2 = {} & (N\ I_{n-p})[A\begin{pmatrix} \boldsymbol{y} \\ \boldsymbol{x}_2 \end{pmatrix} + G\boldsymbol{\gamma}(H_1\boldsymbol{y} + H_2\boldsymbol{x}_2) + \boldsymbol{\rho}(\boldsymbol{y},\boldsymbol{u})] - \\ & N\dot{\boldsymbol{x}}_1 = (A_{22} + NA_{12})\boldsymbol{x}_2 + (NG_1 + G_2)\boldsymbol{\gamma}(H_1\boldsymbol{y} + H_2\boldsymbol{x}_2) + \\ & (NA_{11} + A_{21})\boldsymbol{y} + (N\ I_{n-p})\boldsymbol{\rho}(\boldsymbol{y},\boldsymbol{u}) - N\dot{\boldsymbol{y}}\end{aligned} \tag{8}$$

The reduced-order observer error dynamic system can be obtained by (7)~(8)

$$\dot{\tilde{\boldsymbol{x}}}_2 = (A_{22} + NA_{12})\tilde{\boldsymbol{x}}_2 + (NG_1 + G_2)[\boldsymbol{\gamma}(\boldsymbol{v}) - \boldsymbol{\gamma}(\boldsymbol{w})] \tag{9}$$

where $\tilde{\boldsymbol{x}}_2 = \boldsymbol{x}_2 - \hat{\boldsymbol{x}}_2$, $\boldsymbol{v} = H_1\boldsymbol{y} + H_2\boldsymbol{x}_2$, and $\boldsymbol{w} = H_1\boldsymbol{y} + H_2\hat{\boldsymbol{x}}_2$. If we regard $\boldsymbol{\gamma}(\boldsymbol{v}) - \boldsymbol{\gamma}(\boldsymbol{w})$ as a function of $\boldsymbol{v}$ and $\bar{\boldsymbol{\xi}} = \boldsymbol{v} - \boldsymbol{w} = H_2\tilde{\boldsymbol{x}}_2$, *i.e.*, $\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) = \boldsymbol{\gamma}(\boldsymbol{v}) - \boldsymbol{\gamma}(\boldsymbol{w})$, the error dynamic system (9) can be rewritten as

$$\dot{\tilde{\boldsymbol{x}}}_2 = (A_{22} + NA_{12})\tilde{\boldsymbol{x}}_2 + (NG_1 + G_2)\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) \tag{10}$$

Consider Lyapunov function candidate $\bar{V} = \tilde{\boldsymbol{x}}_2^{\mathrm{T}}P_3\tilde{\boldsymbol{x}}_2$ and the derivative of it along with the trajectories of (10) is

$$\begin{aligned}\dot{\bar{V}} = {} & \tilde{\boldsymbol{x}}_2^{\mathrm{T}}[(A_{22} + NA_{12})^{\mathrm{T}}P_3 + P_3(A_{22} + NA_{12})]\tilde{\boldsymbol{x}}_2 + \\ & 2\tilde{\boldsymbol{x}}_2^{\mathrm{T}}P_3(NG_1 + G_2)\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}})\end{aligned} \tag{11}$$

From LMI (3), we know that

$$(\mathbf{0}^{\mathrm{T}}\ \tilde{\boldsymbol{x}}_2^{\mathrm{T}}\ \boldsymbol{\phi}^{\mathrm{T}}(\boldsymbol{v},\bar{\boldsymbol{\xi}}))$$

$$\begin{pmatrix} (A+LC)^{\mathrm{T}}P + P(A+LC) + \zeta I & PG + (H+KC)^{\mathrm{T}} \\ G^{\mathrm{T}}P + (H+KC) & 0 \end{pmatrix}\cdot$$

$$\begin{pmatrix} \mathbf{0} \\ \tilde{\boldsymbol{x}}_2 \\ \boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) \end{pmatrix} \leq 0$$

Here, $\mathbf{0} \in \mathbf{R}^p$ is used to denote a zero vector. Extending above inequality, we have

$$\begin{aligned} & \begin{pmatrix} \mathbf{0}^{\mathrm{T}} & \tilde{\boldsymbol{x}}_2^{\mathrm{T}} \end{pmatrix}((A+LC)^{\mathrm{T}}P + P(A+LC))\begin{pmatrix} \mathbf{0} \\ \tilde{\boldsymbol{x}}_2 \end{pmatrix} + \\ & 2\begin{pmatrix} \mathbf{0}^{\mathrm{T}} & \tilde{\boldsymbol{x}}_2^{\mathrm{T}} \end{pmatrix} PG\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) \leq -\zeta\tilde{\boldsymbol{x}}_2^{\mathrm{T}}\tilde{\boldsymbol{x}}_2 - \\ & 2\begin{pmatrix} \mathbf{0}^{\mathrm{T}} & \boldsymbol{x}_2^{\mathrm{T}} \end{pmatrix}(H+KC)^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}})\end{aligned} \tag{12}$$

From (5)

$$\begin{aligned} & 2\begin{pmatrix} \mathbf{0}^{\mathrm{T}} & \boldsymbol{x}_2^{\mathrm{T}} \end{pmatrix}(H+KC)^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) = 2\,(H_2\tilde{\boldsymbol{x}}_2)^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) = \\ & 2\bar{\boldsymbol{\xi}}^{\mathrm{T}}\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) \geq 0\end{aligned}$$

Thus by (12), we obtain

$$\begin{aligned} & \begin{pmatrix} \mathbf{0}^{\mathrm{T}} & \tilde{\boldsymbol{x}}_2^{\mathrm{T}} \end{pmatrix}((A+LC)^{\mathrm{T}}P + P(A+LC))\begin{pmatrix} \mathbf{0} \\ \tilde{\boldsymbol{x}}_2 \end{pmatrix} + \\ & 2\begin{pmatrix} \mathbf{0}^{\mathrm{T}} & \tilde{\boldsymbol{x}}_2^{\mathrm{T}} \end{pmatrix} PG\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) \leq -\zeta\tilde{\boldsymbol{x}}_2^{\mathrm{T}}\tilde{\boldsymbol{x}}_2 < 0\end{aligned} \tag{13}$$

Note that $C = (I_p \quad 0)$,

$$(A+LC)^{\mathrm{T}}P + P(A+LC) = \begin{pmatrix} * & * \\ * & (A_{22} + NA_{12})^{\mathrm{T}}P_3 + P_3(A_{22} + NA_{12}) \end{pmatrix}$$

Inserting above equation into (13) and then extending by block matrices leads to

$$\begin{aligned} & \tilde{\boldsymbol{x}}_2^{\mathrm{T}}[(A_{22} + NA_{12})^{\mathrm{T}}P_3 + P_3(A_{22} + NA_{12})]\tilde{\boldsymbol{x}}_2 + \\ & 2\tilde{\boldsymbol{x}}_2^{\mathrm{T}}P_3(NG_1 + G_2)\boldsymbol{\phi}(\boldsymbol{v},\bar{\boldsymbol{\xi}}) \leq -\zeta\tilde{\boldsymbol{x}}_2^{\mathrm{T}}\tilde{\boldsymbol{x}}_2 < 0\end{aligned} \tag{14}$$

(11) together with inequality (14) means

$$\dot{V} \leq -\zeta \tilde{\boldsymbol{x}}_2^{\mathrm{T}} \tilde{\boldsymbol{x}}_2 < 0$$

and this means that system (7) is a reduced-order observer of system (1). □

It should be noted that the derivative of output $\dot{\boldsymbol{y}}$ appearing in the reduced-order observer is not the case we expect because it will enhance the noise of high frequency of output $\boldsymbol{y}$. In order to cancel $\dot{\boldsymbol{y}}$, we make a transformation of $\hat{\boldsymbol{z}}_2 = \hat{\boldsymbol{x}}_2 + N\boldsymbol{y}$, then (7) is transformed into

$$\left\{\begin{aligned} \dot{\hat{\boldsymbol{z}}}_2 = {} & (A_{22} + NA_{12})\hat{\boldsymbol{z}}_2 + (NG_1 + G_2)\boldsymbol{\gamma}(H_2\hat{\boldsymbol{z}}_2 + \\ & (H_1 - H_2N)\boldsymbol{y}) + (NA_{11} + A_{21} - (A_{22} + \\ & NA_{12})N)\boldsymbol{y} + (N \ I_{n-p})\,\boldsymbol{\rho}\,(\boldsymbol{y}, \boldsymbol{u}) \\ \hat{\boldsymbol{x}}_2 = {} & \hat{\boldsymbol{z}}_2 - N\boldsymbol{y} \end{aligned}\right. \tag{15}$$

By this way, the $\dot{\boldsymbol{y}}$ is eliminated from the reduced-order observer. We should also point out that the assumption of C being a special form of $(I_p \quad 0)$ is not crucial for getting the result of Theorem 2. That is, we can obtain the similar conclusion to that of Theorem 2 for the general form of C.

4 Numerical simulation

Consider the system

$$\left\{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_2 - \frac{1}{3}x_2^3 - x_2x_3^2 \\ \dot{x}_3 &= x_2 - x_3 - \frac{1}{3}x_3^3 - x_3x_2^2 \\ y &= x_1 \end{aligned}\right.$$

The system can be written as in the form (1) with

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}, \ \boldsymbol{C} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$G = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}, \ H = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\boldsymbol{\gamma}(\boldsymbol{v}) = \begin{pmatrix} \frac{1}{3}x_2^3 + x_2x_3^2 \\ x_2^2x_3 + \frac{1}{3}x_3^3 \end{pmatrix}$$

where $\boldsymbol{v} = (x_2 \ x_3)^{\mathrm{T}}$. It is easy for us to vary that $\boldsymbol{\gamma}(\boldsymbol{v})$ satisfies monotone property. If we choose $\boldsymbol{L} = \begin{pmatrix} -3 & -8 & -4 \end{pmatrix}^{\mathrm{T}}$, $\boldsymbol{K} = \begin{pmatrix} -2 & -1 \end{pmatrix}^{\mathrm{T}}$, and $\zeta = 0.7$, the LMI has positive definite matrix solution, *i.e.*,

$$P = \begin{pmatrix} 10 & -2 & -1 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

For this example, the full-order observer given by the paper [12] (*i.e.*, Theorem 1 of present paper) is

$$\left\{\begin{aligned} \dot{\hat{x}}_1 = {} & \hat{x}_2 - 3(\hat{x}_1 - y) \\ \dot{\hat{x}}_2 = {} & \hat{x}_2 - 8(\hat{x}_1 - y) - \frac{1}{3}(\hat{x}_2 - 2(\hat{x}_1 - y))^3 - \\ & (\hat{x}_2 - 2(\hat{x}_1 - y))(\hat{x}_3 - (\hat{x}_1 - y))^2 \\ \dot{\hat{x}}_3 = {} & \hat{x}_2 - \hat{x}_3 - 4(\hat{x}_1 - y) - \frac{1}{3}(\hat{x}_3 - (\hat{x}_1 - y))^3 - \\ & (\hat{x}_3 - (\hat{x}_1 - y))(\hat{x}_2 - 2(\hat{x}_1 - y))^2 \end{aligned}\right.$$

The reduced-order observer gain matrix is computed by (6) and it is $\boldsymbol{N} = (-2 \ -1)^{\mathrm{T}}$. Based on (15), the reduced-order observer is

$$\left\{\begin{aligned} \dot{\hat{z}}_2 &= -\hat{z}_2 - \frac{1}{3}(\hat{z}_2 + 2y)^3 - (\hat{z}_2 + 2y)(\hat{z}_3 + y)^2 - 2y \\ \dot{\hat{z}}_3 &= -\hat{z}_3 - (\hat{z}_2 + 2y)^2(\hat{z}_3 + y) - \frac{1}{3}(\hat{z}_3 + y)^3 - y \\ \hat{x}_2 &= \hat{z}_2 + 2y \\ \hat{x}_3 &= \hat{z}_3 + y \end{aligned}\right.$$

The simulation effectiveness are shown in Figs. 1~2, where the initial states of original system and observers are set as $\boldsymbol{x}(0) = (1 \ 2 \ 3)^{\mathrm{T}}$ and $\hat{\boldsymbol{x}}(0) = (4 \ 5 \ 6)^{\mathrm{T}}$, respectively. From the figures we know that the state estimation effect of reduced-order observer is better than that of full-order observer.

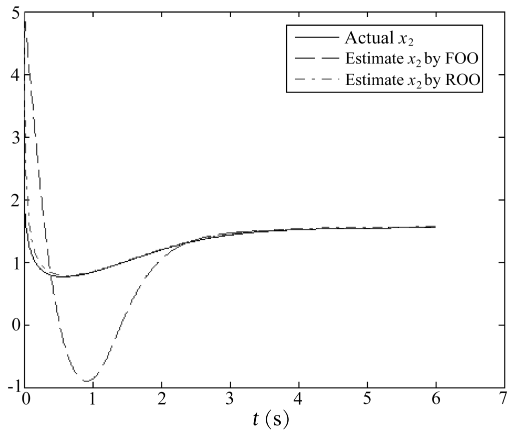

Fig. 1 The estimations of x_2

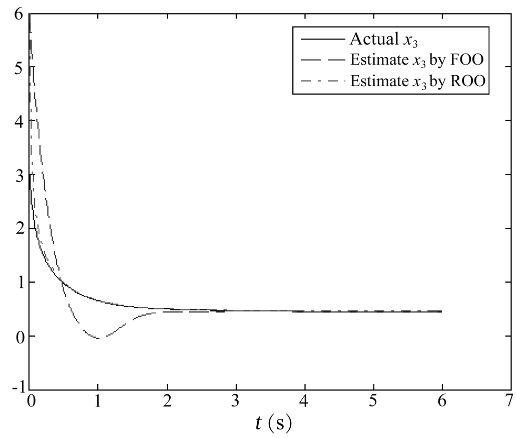

Fig. 2 The estimations of x_3

5 Conclusion

In this paper, the observer design methods for system with monotone nonlinearities are discussed. First, a full-order observer is offered by summarizing other people′s work. Second, under the same assumptions, a reduced-order observer design method is developed and the main result is given by Theorem 2. The reduced-order observer is based on the solution of a LMI and the gain matrix is computed by it. Finally, we make numerical simulation based not only on full-order but also reduced-order observers for a system. We show the superiority of reduced-order observer by comparing the simulation results.

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