Adaptive Observer for a Class of Nonlinear Systems

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Abstract This paper investigates the means to design the observer for a class of nonlinear systems with Lipschitz conditions and unknown parameters. A new design approach of full-order state adaptive observer is proposed. The constructed observer could guarantee the error of state and the error of parameter estimation to asymptotically converge to zero. Furthermore, a numerical example is provided to verify the effectiveness of the observer.

Key words Adaptive observer, nonlinear systems, state and parameters estimation

1 Introduction

The means to design adaptive observers through estimation of states and parameters in linear and nonlinear systems has been actively studied in recent years[1~10]. Some early work on adaptive observers for linear systems can be found in [1, 2], and the design of an adaptive observer for a linear time invariant system has been well analyzed in [3]. In addition, adaptive observers of nonlinear systems have attracted much attention due to their wide uses in theory and practice. Some results on design of adaptive observers for nonlinear systems have been reported in [4~6]. However, the nonlinear systems in the above mentioned works must satisfy the exact linearizable condition. The means to design robust adaptive observers for a class of nonlinear systems was presented in [7], in which some sufficient conditions for state estimate to converge asymptotically were given. A systematic algorithm for adaptive observer synthesis for nonlinear systems was presented in [8], and a numerically efficient interior point method was used to solve an inequality obtaining observer gains. A novel recursive design scheme of state observer for a class of nonlinear systems was proposed in [9]. Using nonlinear state transformation, a nonlinear canonical form observer design approach for general multi-input and multi-output systems was developed in [11]. The nonlinear observer design technique by dynamic observer error linearization was presented in [12]. But, in all these papers[9,11,12], unknown parameters could not be involved.

To overcome such limitations in this paper, the means to design the adaptive observers design for a class of nonlinear systems with unknown parameters is investigated that satisfies Lipschitz condition but cannot be linearized exactly. A new approach of adaptive observe design is proposed, and some new sufficient conditions for estimate error to converge to zero asymptotically are presented. Finally, a numerical example is given to show the validity of the proposed method.

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2 Problem statement

Consider the nonlinear system

$$\begin{aligned} \dot{\boldsymbol{x}} &= A_0\boldsymbol{x} + f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} + \psi(t)\boldsymbol{\theta} \\ y &= \boldsymbol{C}_0\boldsymbol{x} \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$, $\boldsymbol{u}(t) \in \mathbf{R}^l$, and $y(t) \in \mathbf{R}$, are system state, input, and output, respectively.

$$A_0 = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \boldsymbol{C}_0 = [1\ 0\ \cdots 0], \tag{2}$$

$f : \mathbf{R}^n \to \mathbf{R}^n$, $g : \mathbf{R}^n \to \mathbf{R}^n \times \mathbf{R}^l$ are two nonlinear functions in the triangular form

$$\begin{aligned} f(\boldsymbol{x}) &= \begin{bmatrix} f_1(x_1) \\ f_2(x_1, x_2) \\ \vdots \\ f_n(x_1, x_2, \cdots, x_n) \end{bmatrix} \\ g(\boldsymbol{x}) &= \begin{bmatrix} g_1(x_1) \\ g_2(x_1, x_2) \\ \vdots \\ g_n(x_1, x_2, \cdots, x_n) \end{bmatrix} \end{aligned} \tag{3}$$

$\psi(t) \in \mathbf{R}^n \times \mathbf{R}^n$ is a matrix of known signals, and $\boldsymbol{\theta} \in \mathbf{R}^n$ is an unknown constant parameter. It is assumed that $\boldsymbol{u}(t)$ and $\psi(t)$ are both uniformly bounded.

System (1) is very important because its observer design has important roles in theory and in the practical engineering areas such as power system and robot. Especially in theory, system (1) can be transformed into the following more general affine system under some conditions. That is, if

$$\begin{aligned} \dot{\boldsymbol{\xi}} &= F(\boldsymbol{\xi}) + G(\boldsymbol{\xi})\boldsymbol{u},\ \boldsymbol{\xi} \in \mathbf{R}^n,\ \boldsymbol{u} \in \mathbf{R}^l \\ y &= H(\boldsymbol{\xi}),\ y \in \mathbf{R} \end{aligned} \tag{4}$$

is observable, and the transformation

$$\Phi(\boldsymbol{\xi}) = [H(\boldsymbol{\xi})\ L_F H(\boldsymbol{\xi})\ \cdots\ L_F^{n-1} H(\boldsymbol{\xi})]^{\mathrm{T}}$$

and its inverse transformation are global Lipschitz diffeomorphism[10], then by global coordinate transformation, system (4) can be transformed into a new system described in

$$\begin{aligned} \dot{\boldsymbol{x}} &= A_0\boldsymbol{x} + f(\boldsymbol{x}) + g(\boldsymbol{x})\boldsymbol{u} \\ y &= \boldsymbol{C}_0\boldsymbol{x} \end{aligned} \tag{5}$$

where $A_0, \boldsymbol{C}_0, f(\boldsymbol{x})$, and $g(\boldsymbol{x})$have the same form as (2) and (3).

System (1) has one more term $\psi(t)\boldsymbol{\theta}$ than system (3), where $\boldsymbol{\theta}$ is an unknown constant parameter.

In a word, the aim of this work is to design an adaptive observer for system (1) which guarantees that the errors $\hat{\boldsymbol{x}} - \boldsymbol{x}$ tend to zero when $t \to \infty$.

It should be pointed that the premise of this work is that system (1) cannot be linearized exactly by coordinate change and feedback, otherwise the classical method could be applied to solve the design problem of observer.

3 Main result

For any positive real number ρ, define

$$\Lambda = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \rho^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \rho^{-(n-1)} \end{bmatrix} \tag{6}$$

Let S be the solution of the matrix equation

$$A_0^{\mathrm{T}} S + S A_0 + S = \boldsymbol{C}_0^{\mathrm{T}} \boldsymbol{C}_0 \tag{7}$$

and

$$K = \frac{1}{2} S^{-1} \boldsymbol{C}_0^{\mathrm{T}} \tag{8}$$

Remark 1. According to [10], S is a positive definite matrix for any $n \geq 1$.

The following assumptions are required for the design of observer.

Assumption 1. The functions $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are globally Lipschitz, and $\psi(t)$ is a uniformly continuously bounded function.

Assumption 2. The input $\boldsymbol{u}$ stays in a bounded subset U of $\mathbf{R}$, and the unknown parameter vector $\boldsymbol{\theta}$ is bounded.

The design of adaptive observer is stated as follows.

Theorem 1. Consider system (1) and let Assumption 1 and Assumption 2 hold. If there exist matrices $E(t) \in \mathbf{R}^{n \times n_1}$ and $G(t) \in \mathbf{R}^{n_1}$ such that $\psi^{\mathrm{T}}(t) \Lambda S^{\mathrm{T}} = E(t) G(t) \boldsymbol{C}_0$, then for sufficiently large $\rho > 0$, the following conclusions are reached.

1) The adaptive observer

$$\dot{\hat{\boldsymbol{x}}} = A_0 \hat{\boldsymbol{x}} + f(\hat{\boldsymbol{x}}) + g(\hat{\boldsymbol{x}}) \boldsymbol{u} + \psi(t) \hat{\boldsymbol{\theta}}(t) + \rho \Lambda^{-1} K [y(t) - \boldsymbol{C}_0 \hat{\boldsymbol{x}}(t)] \tag{9}$$

$$\dot{\hat{\boldsymbol{\theta}}} = -2EG(\boldsymbol{C}_0 \hat{\boldsymbol{x}} - y) \tag{10}$$

is convergent, $\tilde{\boldsymbol{x}} = \hat{\boldsymbol{x}} - \boldsymbol{x} \to \boldsymbol{0}$, when $t \to \infty$, and $\psi(t)(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \to 0$, when $t \to \infty$;

2) if there exist $\delta > 0,\ T > 0$ such that, for all $t \geq 0$, the following inequality holds:

$$\int_t^{t+T} \psi^{\mathrm{T}}(t) \psi(t) \mathrm{d}t \geq \delta I$$

then the error $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ tends to zero when $t \to \infty$.

Proof. Set coordinate transformation $\boldsymbol{z} = \Lambda \boldsymbol{x}$, and notice that

$$\Lambda A_0 = \rho A_0 \Lambda, \quad \boldsymbol{C}_0 \Lambda = \boldsymbol{C}_0$$

Then, system (1) becomes

$$\begin{aligned} &\dot{\boldsymbol{z}} = \rho A_0 \boldsymbol{z} + \Lambda f(\Lambda^{-1} \boldsymbol{z}) + \Lambda g(\Lambda^{-1} \boldsymbol{z}) \boldsymbol{u} + \Lambda \psi(t) \boldsymbol{\theta} \\ &y = \boldsymbol{C}_0 \boldsymbol{z} \end{aligned} \tag{11}$$

Let $\hat{\boldsymbol{z}} = \Lambda \hat{\boldsymbol{x}}$. Then

$$\dot{\hat{\boldsymbol{z}}} = \rho A_0 \hat{\boldsymbol{z}} + \Lambda f(\Lambda^{-1} \hat{\boldsymbol{z}}) + \Lambda g(\Lambda^{-1} \hat{\boldsymbol{z}}) \boldsymbol{u} + \Lambda \psi(t) \hat{\boldsymbol{\theta}}(t) + \rho K \boldsymbol{C}_0 (\boldsymbol{x} - \hat{\boldsymbol{x}}) \tag{12}$$

Denote $\tilde{\boldsymbol{x}} = \hat{\boldsymbol{x}} - \boldsymbol{x},\ \tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta},\ \tilde{\boldsymbol{z}} = \hat{\boldsymbol{z}} - \boldsymbol{z}$. Then

$$\dot{\tilde{\boldsymbol{z}}} = \rho (A_0 - K \boldsymbol{C}_0) \tilde{\boldsymbol{z}} + \boldsymbol{\eta} + \Lambda \psi(t) \tilde{\boldsymbol{\theta}} \tag{13}$$

where

$$\boldsymbol{\eta} = \Lambda (f(\Lambda^{-1} \hat{\boldsymbol{z}}) - f(\Lambda^{-1} \boldsymbol{z})) + \Lambda (g(\Lambda^{-1} \hat{\boldsymbol{z}}) - g(\Lambda^{-1} \boldsymbol{z})) \boldsymbol{u} \tag{14}$$

Furthermore, the Lyapunov function candidate is defined as

$$V = \tilde{\boldsymbol{z}}^{\mathrm{T}} S \tilde{\boldsymbol{z}} + \frac{1}{2} \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \tilde{\boldsymbol{\theta}}$$

Then, the derivative of V along the trajectories of system (13), (10) is given by

$$\begin{aligned} \dot{V} \ = \ & \rho \tilde{\boldsymbol{z}}^{\mathrm{T}} (A_0^{\mathrm{T}} S + S A_0 - \boldsymbol{C}_0^{\mathrm{T}} \boldsymbol{C}_0) \tilde{\boldsymbol{z}} + 2 \tilde{\boldsymbol{z}}^{\mathrm{T}} S \boldsymbol{\eta} + \tilde{\boldsymbol{\theta}}^{\mathrm{T}} \dot{\tilde{\boldsymbol{\theta}}} + \\ & 2 \tilde{\boldsymbol{z}}^{\mathrm{T}} S \Lambda \psi(t) \tilde{\boldsymbol{\theta}} = \\ & \rho \tilde{\boldsymbol{z}}^{\mathrm{T}} (A_0^{\mathrm{T}} S + S A_0 - \boldsymbol{C}_0^{\mathrm{T}} \boldsymbol{C}_0) \tilde{\boldsymbol{z}} + 2 \tilde{\boldsymbol{z}}^{\mathrm{T}} S \boldsymbol{\eta} + \\ & (\dot{\tilde{\boldsymbol{\theta}}}^{\mathrm{T}} + 2 \tilde{\boldsymbol{z}}^{\mathrm{T}} S \Lambda \psi(t)) \tilde{\boldsymbol{\theta}} \end{aligned}$$

Because

$$\begin{aligned} \dot{\tilde{\boldsymbol{\theta}}} \ = \ & \dot{\hat{\boldsymbol{\theta}}} = -2EG(\boldsymbol{C}_0 \hat{\boldsymbol{x}} - y) = -2EG \boldsymbol{C}_0 (\hat{\boldsymbol{x}} - \boldsymbol{x}) = \\ & -2EG \boldsymbol{C}_0 \Lambda (\hat{\boldsymbol{x}} - \boldsymbol{x}) = -2 \psi^{\mathrm{T}}(t) \Lambda S^{\mathrm{T}} \tilde{\boldsymbol{z}} \end{aligned}$$

we have

$$\dot{V} = -\rho \tilde{\boldsymbol{z}}^{\mathrm{T}} S \tilde{\boldsymbol{z}} + 2 \tilde{\boldsymbol{z}}^{\mathrm{T}} S \boldsymbol{\eta}$$

Because f and g are globally Lipschitz and have the triangular form, we have

$$\|\boldsymbol{\eta}\| \leq \beta(\rho^{-1}) \|\tilde{\boldsymbol{z}}\|$$

where $\beta(\rho^{-1})$is a polynominal in ρ^{-1}depending on the Lipschitz constants of the function f and g and on the bounds of $\boldsymbol{u}$. Then

$$\begin{aligned} \dot{V} \ \leq \ & -\rho \tilde{\boldsymbol{z}}^{\mathrm{T}} S \tilde{\boldsymbol{z}} + \lambda(\rho^{-1}) \tilde{\boldsymbol{z}}^{\mathrm{T}} S \tilde{\boldsymbol{z}} = \\ & -[\rho - \lambda(\rho^{-1})] \tilde{\boldsymbol{z}}^{\mathrm{T}} S \tilde{\boldsymbol{z}} \end{aligned}$$

where $\lambda(\rho^{-1})$ is a polynominal in ρ^{-1}.

For a sufficiently large ρ, there exists $\varepsilon > 0$ such that the following inequality holds:

$$\dot{V} \leq -\varepsilon \tilde{\boldsymbol{z}}^{\mathrm{T}} \tilde{\boldsymbol{z}}.$$

Integrating the above inequality, we obtain

$$V(t) \leq V(0) - \varepsilon \int_0^t \tilde{\boldsymbol{z}}^{\mathrm{T}} \tilde{\boldsymbol{z}} \mathrm{d}t$$

Because $V(t) \in L_\infty$ and $V(0)$ is finite, we have $\tilde{\boldsymbol{z}} \in L_2$. From (13), we obtain $\dot{\tilde{\boldsymbol{z}}} \in L_\infty$. Hence $\tilde{\boldsymbol{z}} \in L_\infty,\ \tilde{\boldsymbol{z}} \in L_2,\ \dot{\tilde{\boldsymbol{z}}} \in L_\infty$. By Barbalat′s lemma [3], $\tilde{\boldsymbol{z}} \to \boldsymbol{0}$, when $t \to \infty$. So when $t \to \infty$, $\tilde{\boldsymbol{x}} \to \boldsymbol{0}$.

Because

$$\int_0^\infty \dot{\tilde{\boldsymbol{z}}} \mathrm{d}t = \lim_{t \to \infty} \tilde{\boldsymbol{z}}(t) - \tilde{\boldsymbol{z}}(0) = -\tilde{\boldsymbol{z}}(0)$$

is bounded, by (13), using the Lipschitz continuity of f and g, $\dot{\tilde{\boldsymbol{z}}}$ is uniformly continuous. According to Barbalat′s lemma [3], we have $\dot{\tilde{\boldsymbol{z}}} \to \boldsymbol{0}$. From (13), we obtain $\psi(t) \tilde{\boldsymbol{\theta}} \to \boldsymbol{0}$. Hence, $\psi^{\mathrm{T}}(t) \psi(t) \tilde{\boldsymbol{\theta}} \to \boldsymbol{0}$.

Define $D(\tau) = \int_t^\tau \psi^{\mathrm{T}}(s) \psi(s) \mathrm{d}s$. Using integration by part, we have

$$\begin{aligned} \textstyle\int_t^{t+T} \psi^{\mathrm{T}}(s) \psi(s) \tilde{\boldsymbol{\theta}}(s) \mathrm{d}s \ = \ & D(t+T) \tilde{\boldsymbol{\theta}}(t+T) - D(t) \boldsymbol{\theta}(t) - \\ & \textstyle\int_t^{t+T} D(s) \dot{\tilde{\boldsymbol{\theta}}}(s) \mathrm{d}s \end{aligned}$$

Noticing $D(t)=0$, we obtain

$$\int_t^{t+T}\psi^{\mathrm{T}}(s)\psi(s)\tilde{\boldsymbol{\theta}}(s)\mathrm{d}s = D(t+T)\tilde{\boldsymbol{\theta}}(t+T)- \int_t^{t+T} D(s)\dot{\tilde{\boldsymbol{\theta}}}(s)\mathrm{d}s \tag{15}$$

Because $\psi^{\mathrm{T}}(t)\psi(t)\tilde{\boldsymbol{\theta}}\to 0$, for any finite T, we have

$$\int_t^{t+T}\psi^{\mathrm{T}}(s)\psi(s)\tilde{\boldsymbol{\theta}}(s)\mathrm{d}s\to 0$$

Moreover, $\psi(\tau)$ is bounded for any finite T and $t\le\tau\le t+T$. Because $t\to\infty$, $\dot{\tilde{\boldsymbol{\theta}}}\to\mathbf{0}$, we obtain

$$\int_t^{t+T} D(s)\dot{\tilde{\boldsymbol{\theta}}}(s)\mathrm{d}s\to\mathbf{0},\ t\to\infty$$

From (15), we obtain $D(t+T)\tilde{\boldsymbol{\theta}}(t+T)\to\mathbf{0}$. By assumption

$$D(t+T)=\int_t^{t+T}\psi^{\mathrm{T}}(s)\psi(s)\mathrm{d}s\ge\delta I$$

for some $\delta>0$; therefore, $\tilde{\boldsymbol{\theta}}(t+T)\to\mathbf{0}$, implying $\tilde{\boldsymbol{\theta}}(t)\to\mathbf{0}$. □

4 A numerical example

Consider the following nonlinear system:

$$\dot{x}_1 = x_2+\varphi_1(t)\theta_1+\tfrac{1}{2}\rho\varphi_1(t)\theta_2$$
$$\dot{x}_2 = \cos x_2+2\rho^{-1}\varphi_2(t)\theta_1+\varphi_2(t)\theta_2$$

where $\varphi_1(t)=\cos 10t+\sin 2t,\ \varphi_2(t)=\sin 3t+\cos 20t$, and $\rho=30$. It is obvious that

$$A_0=\begin{bmatrix}0&1\\0&0\end{bmatrix},\boldsymbol{C}_0=[1\ \ 0]$$

From $A_0^{\mathrm{T}}S+SA_0+S=\boldsymbol{C}_0^{\mathrm{T}}\boldsymbol{C}_0$ it follows that $S=\begin{bmatrix}1&-1\\-1&2\end{bmatrix}$. Hence, we obtain $\boldsymbol{K}=\begin{bmatrix}1\\0.5\end{bmatrix}$. The adaptive observer is given by

$$\dot{\hat{x}}_1=\hat{x}_2+\varphi_1(t)\hat{\theta}_1+\tfrac{1}{2}\rho\varphi_1(t)\hat{\theta}_2+\rho(y-\boldsymbol{C}_0\hat{\boldsymbol{x}})$$
$$\dot{\hat{x}}_2=\cos\hat{x}_2+2\rho^{-1}\varphi_2(t)\hat{\theta}_1+\varphi_2(t)\hat{\theta}_2+\tfrac{\rho^2}{2}(y-\boldsymbol{C}_0\hat{\boldsymbol{x}})$$
$$\dot{\hat{\theta}}_1=\varphi_1(t)(y-\boldsymbol{C}_0\hat{\boldsymbol{x}})$$
$$\dot{\hat{\theta}}_2=2\rho^{-1}\varphi_2(t)(y-\boldsymbol{C}_0\hat{\boldsymbol{x}})$$

Set $\theta_1=1,\ \theta_2=1.5$, and set the initial values at

$$x_1(0)=50,\quad x_2(0)=32,\quad \hat{x}_1(0)=-24$$
$$\hat{x}_2(0)=-20,\quad \hat{\theta}_1(0)=0.8,\quad \hat{\theta}_2(0)=1.4$$

In Figs. 1 and 2, the plotted curves show the state estimation errors. The results show that state estimation errors converge to zero when $t\to 0$.

5 Conclusion

This paper has addressed the design problem of a class of nonlinear systems with unknown parameters, and proposed a novel constructing method of full state adaptive observers. Under certain conditions, the constructed observer enables the state and parameter estimation errors to converge to zero asymptotically. This method will find wide applications because the studied nonlinear system cannot be linearized exactly. In addition, a numerical example has been presented to show the validity of the results.

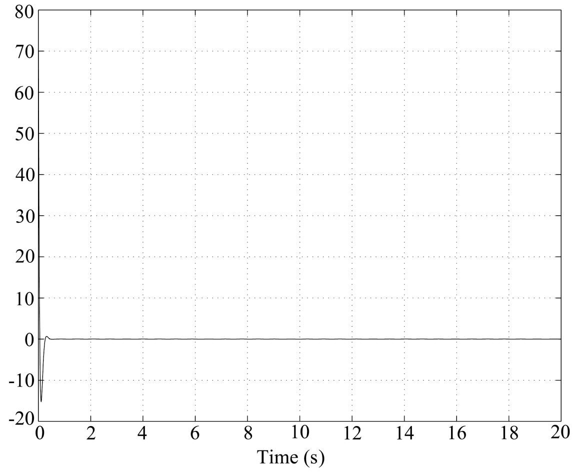

Fig. 1 Error estimation $x_1-\hat{x}_1$

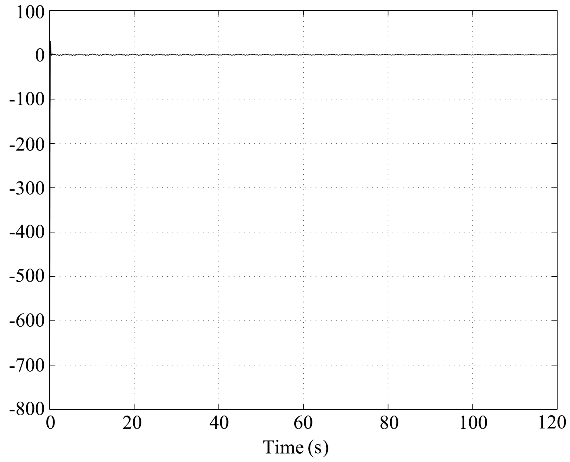

Fig. 2 Error estimation $x_2-\hat{x}_2$

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