Dual-stage Optimal Iterative Learning Control for Nonlinear Non-affine Discrete-time Systems

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Abstract On the basis of a new dynamic linearization technology along the iteration axis, a dual-stage optimal iterative learning control is presented for nonlinear and non-affine discrete-time systems. Dual-stage indicates that two optimal learning stages are designed respectively to improve control input sequence and the learning gain iteratively. The main feature is that the controller design and convergence analysis only depend on the I/O data of the dynamical system. In other words, we can easily select the control parameters without knowing any other knowledge of the system. Simulation study illustrates the geometrical convergence of the presented method along the iteration axis, in which an example of freeway traffic iterative learning control is noteworthy for its intrinsic engineering importance.

Key words Dual-stage, model free adaptive control, iterative learning control, nonlinear and non-affine systems, ramp metering

1 Introduction

The iterative learning control (ILC) was originally presented for robot applications[1]. It involves systems that repetitively perform the same task, aiming to improve their tracking accuracy. Till now, the convergence analysis of ILC has mainly focused on linear systems, and studies on the nonlinear systems[2~5] and for the nonlinear non-affine systems are few[6].

Furthermore, most of the current ILC methods adopt fixed learning gain laws. Only control input sequence is modified, the mapping relationship remains the same during the learning iterations[7]. Clearly, these methods are not effective in a varying control environment. Thus, an adaptive learning control scheme was presented to improve the learning scheme itself as well as the control input sequence[7~9]. However, till now only a limited number of results are available even for linear cases of adaptive ILC.

Another obstacle to the further application of ILC lies in the selection of learning gain. In theory, just based on the I/O data and the desired signal of the controlled system, an iterative learning controller can operate well without any other a *priori* of the dynamic process. However, for the controller design and convergence analysis, some knowledge about the system is required, such as the Jacobin matrix of the controlled systems, though it is not necessary to know their values precisely. If there is not any knowledge of the dynamical system, we most commonly select a proper learning gain case by case. So how to achieve the model-free property of ILC to avoid using any a *priori* of the system becomes a challenging and opening problem.

In recent years, a constructive model-free adaptive controller has been presented for a class of nonlinear and non-affine systems[10,11] based on a new dynamical linearization method and a new concept called pseudo-partial derivative (PPD). The design and analysis are only depended on the input and output measurements without any a *priori* of the system. As a result, it overcomes the limitation of model uncertainties in nature.

In this paper, we explore the possibility of extending the model-free adaptive control[10,11] to ILC tasks coping with nonlinear and non-affine discrete-time systems. By introducing the concept of PPD, a new dynamical linearization method in the iteration domain is developed, which is free of the unmatched dynamical uncertainty. Then, we present a dual-stage optimal ILC. It is model-free, and the design and convergence analysis are directly based on the I/O data. Dual-stage indicates that there are two learning stages: one is the control input learning stage for improving the control input sequence and the other is the parameter-updating stage for estimating the PPD values iteratively. Thus, the learning gain can be tuned iteratively with PPD estimation values. Hence, the learning law itself can be improved iteratively, and it overcomes the difficulty in selecting proper learning gains.

This paper is organized as follows. Section 2 deals with the model transformation, where a new dynamic linearization method is developed. Section 3 copes with the detailed design of the novel dual-stage optimal ILC method. The convergence analysis is provided in Section 4. Section 5 shows the simulation results and Section 6 is some conclusions.

2 Model transformation

To clearly demonstrate the main idea, we consider the following repeatable system

$$y_k(t+1) = f(y_k(t), y_k(t-1), \cdots, y_k(t-n_y), u_k(t), u_k(t-1), \cdots, u_k(t-n_u)) \tag{1}$$

where $y_k(t)$ and $u_k(t)$ are the output and input at time t of the kth iteration, respectively. $t \in \{0, 1, \cdots, T\}$ and $k = 0, 1, 2, \cdots$. n_y and n_u are unknown orders, and $f(\cdots)$ is an unknown nonlinear scalar function.

Assumption 1. The partial derivative of $f(\cdots)$ with respect to control input $u_k(t)$ is continuous.

Assumption 2. Suppose that for all $t \in \{0, 1, \cdots, T\}$ and $k = 0, 1, 2, \cdots$, if $|\triangle u_k(t)| \geq \varepsilon > 0$, then system (1) is generalized Lipschitz, *i.e.*,

$$|\triangle y_k(t+1)| \leq b|\triangle u_k(t)|$$
$$\forall t \in \{0, 1, \cdots, T\} \text{ and } \forall k = 0, 1, 2, \cdots \tag{2}$$

where $\triangle y_k(t+1) = y_k(t+1) - y_{k-1}(t+1)$, $\triangle u_k(t) = u_k(t) - u_{k-1}(t)$, b is a finite positive constant, and ε is an arbitrary small positive constant.

Remark 1. In Assumption 2, the condition $|\triangle u_k(t)| \geq \varepsilon > 0$ for all $t \in \{0, 1, \cdots, T\}$ and $k = 0, 1, 2, \cdots$ can be guaranteed by a reset algorithm (15) given in the following

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section. From a mathematical point of view, provided that $|\triangle u_k(t)| \geq \varepsilon > 0$, there must exist a finite constant b such that (2) can be satisfied. Furthermore, we just need the existence of such a constant b without requiring the exact value.

Lemma. For nonlinear system (1), if Assumptions 1 and 2 hold, there must exist $\theta_k(t)$, called PPD. When $|\triangle u_k(t)| \geq \varepsilon > 0$ for all $t \in \{0, 1, \cdots, T\}$ and $k = 0, 1, 2, \cdots$, such that

$$\triangle y_k(t+1) = \theta_k(t)\triangle u_k(t) \tag{3}$$

and $|\theta_k(t)| \leq b$ with b defined in Assumption 2.

Proof. From system (1), we have

$$\begin{aligned}\triangle y_k(t+1) = {} & f(y_k(t), y_k(t-1), \cdots, y_k(t-n_y),\\ & u_k(t), u_k(t-1), \cdots, u_k(t-n_u)) -\\ & f(y_{k-1}(t), y_{k-1}(t-1), \cdots, y_{k-1}(t-n_y),\\ & u_{k-1}(t), u_{k-1}(t-1), \cdots, u_{k-1}(t-n_u)) =\\ & f(y_k(t), y_k(t-1), \cdots, y_k(t-n_y),\\ & u_k(t), u_k(t-1), \cdots, u_k(t-n_u)) -\\ & f(y_k(t), y_k(t-1), \cdots, y_k(t-n_y),\\ & u_{k-1}(t), u_k(t-1), \cdots, u_k(t-n_u)) +\\ & f(y_k(t), y_k(t-1), \cdots, y_k(t-n_y),\\ & u_{k-1}(t), u_k(t-1), \cdots, u_k(t-n_u)) -\\ & f(y_{k-1}(t), y_{k-1}(t-1), \cdots, y_{k-1}(t-n_y),\\ & u_{k-1}(t), u_{k-1}(t-1), \cdots, u_{k-1}(t-n_u))\end{aligned} \tag{4}$$

Using Assumption 1 and the differential mean value theorem, from (4), we have

$$\triangle y_k(t+1) = \partial f^*/\partial u_k(t)(u_k(t) - u_{k-1}(t)) + \xi_k(t) \tag{5}$$

where $\partial f^*/\partial u_k(t)$ is the proper partial derivative value of $f(\cdots)$ with respect to $u_k(t)$, and $\xi_k(t) = f(y_k(t), y_k(t-1), \cdots, y_k(t-n_y), u_{k-1}(t), u_k(t-1), \cdots, u_k(t-n_u)) - f(y_{k-1}(t), y_{k-1}(t-1), \cdots, y_{k-1}(t-n_y), u_{k-1}(t), u_{k-1}(t-1), \cdots, u_{k-1}(t-n_u))$.

Consider the following equation with a variable $\eta_k(t)$:

$$\xi_k(t) = \eta_k(t)\triangle u_k(t) \tag{6}$$

Because $|\triangle u_k(t)| \geq \varepsilon > 0$ for all $t \in \{0, 1, \cdots, T\}$ and $k = 0, 1, 2, \cdots$, clearly (6) must have a solution $\eta_k(t)$.

Let $\theta_k(t) = \partial f^*/\partial u_k(t) + \eta_k(t)$, so (3) can be obtained from (5). Apparently, under Assumption 2, we can get that $|\theta_k(t)| \leq b$. □

Remark 2. Throughout this paper, all discussions are based on the assumption that $|\triangle u_k(t)| \geq \varepsilon > 0$ for all $t \in \{0, 1, \cdots, T\}$ and $k = 0, 1, 2, \cdots$. Hence, system (1) can be rewritten as (3).

Remark 3. By virtue of the definition of $\theta_k(t)$, we can see that all effects of the past inputs and system states on the system output can be fused into $\theta_k(t)$. In fact, $\theta_k(t)$ is one of the most complicated unknown functions about the past inputs and system states. In this paper, we give the prediction or estimation values of the current iteration in batch by using the information of previous tries.

3 Dual-stage optimal iterative learning controller design

Given a desired trajectory $y_d(t)$, $t \in \{0, 1, \cdots, T\}$, the control target is to find a sequence of appropriate control inputs $u_k(t)$ such that the tracking error $e_k(t+1) = y_d(t+1) - y_k(t+1)$ converges to zero as the iteration number k approaches infinity.

Rewrite (3) as

$$y_k(t+1) = y_{k-1}(t+1) + \theta_k(t)\triangle u_k(t) \tag{7}$$

Define the index function of control input as

$$J(u_k(t)) = |e_k(t+1)|^2 + \lambda|\triangle u_k(t)|^2 \tag{8}$$

where λ is a positive weighting factor.

According to (7) and the definition of $e_k(t+1)$, $J(u_k(t))$ can be rewritten as follows.

$$\begin{aligned}J(u_k(t)) = {} & |y_d(t+1) - y_{k-1}(t+1) - \theta_k(t)(u_k(t) -\\ & u_{k-1}(t))|^2 + \lambda|u_k(t) - u_{k-1}(t)|^2\end{aligned} \tag{9}$$

Using the optimal condition $\partial J/\partial u_k(t) = 0$, we have

$$u_k(t) = u_{k-1}(t) + \rho\theta_k(t)e_{k-1}(t+1)/(\lambda + |\theta_k(t)|^2) \tag{10}$$

where ρ is a step-size constant series, which is added to make the generality of algorithm (10) and used in the analytical stability proof later.

Since PPD, $\theta_k(t)$, is not available, here we present the learning control law as

$$u_k(t) = u_{k-1}(t) + \rho\theta'_k(t)e_{k-1}(t+1)/(\lambda + |\theta'_k(t)|^2) \tag{11}$$

where $\lambda > 0$, $\rho \in (0, 2)$. $\theta'_k(t)$ is to learn the PPD parameter $\theta_k(t)$ and updated iteratively in terms of the optimal solution of the following criterion index function:

$$J(\theta'_k(t)) = |\triangle y_{k-1}(t+1) - \theta'_k(t)\triangle u_{k-1}(t)|^2 + \mu|\theta'_k(t) - \theta'_{k-1}(t)|^2 \tag{12}$$

where μ is a positive weighting factor. Rewriting (12),

$$\begin{aligned}J(\theta'_k(t)) = {} & |y_{k-1}(t+1) - (y_{k-2}(t+1) + \theta'_{k-1}(t)\triangle u_{k-1}(t)) -\\ & (\theta'_k(t) - \theta'_{k-1}(t))\triangle u_{k-1}(t)|^2 +\\ & \mu|\theta'_k(t) - \theta'_{k-1}(t)|^2\end{aligned} \tag{13}$$

Using the optimal condition $\partial J/\partial\theta'_k(t) = 0$, we obtain the parameter updating law as follows:

$$\begin{aligned}\theta'_k(t) = {} & \theta'_{k-1}(t) + \eta\triangle u_{k-1}(t)/(\mu + |\triangle u_{k-1}(t)|^2) \times\\ & (\triangle y_{k-1}(t+1) - \theta'_{k-1}(t)\triangle u_{k-1}(t))\end{aligned} \tag{14}$$

where $\mu > 0$ is the positive weighting factor in (12) and $\eta \in (0, 2)$ is a step-size constant series added to make the generality of algorithm (14). $\theta'_0(t)$ can be chosen arbitrarily.

To make the condition $|\triangle u_k(t)| \geq \varepsilon > 0$ be satisfied for all $t \in \{0, 1, \cdots, T\}$ and $k = 0, 1, 2, \cdots$, and to make the parameter estimation algorithm (14) have stronger ability in tracking variable parameter, we present a reset algorithm as follows:

$$\theta'_k(t) = \theta'_0(t), \text{ if } \quad \theta'_k(t) \leq \varepsilon, \text{ or } \quad |\triangle u_k(t)| \leq \varepsilon \tag{15}$$

where ε is a small positive constant.

Remark 4. It is worth pointing out that the PPD parameters act as the learning gains in learning control law (11) virtually and can be iteratively tuned by (14), depending only on the I/O data. This is quite different from the traditional ILC.

Remark 5. For the proposed ILC scheme, in the practical operation, what we need is to tune the parameters ρ and η in a small range with properly fixed values of λ and μ, without requiring any a *priori* knowledge of the nonlinear dynamic system. This is the major difference from the traditional ILC, in which the design of learning gain requires some knowledge of the nonlinear dynamic system, *e.g.*, the upper and lower boundary of the nonlinear system gradient.

4 Learning convergence analysis

For the rigorous analysis of the following discussion, we give another assumption on PPD parameter as follows.

Assumption 3. The PPD parameter $\theta_k(t)$ satisfies the assumption that $\theta_k(t) \geq 0$ (or $\theta_k(t) \leq 0$), $\forall t \in \{0, 1, \cdots, T\}$ and $\forall k \in \{0, 1, 2, \cdots\}$, and $\theta_k(t) = 0$ holds only at finite points.

Remark 6. This assumption is similar to the limitation of control input direction. In fact, many practical systems can satisfy this assumption such as the temperature control, the pressure control and so on.

Theorem. For a nonlinear and non-affine discrete-time system governed by (1), if Assumptions 1 $\sim$ 3 hold, then the learning control law (11), the PPD parameter updating law (14), and the reset algorithm (15) guarantee that

1) the PPD parameter estimation value $\theta'_k(t)$ is bounded over the finite time interval $0, 1, \cdots, T$;

2) the tracking error converges to zero pointwisely over the finite time interval as k approaches to infinity;

3) the control signals are bounded, that is, $u_k(t) \in l^{\infty}$ for $t \in \{0, 1, \cdots, T\}$ and $k = 0, 1, 2, \cdots$.

Proof. The proof consists of three parts. Part 1 derives the boundedness of the PPD parameter estimation value $\theta'_k(t)$. Part 2 proves the almost perfect tracking performance. The boundedness of control signals is shown in Part 3.

Part 1. The boundedness of $\theta'_k(t)$.

Case 1. When $|\triangle u_k(t)| \leq \varepsilon$, by (15), $\theta'_k(t)$ is clearly bounded.

Case 2. When $|\triangle u_k(t)| > \varepsilon$, subtracting $\theta_k(t)$ from both sides of (14), we have

$$\begin{aligned}\phi_k(t) =& \phi_{k-1}(t) - (\theta_k(t) - \theta_{k-1}(t)) + \\ & \eta \triangle u_{k-1}(t)/(\mu + |\triangle u_{k-1}(t)|^2) \times \\ & (\triangle y_{k-1}(t+1) - \theta'_{k-1}(t)\triangle u_{k-1}(t))\end{aligned} \tag{16}$$

where $\phi_k(t) = \theta'_k(t) - \theta_k(t)$.

Let $\triangle\theta_k(t) = \theta_k(t) - \theta_{k-1}(t)$. Using the relationship of (3), we can rewrite (16) as

$$\begin{aligned}\phi_k(t) =& \phi_{k-1}(t) - \triangle\theta_k(t) + \eta\triangle u_{k-1}(t)/(\mu + |\triangle u_{k-1}(t)|^2) \times \\ & (\theta_{k-1}(t) - \theta'_{k-1}(t))\triangle u_{k-1}(t)) = \\ & \phi_{k-1}(t)(1 - \eta\triangle u_{k-1}(t)^2/(\mu + |\triangle u_{k-1}(t)|^2)) - \triangle\theta_k(t)\end{aligned} \tag{17}$$

Consider the following inequality

$$0 < |1 - \eta\triangle u_{k-1}(t)^2/(\mu + |\triangle u_{k-1}(t)|^2)| < 1 \tag{18}$$

Solving inequality (18), we have

$$\eta \in D_1 = [0, 2 + 2\mu/|\triangle u_{k-1}(t)|^2] \tag{19}$$

Apparently, because $\mu > 0$, $\eta \in (0, 2)$, then inequality (18) can be guaranteed for $\forall t \in \{0, 1, \cdots, T\}$ and $\forall k \in \{0, 1, 2, \cdots\}$. Although $|\triangle u_{k-1}(t)|$ approaches infinity, we still get $|1 - \eta| < 1$. Hence, we can find a constant $d_1 = \sup_{k\in\{0,\infty\}} \sup_{t\in\{0,T\}} |1 - \eta\triangle u_{k-1}(t)^2/(\mu + |\triangle u_{k-1}(t)|^2)|$ such that $0 < d_1 < 1$ for all $t \in \{0, 1, \cdots, T\}$ and $k \in \{0, 1, 2, \cdots\}$.

Taking norm for both sides of (17), we have

$$\begin{aligned}|\phi_k(t)| \leq& |1 - \eta\triangle u_{k-1}(t)^2/(\mu + |\triangle u_{k-1}(t)|^2)||\phi_{k-1}(t)| + \\ & |\triangle\theta_k(t)| \leq d_1|\phi_{k-1}(t)| + 2b \leq d_1^k|\phi_0(t)| + 2b/(1 - d_1)\end{aligned} \tag{20}$$

Hence $\phi_k(t)$ is bounded. Because $|\theta_k(t)| \leq b$, $\theta'_k(t)$ is bounded for all $t \in \{0, 1, \cdots, T\}$ and $k \in \{0, 1, 2, \cdots\}$.

Part 2. Almost perfect tracking performance.

By (7), the error dynamics is

$$\begin{aligned}e_k(t+1) =& y_d(t+1) - y_k(t+1) = \\ & y_d(t+1) - y_{k-1}(t+1) - \theta_k(t)\triangle u_k(t) = \\ & e_{k-1}(t+1) - \theta_k(t)\triangle u_k(t)\end{aligned} \tag{21}$$

Substituting control law (11) into (21), we obtain

$$e_k(t+1) = [1 - \rho\theta_k(t)\theta'_k(t)/(\lambda + |\theta'_k(t)|^2)]e_{k-1}(t+1) \tag{22}$$

To evaluate the relationship between $e_k(t+1)$ and $e_{k-1}(t+1)$ described by (22), we consider the term $[1 - \rho\theta_k(t)\theta'_k(t)/(\lambda + |\theta'_k(t)|^2)]$ in (22).

Solving the following inequality

$$|1 - \rho\theta_k(t)\theta'_k(t)/(\lambda + |\theta'_k(t)|^2)| < 1 \tag{23}$$

we obtain

$$0 < \rho < 2(\lambda + \theta'_k(t)^2)/\theta_k(t)\theta'_k(t) \tag{24}$$

If $\theta_k(t)\theta'_k(t) \leq \theta'_k(t)^2$, then clearly

$$\begin{aligned}2(\lambda + \theta'_k(t)^2)/\theta_k(t)\theta'_k(t) \geq 2(\lambda + \theta_k(t)\theta'_k(t))/\theta_k(t)\theta'_k(t) = \\ 2 + 2\lambda/\theta_k(t)\theta'_k(t) > 2\end{aligned} \tag{25}$$

Hence, $\rho \in (0, 2) \subset (0, 2 + 2\lambda/\theta_k(t)\theta'_k(t))$.

If $\theta_k(t)\theta'_k(t) \geq \theta'_k(t)^2$, we can choose $\lambda > \lambda_{\min} = \theta_k(t)\theta'_k(t) - \theta'_k(t)^2$ to guarantee that $\rho \in (0, 2) \subset (0, 2(\lambda + \theta'_k(t)^2/\theta_k(t)\theta'_k(t))$.

Hence, by properly choosing the values of λ and ρ, we can always guarantee that inequality (23) holds for all $t \in \{0, 1, \cdots, T\}$ and $k \in \{0, 1, 2, \cdots\}$.

Similarly, we also define a constant $d_2 = \sup_{k\in\{0,\infty\}} \sup_{t\in\{0,T\}} |1 - \rho\theta_k(t)\theta'_k(t)/(\lambda + |\theta'_k(t)|^2)|$. Thus $0 < d_2 < 1$ for all $t \in \{0, 1, \cdots, T\}$ and $k \in \{0, 1, 2, \cdots\}$.

Taking norm for both sides of (22) yields

$$\begin{aligned}|e_k(t+1)| = |1 - \rho\theta_k(t)\theta'_k(t)/(\lambda + |\theta'_k(t)|^2)||e_{k-1}(t+1)| \leq \\ d_2|e_{k-1}(t+1)| \leq \cdots \leq d_2^k|e_0(t+1)|\end{aligned} \tag{26}$$

As a result, it guarantees that $e_k(t+1)$ exponentially converges to zero as the iteration number k approaches infinity.

By reset algorithm (15), if $\theta'_k(t) \leq \varepsilon$ or $\triangle u_{k-1}(t) \leq \varepsilon$, setting $\theta'_k(t) = \theta'_0(t)$, then we have

$$\begin{aligned}|e_k(t+1)| = |1 - \rho\theta_k(t)\theta'_0(t)/(\lambda + |\theta'_0(t)|^2)||e_{k-1}(t+1)| \leq \\ d_2|e_{k-1}(t+1)|\end{aligned} \tag{27}$$

Hence, the tracking error $e_k(t+1)$ still decreases with an increase in the iteration number k.

Part 3. Boundedness of control signal.

From learning law (11), we get

$$\triangle u_k(t) = \rho\theta'_k(t)/(\lambda + |\theta'_k(t)|^2)e_{k-1}(t+1) \tag{28}$$

Because the boundedness of $\theta'_k(t)$ has been guaranteed, there must exist a constant

$$N = \sup_{k\in\{0,\infty\}} \sup_{t\in\{0,T-1\}} \{\rho\theta'_k(t)/(\lambda + |\theta'_k(t)|^2)\} \tag{29}$$

such that

$$|\triangle u_k(t)| \leq Ne_{k-1}(t+1) \tag{30}$$

We know that

$$\begin{aligned}|u_k(t)| = &|u_k(t) - u_0(t) + u_0(t)| \leq \\ &|u_k(t) - u_0(t)| + |u_0(t)| = \\ &|u_k(t) - u_{k-1}(t) + u_{k-1}(t) - \cdots + u_1(t) - u_0(t)| + |u_0(t)| \leq \\ &|\triangle u_k(t)| + |\triangle u_{k-1}(t)| + \cdots + |\triangle u_1(t)| + |u_0(t)|\end{aligned} \tag{31}$$

According to (27), (30), and (31), we have

$$\begin{aligned}|u_k(t)| \leq &\\ &Ne_{k-1}(t+1) + Ne_{k-2}(t+1) + \cdots + Ne_0(t+1) + |u_0(t)| \leq \\ &Nd_2e_0(t+1)/(1-d_2) + |u_0(t)| \leq \\ &Ne_0(t+1) + |u_0(t)|\end{aligned} \tag{32}$$

Clearly, the values of initial input $u_0(t)$ and tracking error $e_0(t+1)$ can be chosen as bounded, so the control input $u_k(t)$ is bounded for $t \in \{0, 1, \cdots, T\}$ and $k \in \{0, 1, 2, \cdots\}$. □

5 Simulation study

Now, we present two examples to show the convergence properties of the presented method. The first example aims at evaluating the validity of the dual-stage optimal ILC. The second one is an example of freeway traffic density control that may be of practical importance.

Example 1.

Consider a nonlinear and non-affine SISO system described as

$$y(t+1) = \begin{cases} y(t)/(1+y(t)^2) + u(t)^3, \quad 0 \leq t \leq 50 \\ y(t)y(t-1)y(t-2)u(t-1)(y(t-1)-1)a(t) \\ u(t)/(1+y(t-1)^2+y(t-2)^2), \quad 50 \leq t \leq 100 \end{cases} \tag{33}$$

where $a(t) = 1 + \text{round}(t/50)$ is a varying parameter. It is notable that the structure, orders and parameters of the controlled system are time-varying.

The expected trajectory of output is described by

$$y_d(t+1) = \begin{cases} 0.5 \times (-1)^{\text{round}(t/10)}, \quad 0 \leq t \leq 30 \\ 0.5\sin(t\pi/10) + 0.3\cos(t\pi/10), \quad 30 < t \leq 70 \\ 0.5 \times (-1)^{\text{round}(t/10)}, \quad 70 < t \leq 100 \end{cases} \tag{34}$$

By choosing $\eta = 0.1$, $\rho = 1$, $\lambda = 1$, $\mu = 0.6$, the learning convergence is shown in Fig. 1. The horizon is the iteration number and the vertical is the maximum absolute values of the output tracking error. The simulation result demonstrates the validity of the presented dual-stage optimal ILC scheme.

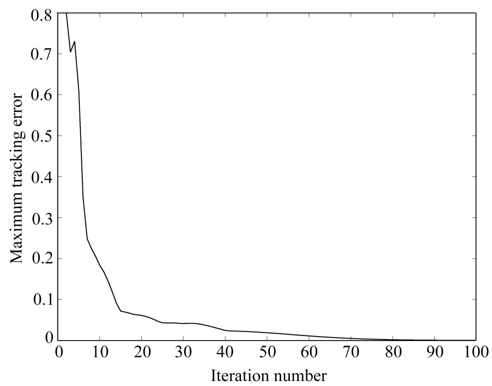

Fig. 1 Convergence of the maximum tracking errors in Example 1

Example 2.

In this example, we address the ramp metering in a macroscopic-level freeway environment. The choice of this example is motivated by its intrinsic engineering importance and high-order strong-nonlinear dynamics.

We refer to the following model originally proposed by Parageorgiou and used in many realms of freeway control[12,13]. The time-discretized traffic flow model for a single freeway with one on-ramp and one off-ramp is given as follows:

$$\rho_i(t+1) = \rho_i(t) + T[q_{i-1}(t) - q_i(t) + r_i(t) - s_i(t)]/L_i \tag{35}$$

$$q_i(t) = \rho_i(t)v_i(t) \tag{36}$$

$$\begin{aligned}v_i(t+1) =& v_i(t) + T[V(\rho_i(t)) - v_i(t)]/\tau + \\ &Tv_i(t)[v_{i-1}(t) - v_i(t)]/L_i - \\ &\nu T[\rho_{i+1}(t) - \rho_i(t)]/\tau l_i[\rho_i(t) + \kappa]\end{aligned} \tag{37}$$

$$V(\rho_i(t)) = V_{free}(1 - [\rho_i(t)/\rho_{jam}]^l)^m \tag{38}$$

where $t \in \{0, 1, \cdots, T\}$, $i \in \{1, \cdots, I\}$.

The freeway is assumed to be divided into I sections of length L_I. T is the sample time interval; $\rho_i(t)$ is the traffic density in section i at time tT (veh/lane/km); $v_i(t)$ is the mean traffic speed in section i at time tT (km/h); $q_i(t)$ is the traffic flow leaving section i and entering section $i+1$ at time tT (veh/h); $r_i(t)$ is on-ramp traffic volume for section i at time tT (veh/h); $s_i(t)$ is off-ramp traffic volume for section i at time tT (veh/h), which is regarded as an unknown disturbance; v_{free} and ρ_{jam} are the free speed and the maximum possible density per lane, respectively; $\tau, \gamma, \kappa, l, m$ are constant parameters, which reflect particular characteristics of a given traffic system. For a real-life network, these parameters are determined by a validation procedure.

It is worthy to point out that the traffic flow patterns are in general repeated every day. From point of view of the system, the repeatability or similarity of the traffic flow implies two conditions: 1) the traffic model is invariant, and 2) the exogenous inputs/disturbances to the freeway system are invariant[13]. In such a circumstance, we apply ILC method to the freeway traffic systems. Before using the presented control method, we first give some assumptions as follows.

Assume that the traffic flow rate entering section 1 during the time period tT and $(t+1)T$ is $q_0(t)$, and the mean speed of the traffic entering section 1 is equal to the mean speed of section 1, *i.e.*, $v_0(t) = v_1(t)$. We also assume that the mean speed and traffic density of the traffic exiting section $I+1$ are equal to those of section I, *i.e.*,

$v_{I+1}(t) = vI(t)$, $\rho_{I+1}(t) = \rho_I(t)$. These assumptions can be summarized as: a) $\rho_0(t) = q_0(t)/v_1(t)$, b) $v_0(t) = v_1(t)$, c) $\rho_{I+1}(t) = \rho_I(t)$, d) $v_{I+1}(t) = v_I(t)$.

Our control objective is to apply the presented ILC scheme, based on the historical freeway traffic data collected from the previous day (week or month), to generate a proper value of $r_i(t)$ that drives traffic density of section i at time tT convergence to the desired traffic density $\rho_{i,desired}(t)$, despite the modeling uncertainties and disturbances occurring at some on-ramp or off-ramp.

For the simulation, we consider a long segment of freeway, which is subdivided into 12 sections. The length of each section is 500 m. The initial entering traffic volume is 1500 veh/h. For all sections, the initial density and the initial mean speed are 30 veh/lan/km and 50 km/h, respectively. Other parameters used in this model are: $V_{free} =$ 80 km/h, $\rho_{jam} = 30$ veh/lane/km, $l = 0.5$, $m = 1.7$, $\kappa = 13$, $\tau = 0.01$, $T = 0.00417$h, $\gamma = 35$, $q_0(t) = 1500$ veh/h, $r_i(t) = 0$ veh/h, $\alpha = 0.95$.

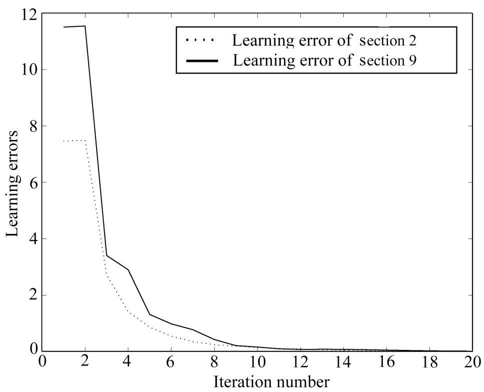

Fig. 2 Learning errors in sections 2 and 9

Fig. 2 shows the learning errors in sections 2 and 9. Here the learning error is defined as the maximum absolute error between the real density and the desired one over the whole period. We can see that the real traffic density converges to the desired density just after a few iterations.

6 Conclusion

The ILC of a nonlinear non-affine system over a finite time interval is one of the most important and difficult problems in the area of control theory. Motivated by the analogy between the model-free adaptive control and ILC, and based on a new dynamical linearization in iteration domain, we work out a dual-stage optimal ILC for improving control input sequence, as well as the learning control scheme itself. The control design and analysis only depend on the input and output information of the dynamic system. The effectiveness of the presented control method is shown by theoretical analysis and intensive simulation results.

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